Non-exclusive Dynamic Contracts, Competition, and the Limits of Insurance *

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Abstract

We study how the presence of non-exclusive contracts limits the amount of insurance provided in a decentralized economy. We consider a dynamic Mirrleesian economy in which agents are privately informed about idiosyncratic labor productivity shocks. Agents sign privately observable insurance contracts with multiple firms (i.e., they are non-exclusive), which include both labor supply and savings aspects. Firms have no restriction on the contracts they can offer, interact strategically. In equilibrium, contrary to the case with exclusive contracts, a standard Euler equation holds, the marginal rate of substitution between consumption and leisure is equated to the worker's marginal productivity. Also, each agent receives zero net present value of transfers. To sustain this equilibrium, more than one firm must be active and must also offer latent contracts to deter deviations to more profitable contingent contracts. In this environment, the non-observability of contracts removes the possibility of additional insurance beyond self-insurance. To test the model, we allow firms to observe contracts at a cost. The model endogenously divides the population into agents that are not monitored and have access to non-exclusive contracts and agents that have access to exclusive contracts. We use US survey data and find that high school graduates satisfy the optimality conditions implied by the non-exclusive contracts while college graduates behave according to the second group.

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1 Introduction

What type of contractual arrangements are available to workers in a decentralized economy when firms compete for the provision of social insurance? In this paper, we study how, in a decentralized economy, the presence of non-exclusive contracts endogenously limits the contracts offered and hence the amount of insurance. We find that competition and non-observability of insurance contracts significantly reduce the amount of insurance provided: the equilibrium allocation in our environment is equivalent to a self-insurance economy and only linear contracts are offered.

Multiple credit and labor relations are an important aspect of everyday life. Survey data shows that individuals and households receive insurance against idiosyncratic risk from a multitude of sources: publicly provided insurance (unemployment, Medicare, Medicaid, disability, food stamps, progressive income taxation); privately provided insurance (employer, between and within family transfers);[1] financial instruments in credit markets; and housing and other large durable goods. The same consideration is true for labor relationships. Paxson and Sicherman (1994) look at the number of concurrent labor relationships held by survey respondents of the Panel Study of Income Dynamics (PSID) between 1977 and 1990 and the Current Population Survey (CPS) of 1991. They find that for any given year, 20% of working males held at least a second job, and during their working life there is at least a 50% probability of holding a second job. However, monitoring all the transactions an agent might engage in with other firms is very costly for an individual firm, especially if these relationships include activities in the informal labor market, private savings, and the ability to transfer leisure into consumption through either home production or shopping time (see Aguiar and Hurst (2005)). Motivated by these considerations, the key friction addressed in this paper is the *non-exclusivity* and *non-observability* of contractual relations. In the first part of the paper, we characterize the optimal contract under the assumption that *none* of

[1]The Panel Study of Income Dynamics reports for the years 1969 to 1985 a measure of income transfer received by households. We find that, in a given year, 24% of the households report receiving a transfer and 67% of the households received a transfer at some stage. These transfers are significant, averaging $1,930 (1983 dollars) and represent between 70% to 90% of total food expenditures.

the labor and credit relations an agent engages in can be observed by an individual firm.[2] In the second part of the paper, we endogeneize the observability of contracts by allowing firms to costly monitor contracts and take the model to the data.

The environment studied is a finite horizon dynamic Mirrleesian economy in which agents are privately informed about idiosyncratic labor productivity shocks that evolve over time. Agents wish to insure this risk by signing contracts with insurance providers (firms). Agents are not limited to a single insurance/labor relationship and can sign contracts with multiple firms. The contracting arrangements are private information of the contracting parties. Given this friction, in general, the communication between agent and firms cannot be limited to the exogenous private shock of agents (firms might also seek information about the other relations the agent has engaged in), as in the case with observable contracts. We extend the results in the common agency literature to our dynamic environment and characterize equilibrium using a menu game.[3] In this game, each firm offers collections of payoff relevant alternatives – menus – and delegates to the agent the choice within these menus. The choice of the agent from a menu can reveal information about his type and the other contractual arrangements in which he might be involved. We impose no restriction on the contracts that firms can offer. A firm can, for example, offer a spot labor contract, a linear intertemporal borrowing and saving contract, a state contingent dynamic insurance contract, and so on.

