Financial Expertise as an Arms Race[*]

Vincent Glode
Wharton School
University of Pennsylvania

Richard C. Green
Tepper School of Business
Carnegie Mellon University

and

Richard Lowery
McCombs School of Business
University of Texas at Austin

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Abstract

We propose a model in which firms involved in trading securities overinvest in financial expertise. Intermediaries or traders in the model meet and bargain over a financial asset. As in the bargaining model in Dang (2008), counterparties endogenously decide whether to acquire information, which improves their bargaining position, even though the information has the potential to create adverse selection. We add to this setting the concept of "financial expertise" as resources invested to lower the cost of later acquiring information about the value of the assets traded. These investments are made before the parties know about their role in the bargaining game, as proposer or responder, buyer or seller. A prisoner's dilemma arises because investments to lower information acquisition costs improve bargaining outcomes given the other party's information costs, even though the information has no social benefit. These investments lead to breakdowns in trade, or liquidity crises, in response to random but infrequent increases in asset volatility. Persistence in the volatility process can produce contractions and expansions resembling employment cycles for financial experts.

1   Introduction

Credit markets "froze up" in 2007-2008 following a drop in housing prices, an increase in uncertainty about the value of asset-backed securities, and the failures or government bailouts of several major financial institutions. Traditional lending relations were disrupted. Businesses, municipalities, and consumers around the world were unable to obtain credit. Economists and other informed observers agree that the negative consequences for real economic activity were dramatic and are likely to persist for some time.

Why were so many financial intermediaries unwilling to trade with each other, despite the apparent gains to trade? Financial economists explain these market failures as a consequence of adverse selection. Firms in the financial sector had invested vast resources transforming relatively straight-forward instruments, such as residential mortgages and credit-card debt, into complex instruments through securitization. They had then created trillions of dollars worth of derivative contracts based on these asset-backed securities. To facilitate this, financial firms hired legions of highly trained and highly compensated experts to design, value, and hedge the complex securities and derivatives. Unfortunately, when asset values fell and volatility rose, the complexity of the financial instruments, and the opacity of the over-the-counter markets where they traded, made it extremely difficult to identify where in the system the most damaged or the most risky liabilities were located. Indeed, the expertise firms had developed may have worked against them in the crises. Their relative advantage in valuing securities may have increased the asymmetric information they faced in dealing with relatively uninformed parties, who were in a position to supply liquidity.

The benign explanation for how the financial system reached this point is that such liquidity crises are an unfortunate consequence of increasing the efficient allocation of risk through more complex derivative securities. Individuals, firms, and regulators must learn about new financial innovations through experimentation, before their risks, and the limitations of models and systems that support their use, are fully understood. There may well be considerable truth to these arguments. We offer an interpretation of these phenomena that is less benign, but which may also help us move to a more complete understanding of them.

We develop a model in which the investments by firms in financial expertise, such as hiring

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Ph.D. graduates to design and value financial instruments of ever increasing complexity, becomes an "arms race." By this phrase we mean two things:

- Investment in financial expertise confers an advantage on any one player in competing for a fixed surplus that is neutralized in equilibrium by similar investment by his opponents.

- Investment in financial expertise is dangerous, in that it creates a risk of destruction of the surplus itself when there is an exogenous shock.

Our model shows that financial firms expecting to be involved in the trading of assets with uncertain value may find optimal to acquire socially undesirable levels of expertise and this might significantly hurt the functioning of financial markets. In the model, agents (or financial intermediaries generally) acquire expertise in processing information about an asset. The resulting efficiency in acquiring information gives them an advantage in subsequent bargaining with competitors. Neither the information, nor the expertise in acquiring and evaluating it, has any social value in our model. Yet intermediaries build such expertise despite the knowledge that it may increase adverse selection in subsequent trading and cause breakdowns in liquidity.

A basic problem in viewing financial expertise as an arms race is addressing the more fundamental question of why anyone would acquire information about a common-value object when doing so creates adverse selection problems that limit gains to trade. In most models with adverse selection in finance, some party is exogenously asymmetrically informed. If they could (publicly) avoid becoming informed, they would do so.

For example, in the classic setting described in Myers and Majluf (1984), an owner-manager-entrepreneur wishes to finance investment in a new project by selling securities to outsiders who know less about the intrinsic value of his existing assets than he does. The positive Net Present Value of this new investment is common knowledge. The entrepreneur is assumed to have acquired his information through his past history managing the firm. This informational advantage, however, is an impediment to the entrepreneur in his dealings with the financial markets, as it costs him gains to trade associated with the NPV of the new investment. If he could manage the firm's assets effectively without acquiring this information, he would do so in order to minimize frictions

associated with financing. Similarly, used-car dealers would not choose to employ expert mechanics if they could manage the car dealership without them and thus avoid the costs of the lemons problem in dealing with customers.

Given the obvious value of precommitting *not* to acquire information, why do we see financial firms, whose major business is intermediating and facilitating trading, investing vast resources in expertise that speeds and improves their ability to acquire and process information about the assets they trade? Indeed, as these entities have hired more Ph.D. graduates in finance, economics, and mathematics, and built up the elaborate information systems required to support their activities, they have increased the complexity and opacity of the financial instruments the experts and expert-systems are used to evaluate. That this has a social cost in the form of adverse-selection problems is quite evident from the massive breakdowns in liquidity associated with the sub-prime mortgage crises.

A recent paper by Dang (2008) offers a model of endogenous information acquisition in alternating-offer bargaining. The model illustrates why traders might decide to acquire costly information about a common-value asset, even though the anticipation of that acquisition may prevent trade to take place in equilibrium. In Dang's model, a buyer and a seller bargain over an asset with uncertain value and symmetric gains to trade. Each party has the opportunity to pay a cost, which is exogenous and symmetric across the buyer and seller, to determine the value of the asset. Such a model of the trading process is suited to an environment where the degree to which the parties trading are informed is a matter of choice, rather than a necessary consequence of doing business. This abstracts from other reasons intermediaries might have private information about the value of the assets they trade, but doing so allows us to highlight incentives to over invest in both information and financial expertise.

In our model, long-lived agents encounter randomly chosen counterparties each period, and bargain over a financial asset in a setting similar to Dang's model.[1] At an initial date, however, agents can invest resources to lower the costs of acquiring information about the financial assets

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[1]This portion of our analysis in Section 4 generalizes Dang's model to allow for asymmetric costs, as this is required to study the impact of investments that lower the costs of information acquisition. Dang's analysis of the situations where the first mover in the bargaining game is informed contains errors that alter the nature of the solution. We provide a more complete characterization of the equilibria, as well as correcting these errors.

they expect to trade. We interpret these initial investments as building financial expertise. We show that intermediaries face a prisoner's dilemma that drives them to invest in expertise just to the point where additional investments would lead trade to break down because of adverse selection. The resulting equilibrium is therefore delicate. Small changes in volatility would lead to market failures. We then show that even when volatility varies stochastically over time, with occasional high-volatility regimes that anticipated ex ante, agents optimally overinvest in expertise even though this leads to market failures and a loss of gains to trade when volatility is high.

When shocks to volatility are persistent, and we allow firms to adjust their investment in expertise through time, increases in volatility trigger breakdowns in trade followed by contractions in spending on expertise. In this way the model can produce dynamics similar to employment cycles on Wall Street, where employment growth is periodically halted by liquidity crises. Firms then respond to these liquidity crises by dramatically cutting back employment.

The model in our paper is naturally interpreted as trading in an over-the-counter market, since trade involves bilateral bargaining rather than intermediation through a specialist or an exchange. Most of the complex securities associated with high levels of financial expertise are traded over the counter—including mortgage- and asset-backed securities, collateralized debt obligations (CDOs), credit default swaps (CDSs), currencies, and fixed-income products such as treasury, sovereign, corporate, and municipal debt. Several models of over-the-counter trading have been proposed in the literature, such as Duffie, Garleanu and Pedersen (2005) and Duffie, Garleanu and Pedersen (2007). In these models search frictions and relative bargaining power are the sources of illiquidity. The search frictions are taken as exogenous. Investments in "expertise" that reduced search frictions would be welfare enhancing, and would lead to greater gains to trade. In contrast, adverse selection is the central friction in our model. Investments in expertise are socially wasteful and put gains to trade at risk.

Other models such as Carlin (2008) view financial complexity as increasing costs to counterparties. In Carlin (2008), however, the financial intermediary directly manipulates search costs to consumers, so these costs are most naturally interpreted as hidden fees for mutual funds, bank accounts, credit cards, and other consumer financial products. Our intent is to model an arms

race among equals—intermediaries trading with each other in the financial markets. We interpret financial expertise as a relative advantage in verifying the value of a common-value financial asset in an environment where the complexity of the security, or the opacity of the trading venue, make this costly.

Economists since Hirshleifer (1971) have recognized that in a competitive equilibrium, private incentives may lead agents to overinvest in information gathering activities that have redistributive consequences but no social value. Our model captures, in addition, the potential these investments have to create adverse selection, and thus destroy value beyond the resources invested directly in acquiring information. In addition, agents in our model behave strategically, rather than competitively, so we can capture the prisoner's dilemma they face, which drives them to invest in expertise in gathering information.

The general notion that economic actors may over-invest in services that help them compete in a zero-sum game goes back at least to Ashenfelter and Bloom (1993), which empirically studies labor arbitration hearings and argues that outcomes are unaffected by legal representation, as long as both parties have lawyers. A party that is not represented, when his or her opponent has a lawyer, suffers from a significant disadvantage. In this setting, however, the investment in legal services is not destructive of value beyond the fees paid to the lawyers. In our setting, expertise in finance has the potential to cause breakdowns in trade since it creates adverse selection.

Baumol (1990) and Murphy, Shleifer, and Vishny (1991) draw parallels between legal and financial services in arguing that countries with large service sectors devoted to such "rent-seeking" activities grow less quickly than economies where talented individuals are attracted to more entrepreneurial careers. They do not directly model the source of rent extraction, as we do.

The paper is organized as follows. In the next section we describe the model. Section 3 informally describes the central tradeoffs that drive the results. Section 4 studies the equilibria of the subgame where financial firms meet and bargain over the price of an asset. In Section 5 we evaluate the decision to invest in financial expertise, and prove our major results. We consider a model where expertise depreciates and firms respond to changes in volatility in Section 6. We argue that the resulting dynamics are similar to employment cycles on Wall Street. Section 7 concludes

and outlines directions for extensions and additional research. Proofs of lemmas and propositions, along with some other technical derivations, are contained in the Appendix.

2   Model

There is a continuum of risk-neutral and infinitely-lived financial intermediaries or traders. In each period t, $t = 1, \ldots, \infty$, agent i meets a randomly chosen counterparty from the set of potential traders, and they have the opportunity to exchange a financial security. When they meet, agent i will be assigned the role of buyer or seller with equal probability, and his counterparty assumes the other role. At $t = 0$ trader i can invest resources, denoted e_i, in financial expertise. This serves to lower the cost of acquiring information about the value of the assets they will exchange in their future trading encounters.

The financial asset has a common-value component, v, and private-values to the buyer and seller that generate gains to trade. The buyer's valuation of the object is $v + \Delta$. The seller's valuation is $v - \Delta$. The gains to trade are common knowledge and constant through time. The common value is independently distributed through time.

The common value can be high, $v = v_h$, or low, $v = v_l$, with equal probability. Suppose i is the seller and j is the buyer. Then at a cost of $c_s = c(e_i)$ and $c_b = c(e_j)$ the seller and buyer, respectively, can engage in research activities to evaluate the asset and determine its value. We will refer to a decision to incur this cost as a decision to "investigate." We assume the cost of investigation, $c(e)$, is positive, continuous, twice continuously differentiable, convex, and monotonically decreasing in the resources invested at date zero in building expertise ($c'(e) < 0, c''(e) > 0$). Expertise, which can be viewed as both human capital and the infrastructure to support it, allows a trader or an institution to more quickly and efficiently value a security. We assume that all agents who do not acquire expertise face the same investigation cost, $c(0) > 0$.

Trading is an ultimatum game where the buyer makes a take-it-or-leave-it offer. He decides, first, whether or not to pay the cost c_b to learn the asset's value, and then makes a take-it-or-leave-it offer to the seller to exchange the asset at price p. The seller, in turn, decides whether to investigate, and determine the value of the asset, and then whether to accept the offer. The seller's

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investigation decision is, thus, conditioned on the buyer's offer. If the buyer's price is accepted, trade occurs and the buyer receives a payoff of

$$v + \Delta - p$$

while the seller gets

$$p - (v - \Delta).$$

We assume that all random variables are drawn independently across time, and that the trading histories of firms are not observable, consistent with the opacity of OTC markets. Levels of expertise, which are the result of investments made at $t = 0$, are known to all counterparties. These assumptions ensure that agent i plays the same trading game in each period, conditional on the expertise of the counterparty. Let $EU^i(e_i, e_j)$ denote the expected payoff to agent i, before he knows if he is a buyer or a seller, given that he has expertise e_i and his opponent has expertise e_j. Then the expected gain before the opponent is revealed is

$$E_0 U^i(e_i) = \int EU^i(e_i, e_j) dF(e_j),$$

where $F(e)$ is the distribution of expertise across the population. All agents discount future profits at rate δ, so agent i chooses his expertise to maximize:

$$\frac{1}{1 - \delta} E_0 U^i(e_i) - e_i.$$

Reversing the role of the buyer and seller, as proposer or responder, has no effect on our results beyond notational complexities, since we assume that agents do not know who will be the buyer and who will be the seller until they meet. They invest in expertise before this, at $t = 0$. We assume what is predetermined is who moves first, buyer or seller, and then allow uncertainty over who becomes buyer and who becomes seller. Alternatively, we could assume that it is predetermined which agent is buyer or seller, and that who moves first is uncertain. This adds notational complexity, since the two cases (buyer moves first or seller moves first) must be tracked through the subgame. What is

important for our results is that, at the point when they decide to invest in expertise, the agents face symmetry with respect to their chances of being the proposer or responder in the ultimatum trading game.

Information acquired by paying c_s or c_b has no social value in this model. It simply serves to increase one's share of a fixed pie, unless it destroys value by shutting down trade due to adverse selection. Similarly, investments in expertise, since they only serve to alter the information acquisition costs, are also socially wasteful. We are abstracting from any broader benefits to expertise and information acquisition, such as improved risk sharing or better coordination of real investment due to more informative prices. This highlights the incentives to engage in an arms race in expertise, despite the costs of adverse selection it engenders. In a more general model, where information and expertise in acquiring and efficiently processing it add value, these incentives would lead to *over*investment in financial expertise.

3 Outline of Central Tradeoffs

In each bargaining subgame, the gains to trade are 2Δ. Under symmetric information, since each party moves first half the time, the expected surplus to each is Δ.

Consider first the two extreme cases.

With $c_s = c_b = 0$ regardless of expertise, it is simple to show that acquiring information is always a best response for the responder. Given this, the first-mover will also acquire information. Trade occurs at two fully informed prices of $v_h - \Delta$ and $v_l - \Delta$ and all gains to trade are consumed. (If the seller moved first, these would be $v_h + \Delta$ and $v_l + \Delta$.)

Similarly, if the costs are very high regardless of expertise, then in equilibrium neither party will acquire information, trade occurs at $p = E(v) - \Delta$, and all gains to trade are consumed.

Most of the intuition driving our results can be illustrated by considering the effect of the seller's information cost on the strategies and payoffs around the point where those costs are so high that the buyer can extract the full surplus. The critical level of the investigation cost for the responder

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that precludes information acquisition is evident from the analysis in Dang's paper. It is

$$\frac{1}{4}(v_h - v_l),$$

which Dang (2008) refers to as the "gains from speculation." The quantity $(v_h - v_l)$ also has a natural interpretation in financial settings as a measure of volatility, or uncertainty about the asset's fundamental value. Thus, the incentive to investigate, which is socially wasteful, is more difficult to resist when volatility is high. Learning whether an asset is worth 50 or 150 dollars is more valuable than learning whether it is worth 99 or 101.

Assume the buyer does not acquire information, and attempts to extract all the surplus by offering $p = E(v) - \Delta$. If the seller accepts the offer, he gets an expected payoff of zero. If he pays the cost to investigate and determine the value, he will only sell if $v = v_l$. Trade will occur with probability 0.5, and the seller's expected payoff at the time he must decide whether to pay the cost will be

$$\frac{1}{2}\left[p - (v_l - \Delta)\right] - c_s = \frac{1}{2}\left[(E(v) - \Delta) - (v_l - \Delta)\right] - c_s$$
$$= \frac{1}{4}(v_h - v_l) - c_s. \tag{1}$$

The seller will not want to investigate when gains from speculation net of the cost of investigation are negative. Thus, if the seller's cost is sufficiently high, the buyer can make a take-it-or-leave-it offer, and extract the full surplus of 2Δ. The buyer will do this irrespective of his own information acquisition cost, since he cannot do better than an expected payoff of 2Δ.

Now suppose c_s is slightly lower than $\frac{1}{4}(v_h - v_l)$, and let

$$\varepsilon \equiv \frac{1}{4}(v_h - v_l) - c_s.$$

If the buyer offers the seller's unconditional reservation price in this situation, he knows the seller will investigate and only sell if the value is low. This subjects the buyer to adverse selection, and

occasions a loss of the gains to trade half of the time. By offering a slightly higher price,

$$p = E(v) - \Delta + \eta,$$

where $\eta > 0$, the buyer may be able to "bribe" the seller to accept the offer unconditionally. If the seller investigates he will have to pay c_s. He will only trade, and collect the "bribe," half of the time. If the seller accepts unconditionally, he earns the full price premium, η, and saves the information acquisition cost.

The unconditional-trade outcome is obviously better for the buyer. If the seller accepts unconditionally he gains the full surplus of 2Δ, less the price premium of η, while if the seller investigates the gains to trade are lost half of the time and, worse, when trade does occur he buys at an average price and loses money because the asset's value is low.

How big must the price premium η be? If the seller investigates, his payoff is

$$
\begin{aligned}
\frac{1}{2}\left[(E(v) - \Delta + \eta) - (v_l - \Delta)\right] - c_s &= \frac{1}{4}(v_h - v_l) - c_s + \frac{1}{2}\eta \\
&= \varepsilon + \frac{1}{2}\eta.
\end{aligned}
\tag{2}
$$

If he accepts unconditionally, the seller receives η. For the latter to be attractive to the seller, we must have $\eta \geq 2\varepsilon$. The buyer wants to pay as little as possible, and so the equilibrium price will be:

$$
\begin{aligned}
p &= E(v) - \Delta + 2\varepsilon \\
&= E(v) - \Delta + 2\left[\frac{1}{4}(v_h - v_l) - c_s\right] \\
&= v_h - \Delta - 2c_s.
\end{aligned}
\tag{3}
$$

The seller receives an expected surplus of

$$2\varepsilon = \frac{1}{2}(v_h - v_l) - 2c_s$$

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while the buyer earns

$$2\Delta - 2\varepsilon = 2\Delta - \frac{1}{2}(v_h - v_l) + 2c_s.$$

Notice the the payoffs of the buyer and seller are symmetric and linear in the seller's cost, and do not depend on the buyer's cost. The lower the seller's cost, the more credible the seller's threat to investigate, which would create adverse selection and destroy gains to trade, and the higher the price premium the buyer must offer to prevent this.

