Beyond Piaget: a Perspective from Studies of Children's Problem Solving Abilities

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The Swiss psychologist Jean Piaget (1896 - 1980) remains unchallenged as the most influential developmental psychologist in history. Indeed, as one prominent researcher put it over 25 years ago, "Before Piaget began his work, no recognizable field of cognitive development existed" (Siegler, 1986 ,pp 21-22). The vast sweep of Piaget's theories, and his ingenious approaches to studying the development of children's minds have profoundly impacted the field. And even though the field has moved well beyond its long period of almost total acceptance of Piagetian stage theory, his detailed analyses of children's behavior at specific points in development remain a source of continued experimental and theoretical inspiration.

Because Piaget published his early work – starting nearly a century ago – in French, his influence on English-speaking developmental psychologists didn't really take off until the late 1950s when his papers and books began to be translated into English. Of particular importance was John Flavell's (1963) interpretive volume which made Piaget (and Flavell!) widely read in English.

Piaget's Empirical Investigations

So, what aspects of children's thought processes did Piaget investigate and what did he discover about them? Well, it seems that he investigated just about everything, and discovered something interesting in every case! The topics include children's developing thinking processes about time, speed, distance, living things, people, space, mathematics, logic, morality, physical causality and psychology (to mention just a few). In many cases, Piaget discovered what Patricia Miller (1993) called the "surprising features of children's thinking" with respect to a wide variety of domains, including, among others:

- Physics: Infants under 8 months old do not expect objects to be permanent: if an object is covered or obscured, it simply doesn't exist in the infant's mind.
- Number: Preschoolers believe that if row of several cookies is spread out, so that they take up more space, that there are now more cookies to eat than before they were spread out.
- Liquid quantity: Four- and five-year-old children believe that when water is poured from a short wide glass into a tall thin glass that there is more water in the latter.
- Morality: Five-year olds believe that wrongness of an act depends on how much damage resulted, rather than the intent of the perpetrator.

- Psychology: Young children don't realize that what they know isn't also known by everyone else, or that someone viewing a scene from a different perspective than their own will see a different relative location of objects in the scene.

"In a discipline that has few real 'discoveries' to rival the discovery of a new planet or the structure of DNA, Piaget's surprises about cognitive development are refreshing and his observations remarkable, considering that they came from seemingly mundane, everyday behavior." (Miller, 1993).

Research style and methods

Piaget's research style has several characteristic features. First and foremost is a very closely linked interaction between what we would currently call the "experimenter" and the "subject". Although there is an overarching goal in each of his investigations – for example to discover how children develop the ability to think about mathematics and logic – most of his studies do not use a detailed "script" that the experimenter follows in *exactly* the same way for every child. Instead, the detailed interactions and specific challenges are adapted to the moment by moment responses of the child. Consequently, no two children are presented with exactly the same sequence of questions, although there is an overarching consistency to the nature of Piaget's interrogation and challenges. Another feature of Piaget's research -- one that may seem surprising until one recalls that Piaget began his investigations of children's thinking in the 1920s -- is that he did not have the luxury of audio- or video-recording devices, so that his data collection is limited to handwritten notes taken in "real time", rather than computer files that can be examined and reexamined long after the data collection is completed so as to correct any mistakes or unintentional biases. A third feature is that the data base for any specific study is typically generated by a relatively small and arbitrary sample of children -- often Piaget's own children– so that generalizations made from these studies are not on very solid statistical ground. Indeed, many of Piaget's pioneering investigations would probably be rejected from most modern journals on methodological grounds of sample size, non-standard measurement, and lack of inter-rater reliability!

Nevertheless, many of Piaget's experiments have been repeated hundreds, if not thousands, of times by investigators all over the world. Quite remarkably, when the procedures are executed in exactly the same way as Piaget described them, the results are almost always the same. However, in many cases, when small changes are made to the procedures, or the materials, one often finds results that challenge

Piaget's theoretical interpretation.

For example, one extensively studied topic of interest to Piaget was the extent to which children understood the logic of classes and subclasses. More specifically, do they understand that the number of objects in a subset cannot exceed the number of objects in a superset of that subset? For example, if there are (only) oaks and pines in a forest, then there can't be more oaks than trees. In a typical investigation of this capacity, Piaget might present children with a collection of 7 toy oaks, and 3 toy pines, and ask the children to count each type of tree. Then he would ask the child if there were more oaks than pines, and the child would answer correctly. Then came the crucial question: "Are there more oaks or more trees? Surprisingly, children under 8 years old typically say that there are more oaks than trees! Piaget interpreted this result as indicating that children at this age are unable to fully understand the logic of class inclusion.