The non-observability of contracts removes the possibility of additional insurance beyond self-insurance and only linear contracts arise in equilibrium. We find that three optimality conditions must hold in equilibrium. First, the intertemporal marginal rate of substitution between consumption at time t and consumption at $t+1$ is equal to the marginal rate of transformation (a standard Euler equation holds).[4] Second, the marginal rate of substitution between consumption and leisure is equated to the marginal productivity for any time and any history.[5] Third, the net present value of the transfers received in equilibrium is equal

[2]The characterization under exclusive contracts is well understood, see Prescott and Townsend (1984).

[3]See Peters (2001), Martimort and Stole (2002), and Epstein and Peters (1999).

[4]If contracts are exclusive, the Euler equation does not hold and agents are savings constrained (see Golosov, Kocherlakota, and Tsyvinski (2003)).

[5]This is also different with respect to the exclusive contracting environment (see, for example, Mirrlees

zero for every agent in the economy. These optimality conditions imply that the unique equilibrium allocation is equivalent to an economy in which agents can trade non-contingent bonds and are paid their marginal productivity and in which there is no redistribution. The intuition for this result is the following. If, for example, a firm offers an intertemporal contract at an implicit rate of return lower than the marginal rate of transformation, it would provide a profitable opportunity for an entrant: it can offer a contract with a return slightly higher and make profits.[6] Such entry cannot be prevented by the first firm by also offering latent contracts because it cannot induce negative profits to the entrant.

These results, linking side trading and linear contracts, are reminiscent of Allen (1985), Hammond (1987), Cole and Kocherlakota (2001). We contribute to this literature by explicitly modeling competition between firms and determining endogenously the market structure. To sustain the equilibrium allocation we show that an incumbent firm must offer latent contracts to deter deviations of other incumbent firms. Moreover, in equilibrium more than one firm must offer the equilibrium allocation. The intuition for this result is that the equilibrium allocation is the most profitable non-contingent contract; however some contingent contracts deliver higher profits. If there is a unique incumbent or no latent contracts, a firm will deviate and offer one of these contracts.

To derive testable implications between non-exclusivity of contracts and the availability of insurance in the data, we generalize the model, relaxing the assumption about the observability of contracts. We assume that at time 0, a firm can pay a cost for each agent which allows the firm to observe all the contracts the agent signs. We assume that agents are heterogeneous with respect to the probability distribution of the productivity shock: some agents draw the productivity shock from a low mean distribution, while others draw from a distribution with higher mean. If the cost is paid, a firm offers the optimal contract under exclusivity (as in Golosov, Kocherlakota, and Tsyvinski (2003) and Albanesi and Sleet

(1971) and Golosov, Tsyvinski, and Werning (2006)), where this relation holds only for the highest skill type, while all of the remaining types face a distortion on the intratemporal margin that discourages consumption and hours provided.

[6] Or similarly, offering a labor contract at an implicit wage lower than marginal productivity.

(2006)). If the cost is not paid, firms offer the contract described in this paper, which implements the self-insurance allocation. With this extension, the model endogenously partitions the population into groups with access to different insurance contracts. Agents with lower average productivity have access to non-exclusive contracts while agents with higher productivity have access to exclusive contracts. We use US survey data to test whether agents' consumptions and hours allocations, when grouped by education attainment, satisfy the optimality conditions under exclusive or non-exclusive contracts. We find that the consumption of college graduates evolves according to the inverse Euler equation, while for individuals with less than college, the consumption satisfies the standard Euler equation. Looking at the static consumption-leisure distortion calculated in the data, we investigate how it evolves as agents age. The model prescribes a constant distortion over age if workers have access to non-exclusive contracts while an increasing distortion in the other case. We find that also in this dimension, we cannot reject the hypothesis that high school graduates behave according to the linear contracts whereas the other group is closer to the constrained efficient contract.

Related Literature

This paper is related the literature on optimal social insurance contracts and its implementation through taxation, commonly referred to as *new dynamic public finance*. [7] In general, the environment studied in these papers assumes that insurance is provided by a unique provider -the government- who perfectly controls both consumption and labor decision of the agents. With respect to this literature, this paper has two distinct implications. Our main result suggests that the constrained efficient allocation cannot be implemented in decentralized environments unless every aspect of the contracting is observable, thus making necessary the provision of insurance via taxes or a centralized institution that makes information public. However, our results also highlight that the presence of hidden and self-enforcing activities (for both consumption and labor) might undo any incentives the government provides through taxes.