Now consider the consequences of allowing the traders to invest resources at an initial date, to build financial expertise that lowers the information acquisition costs they face in subsequent trading. At date zero, when investments are made in financial expertise, an agent has an equal chance of becoming the buyer or seller on encountering a counterparty. Suppose all agents invest zero in expertise, $e_i = e_j = 0$, and that at zero investment the cost of investigation rules out information acquisition by the seller, $c(0) > \frac{1}{4}(v_h - v_l)$. In equilibrium at these costs, the buyer makes an unconditional offer that extracts the full surplus from the seller, $p = E(v) - \Delta$, and earns a surplus of 2Δ, while the seller earns no surplus.

This gives both agents a powerful incentive to lower their respective investigation costs. If agent i can invest to the point where $c(e_i) < \frac{1}{4}(v_h - v_l)$ he is pushed into the region where, whenever he is the seller, he captures part of the surplus. The price the buyer pays him increases linearly as his cost declines. By lowering his costs agent i protects himself in subsequent bargaining whenever he turns out to be the seller. This costs him nothing when he turns out to be the buyer, because the division of surplus depends only on the seller's costs. When agent i is the buyer, his opponent's costs are the result of his opponent's investment choice, which is taken as given in a Nash equilibrium. As long as the resources invested in expertise, e_i, are less than the discounted value of half the seller's surplus in the subgame at the lower cost, it is a dominant strategy to make the investment in expertise.

By symmetry, of course, the same arguments apply to any other agent j, so the resulting prisoner's dilemma will force the agents to invest in expertise to lower investigation costs that, in that particular subgame, they never pay anyway.

Facing agent j in a particular trading encounter, agent i receives a surplus of $\frac{1}{2}(v_h - v_l) - 2c(e_i)$

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as the seller, and a surplus of $-\frac{1}{2}(v_h - v_l) + 2\Delta + 2c(e_j)$ as the buyer. Agent i's expected surplus across the two possibilities is

$$\Delta - [c(e_i) - c(e_j)]. \tag{4}$$

Note that this is half the total surplus of 2Δ, less the excess of his cost over that of the other agent. Taking his counterparty's expertise as given, each agent has an incentive to increase his own expertise, as it helps him when he is the responding party in bargaining, but has no effect on his payoffs when he is the proposer.

Where will the traders stop investing? What, beyond the curvature of the function $c(\cdot)$, limits the arms race? One possibility is that the price premium increases to the point where the seller is absorbing the entire surplus. Then any additional investment will make any offer by the buyer unprofitable, and trade will simply break down. Another possibility is that the buyer's share of the surplus will decline to the point where he will be better off acquiring information himself. The trading game then becomes a signalling game, with only mixed-strategy equilibria that rule out efficient trade. The analysis in the next section is devoted to delineating these boundaries formally.

In either case, the players will have incentives to keep investing until their costs reach such a boundary. At that point any decrease in their cost, or, alternatively, any increase in the volatility $(v_h - v_l)$, will lead to breakdowns in trade. In Section 5 we demonstrate this formally, first, assuming the volatility is constant. In that case, as is clear from expression (4), in a symmetric equilibrium the investigation costs have no effect on the efficiency of trade. These costs are not actually paid in equilibrium. The ex-ante expected surplus from the ultimatum game is the same as when costs exceed $\frac{1}{4}(v_h - v_l)$, because the gains one obtains as a seller are offset by the higher price paid as a buyer. The deadweight loss is limited to the resources expended building expertise, e_i. We then go on to show that if volatility can randomly increase with some sufficiently small probability, it is still an equilibrium for traders aware of that possibility to invest to the point where their costs are on this boundary, and that in the high volatility regime trade breaks down. That is, along with an "arms race," we occasionally have a "war" in which the surplus the players are competing for is either partly or entirely destroyed.

4 The ultimatum subgame

For certain regions of the parameters and investigation costs, the first mover in the ultimatum game may choose to become informed. This creates a signaling game, in which the proposer's offered price can reveal information to the responder. Agents will generally employ mixed strategies, creating an endogenous positive probability of trade breakdown. We will first analyze the subgame assuming the buyer, who is the first mover or proposer in the ultimatum game, never investigates, and then study the subgame where the first mover is informed. The final subsection then evaluates when the buyer will choose to become informed, comparing his payoffs in the various subgames. We assume throughout the analysis that when indifferent between multiple responses that yield the same expected payoff, the responder (seller) chooses the one that is best for the proposer (buyer).

4.1 Subgame with uninformed buyer

Assume the buyer, who makes the take-it-or-leave-it offer, does not acquire information. In this subgame, the equilibrium price will depend only on the seller's investigation cost, c_s.

First, note that we can ignore prices $p < v_l - \Delta$, where the seller would not trade regardless of his information, or $p > E(v) + \Delta$, where the buyer would never trade given that he is uninformed.

For any p between these bounds, if the seller investigates, he sells only if $v = v_l$. The seller's expected surplus at the point he decides to investigate is then

$$\frac{1}{2}[p - (v_l - \Delta)] - c_s \tag{5}$$

and the buyer's expected surplus is

$$\frac{1}{2}(v_l + \Delta - p).$$

If the seller does not investigate, and accepts the offered price, the seller's surplus is

$$p - [E(v) - \Delta] \tag{6}$$

while the buyer receives

$$E(v) + \Delta - p.$$

If the seller does not investigate and refuses the offer both parties get zero.

In any situation where there is trade, regardless of the seller's response, the buyer's payoff decreases in the price. For any given response by the seller, therefore, the buyer will offer the lowest price that sustains that response. Accordingly, we need only consider three possible candidate prices. There is a minimum price that would cause a seller to reject the offer rather than investigating. This price, p_1^*, sets (5) equal to zero and is given by

$$p_1^* = v_l - \Delta + 2c_s. \tag{7}$$

There is a unique price at which the seller is indifferent between investigating and trading without information. This price sets (5) equal to (6), and is given by

$$p_2^* = v_h - \Delta - 2c_s. \tag{8}$$

Finally, there is the lowest price that keeps an uninformed seller from rejecting the offer and earning zero. This sets (6) equal to zero and is given by

$$p_3^* = E(v) - \Delta. \tag{9}$$

We present four lemmas that exhaust the possible outcomes to the subgame when the buyer is uninformed. The logic of the proofs, which are in the Appendix, generally follows Dang (2008), allowing for asymmetric costs. We then provide more intuition about how the seller's investigation cost affects trade and bargaining. The first result shows that if costs are high enough, efficient trade always occurs.

Lemma 1 *When the buyer does not acquire information and $c_s > \frac{1}{4}(v_h - v_l)$, then in the unique Perfect Bayesian Equilibrium to the ultimatum subgame:*

1. *The seller does not investigate.*

2. *Trade always takes place.*

3. *The price is $p_3^* = E(v) - \Delta$.*

4. *The buyer's expected surplus is 2Δ.*

5. *The seller's expected surplus is zero.*

The next result details two sets of conditions when the surplus is split between the two counterparties, and plays a central role in our subsequent analysis. In these regions, the payoffs to the seller are decreasing in his costs of investigation, providing him with incentives to improve his bargaining position by investing in expertise and lowering these costs.

Lemma 2 *When the buyer does not acquire information and either or both of the following conditions hold*

(a.) $\max\{\Delta, \frac{1}{4}(v_h - v_l) - \Delta\} < c_s \leq \frac{1}{4}(v_h - v_l)$,

(b.) $\frac{1}{6}(v_h - v_l) - \frac{1}{3}\Delta < c_s \leq \min\{\Delta, \frac{1}{4}(v_h - v_l)\}$,

then in the unique Perfect Bayesian Equilibrium to the ultimatum subgame:

1. *The seller does not investigate.*

2. *Trade always takes place.*

3. *The price is $p_2^* = v_h - \Delta - 2c_s$.*

4. *The buyer's expected surplus is*

$$-\frac{1}{2}(v_h - v_l) + 2\Delta + 2c_s.$$

5. *The seller's expected surplus is*

$$\frac{1}{2}(v_h - v_l) - 2c_s.$$

The next result shows that when costs are sufficiently low, the buyer investigates and some gains to trade are lost when value is high.

Lemma 3 *When the buyer does not acquire information and $c_s \leq \min\{\Delta, \frac{1}{6}(v_h - v_l) - \frac{1}{3}\Delta\}$, then in the unique Perfect Bayesian Equilibrium to the ultimatum subgame:*

1. *The seller investigates and trades only if $v = v_l$.*

2. *The price is $p_1^* = v_l - \Delta + 2c_s$.*

3. *The buyer's expected surplus is*

$$\Delta - c_s.$$

4. *The seller's expected surplus is zero.*

Finally, we must consider what happens when the buyer cannot make a profitable trade, either by making an offer that leads the seller to investigate or by making an offer that prevents the seller from investigating.

Lemma 4 *When the buyer does not gather information and $\Delta < c_s < \frac{1}{4}(v_h - v_l) - \Delta$, then in the set of perfect Bayesian Equilibria to the ultimatum subgame:*

1. *The seller does not investigate.*

2. *The buyer offers a price $p < v_l - \Delta + 2c_s$.*

3. *Trade never occurs.*

4. *The buyer's expected surplus is zero.*

5. *The seller's expected surplus is zero.*

Figure 1 illustrates the full set of possible outcomes associated with the four lemmas above. While there are five possible cases, depending on the relative magnitudes of the gains to trade, Δ, and the volatility, $v_h - v_l$, all of them share the following structure. For high levels of the seller's information acquisition cost, $c_s \geq \frac{1}{4}(v_h - v_l)$, trade takes place with probability one, and the buyer, as first mover, extracts the full surplus. Below this region is an area described by Lemma 2, where trade takes place with certainty, but the buyer and seller split the surplus. The buyer must share enough of the surplus with the seller to keep him from investigating, in order to preserve the gains to trade when value is high. As is clear from Lemma 2, this payment increases as the seller's cost declines and the threat to investigate becomes more credible. The seller's share of the surplus is

maximized at the lower boundary of this region, which depends on the relative magnitudes of the gains to trade, Δ, and the volatility, $(v_h - v_l)$. Below the region covered by Lemma 2, trade breaks down in whole or in part. In Cases 2-4, for low values of the seller's cost of investigation, the seller investigates and declines the buyer's offer when the value is high. In all these situations the seller earns zero surplus (see Lemma 3). In Case 1 there is also an intermediate region where the surplus the buyer would need to pay the seller to discourage investigation is too high for the buyer to break even, and trade breaks down entirely as shown in Lemma 4.

4.2   The ultimatum game with an informed first mover

In this section, we analyze the game under the assumption that the buyer has acquired information. Our analysis of this situation differs substantially from the results in Dang (2008).[2]

When the first mover in the ultimatum game becomes informed with positive probability, then the ultimatum game becomes a signaling game. We begin the analysis of the buyer's decision about whether to become informed by noting two simple features of the model.

First, if the seller's investigation cost is high enough to discourage him from ever investigating, $c_s > \frac{1}{4}(v_h - v_l)$ as in Lemma 1, then it would never pay for the buyer to become informed about the asset if $c_b > 0$. Since the buyer earns the entire surplus of 2Δ in this situation, he cannot improve his bargaining power in any way by investigating, and investigating involves a positive cost. Thus, outcomes are first-best if the seller's costs are high enough. For this case we have fully characterized the equilibrium.

Lemma 5 *If $c_s > \frac{1}{4}(v_h - v_l)$, then for any value of $c_b > 0$, the buyer chooses not to become informed about the value of the asset and the equilibrium quantities are given in Lemma 1.*

Second, as Dang (2008) shows, once the buyer does become informed, there are no best responses in pure strategies, so any equilibrium to the ultimatum subgame must involve mixed strategies.

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[2]There are several algebraic mistakes in the proofs of Dang (2008) for this situation, which we note in our analysis below and in the Appendix. As a result our expressions for the equilibrium prices are quite different than his, and the equilibrium that is best for the buyer will in some circumstances not exist. In addition, we note that Dang (2008) treats each price offered by the informed first-mover as initiating a proper subgame. In fact, there is a continuum of equilibria that are perfect Bayesian.

If the buyer learns v, and truthfully offers $p_l = v_l - \Delta$ or $p_h = v_h - \Delta$, the seller would always accept without investigating. But given that strategy for the seller, the buyer could, with positive probability, offer p_l when $v = v_h$ and be better off. If, on the other hand, the seller refuses any offer less than $p = v_h - \Delta$, the buyer should always offer that price when $v = v_h$. But given a truthful offer when value is high, the seller should accept a price of $v_l - \Delta$ when value is low, as it reveals the truth. Thus, truthful offers are not an equilibrium, and neither is always offering a high price.

Similar arguments also imply that the seller will never investigate with probability one when the buyer is informed. If the seller always investigates, he will reject any offer that is not truthful. This implies, however, that the buyer will never make a misleading offer, as it costs him gains to trade. If the buyer is always truthful, then the seller has no incentive to incur the costs of investigation.

We now introduce two new critical prices. The first is the lowest price at which a seller will not expect to lose money regardless of his beliefs:

$$p_h^* = v_h - \Delta.$$

The second critical price is :

$$p_l^* = v_l - \Delta + z^*,$$

where

$$z^* \equiv \frac{1}{2}\left[(v_h - v_l) - \sqrt{v_h - v_l}\sqrt{v_h - v_l - 4c_s}\right].$$

This will turn out to be the equilibrium price in the best equilibrium for the buyer when the buyer investigates and the value is low *or* when the value is high and the buyer attempts to exploit his private information by offering a price that will give the seller a negative payoff if he accepts.[3]

It is immediate that in any perfect Bayesian equilibrium no price above p_h^* will ever be offered. As is typical in signaling games, where the informed party moves first, there are many perfect Bayesian equilibria, reflecting the many possible equilibrium beliefs the responding party might have about the buyer's type as a consequence of the offered price. We will focus on the equilibrium most favorable to the buyer. This is equivalent to assuming that the buyer can announce and

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[3]Our expression for z^* is completely different than that in Step 3d in the Appendix of Dang (2008).

commit to a finite set of prices he will charge before observing the true value.

We show that in the subgames where the buyer investigates the equilibria all involve two possible prices. When the buyer observes a low value, he always offers p_l^*. When he observes a high value, however, he mixes between this price and offering the seller p_h^*. When the seller accepts the low price in the high-value state, without investigating, then the buyer is earning information rents. Our proof of the two lemmas below shows that the buyer never mixes over more than two prices. In some circumstances, described by Lemma 7, there is a continuum of equilibria, each involving a different p_l^*, but all of them have the structure that the buyer offers that price when value is low, and mixes across it and p_h^* when value is high.

The following two lemmas describe the optimal equilibrium for the buyer in the subgame in which he decides to become informed.

Lemma 6 *When $c_s \leq \frac{2\Delta(v_h - v_l)^2}{(v_h - v_l + 2\Delta)^2}$ and the buyer acquires information, then in the optimal Perfect Bayesian Equilibrium from the perspective of the buyer:*

1. *If the buyer observes $v = v_l$, the buyer offers the price p_l^*.*

2. *Using $z^* = \frac{1}{2}\left[(v_h - v_l) - \sqrt{v_h - v_l}\sqrt{v_h - v_l - 4c_s}\right]$, if the buyer observes $v = v_h$, he offers p_l^* with probability*
$$\frac{c_s}{2z^*}$$
   and p_h^* otherwise.

3. *The seller buys if the buyer offers p_h^*.*

4. *If the buyer offers p_l^*, the seller investigates with probability*
$$\frac{2\Delta}{v_h - v_l + 2\Delta - c_s}$$
   and sells only if the value is low.

5. *If the seller does not investigate, he always sells.*

6. *The buyer obtains a surplus of*
$$2\Delta - c_b - \frac{1}{2}z^*.$$

7. *The seller receives zero surplus.*

Lemma 7 *When $\frac{1}{4}(v_h - v_l) > c_s > \frac{2\Delta(v_h - v_l)^2}{(v_h - v_l + 2\Delta)^2}$, the least upper bound of the payoffs to the buyer in a perfect Bayesian equilibrium is given by*

$$\Delta - c_b + \frac{2\Delta^2}{v_h - v_l + 2\Delta},$$

while the greatest lower bound for the seller's payoff is zero. In any sequence of Perfect Bayesian Equilibria whose payoff to the buyer converges to the least upper bound:

1. *The price offered by the buyer when $v = v_l$ converges to $v_l - \Delta$.*

2. *The probability that the price offered by the buyer when $v = v_h$ is p_h^* converges to 1.*

3. *The probability that the seller sells after seeing an offer of p_l converges to*

$$\frac{2\Delta}{v_h - v_l + 2\Delta}.$$

4. *The probability that the seller investigates converges to 0.*

Lemma 7 describes the limit of the equilibria that are best for the buyer when the seller's costs lie in an intermediate range. Equilibria exist in which the buyer can drive the seller arbitrarily close to his participation constraint, or a zero expected payoff. He would achieve this in an equilibrium where his offered price always revealed the truth, where the seller accepted unconditionally, and where the price when value is low was $v_l - \Delta$ (i.e., $z^* = 0$). We know, however, from the arguments at the beginning of this section, that truth telling is not an equilibrium. The buyer must, with some positive probability, offer a low price when the value is high, but some surplus will accrue to the seller by making that offered price higher. Hence, the buyer will find optimal to keep the probability of offering a low price when he knows the value is high as small as possible without reaching zero.

4.3 The first mover's information acquisition decision

Given the costs of investigation for the buyer and seller, the buyer will decide whether or not to investigate the value of the asset by comparing his payoffs in the various subgames when he is informed (Lemmas 6 and 7) and when he is not (Lemmas 1-4). A formal derivation of the boundaries between the various lemmas is provided in Part B of the Appendix.

