How should a theory of learning and cognition inform instruction?

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We have developed a set of computer-based cognitive tutors (Anderson, Corbett, Koedinger, & Pelletier, 1995; Koedinger, Anderson, Hadley, & Mark, 1997) for high school mathematics that have been effective in improving learning. These tutors interact with students at the grain size of about 20 seconds during which the student might transform an equation or calculate an angle and the tutor will respond to that. The tutors are able to interact with the student at this grain size on the basis of a cognitive model that can solve these problems in the various ways students are supposed to be able to solve the problems. Compared to most computer-based instruction this is a rather fine grain size. However, compared to current theories of human cognition including our own ACT-R (Anderson & Lebiere, 1998, in press) this is a very gross level of interaction. Temporally, ACT-R postulates primitive acts of cognition taking under 1 second and often as little as 50 msec. Component-wise the theory postulates separate visual, memory, imaginal, and motor components to a student action like transforming an equation. While our focus is on ACT-R, its grain size is typical of modern theories of cognition. The question that this paper will broach is whether there is a role for a theory of cognition at this grain size in impacting something like mastery of high-school algebra. Although not offering any definitive answers to the questions we will review some of our research on eye movements that has attempted to track a finer temporal grain size in mathematical problem solving and we will review some of our research on brain imaging that has attempted to track different cognitive components in mathematical problem solving.

Using Eye Movements to Instruct

Eye movements are behaviors that occur at a temporal grain size of under a second and therefore offer the opportunity of fine-grained tracking of mathematical problem solving. Gluck (1999, Anderson & Gluck, 2001) looked at the instructional leverage one might gain by

monitoring student eye movements while interacting with an algebra tutor. The tutor he developed is a simplified version of the PAT tutor (Koedinger et al., 1997) used in schools and its interface is illustrated in Figure 1. Figure 1a shows a screen display as it appears at the beginning of a problem. The student's task is to fill in the column labels and units, enter a variable and an expression written in terms of that variable, and then answer whatever questions are present. Figure 1b displays the completed problem. The first question in that problem is an example of what is called **result-unknown** problem (where the student must in effect evaluate the expression) and the second is an example of what is called a **start-unknown** problem (where the student must in effect solve for x). The key aspect of the problem is the expression 12 + 45x, which the student has filled in. The principal goal of the lesson is to teach the student how to create such expressions and use them to solve problems.

Gluck (1999) found numerous points where eye movements indicated opportunities for instructional leverage. For instance, about 40% of all tutor messages to students are not read but current tutors proceed on the assumption that the messages have been read. Often students will quickly self-correct when they do not read the message, but if there is a long latency and the students do not read the message, the odds become high that they will make an error.

As another example, eye movements allow us to disambiguate methods of solving problems. Figure 2 shows two examples of students solving result-unknown problems in the tutor. In both cases the student produces the right answer and so there is no bases in the answers they type for suspecting a difference in the solution process. However, the fixation patterns are very different. The critical difference is that in (a) the student is fixating the algebraic expression and using it in the solution of the problem whereas in (b) the student is rereading the verbal statement and ignoring the expression. This reflects a difference in early mastery of algebra

where some students use algebraic expressions while others fall back on pre-algebraic reasoning skills (Koedinger, & McClaren, 1997). Gluck (1999) showed that students solved problems more rapidly and successfully when they displayed the eye movements typified in part (a) than when they displayed the eye movements typified in part (b).

We decided to take Gluck's (1999) research and see if we could not develop instructional interventions for the tutor that were triggered by eye movements. We identified a number of occasions where the eye movements indicated an instructional opportunity and presented brief auditory messages lasting about 2 seconds. For instance, we noted occasions when students did not read instruction and where it seemed that instruction was critical. One such case, discussed above, occurred when students made an error, failed to self correct, and did not read the error message. When 10 seconds had passed without students fixating the error message for more than 1/4 second, they heard "Read the help message." As another example, we identified occasions where student eye movements indicated a deficiency in their problem-solving strategy. For instance, if the student failed to solve a start unknown problem and failed to fixate the expression, we would direct them to use the expression. More precisely, if students made an error and had failed to fixate the expression for at least 1/4 second, they heard the message "Try using the formula to compute your answer."

In a test, where we compared this augmented tutor with a tutor that was otherwise identical (including collection of eye movements), we found that students were able to reach the same level of mastery 20% faster with the augmented instruction. One might argue that this result does not really demonstrate that we are improving instructional effectiveness by attending to the fixation-by-fixation behavior of the student. Perhaps all the leverage simply comes from getting students to read more of the instructional messages. We would like to show that our eye-

movement tutor has impact on behavior at this fine grain size. Figure 3 provides one example of such a refined effect. This shows how the fixation pattern of students changes over the 4 days with respect to amount of time spent encoding the formula. Figure 2b illustrated the curious pattern where students would not use the formula in solving the problem. As noted, the tutor would intervene when they displayed this pattern and made an error and it would suggest that they try to use the formula. This figure plots how much time students spent reading the formula during their first attempt to fill in the cells in the start-unknown and result-unknown cells. In the control condition students show decreased use of the formula, probably reflecting the general speed up in problem solution. However, students in the experimental condition maintained a constant rate of use of the formula. The difference in the linear trends in the two functions is significant ($t(38) = 2.12$, $p < .05$). This finding indicates that indeed detailed monitoring of student's was having an impact on the fine-grain details of their problem-solving behavior.

So this result gives us some reason for optimism that the fine-grained temporal structure of human cognition might be relevant to achieving high-level instructional goals. However, it is important to note that this application did not actually involve the ACT-R theory or any other cognitive theory but rather was just a "common sense" use of the eye tracking data. The ACT-R theory has been developed to address eye movement data (Salvucci, 2001). How such a fine-grained model could be used in instruction remains to be determined.