As noted above, when the task is presented to children exactly as Piaget -- and his life-long collaborator, Barbel Inhelder --presented it (Inhelder & Piaget, 1964), then the results are highly replicable. However, as soon as one introduces small variations in the task (such as varying the relative size of the subsets, using more than two subsets, using other terms for the superset – i.e., "forest" rather than "trees" – then the age at which most children can pass the task varies widely, from 6 yrs old to 10 yrs old. This is a common pattern: first, Piaget invents an ingenious way to investigate some aspect of cognitive development, and produces a surprising and important result. Subsequently, investigations by researchers stimulated by Piaget's findings begin to explore important features of the experimental procedure and the associated theoretical interpretation. Very frequently they find that a slight change in the wording of the problem leads to substantial improvement in children's performance. The general point here is that while Piaget's specific results have withstood the test of time, there are many challenges to his theoretical interpretation of those results that have emerged from systematic variations on the ways that children's knowledge has been assessed.

Piaget's Theory[1]

In addition to his empirical discoveries, Piaget created a theory of cognitive development – a description of the growth of the mind – that was extremely influential for scores of years. He invented a way to characterize children's thinking in terms of mental structures, representations, and processes. He organized his analysis and reporting of children's performance into a set of stages, each with qualitatively different properties, such that they are consistent with his overarching theory of developmental stages from infancy to adolescence that is the hallmark of "the Piagetian approach" (c.f., Piaget, 1983). According to Piaget, children progress systematically through a series of "stages" with distict features and capacities, as follows:

1. Sensorimotor period (birth to 2 years). Infants' understanding of the world derives from their physical actions. Their capacity to interact with the world goes from simple reflexes through several steps to an organized set of behaviors.
2. Preoperational period (2 to 7 years). Children begin to use symbols (mental images, words, gestures) to represent objects and events, and they are able to use symbols in an increasingly organized and logical fashion.
3. Concrete operational period (7 to 11 years). Children acquire certain logical structures that allow them to perform various mental operations, which are internalized actions that can be reversed.
4. Formal operational period (roughly 11 to 15 years). Mental operations are no longer limited to concrete objects; they can be applied to different abstract and formal representations of the physical world, such as verbal or logical statements. In addition, children can reason about the future as well as the present.

Piaget called his research topic "genetic epistemology". "Genetic" because he was interested in the genesis of knowledge: its origins and development (not because he was interested in genes!). "Epistemology" because he was interested in knowledge in a highly abstract sense. And although there are few researchers today who would label themselves as "genetic epistemologists", there is no doubt that

[1] Here I must admit to some anxiety about the audacity of using a heading that says "Piaget's Theory" to summarize, in a handful of paragraphs, seminal contributions that are contained in scores of Piaget's books, each of which has been cited many thousands of times. For a classic study from the master himself, cited over 5000 times, I suggest Piaget (1952). For an excellent review of all of his work, I suggest Scholnick, Nelson, Gelman, & Miller (1999).

Piaget should be viewed as one of the founders of the field of cognitive psychology, because he started his work decades before the "cognitive revolution" that has become the basis of modern psychological research (Miller, 2003). Piaget formulated his theory in a kind of semi-mathematical model, in which each stage of development had increasingly powerful and flexible ways to represent and modify knowledge. Similarly, today's theories of cognitive development are stated in the form of computational models of the mental processes that are implemented in the human brain's neural networks. (Elman, 2005; Klahr, 2004, Rakison & Lupyan (2008).

Children's Problem Solving Ability: a Piagetian and post-Piagetian View

Given the enormous sweep of Piaget's topical and theoretical contributions, it is impossible to provide an account of how the field of cognitive development has moved beyond Piaget's methodological and theoretical approach in each of the vast array of topics and issues that he investigated. Instead, I will focus on a specific topic that is representative of the "beyond Piaget" theme, albeit on a topic that Piaget is not widely known to have explored at all. Nevertheless, this focus will convey the flavor and some of the detail of his approach to a psychological question, and the differences –and similarities -- between Piaget's approach, and the way that contemporary cognitive developmental research approaches a topic. Let's look at some similarities first. An essential feature of "modern" research in cognitive development is the inclusion of an extremely detailed description of the context in which children's thinking is being investigated. The reason for this focus is that unless we can carefully describe the task presented to the child, we cannot begin to understand the processes and methods that the child uses to accomplish the task. One indication of Piaget's revolutionary approach to the field is that his description of the context we will be examining – written over 40 years ago – provides the same level of detail that one finds in today's top journals.