Figure 2 illustrates the set of choices for the buyer and seller described by various lemmas. Across the four panels, we vary the ratio of volatility to gains from trade, $\frac{(v_h - v_l)}{\Delta}$. (For lower values of this ratio, the resulting figures all resemble Panel (a), while for higher values they resemble Panel (d).) In each panel the cost pairs are divided into sections, numbered according to the lemma that describes play in the corresponding subgame. The dashed 45-degree line describes the set of symmetric cost pairs that could result from a symmetric equilibrium in financial expertise, where the potential counterparties' investments at date zero determine their investigation costs. As we show in the next section, the only equilibria that involve pure strategies in the choice of expertise are symmetric ones. The heavy lines separate regions where the first mover, or buyer, investigates. The boundary is derived simply by comparing the payoffs to the buyer from when he does not acquire information (Lemma 1 to Lemma 4, depending on the value of c_s) to the value when he does acquire information (Lemma 6 or Lemma 7). In each panel the lower left-hand corner, where the costs of both the proposer and responder are low, correspond to Lemmas 6 and 7. To the right of and above these cost pairs, the proposer does not pay for information and makes an uninformed offer. That region, in turn, is divided into areas corresponding to Lemma 1, where high costs for the seller and buyer lead to efficient bargaining outcomes, through Lemma 4, where no trade occurs. Note that the buyer's decision to investigate depends on the seller's costs, because he anticipates that party's response. The cost at which he decides not to become informed first rises and then falls in the seller's costs. The rate at which the buyer's payoff changes in the seller's costs varies, as we move between the regions in the figure, leading to this reversal.

Note that in every parameterization in Figure 2 the 45-degree line does not pass through the area associated with Lemma 7. This rules out symmetric equilibria in expertise with subgames in the region where the optimal equilibrium for the buyer does not exist.[4] Rather, the choices over expertise in symmetric equilibria will put the agents at the boundaries between areas governed by Lemmas 2 and 4, where the buyer is uninformed, or Lemmas 2 and 6, where the buyer investigates before making an offer.

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[4]From the expression for the boundary of the buyer's information acquisition decision it is straightforward to verify this for all $\frac{v_h - v_l}{\Delta} > 0$.

5 Investing in expertise

In this section we consider the equilibrium choices of investment in expertise, and show that an arms race occurs. We will first show that in equilibrium, for most parameter values and cost functions, $c(e)$, the equilibrium in expertise is to reduce costs through expertise to the lower boundary of Lemma 2. We then consider the possibility that, with periodic shocks to volatility, the incentives to engage in an arms race can drive the agents into regions where their low costs cause a breakdown in liquidity in the high-volatility regime, and thus a loss in gains to trade.

5.1 Constant volatility

Consider the possibilities depicted in Figure 2. If we can confine our attention to equilibria where investigation costs are symmetric, the analysis of candidate equilibria is simplified considerably. Our first result in this section ensures this is without loss of generality.

Proposition 1 *There are no pure-strategy equilibria with asymmetric levels of expertise investment.*

Accordingly, candidate equilibria must lie on the dashed 45-degree lines in Figure 2. In Panel (c),where $\Delta = \frac{1}{9}(v_h - v_l)$, the boundary between the regions described by Lemmas 2, 4, and 6 intersect the 45-degree line through the origin, where investigation costs are symmetric. The costs at this point define a level of expertise $e^* = c^{-1}(\frac{5}{4}\Delta)$. When volatility is lower (Figure 2, panels (a) and (b)), the diagonal crosses the boundary between Lemma 6 and Lemma 2, while if volatility is higher (Panel (d)) the diagonal passes through the Lemma 4 region, where trade does not occur.

If we consider, first, the case in Panel (d), where volatility is high, It is immediately obvious that no symmetric cost pair in which play proceeds according to Lemma 4 can be an equilibrium with positive investment in expertise. The agents would be investing resources to build expertise but not enjoying any benefit from their investment. Since investment in expertise is costly, they could unilaterally reduce this investment at date zero at no cost to their payoffs in subsequent trading.

The boundary between Lemma 2 and Lemma 4, however, is a natural candidate for an equilibrium. Here each agent gains the full surplus as a seller. Deviations that involve additional investment in expertise will move the agents into the region governed by Lemma 4, where both parties earn zero. Even high levels of investment will move agents into the region governed by Lemma 6. There, the additional expertise benefits the agent when he is the buyer, but because mixed strategies imply some loss in gains to trade, these benefits must be less than the loss of 2Δ when he is the seller.

A similar logic governs the choices when volatility is lower. Then the 45-degree line crosses the boundary between Lemma 2 and Lemma 6. At that point, he does not earn zero as a buyer and 2Δ as a seller. The boundary in this case is such that the buyer's surplus in the subgame where he investigates is equal to his surplus when he does not and play proceeds according to Lemma 2. His expected surplus in Lemma 2 across his roles as both buyer and seller equals the full gains to trade. Under Lemma 6, in contrast, he earns zero as a seller and less than 2Δ as a buyer, because the mixed strategies necessarily imply loss of gains to trade.

Indeed, in all situations where investing in expertise provides a sufficient decrease in costs, both agents will invest in expertise up to the point where any additional investment in expertise would move them out of the Lemma 2 region. This fact is summarized in the following proposition.

Proposition 2 *Define the level of expertise e^* as:*

$$
e^* = \begin{cases} c^{-1}\left(\frac{1}{4}(v_h - v_l) - \Delta\right) & \text{if } \frac{v_h - v_l}{\Delta} \geq 9 \\[2ex] c^{-1}\left(\frac{5}{36}(v_h - v_l)\right) & \text{if } \frac{v_h - v_l}{\Delta} \leq 9 \end{cases}
$$

For any $\frac{v_h - v_l}{\Delta} > 0$, when positive investment in expertise is an equilibrium, there exists an equilibrium in which both agents invest in expertise at the level e^* if and only if $c'(e^*) \leq -(1 - \delta)$. If $c'(e^*) \in [-2(1 - \delta), -(1 - \delta)]$, then e^* is a unique equilibrium level of expertise. If instead $c'(e^*) > -(1 - \delta)$, the level of expertise e' $(< e^*)$ such that $c'(e') = -(1 - \delta)$ is the unique equilibrium.

In the equilibrium with $e = e^*$ agents invest to the point at which any further investment would move them out of Lemma 2 when they are the seller. In the case where $\frac{v_h - v_l}{\Delta} > 9$, the seller

receives 2Δ and the buyer receives 0. For $\frac{v_h - v_l}{\Delta} < 9$, the situation is slightly different. Agents locate at the boundary between Lemma 2 and Lemma 6, but this is *not* the point e^* associated with the full surplus for the seller. As can be seen from Figure 2, costs in this equilibrium are such that the seller's cost exceeds the threshold at which he receives the full surplus, and the surplus is divided between the buyer and the seller. Play in the subgame is efficient in this equilibrium, but the investment in expertise is wasteful.

As the proposition makes clear, for a wide range of cost functions, e^* will be the unique equilibrium that involves pure strategy investment in expertise. However, when the effect of expertise on costs remains very strong until costs are very low, there exists another symmetric equilibrium that involves even greater investment in expertise, with a consequent loss in gains to trade even without shifts in the volatility. This equilibrium has play proceeding according to Lemma 6 and is Pareto dominated by the equilibrium at e^*.

This can be seen in Figure 3, where we parameterize the model and study numerically how the optimal investment in expertise and the resulting costs of investigation will behave in different calibrations. The optimal investment in expertise and the resulting costs when volatility is kept constant across time. We parameterize the problem by setting the discount factor, $\delta = 0.9$. The cost of information acquisition, as a function of investment in expertise, e, and of the initial cost of investigation, C_0, is

$$c(e) = C_0 exp\{-\frac{\lambda e}{C_0}\},$$

where we set $\lambda = 1$. We normalize $\Delta = 1$ and set $v_h - v_l = 10$.

The parameterized function, $c(e)$, is decreasing and strictly convex and satisfies the following regularity condition:

$$\lim_{e \to +\infty} c(e) = 0.$$

This function also has the convenient property that

$$c'(e) = -\lambda \frac{c(e)}{C_0}.$$

This property implies that, at $e = 0$, the "marginal benefit" of expertise, $-c'(e)$, is λ for all possible starting values of C_0. It also implies that the marginal benefit of expertise at a point $c(e)$ is proportional to the ratio of the current cost $c(e)$ to the initial cost C_0. The farther the current investigation cost is from its starting point, the more expensive it will be to further reduce that cost.

The upper panel in Figure 3 shows the best responses in the level of expertise given the initial cost of investigation, C_0, and the cost level of potential counterparties. The bottom panel shows the costs that result from optimal investment in expertise, given the same variables. The highlighted line identifies the cost pairs (agent's initial cost vs. opponent's final cost) that lead to symmetric pure-strategy Nash equilibria in financial expertise. Notice that the costs resulting from optimal investment in expertise are flat, except for very low levels of either the agent's cost or his opponent's cost. The flat level of costs corresponds to $1.5 = c(e^*) = \frac{1}{4}(v_h - v_l) - \Delta$, the lower boundary of the region given in Lemma 2. That this is the cost associated with optimal expertise in equilibrium, for a wide range of initial costs, is evident in the fact that this level of cost results from a best response when the opponent's cost is also 1.5 (the highlighted line where the surface folds up in the figure). For an agent with a starting cost between 1.5 and 6.69, the best response to an opponent's cost of 1.5 is to acquire expertise until his investigation cost is also 1.5. Hence, when both agents have initial costs between 1.5 and 6.69, acquiring expertise until investigation cost is 1.5 is a symmetric pure-strategy Nash equilibrium. This equilibrium is however delicate. Since both agents' costs reach the edge of Lemma 2, even an infinitely small increase in volatility will lead to breakdowns in trade.

Figure 4 illustrates that for some initial cost values multiple symmetric pure-strategy equilibria will exist. The figure shows what panel (b) in Figure 3 would look like from above and the highlighted line identifies the cost pairs (agent's initial cost vs. opponent's final cost) that lead to symmetric pure-strategy Nash equilibria in financial expertise. At very low levels of initial costs, i.e., when initial cost is below 1.89, we can see that the (Pareto-dominated) second pure-strategy equilibrium arises. In such equilibrium, the buyer acquires information before making an offer, and adverse selection eliminates some gains to trade. As Proposition 2 makes clear, the symmetric

equilibrium at e^* with $c(e^*) = 1.5$ will be unique at least for the region of initial costs where $-2(1 - \delta) < c'(e^*) < -(1 - \delta)$. In the current parameterization, that equilibrium is in fact unique for the larger region where initial costs are between 1.89 and 6.69. We provide a formal analysis of the conditions under which multiple equilibria will arise in Part C of the Appendix.

Figure 5 makes clear that expertise acquisition leading to a delicate trading equilibrium is not unique to a particular parameterization. It shows the best response level of investment in expertise, and resulting costs of information acquisition, for parameterizations corresponding to each of Cases 2-5 in Figure 1. We use the same cost function and discount rate and set $\Delta = 1$ as before, but allow $v_h - v_l$ to be 8 (Case 2), 6 (Case 3), 4 (Case 4), and 2 (Case 5). In each case, as in Figure 3, optimal investment in expertise generally pushes the agent to a flat level of costs. The exception occurs when the opponents have very low costs of information acquisition, where investment in expertise spikes and costs drop.

The flat region where the final cost reaches one of the lower bounds defined in Lemma 2 does not allow for a symmetric equilibrium in expertise when $\frac{v_h - v_l}{\Delta} < 9$. Instead, when initial costs are not very low nor very high, the equilibrium in expertise has the final costs of both agents reaching the level where Lemma 2 meets Lemma 6. In all cases, there are symmetric equilibria at such level of costs. Thus, equilibrium investment in expertise is pushing the agents to the boundaries of the areas in Figures 1 and 2, and outcomes will be sensitive to slight increases in volatility. In all cases, there is also an arms race in expertise for very low levels of initial costs. Finally, Lemma 1 prevails when initial costs are sufficiently high, and there is no point in investing in expertise. All the upper portion of the surface are equilibrium outcomes, since no investment in expertise occurs.

5.2 Breakdowns in trade

Proposition 2 describes the equilibrium when the benefits from expertise do not fall too quickly. In that situation, it is a dominant strategy for every agent to invest in expertise to drive their costs to the point where, were they any lower, they would fall into a region where gains to trade were lost some or all of the time. Under the conditions we have considered thus far this outcome is wasteful of resources since firms invest in expertise while not using it in equilibrium. It does not reduce the

total gains to trade that accrue to the agents, however.

Figures 1 and 2 show that the lower boundary of the region determining the equilibrium expertise shifts to the right when volatility, $(v_h - v_l)$, increases. If firms chose their investment in expertise, and then were surprised by an increase in volatility, they would find themselves in a region where their costs were so low that they would be unable to avoid the incentives to investigate, as in Lemma 6, with a loss of gains to trade when values turn out to be high. They might also find themselves in a setting where, as in Lemma 4, the possibility of adverse selection precludes any trade at all.

We now show that even when these possibilities are anticipated ex-ante, firms will invest in expertise as in Proposition 2, as long as the probability of the high-volatility regime is sufficiently small.

Suppose at each date t, with a constant probability π, volatility can rise to $\theta(v_h - v_l)$ where $\theta > 1$. To keep the analysis simple, we assume the regimes are not persistent, so in each period agent i is playing one of two ultimatum games with, again, a randomly selected counterparty. One of these games involves high volatility, and occurs rarely, while in the other, more typical trading game the asset has relatively low volatility. The next proposition shows that firms will invest in expertise as in Proposition 2 as long as the probability of a high-volatility regime is sufficiently small.

Proposition 3 *For every $\frac{v_h - v_l}{\Delta} > 0$ and $\theta > 1$ there is a $\pi^\theta > 0$ such that, for all $\pi < \pi^\theta$, there exists a perfect Bayesian equilibrium where both agents invest in expertise up to e^* and play proceeds according to Lemma 2 in the low-volatility regime. However, in the high-volatility regime, trade breaks down with positive probability as play proceeds according to Lemma 4 if $\frac{v_h - v_l}{\Delta} \geq \frac{28}{3}$ or if $\frac{v_h - v_l}{\Delta} \in (9, \frac{28}{3})$ while $\theta \leq \frac{[(v_h - v_l) - 12\Delta]^2}{(v_h - v_l)[28\Delta - 3(v_h - v_l)]}$ and according to Lemma 6 if $\frac{v_h - v_l}{\Delta} \leq 9$ or if $\frac{v_h - v_l}{\Delta} \in (9, \frac{28}{3})$ while $\theta \geq \frac{[(v_h - v_l) - 12\Delta]^2}{(v_h - v_l)[28\Delta - 3(v_h - v_l)]}$.*

Figure 6 illustrates the equilibrium outcomes when there is a small but non-trivial probability that volatility will increase. It shows the optimal investment in expertise and the resulting costs when $\frac{v_h - v_l}{\Delta} = 10$ and when there is a probability $\pi = 0.05$ in each period that volatility in asset value will be magnified by 1.1. As in the constant-volatility scenario, the costs resulting from

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optimal investment in expertise are flat, except for very low levels of the opponent's costs. If both agents have initial costs between 1.5 and 6.48, both agents will acquire expertise until their investigation costs reach $1.5 = c(e^*) = \frac{1}{4}(v_h - v_l) - \Delta$. However, when the high-volatility regime occurs, both agents will end up playing according to Lemma 4 instead. Trade will therefore break down with probability $\pi = 0.05$ because the buyer prefers not to trade rather than purchasing a "lemon" from an informed seller. Hence, all the gains to trade are lost in the high-volatility regime. Note that if we had parameterized asset volatility to be lower, both agents could end up playing according to Lemma 6 in the high-volatility regime instead of Lemma 4, as stated in Proposition 3. The probability of trade breaking down would then be lower than 5% since trade occurs with positive probability in Lemma 6 compared to with zero probability in Lemma 4. Figure 7 shows that equilibrium outcomes when volatility is stochastic replicate closely those when volatility is constant (Figure 5). Hence, despite the possibility that high volatility causes trade to break down, agents still find optimal to acquire expertise until their investigation costs reach the limit of Lemma 2.

Our model predicts that, in some circumstances, financial intermediaries might find optimal to acquire expertise even though it makes trade fragile when the volatility in fundamental value increases. Adding expertise makes it easier for an intermediary to subsequently acquire information about an asset's value and it amplifies the possibility of creating an adverse selection problem. The threat of facing an informed counterparty might force an intermediary to become informed before actually proposing a trade and the associated investigation cost will sometimes make the idea of a trade unattractive to that intermediary if volatility is expected to be high. If, however, the probability of the high-volatility regime is small enough, the gains to trade lost in the high-volatility regime will not be as important as the increase in profits that added expertise, and the ensuing improved bargaining position, bring in the low-volatility regime. The intermediary will find optimal to acquire expertise that increases expected profits in the more probable low-volatility regime, even though it decreases profits because of trade breakdowns in the less probable high-volatility regime.

6 Employment Cycles

The model we have developed produces periodic breakdowns in liquidity when volatility rises but does not allow for a response on the part of firms. They invest once, at $t = 0$, and the expertise installed at that point lives forever. In an environment where expertise depreciates, and where firms can respond to higher volatility, the tradeoffs that are fundamental to the model will produce cycles in "expertise," which can be interpreted as employment of highly trained and educated labor in the financial sector.

This is straight forward to illustrate, short of a fully dynamic model, by considering the extreme case where expertise fully depreciates each period. Firms discount the future at the market interest rate, and have full access to the capital markets. Since there are no intertemporal linkages between their decisions across periods, they will act to maximize their profits each period. Alternatively, we can view the firms as living for one period. This case is, essentially, the opposite of the one studied in the previous section, where investments in expertise occurred only once but lasted forever. We assume that volatility evolves as a two-state Markov chain, with some persistence in the states. Firms choose their level of expertise at the beginning of each period, before observing the state that will prevail, but conditioning their decision on the previous realization of volatility.

Figure 8 illustrates the resulting dynamics for volatility (dashed line), equilibrium levels of expertise (solid line), and periods where trade breaks down (shaded areas). The parameters are chosen as in our previous examples (see Figure 6) except for the cost-function parameter λ, which is set to 10 rather than 1 to keep the magnitudes comparable to the previous results. The transition probability for moving to high volatility from low volatility is 0.1, and is 0.4 for staying in the high volatility state. These imply an unconditional probability of the high-volatility state of $\frac{1}{7}$. At these conditional probabilities, it is optimal for firms to invest in expertise to e^*, the boundary of the region of costs associated with Lemma 2 in the low-volatility regime. At this point efficient trade occurs. When the high-volatility regime occurs, they lose gains to trade for that period and scale back their levels of expertise because that regime persists with higher conditional probability. Figure 8 shows a typical sample path.