[7] For a review, refer to Kocherlakota (2006) and Albanesi (2008).

Our work is also related to a literature on optimal contract in the presence of hidden trades.[8] In particular, Cole and Kocherlakota (2001) show that, in an private information endowment economy, equilibrium is equivalent to self-insurance when agents can secretly save in a storage technology. In an environment similar to ours, Golosov and Tsyvinski (2007) characterize equilibrium when agents can engage in hidden trades of Arrow-Debreu securities. They show that a standard Euler equation holds and that the decentralized equilibrium is not efficient, since firms do not internalize the effects of the contracts offered on the market rate of return. This paper can be seen as a generalization of the previous two papers, in the sense in that those the recontracting possibilities are assumed exogenously (a market with linear prices or a storage technology) while in this paper the recontracting market is a result of an equilibrium game between insurance providers.

This paper also relates to Bisin and Guaitoli (2004), who analyze a static moral hazard environment under non-exclusive contracting. Their main result shows that latent contracts are used to sustain the equilibrium. However, the nature of the moral hazard environment, differently from our environment, enables latent contracts to prevent any profitable entry by additional insurance providers, thus delivering a positive profit equilibrium to the incumbents.

The quantitative analysis in this paper is related to Townsend (1995) and Ligon (1998). These papers investigate whether the consumption patterns in villages in Thailand and India, respectively, are consistent with the predictions of a constrained efficient allocation or the full information model. Ligon (1998) estimates the inverse Euler equation and the Euler equation for three villages in India. He finds that in two villages the consumption behavior is consistent with the Euler equation while in one village it is consistent with the constrained efficient allocation. Townsend (1995) investigates the consumption in Thai villages and finds that for some the constrained efficient allocation describes accurately the fluctuations while for others the full information model is a good benchmark. The study also emphasizes how villages differ in *information* flows between households (including assets and transactions)

[8] For example Cole and Kocherlakota (2001), Golosov and Tsyvinski (2007) and Abraham and Pavoni (2005).

and how this could be responsible for the different insurance regimes observed.

The paper is organized as follows. In Section 2, we describe the environment and show that any equilibrium can be implemented by a menu game. Section 3 characterizes the equilibrium of our benchmark environment and shows that it is equivalent to self-insurance. We also show that latent contracts are necessary to implement the equilibrium allocation. Section 4 extends the model, allowing firms to observe contracts, and analysis its implications using US survey data. Section 5 is the conclusion.

2 Environment

Consider an economy populated by a continuum of measure one of ex ante identical agents and I firms (insurance providers), with $I \geq 2$. The economy lasts for a finite number T of periods. Agents' period utility is defined over consumption c and labor l and is given by $u(c) - v(l)$. Agents discount future utility at rate $0 < \beta < 1$. Assume $u : \mathbb{R}_+ \to \mathbb{R}$ is twice continuously differentiable, increasing and strictly concave function, $\lim_{c\to 0} u'(c) = \infty$ and $\lim_{c\to\infty} u'(c) = 0$; and $v : \mathbb{R}_+ \to \mathbb{R}$ is twice continuously differentiable, increasing and strictly convex function and $\lim_{l\to\bar{L}} v'(l) = \infty$, where $\bar{L}$ is the maximum feasible number of hours in a period. At every time $t = 1, 2, ..., T$, each agent draws a privately observed productivity shock $\theta_t \in \Theta$, where Θ is a finite set and its smallest element is strictly positive. [9] We assume the law of large numbers holds. The shock is distributed according to probability distribution $\pi(\cdot)$ and is independent and identically distributed over time and across agents. Let $\theta^t = (\theta_1, ..., \theta_t)$ denote the history of uncertainty of an agent up to time t. Given a sequence of consumption and leisure $\{c, l\} = \{c_t, l_t\}_{t=1}^T$, the expected discounted utility of an agent is given by

$$U(\{c, l\}) = \mathbb{E}_0 \sum_{t=1}^{T} \beta^{t-1}[u(c_t) - v(l_t)]. \tag{1}$$

[9] Assume $\min_{\theta\in\Theta} \theta > 0$.