Brain Imaging and the Learning of Algebra

We (Qin, Anderson, Silk, Stenger, & Carter, 2003) have been using fMRI brain imaging to study to learning of algebra. This work started with use of brain imaging to study algebra equation solving in highly competent college students (Anderson. Qin, Sohn, Stenger, & Carter, 2003). That work was able to successfully relate an ACT-R model to the brain imaging data that

we obtained. Figure 4 shows the steps that the ACT-R model goes through in solving an equation. The model involves the ACT-R components for reading the equation, imagining transformations to it, retrieving relevant facts from declarative memory, and programming the output of an answer (indicated by pressing a finger in a data glove in the magnet). Figure 5 illustrates three regions (all in the left cortex) that we found related to three of these components. There was a prefrontal region that seemed to reflect retrieval of relevant facts, a motor region that seems to reflect programming of the right hand, and a parietal region that seemed to reflect the transformation of the equations.

Given this background we decided to see if the same regions would be involved in children just learning algebra equation solving. We looked at children 11-14 years of age whose first instruction in equation solving was in our laboratory and we looked at what happened to their performance over five days of an hour practice each day. We looked at their solution to equations of three levels of complexity in terms of number of steps of transformations to solve:

0-step: e.g., 1x+0=4

1-step: e.g., 3x+0=12, 1x+8=12

2-step: e.g., 7x+1=29

They were scanned in the magnet on the first and fifth day of the experiment. Figure 6 shows their time to solve the equations and the predictions of the ACT-R model in which all parameters where fixed (and set on the basis of the adult data) except for the times for the retrieval and transformation steps in Figure 4 which sped up according the following power function

$$Time = F \bullet Days^{-a}$$

where F was estimated to be .63 sec and a to be .28.

Figure 7 shows the scanning procedure for each trial in the magnet. Note that each trial is followed by a substantial intertribal interval (ITI) so that the sluggish fMRI BOLD (Blood Oxygen Level Dependent) response can rise and drop before the next trial. Anderson et al. (2003) describe how one can take the pattern of component activities in the different cognitive components in Figure 4 and convolve them with a standard gamma form for BOLD signal to predict the BOLD response in each condition. Figures 8 – 10 show the BOLD responses obtained and predictions of the ACT-R model for the three regions of interest. In producing these BOLD functions there are parameters estimated to reflect their magnitude and exact rise and fall times. However, there are two predictions that are parameter-free consequences of the ACT-R model in Figure 4.

1. The relative areas under the BOLD functions for different conditions in a brain region will be directly proportional to the time that the components are active.
2. The point at which the functions peak will reflect the difference in mean time from stimulus onset to the time when the components are active.

It is striking how well the ACT-R model predicts these BOLD functions and this gives us reason for optimism that the brain imaging can track events of significance with respect to the learning of mathematical cognition. It is worth noting qualitatively the different patterns predicted for the three components:

1. The size of the BOLD response in the motor area is constant both across complexity condition and practice. This is as would be expected since it is supposed to reflect the same finger press in all conditions. However, the peak of the functions is delayed as complexity increases. The effect of practice is to move the functions forward and compress the differences among them, reflecting the change in mean times with practice.

2. The size of the BOLD responses in the parietal area directly reflects the number of transformations that are occurring in the imaginal buffer. Note that there is a considerable response even in the zero transformation condition because even in this condition it is necessary to build up a representation of the equation and extract the answer from it. With practice the size of the BOLD responses diminish, reflecting the postulated speed up in the mental transformations of the equation.
3. The BOLD functions for the prefrontal region are similar to those in the parietal region, showing differences in magnitude that reflect both number of transformations and practice. The one difference is that the response in the 0-transformation condition is predicted to be zero and nearly is so. This is because in this condition nothing has to be retrieved from long-term memory to calculate the answer.

The pattern of data that we see in this study is very similar to the pattern of data that we have seen with our adult participants. The one difference is that in studies of learning (not in the domain of equation solving where our college students are already at asymptote) we find only learning-related changes in the prefrontal retrieval region and not in the partietal region that is used for problem representation. This difference suggests a greater plasticity in children's capability for learning new sorts of mental transformations.

This study does indicate that we can find a rigorous mapping from the learning of mathematical problem solving onto brain imaging. It leaves open the issue of how one might use such brain imaging to actually improve mathematics instruction. One will never see an fMRI brain scanner in a classroom. However, we can observe brain patterns while students solve problems with our tutors in a scanner and this information can be used to help improve the tutor

instruction. It is also the case that there is a promising new non-intrusive infrared technology (e.g., Strangman, Boas, & Sutton, 2002) that might someday see actual real-world use.

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Figure Captions

Figure 1 The tutor screen (a) at the beginning of the problem and (b) at the end of the problem.

Figure 2 Eye movements (a) when a student calculates the answer using the expression and (b) when a student calculates the answer using the problem statement. The blotches reflect fixation points that go from dark to light with the passage of time.

Figure 3 Amount of time spent fixating the formula in the control and experimental conditions as a function of the amount of practice.

Figure 4. The steps ACT-R goes through in the solution of the equation $7x + 1 = 29$.

Figure 5. Regions of interest identified by Anderson et al. (2003).

Figure 6. Time to solve equations of differential complexity as a function of practice

Figure 7. The scanner protocol

Figure 8. The observed and predicted BOLD responses in the motor region on Day 1 and Day 5.

Figure 9. The observed and predicted BOLD responses in the parietal region on Day 1 and Day 5.

Figure 10. The observed and predicted BOLD responses in the prefrontal region on Day 1 and Day 5.