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7 Conclusion and Extensions

The model in this paper illustrates the incentives for financial market participants to overinvest in financial expertise. Expertise in finance increases the speed and efficiency with which traders and intermediaries can determine the value of assets when they are negotiating with potential counterparties. The lower costs give them advantages in negotiation, even when the information acquisition has no value to society and even though it can create adverse selection problems that disrupt frictionless trade.

Some extensions to the model may warrant additional research. Financial expertise might also be viewed as allowing intermediaries to increase the costs of information acquisition for their counterparties, as well as lowering their own costs. Investment in expertise permits firms to create, and make markets in, more complex financial instruments. In our notation, we can view the cost of acquiring information for agent i as $c(e_i, e_j)$, which decreases in i's own expertise and increases in that of his counterparty. The logic of our analysis suggests firms benefit from increasing the relative costs of their counterparties. The tension between the incentives to raise others' costs, which would reduce adverse selection, and lower one's own costs, which increases it, may help us better understand innovation and evolution in financial markets.

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Appendix A   Proofs of Lemmas and Propositions

Proof of Lemma 1: We first show that if $c_s > \frac{1}{4}(v_h - v_l)$, then the best response of the seller at p_1^* or p_3^* is accept the offered price without investigating. (At p_2^* the seller is by definition indifferent between accepting without investigating and investigating and accepting if $v = v_l$.)

At an offered price of p_1^* the seller's surplus from accepting unconditionally is

$$
\begin{aligned}
p_1^* - (E(v) - \Delta) &= v_l - \Delta + 2c_s - (\frac{1}{2}v_h + \frac{1}{2}v_l - \Delta) \\
&= 2c_s - \frac{1}{2}(v_h - v_l)
\end{aligned}
$$

This is positive as long as $c_s > \frac{1}{4}(v_h - v_l)$. The seller's expected surplus from investigating is, by construction, zero at p_1^*, so accepting unconditionally is a best response.

At an offered price of p_3^* the seller's surplus from accepting unconditionally is zero by construction. The payoff from investigating is

$$
\begin{aligned}
\frac{1}{2}[p_3^* - (v_l - \Delta)] - c_s &= \frac{1}{2}\left[\frac{1}{2}v_h + \frac{1}{2}v_l - \Delta - (v_l - \Delta)\right] - c_s \\
&= \frac{1}{4}(v_h - v_l) - c_s
\end{aligned}
$$

This is negative as long as $c_s > \frac{1}{4}(v_h - v_l)$, so accepting unconditionally is a best response.

We know that at p_2^*, the seller is indifferent between investigating and accepting unconditionally. However, the seller's surplus at this price is negative if $c_s > \frac{1}{4}(v_h - v_l)$. By accepting unconditionally, the seller gets:

$$
\begin{aligned}
p_2^* - (E(v) - \Delta) &= v_h - \Delta - 2c_s - (\frac{1}{2}v_h + \frac{1}{2}v_l - \Delta) \\
&= \frac{1}{2}(v_h - v_l) - 2c_s \\
&< 0.
\end{aligned}
$$

Therefore, if offered p_2^*, the seller would decline and earn zero, yielding a zero payoff for the buyer as well. Thus, we need only compare the buyer's payoff at p_1^* and p_3^*, where the seller's best response is unconditional acceptance.

At p_3^*, since the seller receives his reservation price, the buyer collects the gains to trade of 2Δ. At p_1^* the buyer receives:

$$
\begin{aligned}
E(v) + \Delta - p_1^* &= \frac{1}{2}v_h + \frac{1}{2}v_l + \Delta - (v_l - \Delta + 2c_s) \\
&= \frac{1}{2}(v_h - v_l) - 2c_s + 2\Delta
\end{aligned}
$$

which is less than 2Δ if $c_s > \frac{1}{4}(v_h - v_l)$.∎

Proof of Lemma 2: The same steps that showed unconditional acceptance to be a best response of the seller to offers of p_1^* and p_3^* when $c_s > \frac{1}{4}(v_h - v_l)$ imply investigating and accepting the offer only when $v = v_l$ is a best response when $c_s \leq \frac{1}{4}(v_h - v_l)$, which is the case under both (a.) and (b.). Similarly, we showed that the seller's surplus was negative when offered p_2^*, which makes him indifferent to acquiring information, as long as $c_s > \frac{1}{4}(v_h - v_l)$. It will be positive when this inequality is reversed.

At p_2^*, the seller is indifferent between investigating and unconditional acceptance, but the buyer prefers the latter. To see this, compare the buyer's surplus when the seller is informed

$$\frac{1}{2}(v_l + \Delta - p_2^*) = -\frac{1}{2}(v_h - v_l) + \Delta + c_s \tag{1}$$

to that when the seller accepts unconditionally,

$$E(v) + \Delta - p_2^* = -\frac{1}{2}(v_h - v_l) + 2\Delta + 2c_s. \tag{2}$$

The latter is obviously larger by $\Delta + c_s > 0$. We also know this expected surplus for the buyer is positive, since $c_s > \frac{1}{4}(v_h - v_l) - \Delta$ under condition (a.). Under the condition in (b.), note that $\Delta > \frac{1}{8}(v_h - v_l)$. Therefore,

$$
\begin{aligned}
-\frac{1}{2}(v_h - v_l) + 2\Delta + 2c_s \ &\geq\ -\frac{1}{2}(v_h - v_l) + 2\Delta + 2(\frac{1}{6}(v_h - v_l) - \frac{1}{3}\Delta) \\
&=\ -\frac{1}{6}(v_h - v_l) + \frac{4}{3}\Delta \\
&\geq\ -\frac{1}{6}(v_h - v_l) + \frac{4}{3}\frac{1}{8}(v_h - v_l) \\
&=\ 0
\end{aligned}
\tag{3}
$$

where the first inequality follows by substituting the lower bound on c_s from condition (b.).

We now need to evaluate the buyer's expected payoff at p_1^* and p_3^*. At p_1^*, given the seller's best response, the buyer gets
$$\frac{1}{2}(v_l + \Delta - p_1^*) = \Delta - c_s, \tag{4}$$

which is negative since $\Delta < c_s$ under condition (a.), but may be positive under (b.). Under condition (b.), the buyer's surplus at p_2^* is greater than that at p_1^* if

$$-\frac{1}{2}(v_h - v_l) + 2\Delta + 2c_s > \Delta - c_s$$

or if

$$-\frac{1}{6}(v_h - v_l) + \frac{1}{3}\Delta + c_s > 0,$$

which follows from the lower bound on c_s in (b.) We can therefore eliminate p_1^* as a candidate equilibrium price.

At p_3^*, where the seller's best response is investigation:

$$\frac{1}{2}(v_l + \Delta - p_3^*) = -\frac{1}{4}(v_h - v_l) + \Delta. \tag{5}$$

The right-hand side is negative under (a.), since $\Delta < c_s \leq \frac{1}{4}(v_h - v_l)$, but it may be positive under (b.). The buyer's surplus at p_2^* is greater than that at p_3^* if

$$-\frac{1}{2}(v_h - v_l) + 2\Delta + 2c_s > -\frac{1}{4}(v_h - v_l) + \Delta,$$

or if

$$-\frac{1}{4}(v_h - v_l) + \Delta + 2c_s > 0.$$

Using the lower bound on c_s from (b.),

$$
\begin{aligned}
-\frac{1}{4}(v_h - v_l) + \Delta + 2c_s &\geq -\frac{1}{4}(v_h - v_l) + \Delta + \frac{1}{3}(v_h - v_l) - \frac{2}{3}\Delta \\
&= \frac{1}{12}(v_h - v_l) + \frac{1}{3}\Delta \\
&> 0.
\end{aligned}
$$

This eliminates p_3^* as a candidate equilibrium. ∎

Proof of Lemma 3: We have already established that with $c_s < \frac{1}{4}(v_h - v_l)$, which is true here, the seller's best response at p_1^* and p_3^* is to investigate and sell only if $v = v_l$. The buyer's surplus at p_1^* is

$$\frac{1}{2}(v_l + \Delta - p_1^*) = \Delta - c_s,$$

which is positive since $c_s < \Delta$.

At p_3^* the buyer's surplus is

$$\frac{1}{2}(v_l + \Delta - p_3^*) = \Delta - \frac{1}{4}(v_h - v_l).$$

Since $c_s < \frac{1}{4}(v_h - v_l)$, the buyer prefers p_1^*, eliminating p_3^* as a candidate for an equilibrium.

At p_2^*, we know the seller is indifferent between investigating and accepting unconditionally, but that the buyer prefers unconditional acceptance, from the comparison of equations (1) and (2). The buyer's surplus from offering p_2^* is given by equation 2 and exceeds the payoff to offering p_1^*, $\Delta - c_s$, only if

$$c_s > \frac{1}{6}(v_h - v_l) - \frac{1}{3}\Delta. \tag{6}$$

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Note also that this surplus must be positive and therefore preferred to no trade, since it exceeds $\Delta - c_s$, which in turn exceeds 0 by the assumption that $c_s < \Delta$. ∎

Proof of Lemma 4: From the analysis associated with equations (4) and (5) we know that if $c_s > \Delta$, the best payoff that the buyer can achieve when inducing the seller to investigate is less than zero. But, from (2), we can also see that the buyer can achieve a positive profit when preventing the seller from acquiring information only if

$$c_s > \frac{1}{4}(v_h - v_l) - \Delta.$$

But, the bounds on c_s imply that $\Delta < \frac{1}{4}(v_h - v_l) - \Delta$, which in turn implies the existence of a region where trade is not profitable either with or without the seller gathering information. ∎

Proof of Lemma 5: The arguments in the text immediately preceding the statement of the lemma prove the result. ∎

Proof of Lemma 6 and Lemma 7: We prove these two lemmas together. We first posit a two-price equilibrium in which the buyer offers one price, p_l, if the value is low and mixes between p_l and another price, p_h, if the value is high. We will then derive the payoff from the optimal equilibrium in the case of Lemma 6 and the supremum of the payoffs in the case of Lemma 7, from the perspective of the buyer under this assumption. We will then demonstrate that the buyer will not do better in any equilibrium that uses more than two prices.

Assuming that there are two prices, it is immediate that the higher price must never be offered by the buyer after observing the low value. If it were, this would imply that both prices are offered with positive probability for both types, which would imply that the buyer is indifferent between the two prices for both realizations of the value, which is impossible.[5] We can also conclude that in any Perfect Bayesian equilibrium, the high price must be $v_h - \Delta$, since any lower price would admit a profitable deviation to a higher price when $v = v_h$. Finally, we can rule out any pure strategy equilibria; the buyer must mix between the high and the low price when the value is high, and the seller must reject the low price with positive probability.[6] The seller will reject the low price under two circumstances. Either he does not investigate and rejects the low price anyway, or he

---

[5]For the buyer to be indifferent between the two prices following both realizations of the value, we require that the probability of trade, conditional on the price, times the expected payoff to the buyer conditional on trade occuring, be equal. This implies the ratio of expected payoffs, given trade, in each state be equal to the ratio of the probability of trade, so that:

$$\frac{v_h + \Delta - p_l}{v_h + \Delta - p_h} = \frac{v_l + \Delta - p_l}{v_l + \Delta - p_h},$$

which is clearly false for $p_h \neq p_l$.

[6]If the seller always accepts the low price, the buyer will always offer the low price and the seller will have negative expected profits. If the seller never accepts the low price, the buyer will never have an incentive to offer the low price when the true value is high (since he can obtain 2Δ by offering a price of $v_h - \Delta$), and the seller then must put probability one on the value being low following a low offer and therefore must accept with probability 1.

investigates and rejects the low price when the true value is high. We define the mixed strategies as follows. Let the probability that the buyer offers the low price when $v = v_h$ be α, let the probability that the seller does *not* investigate following a low offer be β, and let the probability that an uninformed seller sells at the low price be γ. We will also define $p_l = v_l - \Delta + z$ and $p_h = v_h - \Delta$ for convenience.

To construct the equilibrium, we consider two situations. First, if $z < c_s$, the seller will never investigate since his payoff to investigating is negative regardless of his beliefs.[7] Since the seller must reject the low price with positive probability, the seller must mix between unconditional acceptance and rejecting without investigation when the offer is low. The requirement for this to happen is then:

$$\frac{1}{2}z + \frac{1}{2}\alpha(v_l - v_h + z) = 0$$

where the left hand side is the payoff to the strategy of accepting either p_l or p_h and the right hand side is the payoff for only accepting p_h. This gives:

$$\alpha = \frac{z}{v_h - v_l - z} \in (0, \frac{c_s}{v_h - v_l - c_s}) \subset (0, \frac{1}{3}).$$

The subset restriction comes from the assumption that $c_s < \frac{1}{4}(v_h - v_l)$ and guarantees that we do not need to check whether the conditions for the seller to mix exist.

In order for the buyer to mix, we must have:

$$2\Delta = \gamma(v_h - v_l + 2\Delta - z)$$

which gives:

$$\gamma = \frac{2\Delta}{v_h - v_l + 2\Delta - z}$$

which will also fall in the interval $(0, 1)$ for all parameters under consideration, i.e., for $p_l < p_h$.

---

[7]In step 3c of Dang (2008), the claim is that the seller never chooses to acquire information if (in our notation) $z < 2c_s$. This is based on an incorrect calculation of the expected payoff to the seller for using a strategy of investigating following an offer of p_l. The calculation fails to take into account that the seller will not pay the costs of investigating when v is high and the buyer offers p_h, an event that occurs with probability $\frac{1}{2}(1 - \alpha)$. Specifically, Dang (2008) states that the payoff to the seller of investigating when $p = p_l$ is:

$$\frac{1}{2}z - c_s,$$

whereas the correct value is

$$\frac{1}{2}z - \frac{1}{2}(1 + \alpha)c_s.$$

The payoff to the buyer in an equilibrium with a given z can be written as:

$$
\frac{1}{2}(1-\alpha)2\Delta + \frac{1}{2}\alpha\gamma[v_h + \Delta - (v_l - \Delta + z)] + \frac{1}{2}\gamma[v_l + \Delta - (v_l - \Delta + z)] + \frac{1}{2}(1-\gamma)0 - c_b
$$
$$
= \frac{1}{2}(1-\alpha)2\Delta + \frac{1}{2}\alpha 2\Delta + \frac{1}{2}\gamma(2\Delta - z) - c_b
$$
$$
= \Delta - c_b + \frac{1}{2}\gamma(2\Delta - z).
$$

This expression becomes

$$
\Delta - c_b + \frac{\Delta(2\Delta - z)}{v_h - v_l + 2\Delta - z},
$$

which is maximized at $z = 0$. But, $z = 0 \Rightarrow \alpha = 0$, which is not an equilibrium. We can thus conclude that there does not exist an optimal equilibrium from the perspective of the buyer with $z < c_s$, and we can also conclude that the sup of the payoffs to equilibria with $z < c_s$ is

$$
\Delta - c_b + \frac{2\Delta^2}{v_h - v_l + 2\Delta} \tag{7}
$$

and that the buyer always prefers a smaller z and consequently must choose α very small. The limiting value for γ is also immediate, but note that it does *not* go to zero as z disappears. We have now described all of the characteristics of the equilibria described in Lemma 7.

Alternatively, the buyer could offer a price that could induce the seller to investigate with positive probability. This price must have $z \geq c_s$, but while the offer price is higher, and therefore worse for the buyer, the equilibrium may be preferable because if the seller investigates with positive probability trade may occur more frequently when $v = v_l$. In this situation, we require that

$$
\beta\gamma = \frac{2\Delta}{v_h - v_l + 2\Delta - z}
$$

where $\beta\gamma$ is the probability that the seller does not investigate following an offer of p_l *and* the seller buys anyway. The payoff to the buyer is then:

$$
\Delta - c_b + \frac{\Delta(2\Delta - z)}{v_h - v_l + 2\Delta - z} + \frac{1}{2}(2\Delta - z)(1 - \beta).
$$

The final term is the contribution to the payoff of the event that $v = v_l$ and the seller investigates, necessarily buying after discovering v.[8] It is straightforward to verify that this expression is strictly decreasing in both β and z. However, we face several additional constraints in order for z and β to

---

[8]This term is one of the elements missing from the analysis in Dang (2008) (Step 3d. of the appendix). As a consequence of including it, β and γ are not separately identified by the conditions ensuring indifference in a mixed-strategy equilibrium.

constitute a mixed strategy equilibrium. First, we know that:

$$\beta\gamma = \frac{2\Delta}{v_h - v_l + 2\Delta - z}.$$

Since γ does not enter the equilibrium payoff separately from $\beta\gamma$, it is immediate that at the equilibrium that provides the highest payoff to the buyer we will have $\gamma^* = 1$. This condition will allow β to be set as small as possible, but we still must have $\beta \geq \frac{2\Delta}{v_h - v_l + 2\Delta - z}$ when $\gamma \leq 1$. Note that this constraint depends on the value of z, but decreasing z relaxes the constraint (i.e. allows a smaller β to be chosen). Thus, the payoff to the buyer will be maximized by setting z as low as possible given the other constraints. The constraints of concern are the following:

$$\frac{1}{2}(z - c_s) + \frac{1}{2}\alpha(-c_s) \geq 0 \tag{8}$$

$$\frac{1}{2}(z - c_s) + \frac{1}{2}\alpha(-c_s) \geq \frac{1}{2}z + \frac{1}{2}\alpha(v_l - v_h + z). \tag{9}$$

These constraints guarantee that the seller at least weakly prefers to use a strategy calling for investigation after an offer of p_l over a strategy calling for rejecting p_l and over a strategy calling for accepting p_l without investigation, respectively. In any equilibrium, at least one of these constraints must bind in order for the seller to mix as required. In the equilibrium we wish to construct, with $\beta \in (0, 1)$, the second inequality must bind. The first inequality may not bind as we have shown that $\gamma^* = 1$ and therefore the seller may strictly prefer investigating to not investigating and refusing to sell at p_l, but it will turn out that in the buyer's preferred equilibrium both inequalities will bind.