For a given realization of the labor productivity shock θ, an agent can produce y units of effective output according to $y = \theta l$, where l denotes his labor input. We assume the labor input is private information of the agent while output y is publicly observable to each firm for which the agent is producing output y.

Each firm $i \in \{1, ..., I\}$ offers labor and credit contracts to agents to insure against productivity shocks. A contract prescribes, at every time t, output requirement y_t^i and consumption transfer $y_t^i + b_t^i$. The period profit of firm i is given by $V^i(b^i) = -b^i$. Firms can transfer resources over time at constant rate q.[10]

An important feature of our environment is that agents can sign contracts simultaneously with more than one firm, and the terms of the contract between an agent and a firm i are not observed by other firms.[11] We do not impose any restriction on the contracts offered by each firm. For example, a firm can offer a contract for the entire time horizon $t = 1, ..., T$; for a particular set of dates; only credit contracts ($y_t = 0, \ \forall t$); only labor contracts, or both. We also do not impose any specific contingency on the contracts; in particular, we do not restrict to linear contracts.

At time 0, before any uncertainty is realized, agents sign a contract with each firm i. To take into account the voluntary participation of agents, every firm is required to offer at time 0 a null contract that determines no output requirement and no consumption transfers in every period. The contracts offered by a firm at time 0 are contingent on the future communication between that firm and the agent. We assume that contracts must be honored and neither firms nor agents can renege on them.[12]

[10] This fixed interest can be interpreted as the firm having access to external credit markets.

[11] We assume that each agent is atomless and no interaction between agents is allowed.

[12] We interpret contracts as self-enforcing in the following way. Both agents and firms have access to an enforcement mechanism ("court") upon the payment of a cost, whenever one of the parties reneges on a contract. If this cost is paid, the terms of the contract between the two parties in consideration become public, and this court can enforce a punishment to the party that reneged on the contract. If either firms or agents falsely report a breach on the contracts, they can also be punished by court. We assume this punishment can be made large enough so that in equilibrium neither firms nor agents will renege the contracts signed.

2.1 Communication and Menu Games

Communication

Firms and agents communicate according to communication mechanism,[13] which consists of message spaces $\mathcal{R}^i$ for time 0 and message spaces $\mathcal{M}^i_t$ for each $t \in \{1, ..., T\}$, for each firm $i \in \{1, ..., I\}$. Denote the set of all possible messages that can be exchanged by an agent and firm i up to time t by $\mathcal{M}^{i,t} = \mathcal{M}^i_1 \times ... \times \mathcal{M}^i_t$. Each firm chooses allocation functions $g^i_t : \mathcal{M}^{i,t} \to \mathbb{R}^2$, which specify transfers of consumption and output at time t, and $\phi^i : \mathcal{R}^i \to G^i_1(\mathcal{M}^{i,1}) \times ... \times G^i_T(\mathcal{M}^{i,T})$, where $G^i_t(\mathcal{M}^{i,t})$ is the set of all measurable mappings from message space $\mathcal{M}^{i,t}$ to the allocation space $\mathbb{R}^2$. Let $(b(m^{i,t}), y(m^{i,t})) = g^i_t(m^{i,t})$ denote the allocation received by an agent who sends messages $m^{i,t} = (m^i_1, ..., m^i_t)$ to firm i. The function ϕ^i determines the contracts an agent will face in all subsequent periods. Denote by $G^i(\mathcal{M}^i) = G^i_1(\mathcal{M}^{i,1}) \times ... \times G^i_T(\mathcal{M}^{i,T})$ and $\mathcal{M}^i = \mathcal{M}^i_1 \times ... \times \mathcal{M}^i_T$. Let $\Phi^i(\mathcal{R}^i, \mathcal{M}^i)$ be the set of all measurable mappings from message space $\mathcal{R}^i$ to the set G^i and note that $\phi^i \in \Phi^i(\mathcal{R}^i, \mathcal{M}^i)$. Let $\mathcal{M} = \times^I_{i=1}\mathcal{M}^i$ and $\mathcal{R} = \times^I_{i=1}\mathcal{R}^i$. Denote the game associated with the communication mechanism $(\mathcal{M}, \mathcal{R})$ by $\Gamma_{\mathcal{M},\mathcal{R}}$.