Figure 1a

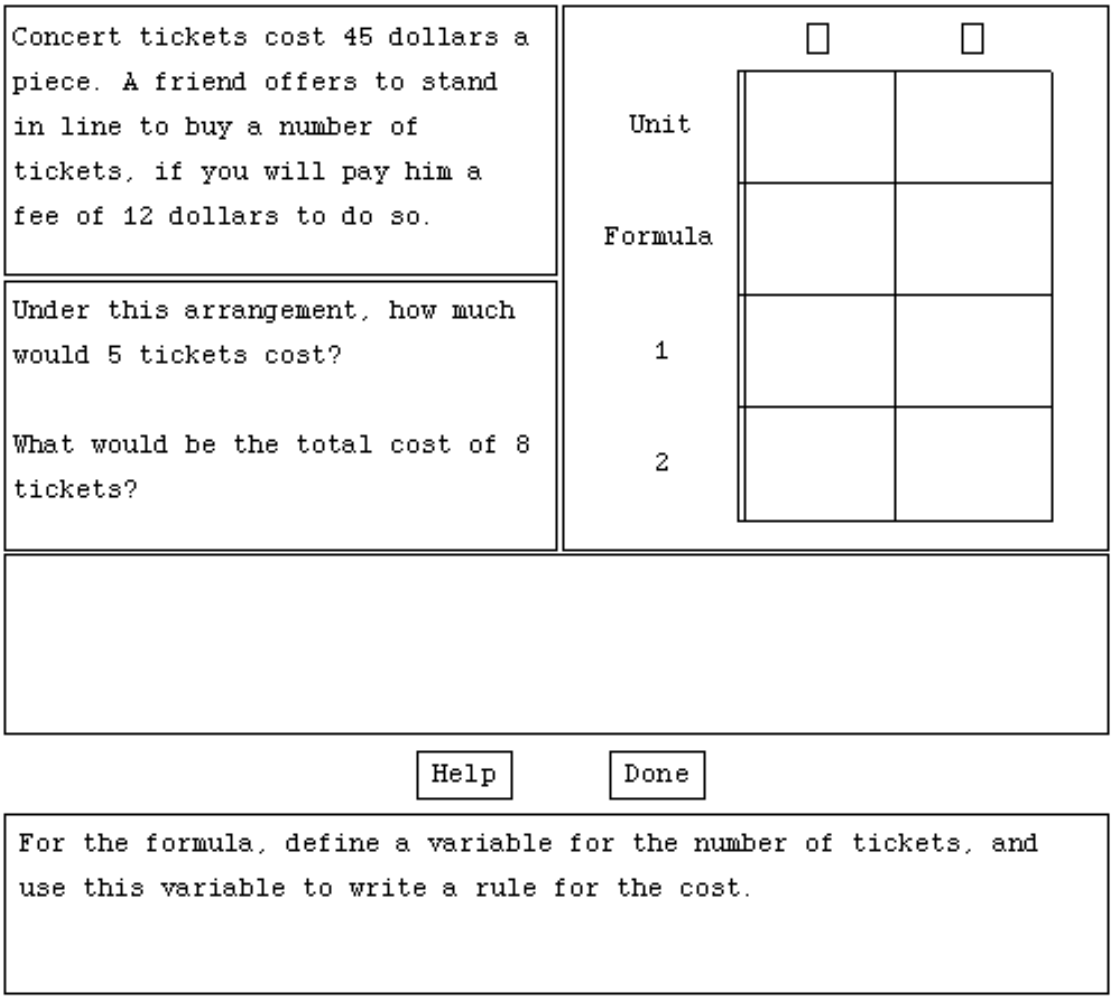

Concert tickets cost 45 dollars a piece. A friend offers to stand in line to buy a number of tickets, if you will pay him a fee of 12 dollars to do so.
Under this arrangement, how much would 5 tickets cost?
What would be the total cost of 8 tickets?
Unit
Formula
1
2
Help
Done
For the formula, define a variable for the number of tickets, and use this variable to write a rule for the cost.