Inequality (8) reduces to

$$\alpha \leq \frac{z - c_s}{c_s}.$$

Inequality (9) reduces to

$$\alpha(v_h - v_l - c_s - z) \geq c_s.$$

Now suppose that $v_l - v_h + z > -c_s$. The payoff to being lied to when not investigating and the true value is high is not as negative as the cost of investigating, so investigation is dominated and could not be an equilibrium when $v_h - v_l - c_s - z$ is negative. Consequently, we have that $v_h - v_l - c_s - z > 0$ and inequality (9) becomes:

$$\alpha \geq \frac{c_s}{v_h - v_l - c_s - z},$$

which tells us:

$$\frac{c_s}{v_h - v_l - c_s - z} \leq \alpha \leq \frac{z - c_s}{c_s}.$$

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In order for an equilibrium to call for mixing on the part of the seller, we must then have that:

$$\frac{c_s}{v_h - v_l - c_s - z} \leq \frac{z - c_s}{c_s},$$

which, noting that $z > c_s$, becomes:

$$z^2 - (v_h - v_l)(z - c_s) \leq 0.$$

Therefore, to maximize the payoff to the buyer, we minimize z subject to the above constraint. The constraint is clearly violated at $z = c_s$ and for very large or very negative z, and there can be no root of the right hand side expression between for z between 0 and c_s. The admissible z's, if any, must then fall between the roots of $z^2 - (v_h - v_l)(z - c_s) = 0$, and therefore z is minimized subject to the constraint at

$$z^* = \frac{1}{2}\left[(v_h - v_l) - \sqrt{v_h - v_l}\sqrt{v_h - v_l - 4c_s}\right]$$

whenever

$$c_s \leq \frac{1}{4}(v_h - v_l).$$

Again, the buyer can always obtain the full surplus 2Δ without investigating when $c_s \geq \frac{1}{4}(v_h - v_l)$, so we can confine attention to those situations where c_s is smaller.

To recap, if $c_s < \frac{1}{4}(v_h - v_l)$, z^* is the smallest value for z such that the seller could, for some range of α, weakly prefer investigating following p_l to both buying outright and refusing to buy at all. Choosing this z and letting:

$$\alpha^* = \frac{c_s}{v_h - v_l - c_s - z^*},$$

we have the equilibrium preferred by the buyer, assuming he gathers information. Note that $\alpha^* > 0$ since $c_s + z < v_h - v_l$.

But, we can now solve for:

$$\alpha^* = \frac{2c_s}{(v_h - v_l) + \sqrt{v_h - v_l}\sqrt{v_h - v_l - 4c_s} - 2c_s}.$$

It is immediate to verify that, for $c_s < \frac{1}{4}(v_h - v_l)$, $\alpha^* \in (0, 1)$. We can also obtain

$$\beta^* = \frac{4\Delta}{v_h - v_l + \sqrt{v_h - v_l}\sqrt{v_h - v_l - 4c_s} + 4\Delta}.$$

The payoff to the buyer is then:

$$\Delta - c_b + \frac{1}{2}(2\Delta - z^*)$$

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since the seller always either buys outright or investigates.

We can ignore equilibria in which the seller mixes between investigating and rejecting without investigation. These equilibria provide payoffs of:

$$\Delta - c_b + \frac{1}{2}\beta\gamma(2\Delta - z),$$

with $z > z^*$ and $\beta\gamma < 1$, so they are clearly dominated from the perspective of the buyer by the equilibrium with mixing between investigation and outright purchase. A three-way mix can occur only at $z = z^*$ and is clearly worse than the equilibrium with mixing only between acceptance and investigation.

So, the maximum payoff to the buyer is given by:

$$2\Delta - c_b - \frac{1}{4}\left[(v_h - v_l) - \sqrt{v_h - v_l}\sqrt{v_h - v_l - 4c_s}\right]. \tag{10}$$

Note finally that for $c_s \in (0, \frac{1}{4}(v_h - v_l))$, this expression decreases from $2\Delta - c_b$ to $2\Delta - c_b - \frac{1}{4}(v_h - v_l)$.

The seller receives zero surplus since he is indifferent among all alternatives at z^*, α^*.

Finally, we can compare the payoff to the equilibrium in which the seller obtains information with positive probability to the upper bound on the payoffs when the seller never obtains information to find the separation point between the two lemmas. As long as $\Delta < \frac{1}{2}(v_h - v_l)$ (leading any positive z to be accepted), the condition for the seller not to gather information with positive probability becomes:

$$c_s > \frac{2\Delta(v_h - v_l)^2}{(v_h - v_l + 2\Delta)^2}.$$

Note that the inequality is strict because the buyer cannot obtain the sup of the payoffs associated with equilibria that involve no investigation by the seller.

To complete the proof, we demonstrate that no PBE can have the buyer mixing over more than two prices.

Suppose the buyer is informed and plays a mixed strategy with K prices when $v = v_h$. Given the gains to trade, it would be suboptimal for the buyer to offer a price that would be accepted by the seller with zero probability. Under the same logic as in the two-price equilibrium (see footnote 5), the buyer cannot be indifferent between a particular pair of the K prices, simultaneously when $v = v_h$ and when $v = v_l$. Hence, only one price among the K prices can be chosen when $v = v_l$. Let p_l denote this price. In a PBE, any of the $K - 1$ prices that are chosen when $v = v_l$ will be interpreted by an uninformed seller as a perfect signal that $v = v_h$. Therefore, the seller will accept with probability one any price that is higher than or equal to $v_h - \Delta$. The buyer prefers the lowest price possible, and $v_h - \Delta$ will be the only price chosen when $v = v_h$, other than p_l. Hence, in a PBE, the buyer will mix over no more than two prices when $v = v_h$. The only remaining way to

have a PBE with more than two prices, therefore, is to have the buyer playing a mixed strategy involving additional prices when $v = v_l$, other the price played when $v = v_h$. Any prices the buyer choses when $v = v_l$ that are not p_l, an uninformed seller will interpret as a perfect signal that $v = v_l$. As in the case with $v = v_h$, only one price other than p_l can be chosen by the buyer with positive probability, and accepted by the seller with positive probability, when $v = v_l$ in a PBE. That price is $v_l - \Delta$, which is the minimum price the seller would accept in the model. At that price, the seller will know for sure that $v = v_l$ and will accept the offer. Since we need $p_l > v_l - \Delta$ to have mixing over more than two prices and acceptance by the seller with positive probability, the buyer will find optimal to deviate and always offer $v_l - \Delta$, rather than p_l, which rules out the possibility of another price coexisting with p_l in the low-value state. Hence, only two-price PBEs will exist. ∎

Proof of Proposition 1 In order for a cost pair to be part of an asymmetric equilibrium, we require that both agents prefer to maintain their current level of expertise investment rather than to reduce their investment to save on the costs of expertise. Thus, the payoffs of both agents must be sensitive to expertise, or one agent must invest zero in expertise.

Payoffs in the trading subgame are only sensitive to an agent's own expertise when that agent is the seller if the game proceeds according to Lemma 2. Payoffs are sensitive to own expertise when the agent is the buyer only if play proceeds according to Lemma 6 or Lemma 7.

In Lemmas 6 and 7, the marginal benefit of reduced costs is (locally) constant at 1 in both cases, and, in particular, not a function (again locally) of the opponent's costs. In order to have an asymmetric equilibrium we require that play proceeds according to different lemmas depending on who is the buyer and who is the seller and that the marginal benefit of reduced costs is different in those lemmas. But, in every situation where that could hold, we have payoffs sensitive to only one agent's costs regardless of whether he is the buyer or the seller, so the other agent has an incentive to reduce investment in expertise.

We are left to consider exclusively situations on the boundary between two lemmas, where a small change in costs leads to a change in the governing lemma and a discontinuous change in payoff for the deviator.

At every point the buyer's payoff is continuous in his own costs, so we can confine attention to situations where the payoff to one of the agents when he is the *seller* is located at a discontinuity point of his payoffs. This requires that any asymmetric equilibrium locates the cost pair at the boundary between Lemma 2 and either Lemma 4 or Lemma 3 (depending on whether $\frac{(v_h - v_l)}{\Delta}$ is such that a Lemma 4 region exists) or between Lemma 2 and Lemma 7.

Suppose that one agent invests in expertise until his payoff is at the border between Lemma 3 and Lemma 2 or Lemma 4 and Lemma 2, when he is the seller. The payoff in the subgame to the other agent is invariant in that agent's own costs when he is the buyer. When he is the seller, costs only matter if play proceeds according to Lemma 2. Since the other agent chooses

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to locate on the Lemma 4/Lemma 3 boundary with the Lemma 2 region, it must be the case that returns to expertise in the Lemma 2 region are strictly higher than the costs. So, the only possible expertise choice for the second agent is on the boundary between Lemma 6/Lemma 7 and Lemma 2 (since the Lemma 3/Lemma 4 vs. Lemma 2 boundary is a vertical line). But, since the slope of the Lemma 6/Lemma 7 to Lemma 2 boundary is ≤ -2, any point on the boundary between Lemma 3/Lemma 4 and Lemma 2 is reflected into the interior of Lemma 2 if said point is above the diagonal, a contradiction. Reversing this argument shows that agent i locating on the Lemma 6/Lemma 7 boundary also cannot be an asymmetric equilibrium.

This leaves only the situation where, say, agent i locates on the Lemma 4 to Lemma 2 boundary but at a point below the diagonal, which can only occur when $\frac{(v_h - v_l)}{\Delta} > 9$. If returns to expertise are high, at least initially, an agent will have an incentive to invest in expertise to move play into the interior, but never to the boundary, of Lemma 6 as this increases this agent's payoffs when he is the buyer. Such a deviation, however, moves play when the deviator is the seller into the Lemma 6 region as well, as can be directly verified from Lemma 4 and Lemma 6. Now, either the agent who did not invest has an incentive to deviate to exactly the investment of the other agent, or the investing agent has an incentive to deviate if he did not choose the level of investment that made $c'(e) = -2(1 - \delta)$.

We can thus conclude that there is never an equilibrium with pure strategy investment in expertise where agents choose different levels of expertise. ∎

Proof of Proposition 2 We will first consider the case when $\Delta \leq \frac{1}{9}(v_h - v_l)$.

The proposed equilibrium places the agents at the intersection of Lemma 2 (part a) and Lemma 4, and, for $\frac{v_h - v_l}{\Delta} = 9$, at the intersection of these two areas with that governed by Lemma 6. Any deviation to a higher investment in expertise reduces payoffs to the deviator when he is the seller from 2Δ to 0. While payoffs increase when the deviator is the buyer (from 0), 2Δ is an upper bound[9] on the increase. Therefore, since the additional expertise investment is costly, the deviation cannot be profitable. If $c'(e^*) \leq -(1 - \delta)$, the payoffs given by Lemma 2(a), and the convexity of the cost function imply that deviations to a lower level of expertise investment are unprofitable. Then, no $e < e^*$ can be an equilibrium because there would be a profitable local deviation to a higher level of expertise as such an equilibrium would mean the agents were in the interior of the Lemma 2(a) region.

To see that these choices are unique under the restrictions on the marginal cost, suppose an agent chooses $e > e^*$. Then there is a profitable deviation to less expertise since $c'(e) > -2(1 - \delta)$ for all $e > e^*$, because the payoff to the seller is invariant in his own costs in the Lemma 6 region while the payoff to the seller is decreasing linearly with slope $-\frac{1}{1-\delta}$ in his own costs.

If $c'(e^*) > -(1 - \delta)$, deviating to less expertise is profitable at e^*, and there is no profitable deviation to more expertise from $\hat{e}$. (If $c'(\hat{e}) = -(1-\delta) \Rightarrow c(\hat{e}) > \frac{1}{4}\frac{v_h - v_l}{\Delta}$, there will be no investment

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[9]The value 2Δ is not the sup, but the fact that it is an upper bound is sufficient for our purposes.

in expertise, while if $c'(\hat{e}) = -1 \Rightarrow c(\hat{e}) < \frac{1}{4}\frac{v_h - v_l}{\Delta}$, but $c(0) > \frac{1}{4}\frac{v_h - v_l}{\Delta}$, the equilibrium *may* call for no investment in expertise since it may not be profitable to pay the fixed costs associated with increasing expertise up to $c(e) = \frac{1}{4}\frac{v_h - v_l}{\Delta}$, the point at which expertise begins to pay off at the margin.)

The final step is to show that agents will always play according to Lemma 2(a) in the subgame. Play with the characteristics of Lemma 4 (and, for $\frac{v_h - v_l}{\Delta} = 9$, Lemma 6) can also be supported in a subgame following investment in expertise of e^*. However, if play proceeds according to either Lemma 4 or Lemma 6 with positive probability there is a profitable deviation from e^* since by unilaterally and infinitesimally reducing investment in expertise either party can move the agents into the interior of the Lemma 2(a) region. Since Lemma 2(a) provides efficient payoffs and Lemmas 4 and 6 do not, there would always be some small reduction in expertise investment that would be profitable.

We now consider the case where $\frac{v_h - v_l}{\Delta} < 9$.

The discussion relating to $e < e^*$ from above applies effectively unchanged, so it remains only to show that there is no profitable increase in expertise when $c'(e^*) < -(1 - \delta)$.

To show that there is no profitable increase in expertise, observe that $\{c(e^*), c(e^*)\}$ defines the symmetric cost point at which the buyer is just indifferent between gathering information and not. Any increase in expertise by one agent moves the game to either the Lemma 6 region or the Lemma 3 region when the deviator is the seller. This yields zero payoff to the seller in the subgame. The increase in the payoff when the deviator is the buyer is bounded above by $\left(2 - \frac{1}{12}\frac{v_h - v_l}{\Delta}\right) - \left(2 - \frac{2}{9}\frac{v_h - v_l}{\Delta}\right)$. The expression in the first set of parentheses is the payoff to adhering that accrues when the agent turns out to be the buyer, which involves positive surplus to the buyer because $\frac{5}{36}\frac{v_h - v_l}{\Delta}$ exceeds the lower bound for the sellers cost for the Lemma 2 region when the buyer does not gather information whenever $\frac{v_h - v_l}{\Delta} < 9$, as assumed. That is, the point at which Lemma 3 transitions to Lemma 2 occurs where the buyer and the seller are sharing the surplus, not where the seller gets all of the surplus. The expression in the second set of parentheses is the payoff when the deviator is the buyer, assuming he deviates to $c(e) = 0$, but does not accrue any additional expenses with respect to expertise (an upper bound on the payoff to the deviation. Direct calculation then shows that the payoff to a deviation is no greater than $-\frac{1}{24}\frac{v_h - v_l}{\Delta}$, so there is no profitable deviation to more expertise. ∎

Proof of Proposition 3 In this proof it is convenient to introduce a normalization on the costs. We define $k_i \equiv \frac{c_i}{\Delta}$ for $i \in \{b, s\}$. All descriptions of boundaries for costs will be in terms of these normalized costs, and the ratio of volatility to gains to trade, denoted $A = \frac{v_h - v_l}{\Delta}$.

First, we note the following facts:

The buyer's (normalized) cost, as a function of the seller's cost, that sets the buyer's payoff equal in Lemma 2 and in Lemma 6 is increasing in A when $k_s \in (0, \frac{1}{4}A]$. The derivative of that

boundary with respect to A is:

$$\frac{A + \sqrt{A(A - 4k_s)} - 2k_s}{4\sqrt{A(A - 4k_s)}},$$

and is real and positive when $k_s < \frac{1}{4}A$, or equivalently when $c_s < \frac{1}{4}(v_h - v_l)$. To see this, note that the buyer's payoffs for acquiring information are equal to the payoffs for not acquiring information when

$$-\frac{1}{2}(v_h - v_l) + 2\Delta + 2c_s = 2\Delta - c_b - \frac{1}{4}\left((v_h - v_l) - \sqrt{(v_h - v_l)(v_h - v_l - 4c_s)}\right)$$

which gives

$$c_b = \frac{1}{4}\left(v_h - v_l + \sqrt{(v_h - v_l)(v_h - v_l - 4c_s)} - 8c_s\right).$$

Dividing both sides by Δ gives the boundary in terms of normalized costs and A:

$$k_b = \frac{1}{4}\left(A + \sqrt{A(A - 4k_s)} - 8k_s\right).$$

Differentiating this expression with respect to A gives the desired expression. Note that in taking this derivative it is *not* appropriate to treat k_s as a function of Δ and therefore of A because we are considering where the buyer is indifferent for k_b, k_s pairs.

The buyer's (normalized) cost, as a function of the seller's cost, that sets the buyer's payoff equal in Lemma 4 and in Lemma 6 is also increasing in A. This boundary is given by:

$$k_b = \frac{1}{4}\left[-A + \sqrt{A(A - 4k_s)} + 8\right].$$

The derivative of that boundary with respect to A is:

$$\frac{A - \sqrt{A(A - 4k_s)} - 2k_s}{4\sqrt{A(A - 4k_s)}},$$

which is positive whenever $k_s \in (0, \frac{1}{4}A]$, or equivalently $c_s \in (0, \frac{1}{4}(v_h - v_l)]$.

The seller's (normalized) cost that divides Lemma 2 and Lemma 4 is also increasing in A. This is obvious from Lemma 4, noting $c_s \leq \frac{1}{4}(v_h - v_l) - \Delta$ is equivalent to $k_s \leq \frac{1}{4}A - 1$.

Now suppose that both agents invest in expertise up to $e^*(A)$, hence play proceeds according to Lemma 2 in the low-volatility regime.[10] For $A < 9$, $e^*(A)$ will be on the boundary between Lemma 6 and Lemma 2. Thus, an increase in volatility to θA will move play into the Lemma 6

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[10]Note that the condition for e^* from Proposition 2 can be rewritten in terms of k (with k, of course, defined as $\frac{c}{\Delta}$) as

$$e^* = \begin{cases} k^{-1}\left(\frac{1}{4}A - 1\right) & \text{if } A \geq 9 \\ k^{-1}\left(\frac{5}{36}A\right) & \text{if } A \leq 9 \end{cases},$$

which permits us to write e^* as a function of A.

region, as is clear from the above three facts.

What happens when $A \geq 9$ is somewhat less obvious. When A increases, play can no longer remain in the Lemma 2 region because the seller's costs now imply that play proceeds according to Lemma 4 if the buyer is informed. However, since the boundary between Lemmas 4 and 6 boundary is increasing in A, we cannot immediately rule out the possibility that play proceeds according to Lemma 6, particularly for very large shocks. However, we observe, using l'Hospital rule, that

$$\lim_{A \to \infty} \frac{1}{4} \left(-A + \sqrt{A(A - 4k_s)} + 8 \right) = \lim_{A \to \infty} \frac{1}{4} \left(-A + \sqrt{A(A - 4k_s)} + 8 \right) \frac{1/A}{1/A} = -\frac{k_s}{2} + 2.$$

That is, the function defining the boundary between Lemma 4 and Lemma 6 will converge to a finite function. Since we know that, for $A \geq 9$, the symmetric equilibrium is where the diagonal intersects the vertical line at $\frac{1}{4}A - 1$, we know that play will always proceed according to Lemma 4 regardless of the magnitude of θ if and only if

$$-\frac{\frac{1}{4}A - 1}{2} + 2 < \frac{1}{4}A - 1,$$

which means that the limit, as A goes to $+\infty$, of the threshold on c_b for Lemma 4 is smaller than the smallest cost level in Lemma 2 and reduces to the condition $A > \frac{28}{3}$.