The Task

Piaget was interested in whether young children, around 5 years old, could "think ahead", in their everyday lives. He decided that rather than just observe what children did in the normal course of events, he would present them with a puzzle, and then carefully record and analyze the ways in which they solved it. He used a simple version of a popular puzzle, known as the Tower of Hanoi (TOH). The puzzle involves moving a stack of disks from one peg to another, subject to two rules. (1) Only move one disk at a time and (2) never put a larger disk above a

smaller disk. The 3-disk version of the puzzle is shown in Figure 1a. The minimum number of moves to solution is 7, as shown in Figure 1b. Piaget wanted to chart the full course of children's developing ability to solve this problem, so he started with a very simple version (shown in Figure 1c), that only had 2 disks, and could be solved in three moves (Disk 2 to Peg B, Disk 1 to Peg C, and finally Disk 2 to Peg C.

The following paragraphs are from Piaget's investigation of children's ability to solve this puzzle (Piaget, 1976, Chapter 14). I have included direct quotations[2] here to convey a sense of Piaget's characteristic approach to presentation of data and theoretical interpretation of those data. Here is an example, in Piaget's words, of what he observed with a 5 year old child (named "Mar"):

(Child's comments in italics) Mar (5,4), with .. two disks starts off by just moving Disk 2 from *A* to C to *B* to *A* to C.

Piaget: But I wanted the whole tower to be here.

(Mar again moves Disk 2 from *A* to C to *B* to *A* to C, and then Disk 1 from *A* to C to *B* to *A* to C, which results in the tower's being upside down.)

Piaget: I wanted a tower the right way up.

(Mar, starts off again by moving Disk 2 from *A* to C to *B* to *A* to C, and next to *B,* and then makes Disk 1 follow the same circuit, that is, from *A* to C to *B* to *A* to C. So Disk 1 ends up on C and Disk 2 on *B.)*

Piaget: What have you got to do now?

(Mar puts Disk 2 on top of Disk 1 on C, thus success, by chance and after a corrected inversion.)

Piaget: Very good. Could you have done it more quickly?

(Mar takes both discs at the same time.)

Piaget: No, one at a time.

(Mar moves Disk 2 from *A* to C and puts Disk 1 on top of it, but seeing the mistake, puts them on the table and reestablishes the order Disk 2 on top of Disk 1.)

Piaget: No. How about another way?

Child: No. I want to take the big one first; that's better.

Piaget: Try again.

(Mar moves Disk 2 from *A* to *B* and Disk 1 from *A* to C.)

Piaget: Have you finished?

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[2] With minor formatting changes for clarity.

Child: Yes. Oh, no.

(He puts Disk 2 on top of Disk 1 on C (thus success).

Piaget: Good, can you do it again?

Piaget concludes his analysis of the child's behavior by saying:

> "The striking finding at this stage is the difficulty in solving such an easy problem as that involving only two discs. The length of the trial-and-error period varies (sometimes longer and sometimes shorter than in the case of Mar). However, none of these subjects make a plan or even understand how they are going to move the *tower,* they only know that the two discs must be moved from *A* to C."

In true "Piagetian fashion", Piaget reports many more pages of this kind of detailed interaction between the Experimenter and the child, based on this somewhat informal, albeit systematic, exploration by the Experimenter of the child's ability to solve increasingly difficult problems. Of particular relevance for the "modern" investigation of this kind of problem-solving ability is Piaget's conclusion from this series of studies:

> "Stage III: This level, which starts at eleven to twelve years, is characterized by rapid and stable success in the problem of the three-disc tower and by an increasingly inferential anticipation in the case of towers with more discs, together with an explicit use of earlier experience."

In other words, Piaget is claiming that not until children are between 11 and 12 years old can they reliably execute the 7-move solution to the 3-disk problem (i.e., the solution shown in Figure 1b).