Figure 1b

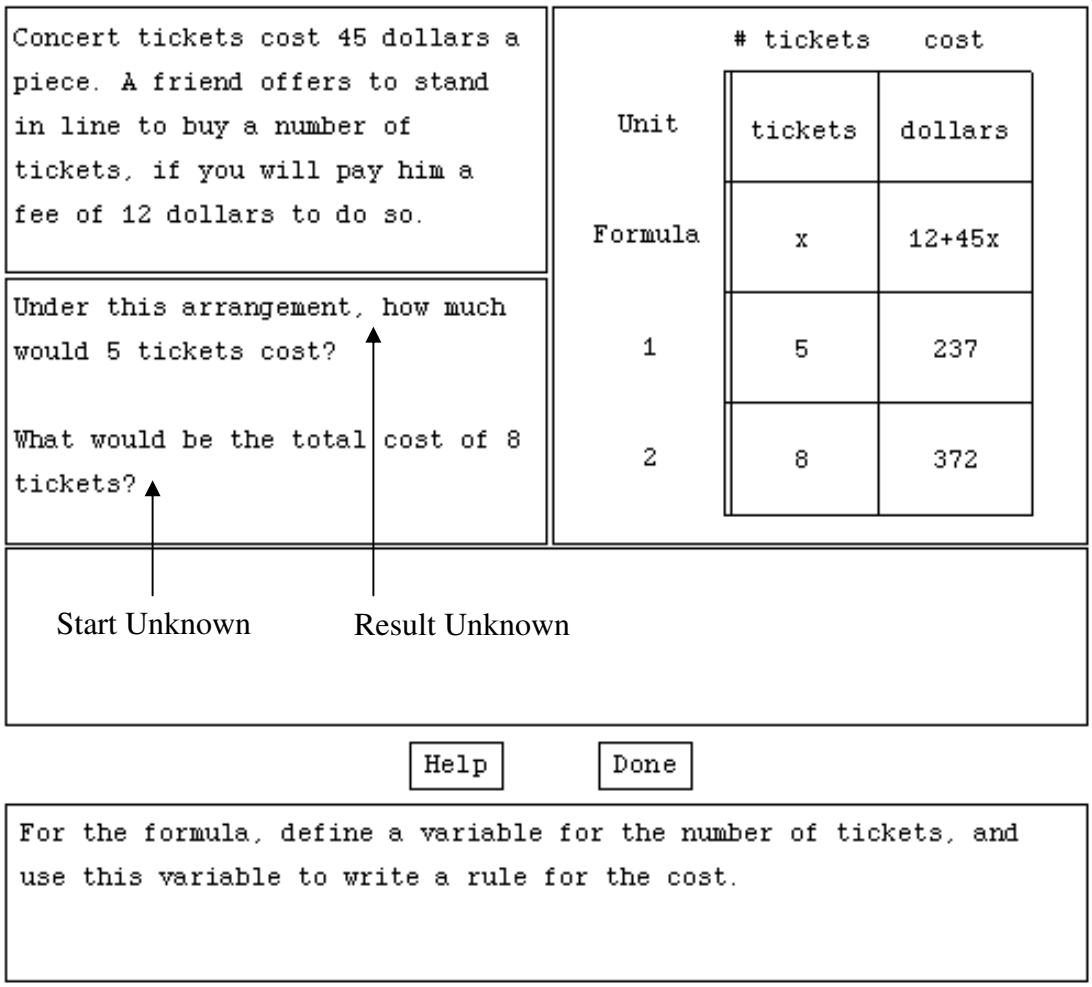

Concert tickets cost 45 dollars a piece. A friend offers to stand in line to buy a number of tickets, if you will pay him a fee of 12 dollars to do so.
Under this arrangement, how much would 5 tickets cost?
What would be the total cost of 8 tickets?
tickets
cost
Unit
tickets
dollars
Formula
x
12+45x
1
5
237
2
8
372
Start Unknown
Result Unknown
Help
Done
For the formula, define a variable for the number of tickets, and use this variable to write a rule for the cost.

Figure 2a

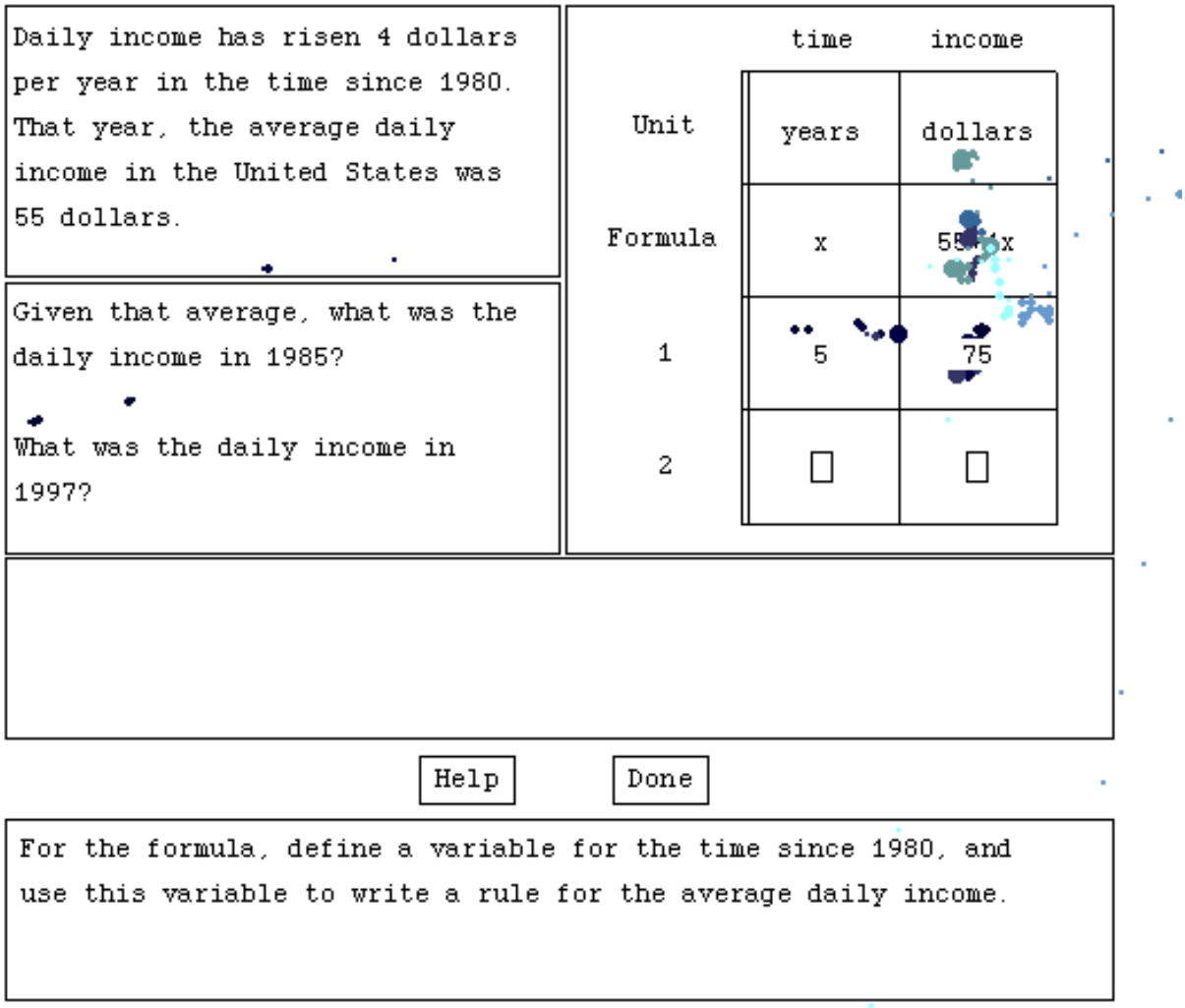

Daily income has risen 4 dollars per year in the time since 1980. That year, the average daily income in the United States was 55 dollars.
Given that average, what was the daily income in 1985?
What was the daily income in 1997?
time
income
Unit
years
dollars
Formula
x
55 + 4x
1
5
75
2
Help
Done
For the formula, define a variable for the time since 1980, and use this variable to write a rule for the average daily income.