The magnitude of the shock required is given by finding the shock θ that moves the boundary for the buyer's decision to acquire information, evaluated at the normalized costs associated with $e^*(A)$ (which are given by $\frac{1}{4}A - 1$ from Proposition 2), to that same cost level. That is, we find the threshold for the shock to lead to play according to Lemma 6 by finding the increase in volatility that makes the buyer just indifferent between acquiring and not acquiring information at the equilibrium cost level when the seller's costs are the equilibrium costs.

$$\frac{1}{4} \left(-A\theta + \sqrt{A\theta(A\theta - 4\left(\frac{1}{4}A - 1\right))} + 8 \right) = \frac{1}{4}A - 1$$

$$\theta = \frac{(A - 12)^2}{A(28 - 3A)}$$

Since the boundary is monotonically increasing in A (and therefore θ), this implies that we enter the Lemma 6 region if and only[11] if $\theta \geq \frac{(A-12)^2}{A(28-3A)}$. Otherwise, play proceeds according to Lemma 4.

We need to show that there does not exist a unilateral deviation in expertise that would be optimal for one agent. We first study what would happen if one agent were to increase expertise above e^* and then we study what would happen if one agent were to decrease expertise below e^*.

Suppose one agent were to increase his investment in expertise enough to move to the Lemma 6 region when he turns out to be the buyer. When he is the seller, the game would either remain

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[11]Since we assume the choice preferred by the buyer is made if the seller is indifferent.

in the region of Lemma 4 for small increases or it would move to Lemma 6 or Lemma 3 for larger increases. All of these regions would give him zero payoffs. The only gain to an increase in expertise would have to accrue when he is the buyer. But using Lemma 6 we can show that, keeping the seller's cost fixed and below $\frac{1}{2}(v_h - v_l)$, the payoff to the buyer at any level of his own costs will be decreasing in A. Therefore, the buyer's payoff will be strictly lower in the high-volatility regime than in the low-volatility regime. So the benefits from increasing expertise in the stochastic-volatility setting will be smaller than the benefits from increasing expertise in the constant-volatility setting (where the low-volatility regime always occurs). Therefore, since increasing expertise above e^* is not a profitable unilateral deviation in the constant-volatility setting, such deviation will not be profitable when volatility is stochastic either.

If $A \geq \frac{28}{3}$ or $9 < A < \frac{28}{3}$ and $\theta < \frac{(A-12)^2}{A(28-3A)}$, an agent could consider reducing his investment in expertise such that play is on the boundary between Lemma 4 and Lemma 2 and trade occurs with probability one in the high-volatility regime when he is the seller. This deviation clearly dominates all the other possible deviations where expertise is lower than e^*. Such deviation would be costly in the low-volatility regime because the deviator's payoffs would be reduced at a rate of $\frac{2}{1-\delta}$ as his cost increases, until he reaches the region of Lemma 1 where his payoff is zero. On the other hand, his payoff would increase to 2Δ in the high-volatility regime. A deviation to such a lower level of expertise (or anything lower) would not be profitable when π is low enough. To see this, let $\tilde{e}(\theta)$ be the level of expertise such that the seller gets a payoff of 2Δ in the high volatility state.

Then, the difference in payoffs from picking e^* rather than $\tilde{e}(\theta)$, when the probability of the high-volatility state is denoted π is given by:

$$\frac{\pi}{1-\delta}(-\Delta) + \frac{1-\pi}{1-\delta}[c(\tilde{e}(\theta)) - c(e^*)] - [e^* - \tilde{e}(\theta)] \tag{11}$$

If $\pi = 0$, then we get from property P2 that the expression in (11) is positive and e^* is the optimal level of expertise.

Now, if we take the derivative of the expression in (11) with respect to π, we get:

$$-\frac{1}{1-\delta}\Delta - \frac{1}{1-\delta}[c(\tilde{e}(\theta)) - c(e^*)],$$

which is bounded from below by $-\frac{1}{1-\delta}[\Delta + c(0)]$, which is finite. Thus, by continuity, for π close enough to zero, the expression in (11) is positive and e^* provides the seller with a higher expected payoff ex ante than any other level of expertise.

If $A \leq 9$ or $9 < A < \frac{28}{3}$ and $\theta \geq \frac{(A-12)^2}{A(28-3A)}$, the analogous reduction in expertise would lead play to the boundary between Lemma 2 and Lemma 6 (or sometimes Lemma 7) in the high-volatility regime. In this case, the above discussion applies with the adjustment that 11 represents a lower bound on the excess value of choosing e^* over a lower level of expertise since payoffs to the seller will be less than 2Δ at $\tilde{e}(\theta)$ in the high volatility state and payoffs to the buyer at e^* are not zero

in the high volatility state (since play proceeds according to Lemma 6 rather than Lemma 4), and in fact are higher at e^* than at $\tilde{e}(\theta) < e^*$ since the buyer's payoff in the subgame must be (weakly) decreasing in his own costs regardless of the costs of the seller. ∎

Appendix B   Deriving the buyer's information acquisition boundary

Here, we the denote the ratio of volatility to gains to trade as $A = \frac{v_h - v_l}{\Delta}$, and normalize $\Delta = 1$ without loss of generality.

To determine whether the buyer will acquire information for a particular cost pair $\{c_s, c_b\}$ at a particular A, we simply find the cost to the buyer that makes him indifferent between acquiring information and not acquiring information, exploiting the fact that his information acquisition decision induces a proper subgame. Thus, we can simply compare the payoffs to the buyer for a given A and c_s under the appropriate lemma when the buyer does and does not acquire information. These thresholds are all derived in the lemmas in the text and produce the following boundary segments:

For Lemma 3 versus 6, the buyer acquires information if his costs satisfy:

$$
\begin{aligned}
1 - c_s &< \frac{1}{4}\left(-A + \sqrt{A(A - 4c_s)}\right) + 2 - c_b \\
c_b &< \frac{1}{4}\left(-A + \sqrt{A(A - 4c_s)} + 4c_s + 4\right).
\end{aligned}
$$

For Lemma 4 versus 6, the buyer acquires information if his costs satisfy:

$$
\begin{aligned}
0 &< \frac{1}{4}\left(-A + \sqrt{A(A - 4c_s)}\right) + 2 - c_b \\
c_b &< \frac{1}{4}\left(-A + \sqrt{A(A - 4c_s)} + 8\right).
\end{aligned}
$$

For Lemma 2 versus 6, the buyer acquires information if his costs satisfy:

$$
\begin{aligned}
2c_s + 2 - \frac{A}{2} &< \frac{1}{4}\left(-A + \sqrt{A(A - 4c_s)}\right) + 2 - c_b \\
c_b &< \frac{1}{4}\left(-A + \sqrt{A(A - 4c_s)} + 8c_s\right).
\end{aligned}
$$

For Lemma 2 versus 7, the buyer acquires information if his costs satisfy:

$$
\begin{aligned}
2c_s + 2 - \frac{A}{2} &\leq 1 - c_b + \frac{2}{A + 2} \\
c_b &< \frac{A^2 - 4Ac_s - 8c_s}{2(A + 2)}.
\end{aligned}
$$

These four cases exhaust all possible relevant comparisons. If the game proceeds according to Lemma 1 when the buyer does not acquire information, he will clearly not acquire information if his costs are even weakly positive since he can capture the full surplus while remaining uninformed. Comparisons between Lemma 7 and any lemma other than Lemma 2 are irrelevant since $c_s > \frac{2A^2}{(A+2)^2} > 0$ (the threshold for Lemma 7 to be relevant) implies $c_s > \max\{\frac{1}{4}A - 1, \frac{1}{6}A - \frac{1}{3}\}$ (the threshold above which Lemma 2 becomes relevant when the buyer does not acquire information).

From the above 4 conditions, we can determine whether the buyer acquires information for any pair $\{c_s, A\}$. Of course for some values of A certain of the above conditions will be irrelevant for all values of c_s, as is obvious from Figure 1.

Appendix C   Condition for the possible existence of multiple equilibria

We denote the ratio of volatility to gains to trade as $A = \frac{v_h - v_l}{\Delta}$ and normalize $\Delta = 1$ without loss of generality. Additionally, we normalize costs in terms of the discount factor by defining $C(e) = \frac{c(e)}{(1-\delta)}$.

Proposition 4 *Define:*

$$F(A) = \begin{cases} f_0(A) & \text{if } A \leq R_1 \\ f_1(A) & \text{if } A \in [R_1, R_2] \\ f_2(A) & \text{if } A > R_2 \end{cases}$$

where

$f_0(A) \equiv \frac{1}{200}(3\sqrt{41} - 13)A$

$f_1(A):$ *The root in x equal to 0 at $A = 8$ of the polynomial*[12]

$$256 - 576A + 436A^2 - 126A^3 + 10A^4 + (-1024 + 1792A - 944A^2 + 141A^3)x$$

$$+(1536 - 1856A + 544A^2)x^2 + (-1024 + 640A)x^3 + 256x^4$$

$f_2(A) \equiv \frac{16 - 6A - 3A^2}{8(2+A)} + \frac{1}{8}\sqrt{\frac{-32A + 18A^2 + 9A^3}{2+A}}$

$R_1 \equiv \frac{4861 + 173\sqrt{3} + \sqrt{41}(91 + 263\sqrt{3})}{1403}$

$R_2:$ *The largest real root of the polynomial:*

$$128 + 64x + 128x^2 - 56x^3 - 12x^4 - 30x^5 + 5x^6$$

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[12]A direct representation of this function can, of course, be obtained, but in the interest of space we do not include it here.

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There exists a decreasing, convex cost function such that there is a second equilibrium that is pure in investment in expertise and involves investment in expertise of e if and only if $e < F(A)$. When such an equilibrium exists, $C'(e) = -2$.

Proof: In the following, we will frequently refer to regions governed by certain lemmas and boundaries between such regions. Unless otherwise stated, these refer to the regions and boundaries for graphs of the type presented in Figure 2, where the x axis is seller's cost and the y axis is buyer's cost. Boundaries between regions governed by lemmas numbered 1-4 and regions governed by lemmas numbered 6 or 7 are determined by the function mapping seller's costs into the lowest cost to the buyer at which the buyer would not acquire information (henceforth the buyer's information acquisition boundary). Boundaries between lemmas within either of the two ranges are vertical rays at the points defined by the thresholds in the lemmas (as described for Lemmas 1- 4 in Figure 1, for example), extending above or below the buyer's information acquisition boundary depending on whether the lemmas in question address behavior when the buyer does or does not acquire information.

The construction of a threshold below which a cost c will represent a symmetric equilibrium for some cost function proceeds by identifying all c that lead to play in the region of Lemma 6 and from which there is no profitable deviation to a higher cost when we ignore the increase in payoffs resulting *directly* from the decrease in expertise investment. That is, we consider payoffs only in the trading subgames. We will refer to these points as *potential equilibria*. For any such point, it is easy to identify the best possible deviation that could possibly result in an improvement over adhering to c. This will not, in general, be the deviation with the highest payoff since in any case where there turns out to be no profitable deviation an infinitesimal increase in costs will be more profitable than the deviation we describe. However, from the payoffs to Lemma 6 it is immediate that a small increase in costs cannot improve payoffs in the sense described here.

The deviation that we must consider, then, is a deviation to the lowest cost that leads to play according to Lemma 2 when the deviator is the seller. We will refer to this, with some imprecision, as the *locally optimal deviation*. For c to be part of an equilibrium, it must be the case that $C'(e) = -2$ at said c, so, since we are concerned exclusively with decreasing and convex cost functions, $C'(e) = -2$ for all e that lead to play in the Lemma 2 region, so it is impossible for a deviator to do better (as the seller) by moving further into the interior of the region governed by Lemma 2 since the payoff as the seller is decreasing at rate -2 in the seller's cost.

This deviation can take two forms. If $A < 6$ or $A > 6$ but $c \geq \frac{A^2(A-6)}{2(A+2)^2}$, the deviation is to the boundary between the Lemma 2 region an the Lemma 6 region. Otherwise, the deviation is to the boundary between Lemma 2 and Lemma 7, again when the deviator is the seller. This threshold is derived as follows:

Recall that, under the normalization described, the threshold in seller's costs separating Lemma 6 and Lemma 7 is

$$\rho(A) \equiv \frac{2A^2}{(A+2)^2}.$$

Evaluating the buyer's information acquisition boundary at this point gives

$$c_b = \frac{1}{4}\left(A - \frac{16A^2}{(A+2)^2}\right)$$

which reduces to

$$c_b = \begin{cases} \frac{A(2-3A)}{(A+2)^2} & \text{if } A \leq 2 \\ \frac{A^2(A-6)}{2(A+2)^2} & \text{if } A \geq 2. \end{cases}$$

But, $\frac{A(2-3A)}{(A+2)^2} < 0$ for all $A \leq 2$ and $\frac{A^2(A-6)}{(A+2)^2} < 0$ for all $A < 6$, so, for all $A < 6$, there is no c such that the locally optimal deviation takes play to the boarder of Lemma 7. This is, of course, because Lemma 7 does not arise for any cost pair since the boundary between Lemma 2 and Lemma 6 strikes the origin *before* $c_s = \rho(A)$. For $A > 6$, Lemma 7 is relevant and $c < \frac{A^2(A-6)}{2(A+2)^2}$ implies that the locally optimal deviation is to the boundary between Lemma 2 and Lemma 7.

We will for now ignore the possibility that $c < \frac{A^2(A-6)}{2(A+2)^2}$. Specifically, we will proceed *as if* the locally optimal deviation is to the boundary between Lemma 2 and Lemma 6. If such a deviation is *not* profitable (using the payoffs described in Lemma 2 when the deviator is the seller) then the deviation to the true locally optimal deviation will not be profitable because the seller's payoff would be lower (since, for all $c_s > \rho$, the boundary between Lemmas 2 and 7 is "northeast" of the Lemma 2-6 boundary, and thus the bargaining power of the seller at the 2-7 boundary is less than at the 2-6 boundary, assuming the buyer does not acquire information: see Figure 2, panels (b) and (c)).

If it turns out that the largest c for which which there is no profitable locally optimal deviation under this assumption exceeds $\frac{A^2(A-6)}{2(A+2)^2}$, then Lemma 7 effectively drops out. If, however, this largest c falls below $\frac{A^2(A-6)}{2(A+2)^2}$, we have then *not* shown that there is a profitable deviation at or below c because the positive payoff for the deviation does not arise along the equilibrium path of the subgame induced by the deviation. In this case, we must instead find the largest c for which there is no profitable deviation to the boundary between Lemma 2 and Lemma 7.

We will then proceed by ignoring Lemma 7, deriving the threshold for c, and finding (for $A > 6$) where that threshold falls below $\frac{A^2(A-6)}{2(A+2)^2}$. We end the proof by deriving the appropriate adjustment in this region, which will complete the derivation of $F(A)$.

The discussion to the point has described a deviation in terms of where play proceeds when the deviator is the seller. In the posited deviation, the seller clearly does better than he does when adhering. On the other hand, when the the deviator is the buyer he does worse than he would have by adhering since his costs are higher (recall that we ignore savings from reduced expertise

expenditure). Whether the deviation is profitable depends on which of these effects dominates, but the appropriate comparison to make depends on whether play continues to proceed according to Lemma 6 when the deviator is the buyer or if play moves to Lemma 3. (It is immediate that an optimal deviation will not move play when the deviator is the buyer into the Lemma 2 region because the slope of the Lemma 2- 6 boundary is always < -2.)

We first must, of course, derive the locally optimal deviation (again, ignoring Lemma 7). Taking the proposed equilibrium costs as c and the optimal deviation as c^d, we have

$$c = \frac{1}{4}\left(A + \sqrt{A(A - 4c^d)} - 8c_d\right)$$

$$c^d = \frac{1}{32}\left(3A - 16c + \sqrt{A(9A + 32c)}\right),$$

which is the value for c_s at which the boundary between Lemma 2 and Lemma 6 equals the original cost c. Note that while there are two solutions to the above equation, only one is positive. We can now find conditions for c_d to leave play in the Lemma 6 region when the deviator is the buyer:

$$\frac{1}{32}\left(3A - 16c + \sqrt{A(9A + 32c)}\right) < \frac{1}{4}\left(4 - A + \sqrt{A(A - 4c)} + 4c\right).$$

The right hand side of this condition is the boundary between Lemma 3 and Lemma 6 evaluated at the original cost, while the left is the locally optimal deviation.

Confining attention to cases where $A \leq 8$ (since $A > 8$ implies that there is some c above which Lemma 4 is relevant rather than Lemma 3 or Lemma 2), this inequality holds if and only if $A \leq \frac{16}{3}$ or $A > \frac{16}{3}$ and $c > g(A)$, where $g(A)$ is defined as the root in x of the polynomial

$$1024 - 1408A + 580A^2 - 66A^3 +$$

$$(6144 - 5888A + 1432A^2 - 77A^3)x + (12824 - 8160A + 924A^2)x^2 + (13824 - 3744A)x^3 + 5184x^4$$

for which $g(\frac{16}{3}) = 0$.[13]

For $A > \frac{16}{3}$ and $c < g(A)$, the optimal deviation moves play to Lemma 3. But, analogous to the treatment of Lemma 7, we can ignore Lemma 3 and calculate whether a deviation is profitable based on the Lemma 6 payoffs. Under this assumption, the locally optimal deviation from c is *not* profitable if, by direct calculation of the payoff to adhering versus the payoff to deviating (calculated as described),

$$2 + \frac{1}{4}\left(-A + \sqrt{A(A - 4c)}\right) - c >$$

$$2 + \frac{1}{4}\left(-A + \sqrt{A(A - 4c)}\right) + \frac{1}{32}\left(-3A + 16c - \sqrt{A(9A + 32c)}\right) + \frac{A}{2} + \frac{1}{16}\left(-3A + 16c - \sqrt{A(9A + 32c)}\right),$$

---

[13]It is of course possible to obtain a closed form solution for this polynomial and thus explicitly express g, but in the interest of space we omit this rather unwieldy expression.