So much for methodology. A more serious question about Piaget's study of children's performance on the TOH puzzle is his conclusion that most 5- and 6-year-old children "cannot move the three-disk tower even after trial and error. They do succeed in moving the two-disk tower, but only after all sorts of attempts to get around the instructions and without being conscious of the logical links." (p. 288) Moreover, "none of these subjects make a plan or even understand how they are going to move the tower" (p. 290), and later, "There is . . . a systematic primacy of the trial- and-error procedure over any attempt at deduction, and no cognizance of any correct solution arrived at by chance." (p. 291) Finally, as noted earlier, Piaget claims that not until the age of 11 or 12 can children routinely solve the three disk problem

Reasons for doubt. This is a curious result, because the 2-disk problem requires only that the subject remove a single obstacle, the small disk, and place it

temporarily on an unused peg in order to move the large disk and then the small disk. It is about the most rudimentary problem that one could pose. The results are also curious because, even an infant can remove a single obstacle to achieve a desired goal (McCarty, Clifton, & Collard, 1999) or use a tool to retrieve a desired object (Chen & Siegler, 2000). Furthermore, casual observation of young children coping with their daily circumstances suggests that they are capable of solving "problems" in familiar environments requiring 3 or 4 "moves" (such as getting a chair to reach a cabinet containing a string to tie on a doll). For example, consider the following:

Scene: Child and father in yard. Child's playmate appears on bike.
Child: Daddy, would *you* unlock the basement door?
Daddy: Why?
Child: 'Cause I want to ride my bike.
Daddy: Your bike is in the garage.
Child: But my socks are in the dryer.

What kind of strange child is this? What could possibly explain such an exchange? Let me propose a hypothetical sequence of the child's mental activity as shown in Table 1:

Table 1 about here

The *example* is real (in fact it is from my own experience) and should be plausible to everyone who has spent time around young children. On the other hand, the *analysis* of the example is less convincing, based as it is on a host of assumptions. Some of these assumptions are easily testable. We could determine whether the child knows *constraints,* such as the one about riding bikes only when shod. Similarly, we could assess the child's knowledge of *facts* about dryer location, shortest route to the basement, and so on. Somewhat more difficult, but still reasonable, would be the job of finding out what sorts of *inferences* the child was capable of making about her day-to-day environment, such as the one about where the socks might be, given that they were not in the drawer. However, the dominant feature of the hypothesized thought sequence is not anyone of these features in isolation. Rather, it is their organization into a systematic means-ends chain. Thus, I am suggesting that by the time the child is old enough to exhibit the sort of behavior just described, she has already acquired some *general problem-solving*

processes. These enable her to function effectively - that is, to achieve desired goals - by noticing relevant features of the environment and organizing a wide range of facts, constraints, and simple inferences in some systematic manner. The TOH provides an ideal context in which to explore these issues.

This kind of behavior suggested to me that, contrary to earlier claims, preschool children do indeed have a greater problem- solving capacity than had yet been revealed, and I decided to explore this hypothesis systematically (rather than depend on personal anecdote). In the rest of this chapter, I will describe an investigation that used a version of the TOH, and a novel procedure to assess pre-schoolers' problem solving abilities. At the same time, I wanted to guard against the problem of false positive interpretations (i.e. attributing an ability to the child that she does not have). The steps I took to increase task sensitivity included modifications of the materials themselves, presentation of partial problems, prior familiarization with the materials, and a motivating cover story. Our attempt to guard against false positive assessment consisted of requiring the child to present a plan for his entire move sequence rather than simply making one move at a time.

Studying Preschoolers' ability to solve problems with multiple sub-goals

As noted earlier, Piaget used the TOH in his investigations of children's problem solving abilities and the puzzle has been used extensively to study adults' problem solving (Simon, 1975; Anzai & Simon, 1979). In the study to be describe here, we modified the standard physical configuration of the puzzle, while maintaining its underlying formal properties (Klahr & Robinson, 1981)[3]

Materials and Procedure

We reversed the size constraint and used a set of nested inverted cans that fit loosely on the pegs. When they were stacked up it was impossible to put a smaller can on top of a larger can (see Figure 2). Even if the child forgot the relative size constraint, the materials provided an obvious physical consequence of attempted violations: smaller cans fell off bigger cans.

Externalization of the goal. In addition to the initial configuration, the goal configuration was always physically present. We arranged the child's cans in a goal configuration and the experimenter's cans in the initial configuration. We did this

[3] Researchers continue to use this puzzle to explore children's thinking processes, but the focus is now on more recently formulated theoretical constructs, such as "executive function", "working memory", and "cognitive inhibition" (Bull, Espy, & Senn, 2004).

because we believed that the children in Piaget's version of the puzzle may have simply forgotten – or even failed to create – a mental representation of what the goal state should look like. We reasoned that if we provided that goal state externally, we would be able to detect their true problem solving ability, without simultaneously taxing their limited memory capacity.