Figure 2b

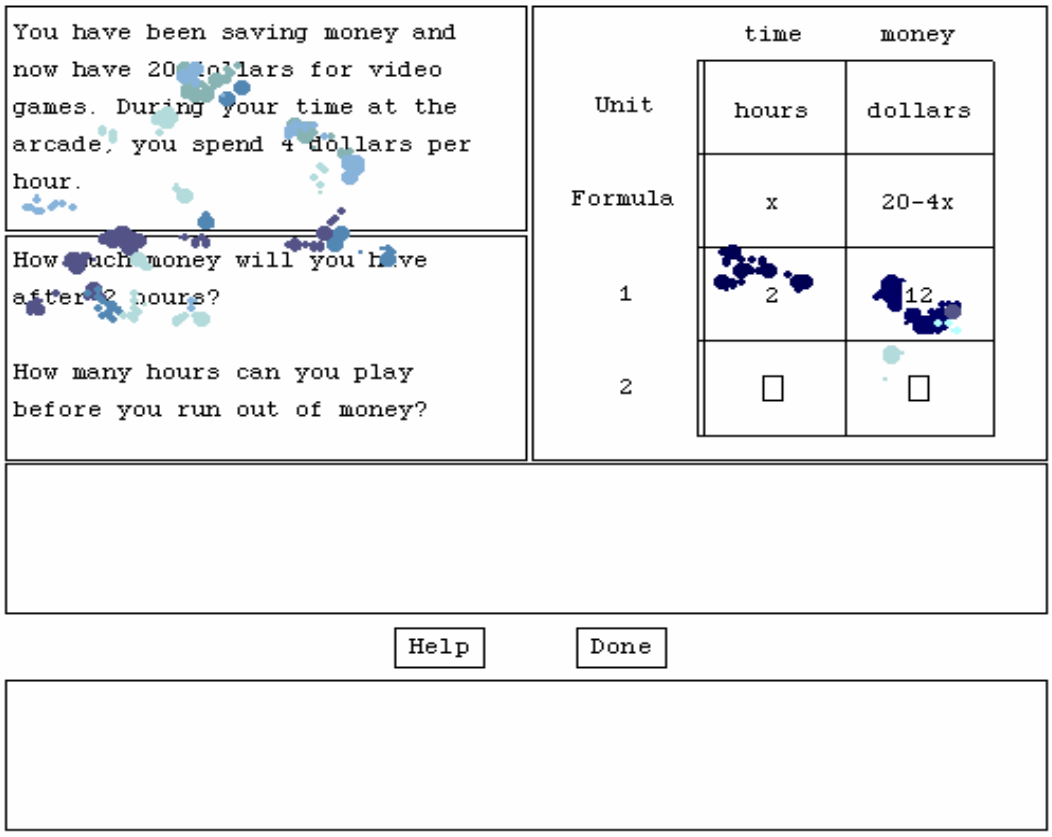

You have been saving money and now have 20 dollars for video games. During your time at the arcade, you spend 4 dollars per hour.
How much money will you have after 2 hours?
How many hours can you play before you run out of money?
time
money
Unit
hours
dollars
Formula
x
20-4x
1
2
12
2
Help
Done

Figure 3

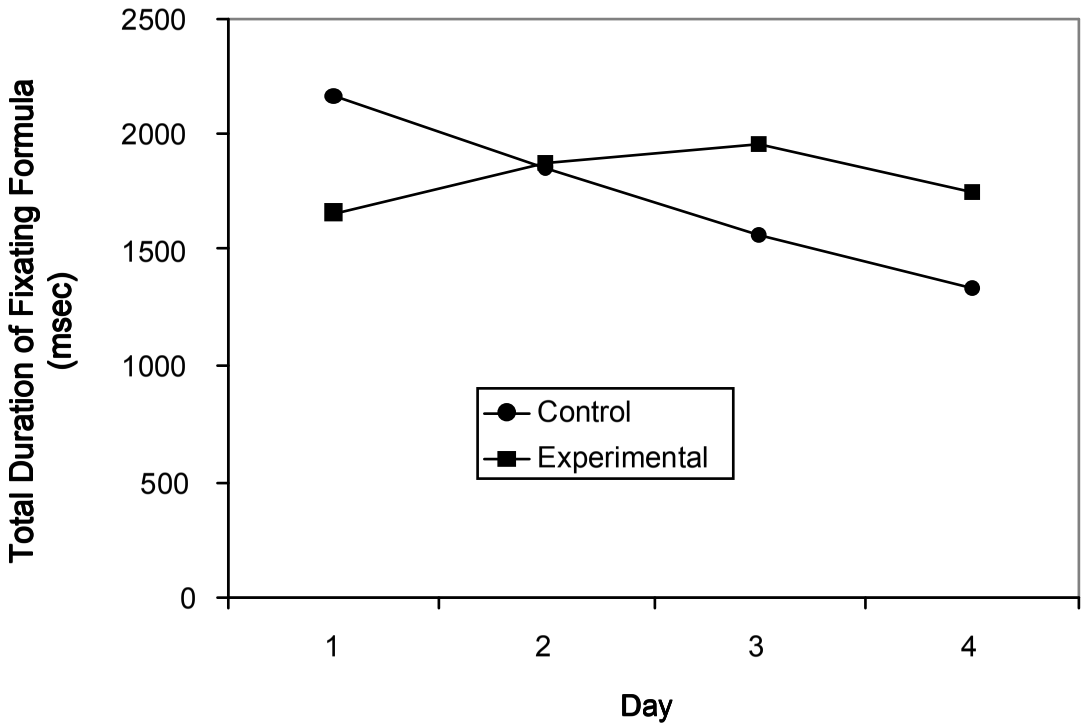

Total Duration of Fixating Formula (msec)
2500
2000
1500
1000
500
0
1
2
3
4
Day
Control
Experimental