At time 0, before any uncertainty is realized, each firm i simultaneously offers a collection of allocation functions ϕ^i, and agents communicate with firms sending a message r^i. This message determines, through ϕ^i, the functions g^i_t at every period t. The timing of the game $\Gamma_{\mathcal{M},\mathcal{R}}$ is the following:

- At time 0:
    1. Each firm i simultaneously offers contract $\phi^i : \mathcal{R}^i \to G^i(\mathcal{M}^i)$;
    2. Agents send a report $r^i \in \mathcal{R}^i$ to each firm i.
- At time t:
    1. Agent learns his private type θ_t;

[13] No communication between firms is allowed.

2. Firm offers allocation rule $g_t^i : \mathcal{M}^{i,t} \to \mathbb{R}^2$ according to $\phi^i(r^i)$;

3. Agent sends a message $m_t^i \in \mathcal{M}_t^i$ to each firm i;

4. Payoffs are realized.

Given messages $(\mathcal{M}, \mathcal{R})$, we consider a static Nash equilibrium played by firms at time 0 when choosing the contracts that are offered in future periods. Given these contracts, agents optimize choosing the report at time 0 and messages in every period $t = 1, ..., T$.

Definition 1 (**Equilibrium of Communication Game**). *A pure strategy equilibrium of* $\Gamma_{\mathcal{M},\mathcal{R}}$ *is* (r^*, m^*, ϕ^*, g^*) *such that:*[14]

1. ***Agent's message*** m_t^**:* $G_t^1 \times ... \times G_t^I \times \Theta^t \to \mathcal{M}_t$ *solves for each* $t \in \{1, ..., T\}$*:*

$$U_t\left(m^{t-1}, \theta_t | g^*\right) = \max_{m_t \in \mathcal{M}_t} u\left(\sum_{i=1}^{I}\left(b^i(m^{i,t}) + y(m^{i,t})\right)\right) - v\left(\frac{\sum_{i=1}^{I} y(m^{i,t})}{\theta_t}\right) + \\ + \beta \sum_{\theta_{t+1}} \pi(\theta_{t+1}) U_{t+1}\left(m^t, \theta_{t+1} | g^*\right),$$

subject to $\sum_{i=1}^{I}\left(b^i(m^{i,t}) + y(m^{i,t})\right) \geq 0, \quad \sum_{i=1}^{I} y(m^{i,t}) \geq 0, \quad \forall t$
where $(b(m^{i,t}), y(m^{i,t})) = g_t^{*,i}(m^{i,t})$.

2. ***Agent's reporting strategy at time 0***, $r^* : G^1 \times ... \times G^I \to \mathcal{R}$ *solves:*

$$\max_{r \in \mathcal{R}} \sum_{\theta_1} \pi(\theta_1) U_1\left(m^0, \theta_1 | g\right)$$

where $g^i = \phi^{i,*}(r_i)$.

3. *For each* $i \in \{1, ..., I\}$*, taking as given the choices of the other firms and the agents' choices,* ***firm's*** i ***allocation function*** $\phi^{i,*}$ *solves:*

$$V^i(\phi^{i,*}, \phi^{-i,*}) \equiv \min \sum_{t=0}^{T} \sum_{\theta^t} \pi(\theta^t) q^t b_t^{i,*}(\theta^t),$$

[14] We do not allow random strategies.

$$b_t^*(\theta^t) = b(m^{t,*}(\theta^t)),\ g^i = \phi^i\left(r^{i,*}\right)\ \textit{and}\ g^{-i,*} = \phi^{-i,*}\left(r^{-i,*}\right).$$

Denote the equilibrium *allocation* of a general communication game by (b^*, y^*).

Menu Games

If contracts are exclusive (or equivalently observable), the environment is equivalent to a standard dynamic Mirrleesian environment as in Golosov, Kocherlakota, and Tsyvinski (2003). In this case, the revelation principle guarantees that without loss of generality, firms can restrict to direct mechanisms that are incentive compatible. However, under non-exclusive contracting, the preference ordering of the agents is influenced not only by their exogenous private information, but also by the set of contracts offered. In particular, the choice of an agent in the contracts offered by firm i depends on the contracts offered by other firms. This implies that restricting to a direct mechanism may not allow a firm to have a rich enough communication with the agent in order to obtain information on the other contracts.