Then the child was asked to tell the experimenter what she (the experimenter) should do in order to get her (experimenter's) cans to look just like the child's. This procedure was used to determine the extent to which the child could create a mental representation about a series of intermediate states (i.e., "imagining" where each can would be after a move: children were asked to describe the complete *sequence* of moves necessary to solve the problem.

Participants. Fifty-one children attending the Carnegie Mellon University Children's School participated in the study. There were 19 children each in the 4-year and 5-year groups and 13 in the 6-year group. The children came predominantly, but not exclusively, from middle-class backgrounds. There were approximately equal numbers of boys and girls at each age level.

Cover story

Children were familiarized with the materials shown in Figure 2, in the context of the following cover story.

> Once upon a time there was a blue river (experimenter points to space between rows of pegs) On your side of the river there were three brown trees. On my side there were also three brown trees. On your side there lived three monkeys: a big yellow daddy (present yellow can and place on peg), a medium size blue mommy (present and place), and a little red baby. The monkeys like to jump from tree to tree [according to the rules]; they live on your side of the river. (Establish legal and illegal jumps) On my side there are also three: a daddy, a mommy and a baby (introduce Experimenter's cans). Mine are copycat monkeys. They want to be just like yours, right across the river from yours. Yours are all stacked up like so (points to goal state on child's side of the table) mine are like so [points to E's side of the table] Mine are very unhappy because they want to look like yours, but right now they are a little mixed up. Can you tell me what to do in order to get mine to look like yours? How can I get my daddy across from your daddy [etc.]?

For each problem the child told the Experimenter the full sequence of proposed moves, and the Experimenter gave supportive acknowledgement *but did not move the cans*. Then the next problem was presented. Children found the cover story easy to comprehend and remember, and they readily agreed to consider the cans as

monkeys. The remaining variations are best described after considering some of the formal properties of this task

Problem set. We used a set of 40 problems that had minimum path lengths of from 1 to 7 moves. (A seven move problem with 3 cans is the "conventional" TOH puzzle that starts with 3 cans on one peg and ends with the 3 cans on a different peg). Path length was set by presenting "partially solved" problems. That is, instead of the convention of having the initial and final states with all the disks stacked on one peg or another, problems were set up so that, for example, only two moves were necessary to complete the stack. (Figure 2 shows a problem in which only one move is necessary, that is, to move the large 'daddy' can from the peg on the experimenter's right to fit on top of the medium size 'mommy' and small 'baby' cans already stacked on the Experimenter's left.) Problems were presented in order of increasing difficulty (i.e., number of moves) until the child appeared to reach his or her upper limit.

Results

The main question of interest is how far into the future a child could "see" in describing move sequences. To avoid overestimating this capacity on the basis of a few fortuitous solutions, we used a very strict criterion: a child was scored as able to solve *n*-move problems only after proposing the minimum path solution for *all four* of the problems of length *n*.

The proportion of children in each age group producing correct solutions for all problems of a given length is shown in Figure 3. It is important to emphasize that the y-axis in Figure 3 is not overall proportion correct, but rather a much more severe measure: the proportion of children with perfect solutions on all problems of a given length. For example, 69% of the 6-year-olds were correct on all four of the five-move problems, while only 16% of the 5-year-olds and 11% of the 4-year-olds produced four flawless five-move solutions.

The absolute level of performance was striking, given Piaget's earlier claims, Over two-thirds of the 5-year-olds and nearly all of the 6-year-olds consistently gave perfect four-move solutions, and over half of the 6-year-olds gave perfect six-move solutions. Almost half of the 4-year-olds could do the three-move problems. Recall that these solutions required that the child manipulate mental representations of future states, because the cans were not moved during or after the child's description of the solution sequence. Furthermore, all intermediate states were different from, but highly confusable with, the two physically present states (the initial and final configurations),

Concluding comments

It is clear that, when presented with an arbitrary and novel problem solving challenge, many 6-year-olds and some 5-year -olds are able to look ahead six moves into the future, much more so than Piaget claimed in his own work on problem solving. This ability appears to result from systematic application of both planning and means-ends analysis, two crucial aspects of the full repertoire of human problem solving abilities (Newell & Simon, 1972).

I have used the domain of problem solving to convey some of the issues raised by "revisiting Piaget". These issues include: his pioneering empirical work, his innovative method of stimulating and then recording children's performance, his attempt to interpret his results in terms of an overarching theoretical model, and the subsequent challenges to his conclusions and interpretations, as manifested in carefully designed and executed experimental situations. There is no doubt that he created the path for thousands of subsequent researchers in cognitive development to follow.