Figure 4

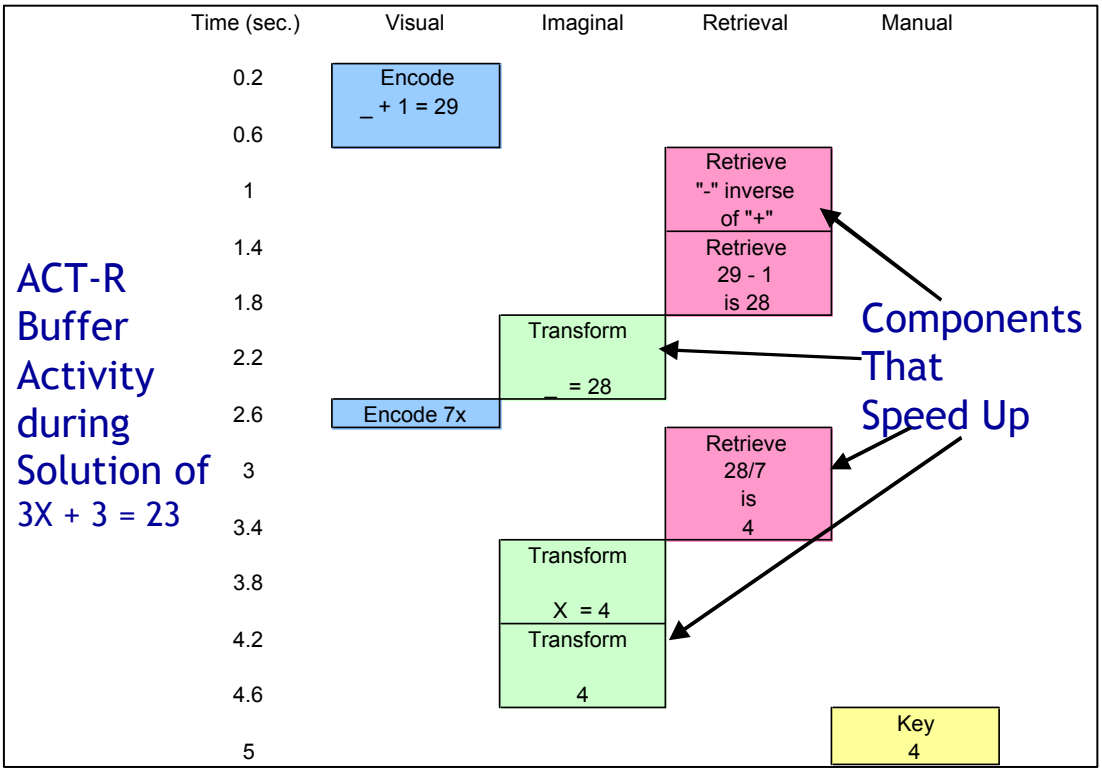


Figure 5

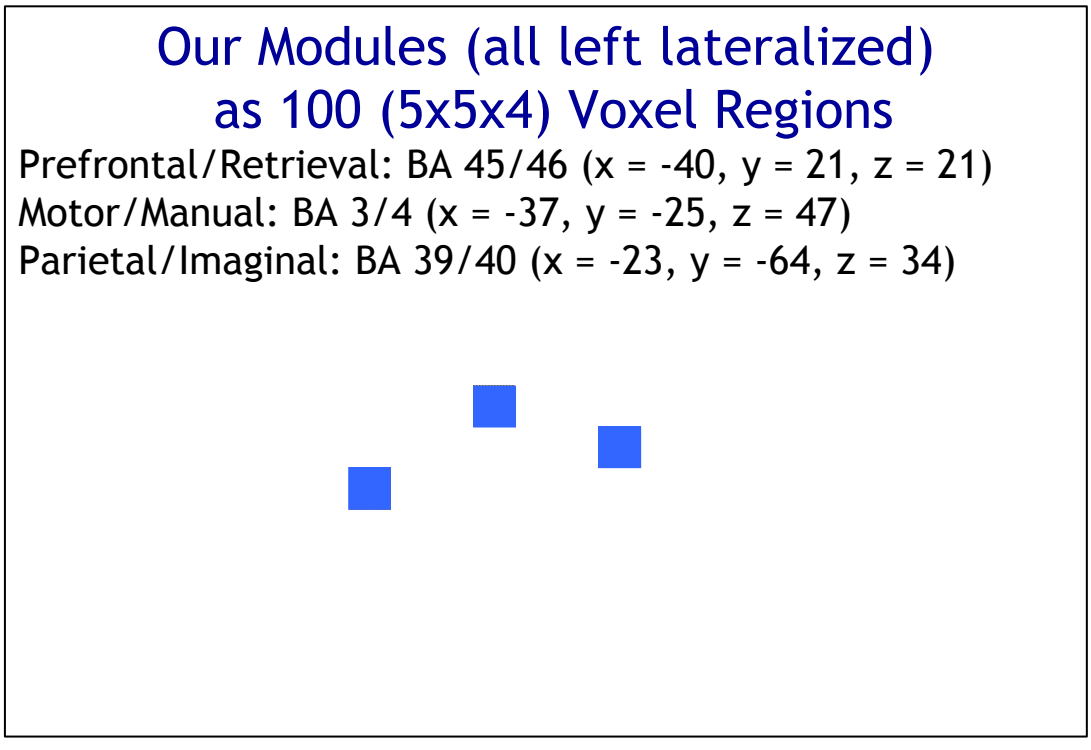


Figure 6

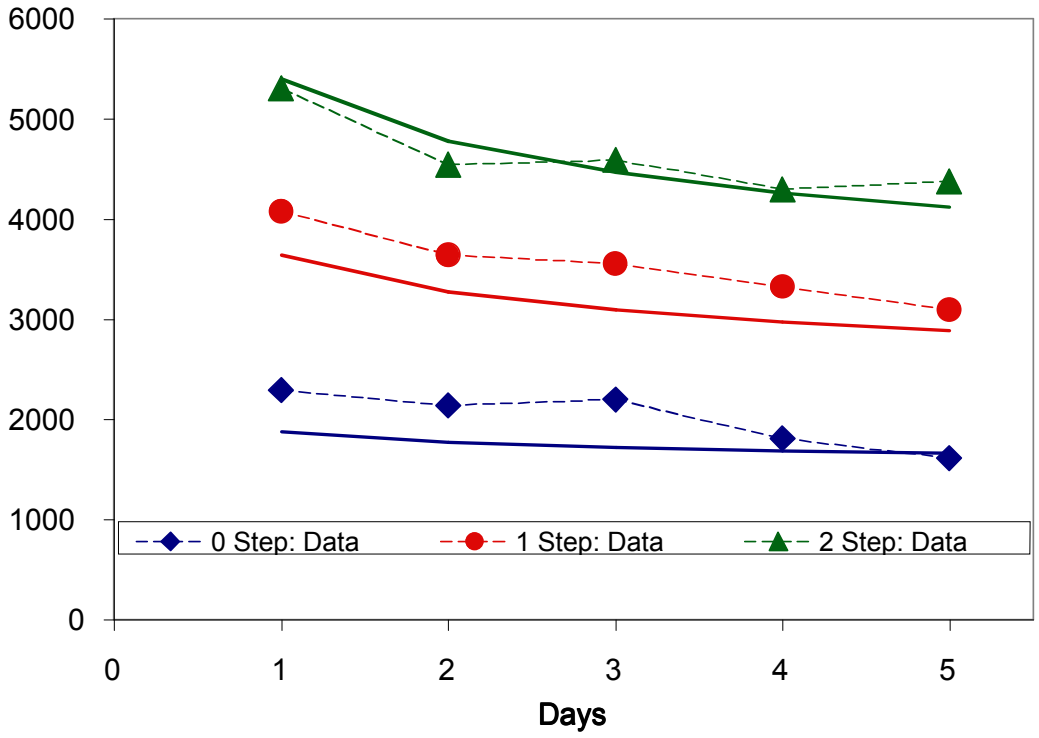


Figure 7

21.6 Second Structure of fMRI Trial