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which simplifies to

$$c < f_0(A) \equiv \frac{A}{200} \left(3\sqrt{41} - 13 \right).$$

Note that the inequality is strict because there is at least an infinitesimal savings associated with reducing expertise. For this case, we are then left to determine if there are any $c < g(A)$ that are *not* potential equilibria. Such a c *will* be a potential equilibrium if:

$$2 + \frac{1}{4} \left(-A + \sqrt{A(A - 4c)} \right) - c > (1 - c) + \frac{A}{2} + \frac{1}{16} \left(-3A + 16c - \sqrt{A(9A + 32c)} \right),$$

where we confine attention to $A > \frac{16}{3}$ and $c < g(A)$, and further confine attention to $A < R_1$, with:

$$R_1 \equiv \frac{4861 + 173\sqrt{3} + \sqrt{41}(91 + 263\sqrt{3})}{1403}$$

as stated in the proposition. This final restriction arises because, for $A > R_1$,

$$g(A) > \frac{A}{200} \left(3\sqrt{41} - 13 \right)$$

and, therefore, computing the threshold as if a deviation kept the game in Lemma 6 when the deviator is the buyer is invalid.

But, under these restrictions, the condition for c to be a potential equilibrium is always true. Thus, for $A < g(A)$, play entering the Lemma 3 region after a the locally optimal deviation, when the deviator is the buyer, is a sufficient condition for c to be a potential equilibrium, but this condition is slack. So, for $A < R_1$, c is a potential equilibrium if and only if

$$c < \frac{A}{200} \left(3\sqrt{41} - 13 \right).$$

We have neglected one step. With $2 < A < 8$, for values of $c > \frac{1}{6}A - \frac{1}{3}$, Lemma 3 is irrelevant (from the boundary between Lemma 3 and Lemma 2). Since the slope of the Lemma 2-Lemma 6 boundary is less than -2, this means that, for $c > \frac{1}{6}A - \frac{1}{3}$, the relevant lemma for the payoff when the deviator is the buyer is Lemma 6. But, for $c > \frac{1}{6}A - \frac{1}{3}$, the expression for the boundary between Lemma 3 and Lemma 6 exceeds the Lemma 2-Lemma 6 boundary (in fact, the statements are equivalent):

$$\frac{1}{4} \left(4 - A + \sqrt{A(A - 4c_s)} + 4c_s \right) > \frac{1}{4} \left(A + \sqrt{A(A - 4c_s)} - 8c_s \right) \Leftrightarrow c > \frac{1}{6}A - \frac{1}{3},$$

so this condition is slack as well.

We now move on to the case where $A > R_1$. In this case, all values of c which would imply that there was no profitable deviation using payoffs calculated from Lemma 6 when the deviator is the buyer fall *below* the threshold under which Lemma 3 becomes the appropriate comparison. Thus,

above R_1, the appropriate threshold for c to be a potential equilibrium is (restricting attention to $A < 8$)

$$2 + \frac{1}{4}\left(-A + \sqrt{A(A - 4c)}\right) - c > (1 - c) + \frac{A}{2} + \frac{1}{16}\left(-3A + 16c - \sqrt{A(9A + 32c)}\right)$$

which reduces to $c > f_1(A)$, where $f_1(A)$ is defined as in the proposition.

Note, however, that $f_1(8) = 0$. But, we know that, for small c and $A > 6$, the locally optimal deviation actually takes us to the Lemma 2-7 boundary, and that such deviations are less profitable than those calculated (inappropriately) using the Lemma 2-6 boundary.

As discussed above, the procedure up to this point has thus identified a subset of potential equilibria. We now determine, for every A, the value for c at which an optimal deviation leads to the intersection of the the Lemma 2-6 and the Lemma 2-7 boundary. If the proposed threshold for c to be a potential equilibrium exceeds this c, then said threshold is valid. Otherwise, we must calculate the value of the locally optimal deviation using the the Lemma 2-7 boundary and find the correct threshold.

We have already found the c for which Lemma 7 becomes relevant:

$$c = \frac{A^2(A - 6)}{2(A + 2)^2}.$$

Note

$$\frac{A^2(A - 6)}{2(A + 2)^2} < \frac{A}{200}\left(3\sqrt{41} - 13\right)$$

for all $A < R_1$, so the relevant condtion is

$$\frac{A^2(A - 6)}{2(A + 2)^2} < f_1(A),$$

which gives as the threshold exactly R_2, which is, of course, greater than R_1.

So, for $A > R_2$, we must calculate c^d as the solution to:

$$c = \frac{A^2 - 8c^d - 4Ac^d}{2(A + 2)}$$

which gives
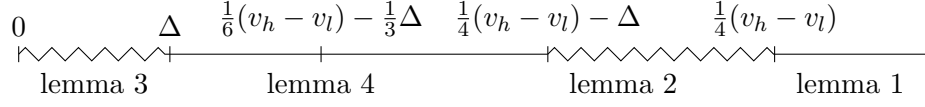
$$c^d = \frac{A^2 - 4c - 2Ac}{4(A + 2)}.$$

Now, since we are already below the threshold below which c^d moves into the Lemma 3 region when the deviator is the buyer when using the Lemma 2-Lemma 6, and the deviation to the Lemma 2-Lemma 7 boundary is strictly larger when said boundary is appropriate (by construction), we know that c^d moves play into the Lemma 3 region when the deviator is the buyer. Thus, for
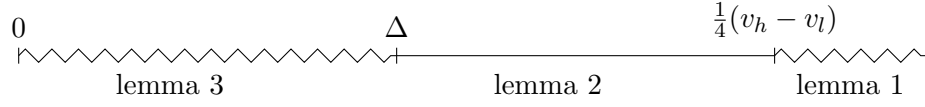
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$A > R_2$, c is a potential equilibrium if

$$c < f_2(A),$$

which is calculated in the same manner as $f_1(A)$ but replacing the payoff to the deviator when he is the seller with the payoff to the seller on the Lemma 2- 7 boundary.■
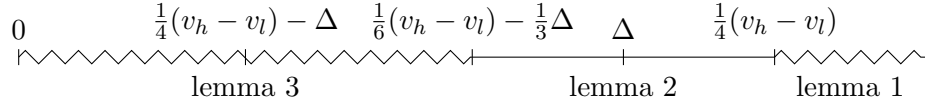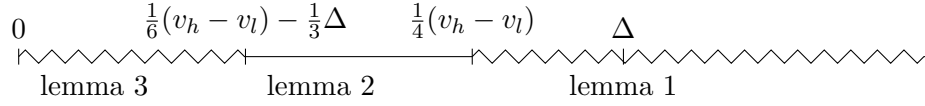
Case 1: $\Delta < \frac{1}{8}(v_h - v_l)$

0 Δ $\frac{1}{6}(v_h - v_l) - \frac{1}{3}\Delta$ $\frac{1}{4}(v_h - v_l) - \Delta$ $\frac{1}{4}(v_h - v_l)$

    lemma 3   lemma 4   lemma 2   lemma 1

Case 2: $\Delta = \frac{1}{8}(v_h - v_l)$

0 Δ $\frac{1}{4}(v_h - v_l)$

       lemma 3   lemma 2   lemma 1

Case 3: $\frac{1}{4}(v_h - v_l) > \Delta > \frac{1}{8}(v_h - v_l)$

0 $\frac{1}{4}(v_h - v_l) - \Delta$ $\frac{1}{6}(v_h - v_l) - \frac{1}{3}\Delta$ Δ $\frac{1}{4}(v_h - v_l)$

       lemma 3   lemma 2   lemma 1

Case 4: $\frac{1}{2}(v_h - v_l) > \Delta \geq \frac{1}{4}(v_h - v_l)$

0 $\frac{1}{6}(v_h - v_l) - \frac{1}{3}\Delta$ $\frac{1}{4}(v_h - v_l)$ Δ

    lemma 3   lemma 2   lemma 1

Case 5: $\Delta \geq \frac{1}{2}(v_h - v_l)$

0 $\frac{1}{4}(v_h - v_l)$ Δ

   lemma 2   lemma 1
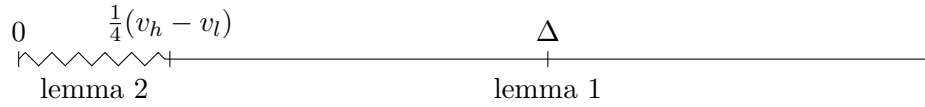
Figure 1: Regions of seller's costs where Lemmas 1-4 apply. The five cases depend on the relative magnitude of the gains to trade, Δ, and the volatility, $v_h - v_l$.
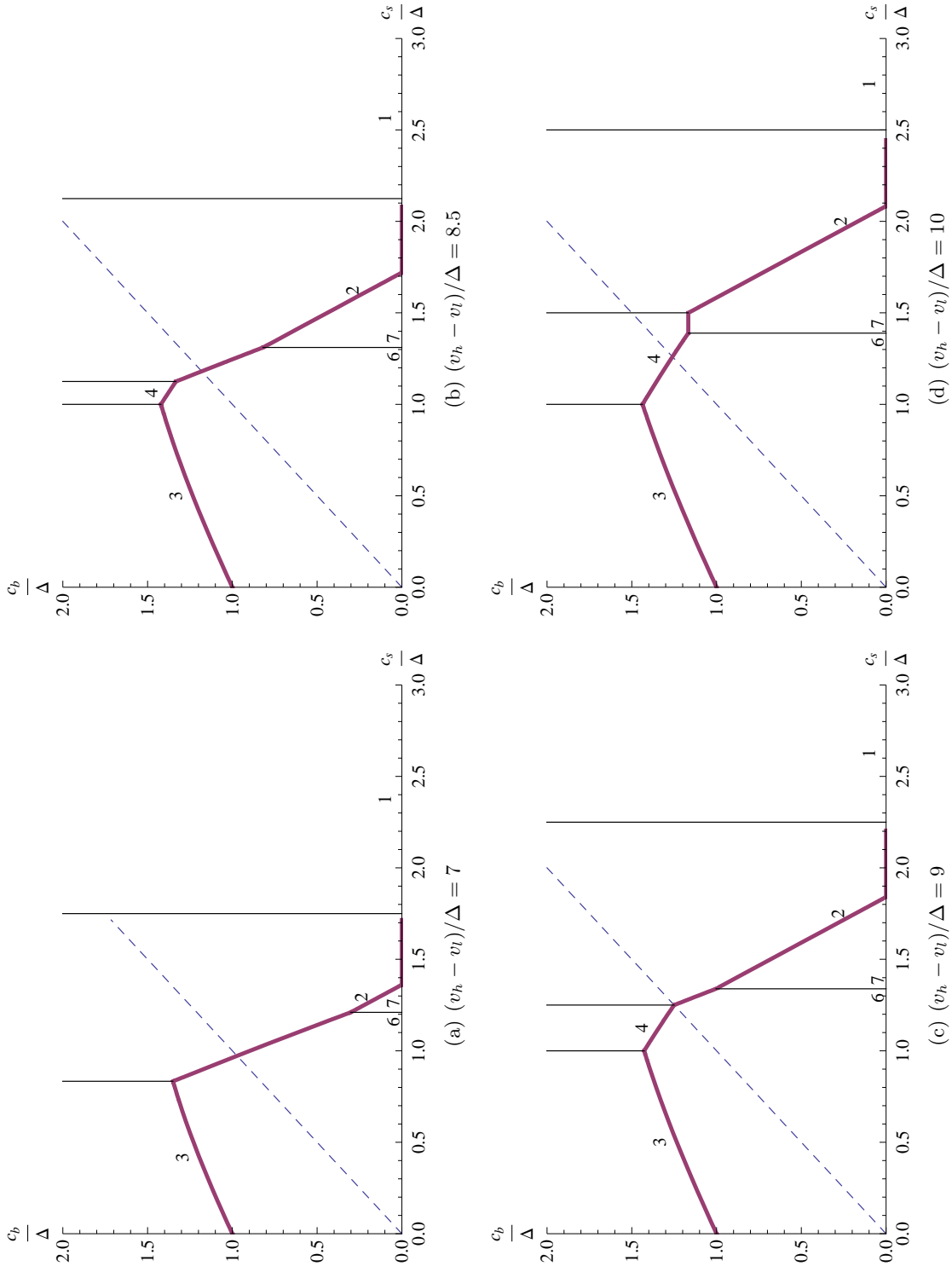
Figure 2: Costs, normalized by gains to trade, at which the seller and buyer will investigate as functions of the ratio of volatility to gains to trade: $(v_h - v_l)/\Delta$. The possible cost pairs are divided into areas covered by Lemmas 1-4 and 6-7. For cost pairs in the lower left-hand corner, the buyer investigates before making an offer to the seller (Lemmas 6 and 7). To the right and above that area, the buyer stays uninformed. The 45-degree dashed line illustrates possible symmetric equilibria in information acquisition.

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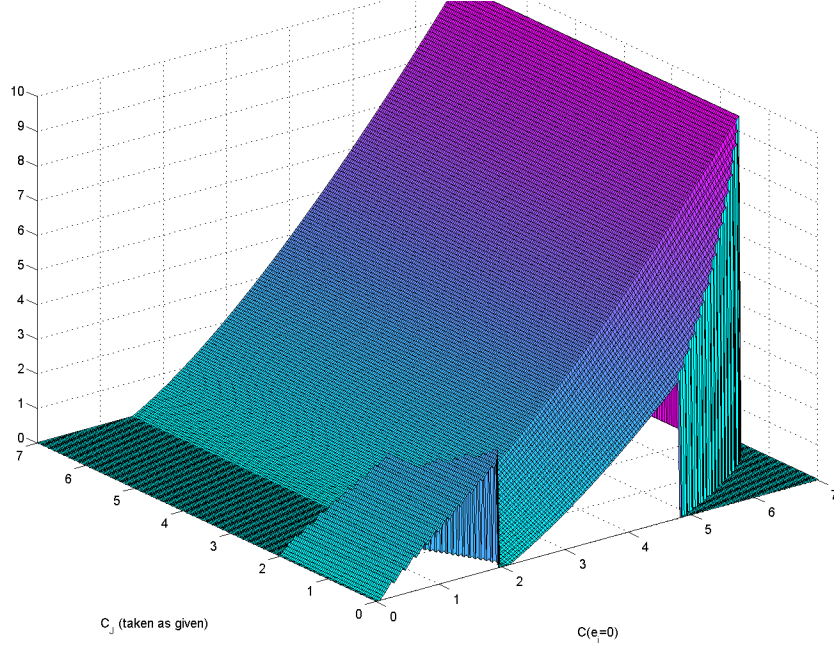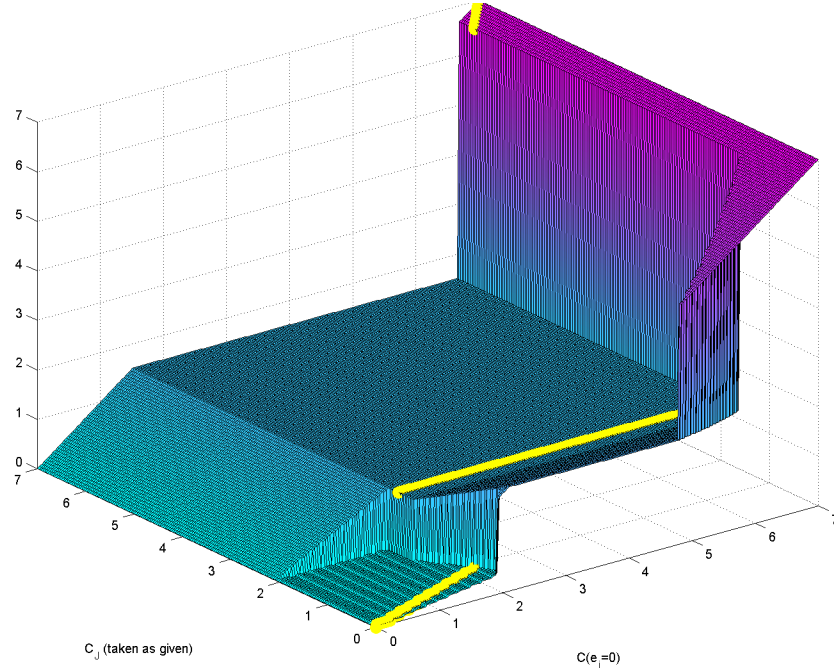
(a) Expertise