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Table 1: Hypothetical sequence of goals, subgoals, and constraints leading to child's request

Top goal: ride bike.
Constraint: shoes or sneakers on.
Fact: feet are bare.
Subgoal 1: get shod.
Fact: sneakers in yard.
Fact: sneakers hurt on bare feet.
Subgoal 2: protect feet (get socks).
Fact: sock drawer was empty this morning.
Inference: socks still in dryer.
Subgoal 3: get to dryer.
Fact: dryer in basement.
Subgoal 4: enter basement.
Fact: long route through house, short route through yard entrance.
Fact: yard entrance always locked.
Subgoal 5: unlock yard entrance.
Fact: Daddies have all the keys to everything.
Subgoal 6: ask daddy.

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Figure Captions

Figure 1 (a) A 3-disk version of the "Tower of Hanoi" Puzzle. The challenge is to move the stack of disks from Peg A to Peg C, subject to two constraints. (1) only move one disk at a time; (2) Never put a larger disk above a smaller disk. The minimum number of moves for the 3-disk problem is 7.

(b)The optimal 7 move solution is shown here.

(c) A 2-disk version of the TOH puzzle, used in the example protocol from one of Piaget's studies.

Figure 2: "Monkey Cans". Instead of disks of increasing size, the materials include inverted cans of decreasing size. The constraints are (1) only move one can at a time; (2) never put a smaller can on top of a larger can. Both the initial state (the Experimenter's cans) and the goal state (the Child's cans) are included in the display. However, none of the intermediate states are physically represented: they must be constructed in the child's mind. This configuration shows a "partially solved" 3-can problem. Only one move is necessary to solve the problem.

Figure 3: Percent of 4-, 5-, and 6-year old children with 4 out of 4 perfect solutions to N move problems

Fig 1a (standard 3-disk problem)