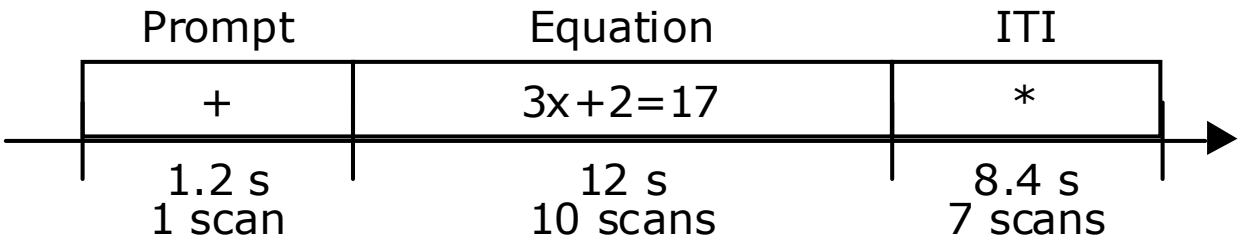


Figure 8

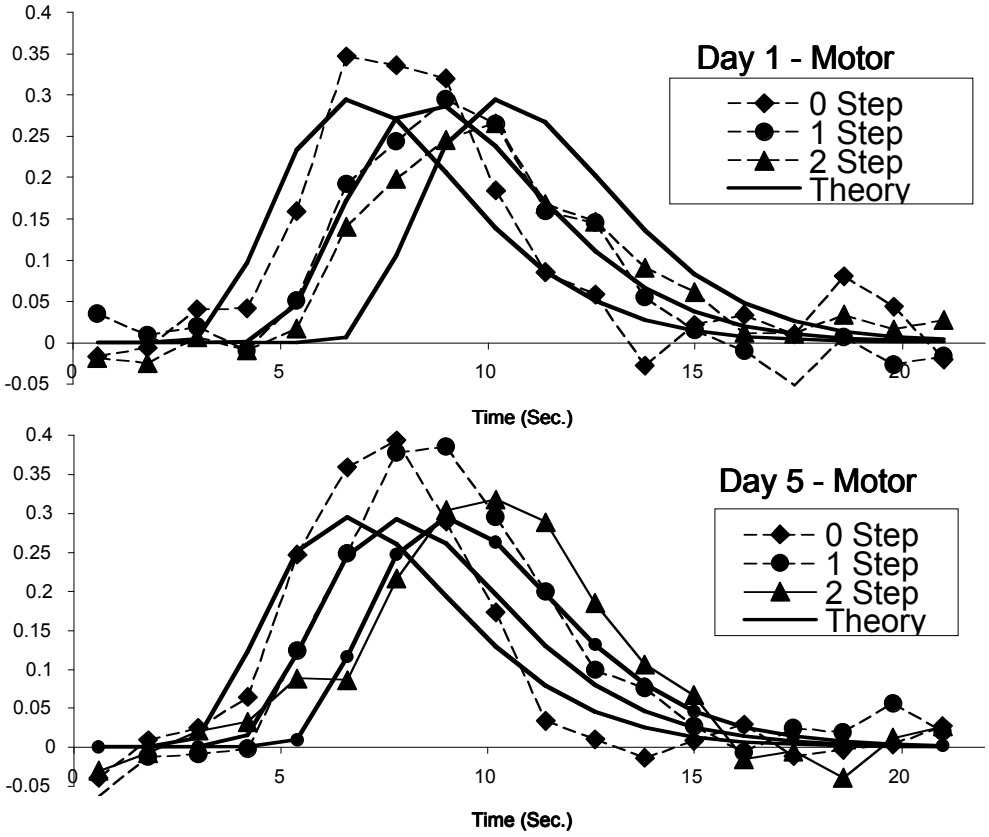

Day 1 - Motor
0 Step
1 Step
2 Step
Theory
Time (Sec.)
Day 5 - Motor
0 Step
1 Step
2 Step
Theory
Time (Sec.)