(b) Cost

Figure 3: Response Surfaces when Volatility is Constant. Panel (a) shows the best responses in the level of expertise given the initial cost of investigation and the cost level of potential counterparties. Panel (b) shows the costs that result from optimal investment in expertise, given the same variables. The highlighted line identifies the cost pairs (agent's initial cost vs. opponent's final cost) that lead to symmetric pure-strategy Nash equilibria in financial expertise. Figures are generated by setting: $\delta = 0.9$, $\lambda = 1$, $\Delta = 1$, and $(v_h - v_l) = 10$.
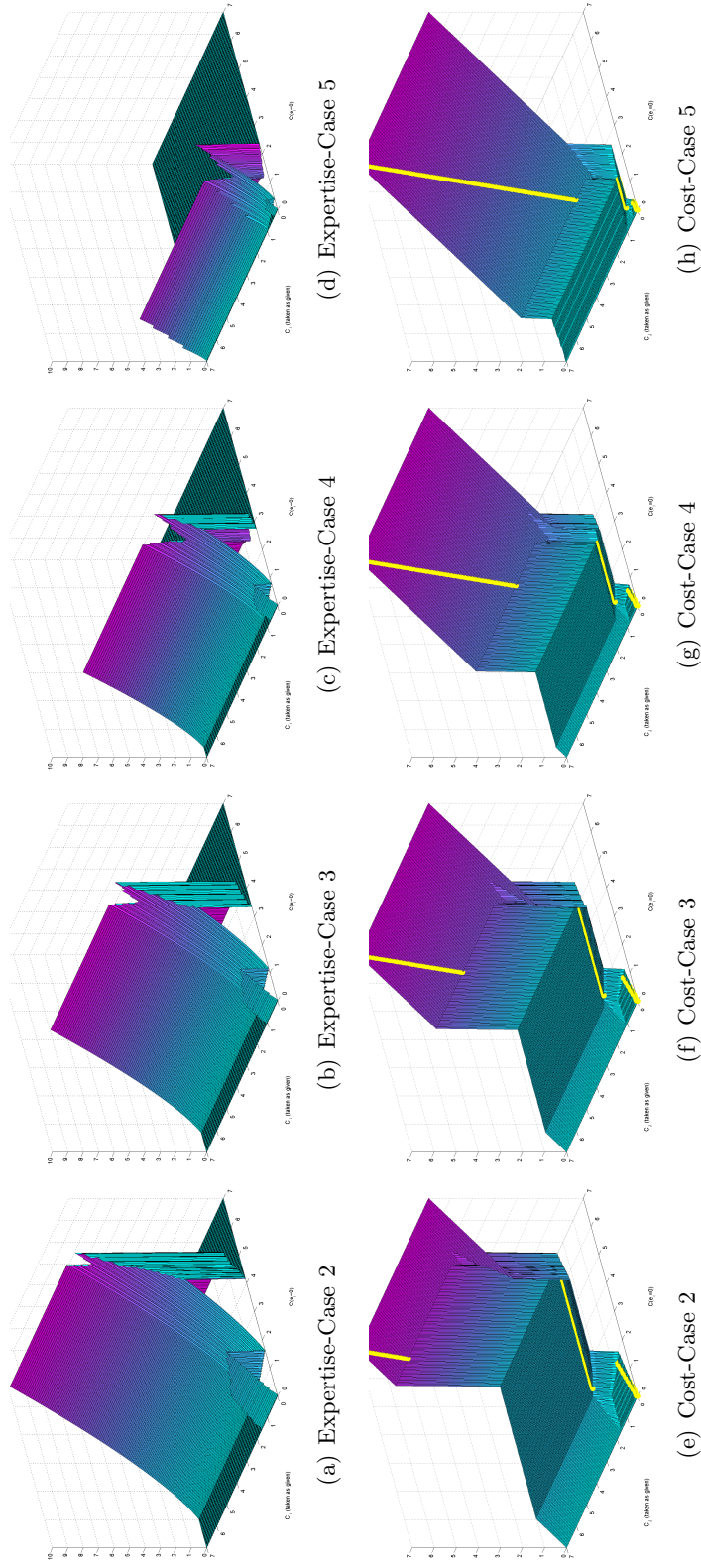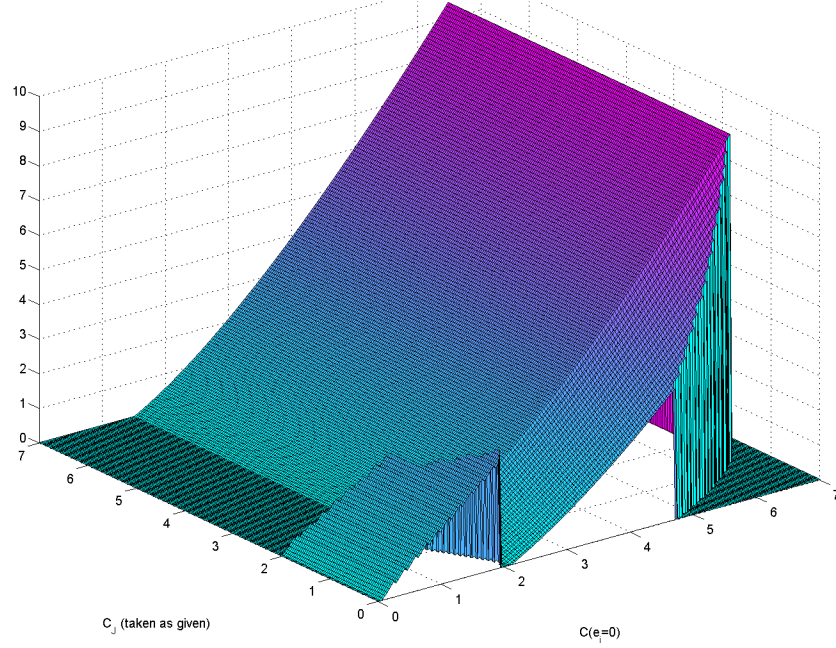
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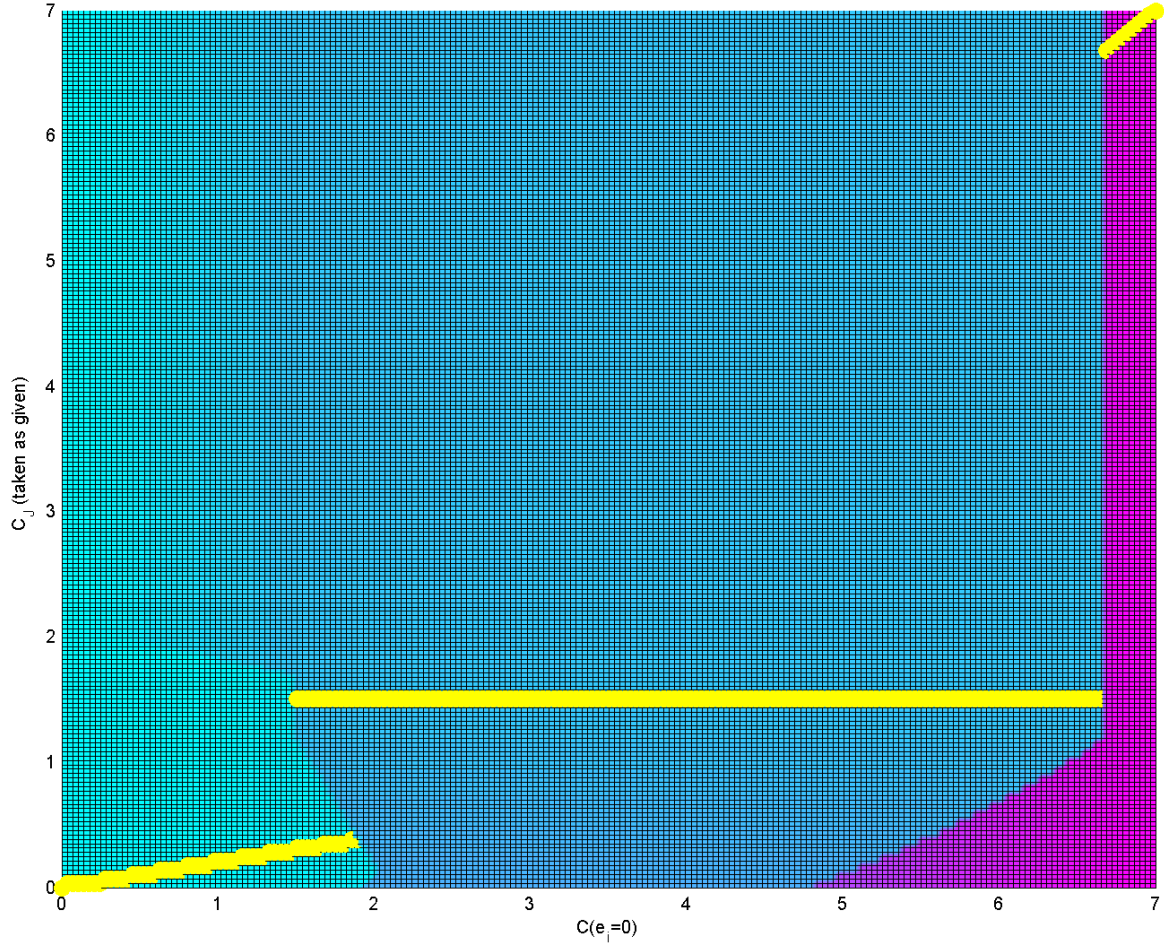
Figure 4: Symmetric Equilibria when Volatility is Constant. The highlighted line identifies the cost pairs (agent's initial cost vs. opponent's final cost) that lead to symmetric pure-strategy Nash equilibria in financial expertise. Figure is generated by setting: $\delta = 0.9$, $\lambda = 1$, $\Delta = 1$, and $(v_h - v_l) = 10$.

(a) Expertise-Case 2 (b) Expertise-Case 3 (c) Expertise-Case 4 (d) Expertise-Case 5

(e) Cost-Case 2 (f) Cost-Case 3 (g) Cost-Case 4 (h) Cost-Case 5

Figure 5: Response Surfaces when Volatility is Constant. Panels (a)-(d) show the best responses in the level of expertise given the initial cost of investigation and the cost level of potential counterparties. Panels (e)-(h) show the costs that result from optimal investment in expertise, given the same variables. The highlighted line identifies the cost pairs (agent's initial cost vs. opponent's final cost) that lead to symmetric pure-strategy Nash equilibria in financial expertise. Figures are generated by setting: $\delta = 0.9$, $\lambda = 1$, $\Delta = 1$, and $(v_h - v_l) = 8$ for Case 2, $(v_h - v_l) = 6$ for Case 3, $(v_h - v_l) = 4$ for Case 4, and $(v_h - v_l) = 2$ for Case 5.

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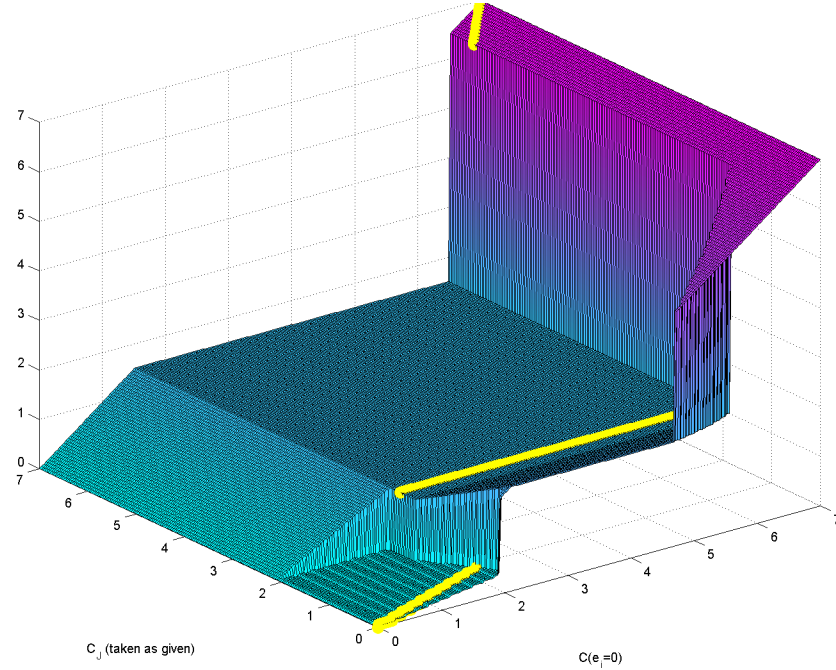
(a) Expertise

(b) Cost

Figure 6: Response Surfaces when Volatility is Stochastic. Panel (a) shows the best responses in the level of expertise given the initial cost of investigation and the cost level of potential counterparties. Panel (b) shows the costs that result from optimal investment in expertise, given the same variables. The highlighted line identifies the cost pairs (agent's initial cost vs. opponent's final cost) that lead to symmetric pure-strategy Nash equilibria in financial expertise. Figures are generated by setting: $\delta = 0.9$, $\lambda = 1$, $\Delta = 1$, $\theta = 1.1$, $\pi = 0.05$, and $(v_h - v_l) = 10$.
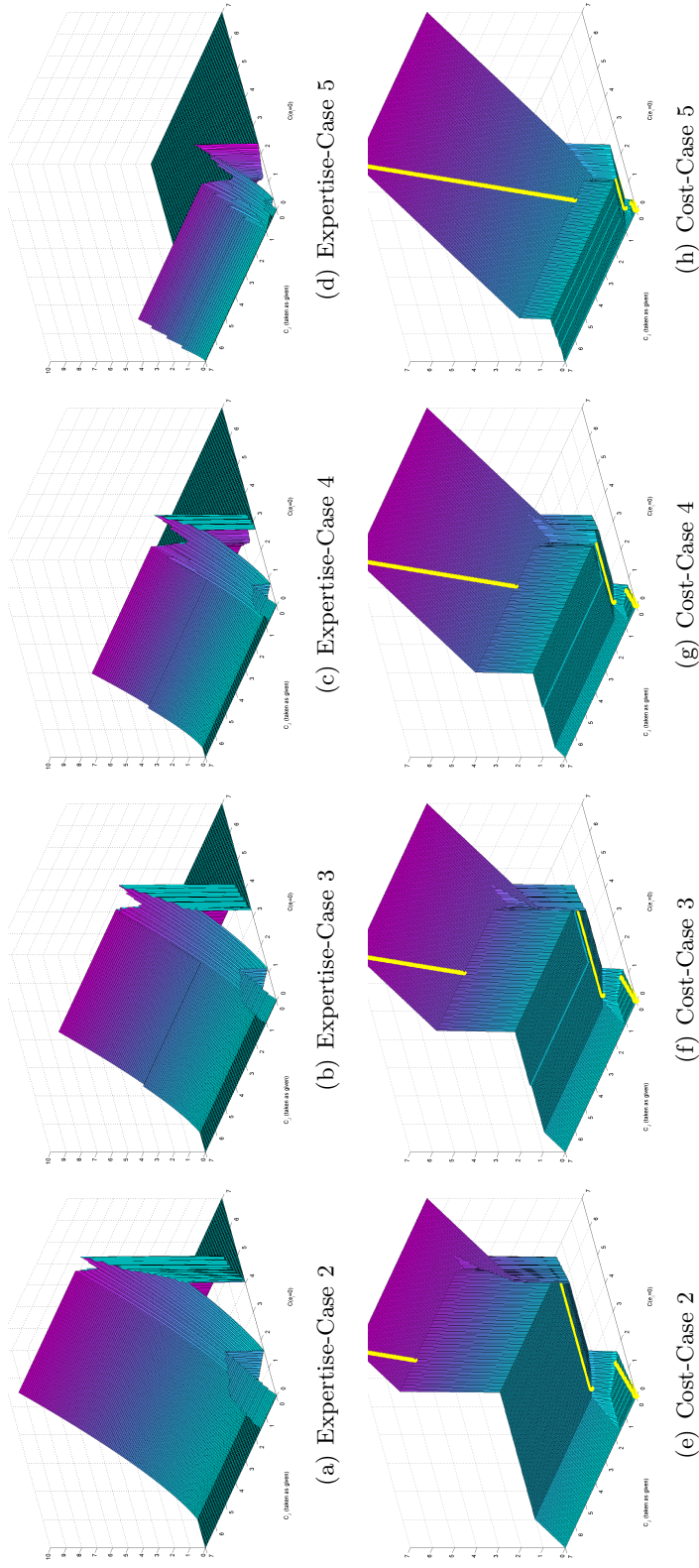
Figure 7: Response Surfaces when Volatility is Stochastic. Panels (a)-(d) show the best responses in the level of expertise given the initial cost of investigation and the cost level of potential counterparties. Panels (e)-(h) show the costs that result from optimal investment in expertise, given the same variables. The highlighted line identifies the cost pairs (agent's initial cost vs. opponent's final cost) that lead to symmetric pure-strategy Nash equilibria in financial expertise. Figures are generated by setting: $\delta = 0.9$, $\lambda = 1$, $\Delta = 1$, $\theta = 1.1$, $\pi = 0.05$, and $(v_h - v_l) = 8$ for Case 2, $(v_h - v_l) = 6$ for Case 3, $(v_h - v_l) = 4$ for Case 4, and $(v_h - v_l) = 2$ for Case 5.
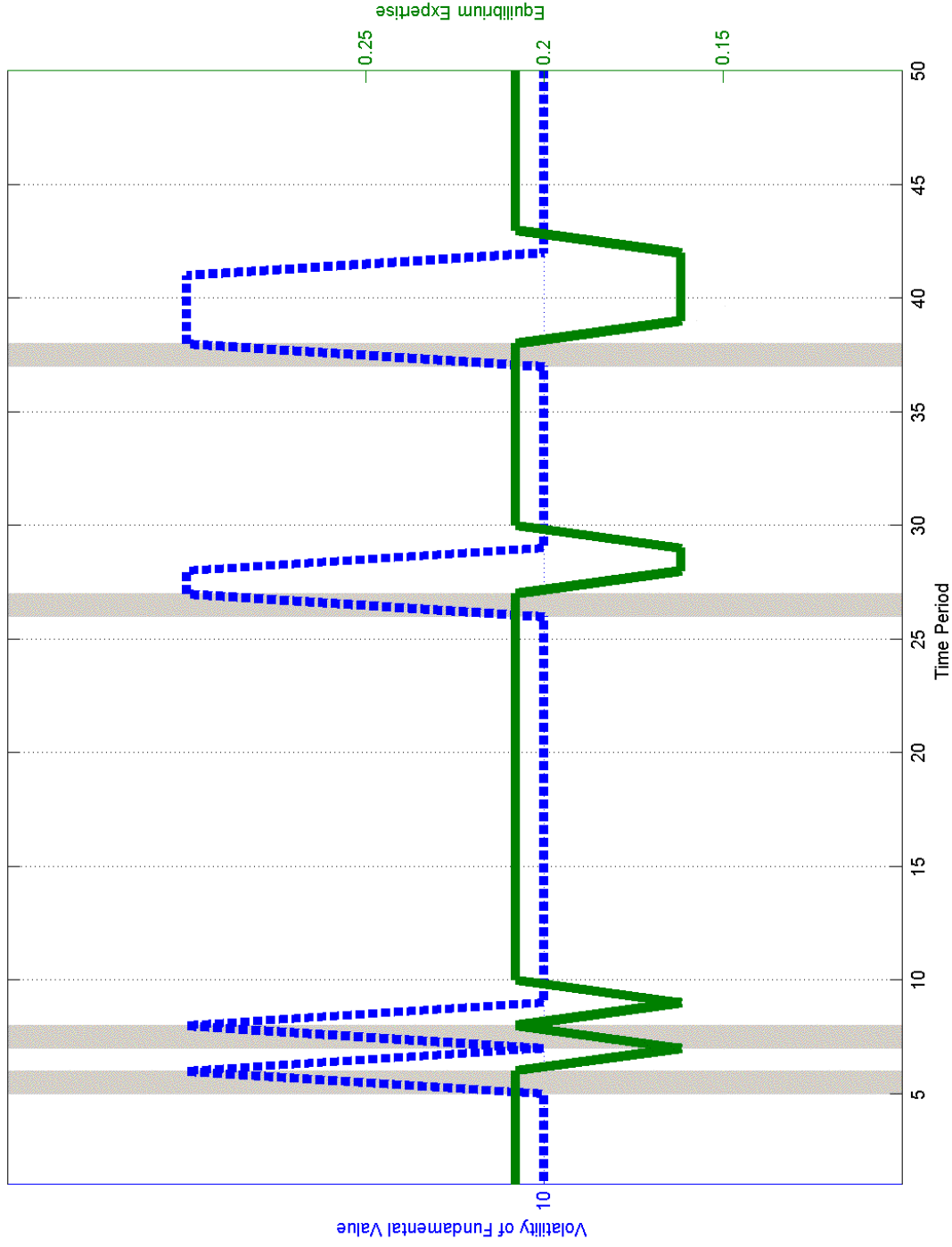
Figure 8: Equilibrium Responses to Volatility Increases. The dashed line represents a sample path for exogenous volatility, which follows a two-state Markov chain. Transition probabilities to the high-volatility state are 0.4 and 0.1 from the high- and low-volatility state, respectively. The solid line shows equilibrium levels of expertise, and the shaded areas represent periods when trade breaks down. The Figure is generated by setting: $\lambda = 10$, $\Delta = 1$, $\theta = 1.1$, and $(v_h - v_l) = 10$.