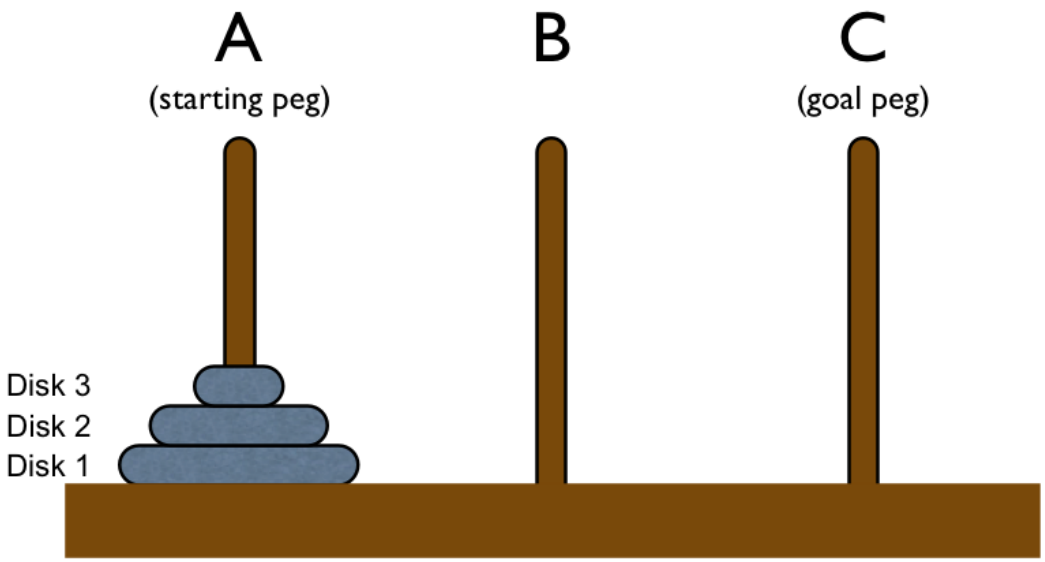


Fig 1b

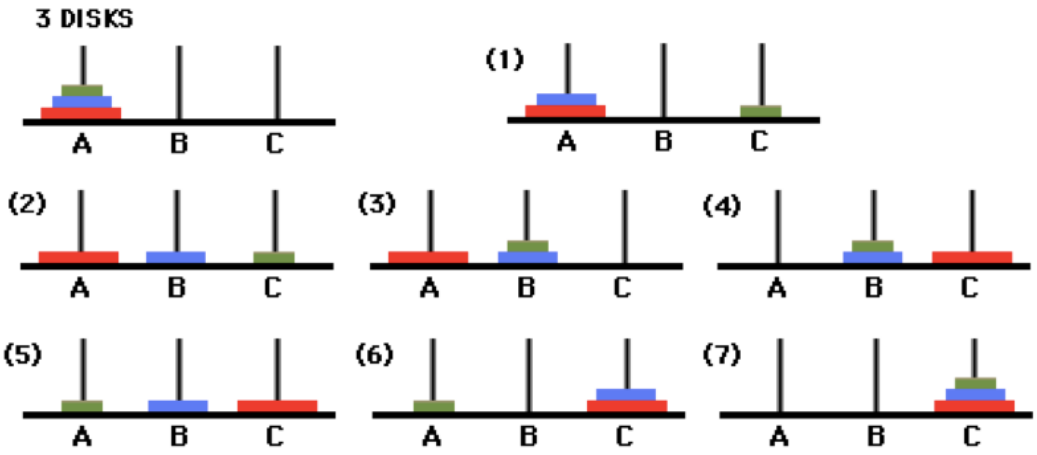


Fig 1c

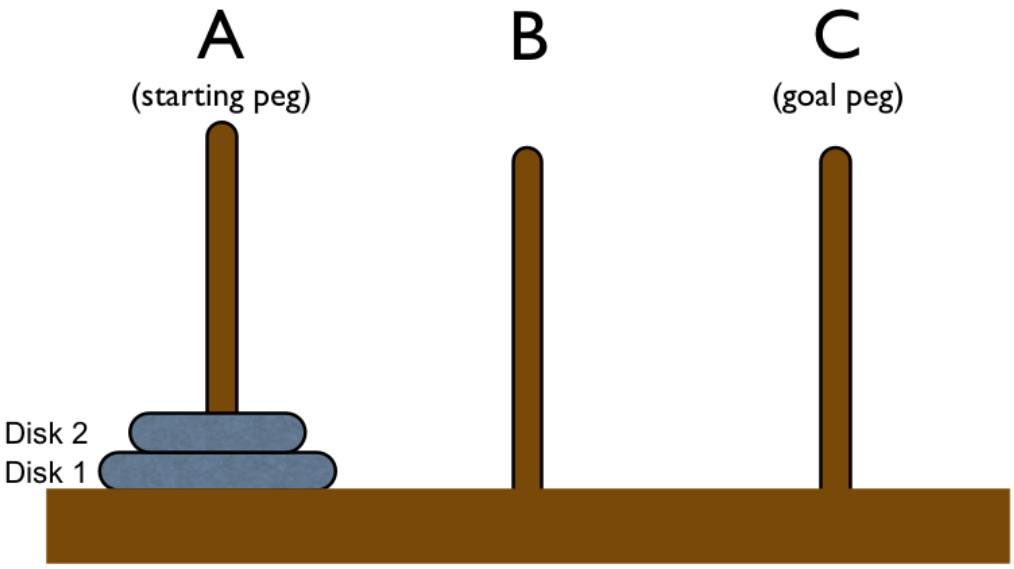


Fig 2

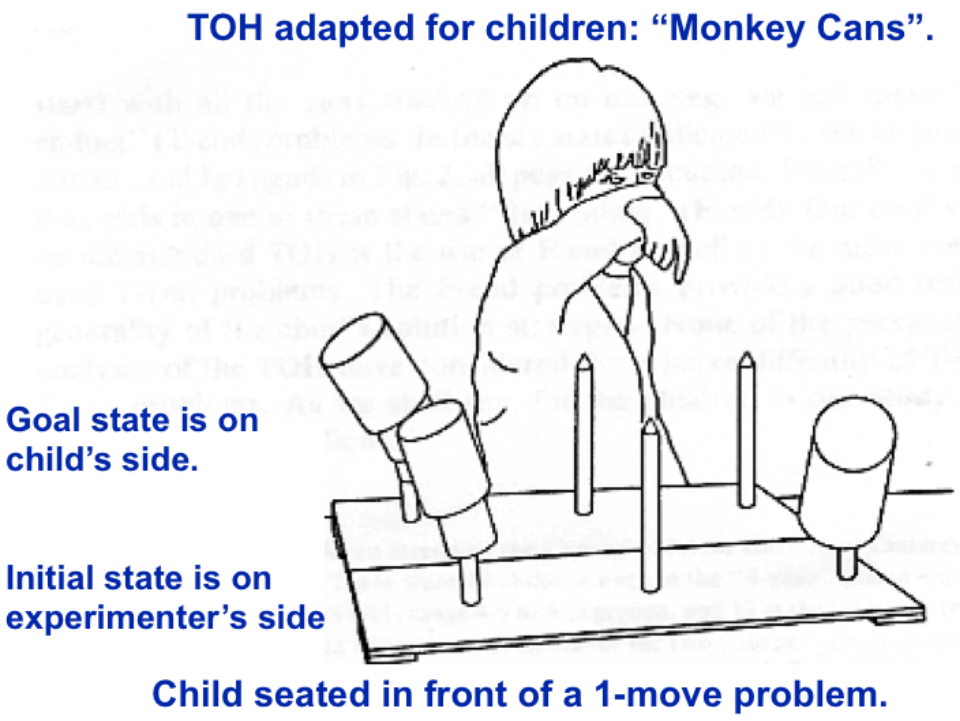
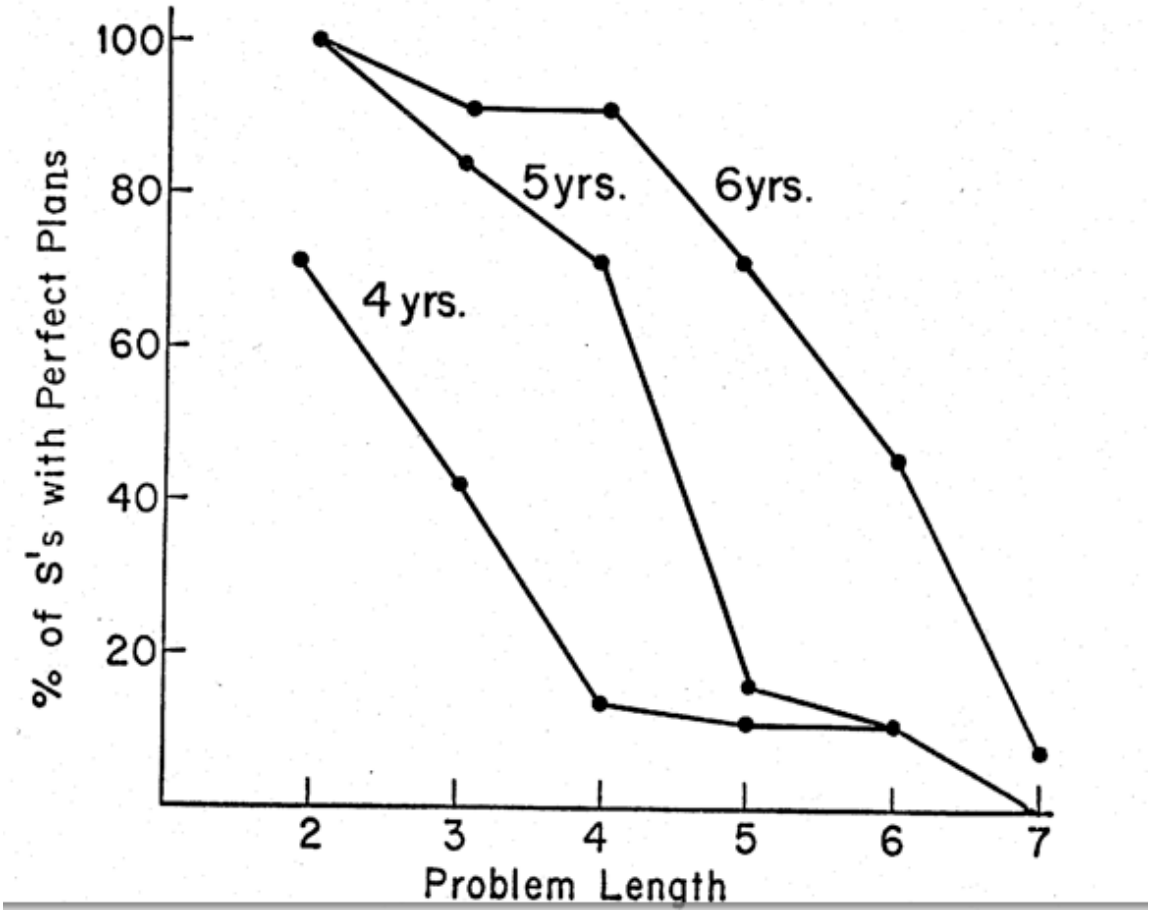

TOH adapted for children: “Monkey Cans”.
Goal state is on child’s side.
Initial state is on experimenter’s side
Child seated in front of a 1-move problem.

fig 3


100
80
60
40
20
% of S's with Perfect Plans
5yrs.
6yrs.
4 yrs.
2
3
4
5
6
7
Problem Length