Figure 9

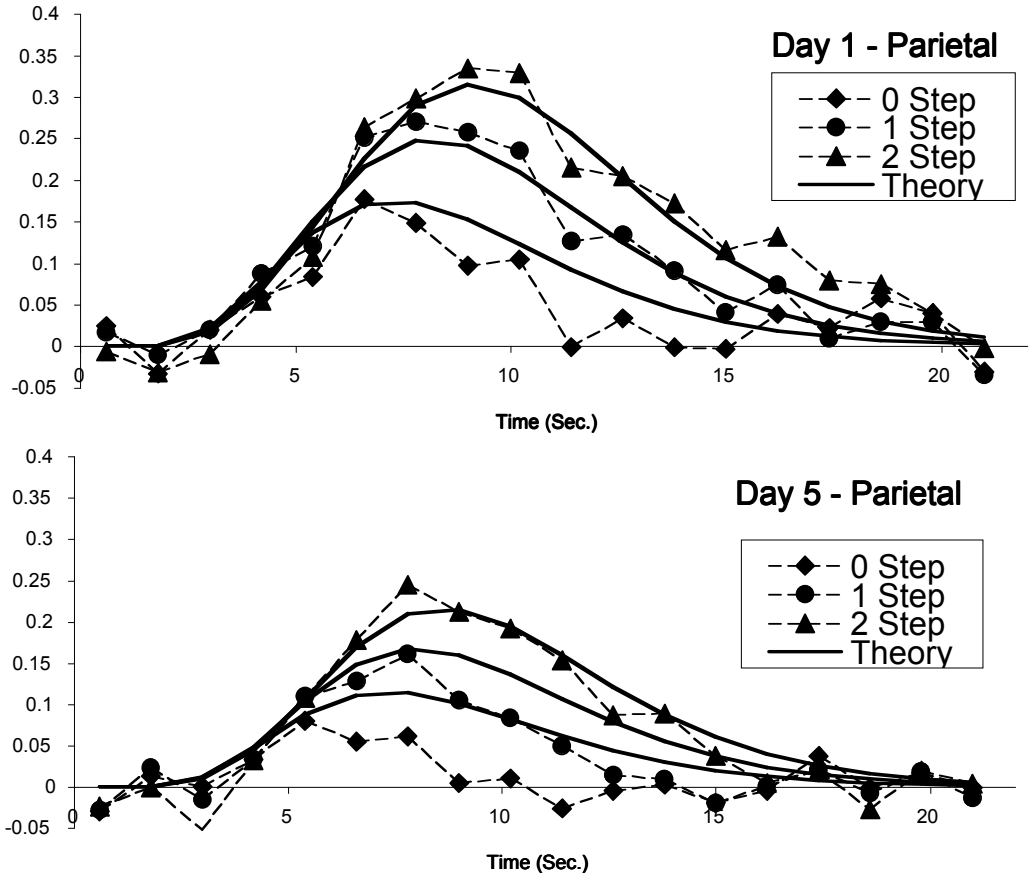
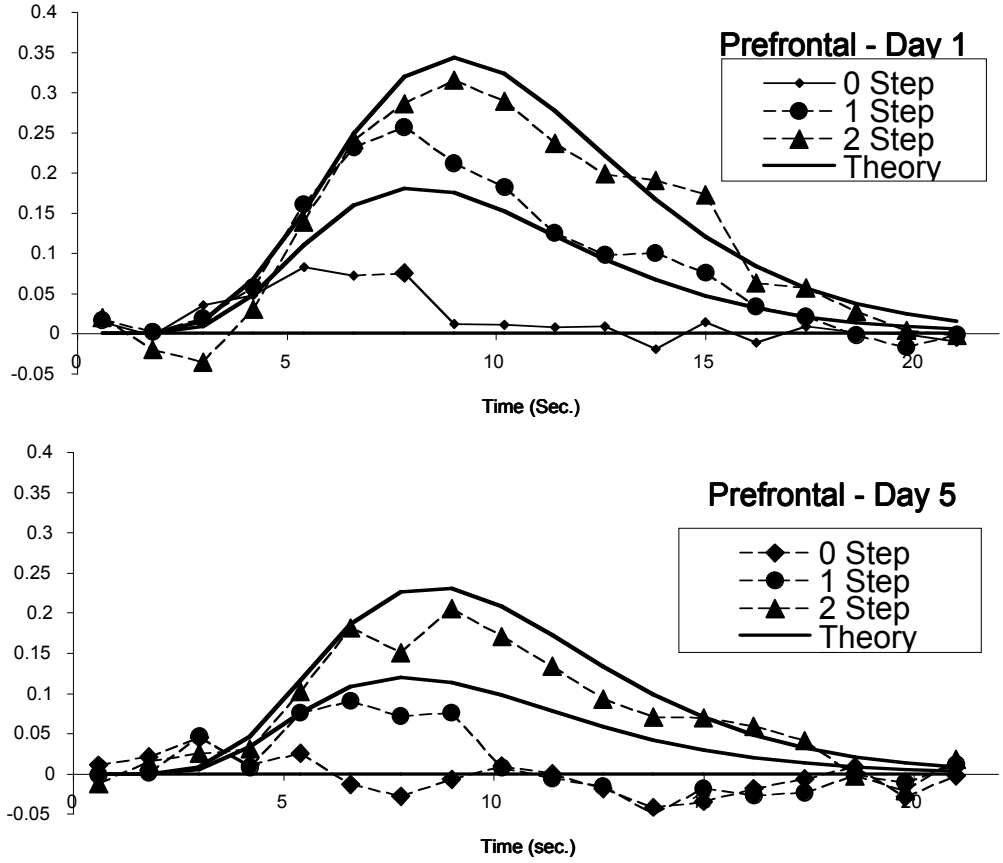

Day 1 - Parietal
0 Step
1 Step
2 Step
Theory
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-0.05
0
5
10
15
20
Time (Sec.)
Day 5 - Parietal
0 Step
1 Step
2 Step
Theory
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-0.05
0
5
10
15
20
Time (Sec.)

Figure 10


Prefrontal - Day 1
0 Step
1 Step
2 Step
Theory
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-0.05
0
5
10
15
20
Time (Sec.)
Prefrontal - Day 5
0 Step
1 Step
2 Step
Theory
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-0.05
0
5
10
20
Time (sec.)