In order to characterize the contracts offered by each firm, we extend the *delegation principle* proved by Peters (2001) and Martimort and Stole (2002) to our environment. This principle states that, without loss of generality, the equilibrium outcomes of any communication game can be implemented as an equilibrium of a menu game. The key idea is that any communication in the original communication mechanism can be replaced by firms offering menus of payoff-relevant alternatives and delegating to the agents the choice within this menu. To incorporate a richer communication between firms and agents, firms might offer menus with elements that are not chosen in equilibrium (latent contracts). As highlighted by Arnott and Stiglitz (1991), offering latent contracts might be necessary to sustain particular equilibria by deterring entry of additional insurance providers and by preventing deviation of the incumbent insurance providers.[15]

A communication mechanism induces allocation functions and, hence, distribution over allocations. This means that to prove the equivalence between the equilibrium allocation of

[15]Our environment differs from the previous literature along two dimensions. First, the environment is dynamic in the sense that the exogenous uncertainty is realized in every period. Second, agents choose a communication-contingent contract from each firm i *before* any uncertainty is realized. This is important since at time 0, agents are identical thus might be possible to extract more information about the contracts being offered by other firms.

a given communication mechanism and the equilibrium of a menu game, it is essential that the menus offered are rich enough to capture the strategies used to implement equilibrium in a communication mechanism. In our environment, a menu is a sequence of sets, where each set is a subset of the allocation space $\mathbb{R} \times \mathbb{R}_+$. For a message space $(\mathcal{M}, \mathcal{R})$, define, for each firm i, the set $C_t^i(m^{i,t-1}, \mathcal{M}_t^i | G_t^i)$ as the menu that can be implemented through a message space $\mathcal{M}_t^i$ at time t given a history of messages $m^{i,t-1}$ and a set of allocation functions G_t^i. Formally, a menu at time t is the following set:

$$C_t^i(m^{i,t-1}, \mathcal{M}_t^i | G_t^i) \equiv \{C_t^i \subseteq \mathbb{R} \times \mathbb{R}_+ | \ \exists \ g_t^i \in G_t^i \subseteq G_t^i(\mathcal{M}^{i,t}) : C_t^i = \mathbf{Im}(g_t^i | m^{i,t-1})\} \ \ \forall t, \ \forall i \quad (2)$$

where

$$\mathbf{Im}(g_t^i | m^{i,t-1}) = \left\{x \in \mathbb{R} \times \mathbb{R}_+ | \ \exists \ m_t^i \in \mathcal{M}_t^i \ : x = g_t^i \left(m^{i,t-1}, m_t^i\right)\right\} \quad \forall t, \ \forall i. \quad (3)$$

Each set defined in (2) contains all subsets of $\mathbb{R}^2$ with cardinality at most $\mathcal{M}_t^i$.

For any subset $G_t^i \subseteq G_t^i(\mathcal{M}^{i,t})$, let $G^i = G_1^i \times ... \times G_t^i$ and define a sequence of menus offered by firms at time 0 as:

$$C\left(G^i\right) = \left\{C_t^i \subseteq C_t^i(m^{i,t-1}, \mathcal{M}_t^i | G_t^i), \ t = 1, ..., T, \ \forall m^{i,t-1} \in \mathcal{M}^{i,t-1}, m_t^i \in \mathcal{M}_t^i\right\}. \quad (4)$$

At time 0, each agent chooses a sequence of menus in the collection offered by firm i. Define $\mathcal{C}^i$ as the collection of menus that are consistent with a communication system $(\mathcal{M}, \mathcal{R})$.

$$\mathcal{C}^i(\mathcal{R}^i, \mathcal{M}^i) \equiv \{C^i \subseteq C^i\left(G^i\right) | \exists \ \phi^i \in \Phi^i(\mathcal{R}^i, \mathcal{M}^i) : G^i = \mathbf{Im}(\phi^i)\}. \quad (5)$$

This set contains all the collections of sets C^i with cardinality less than or equal to the cardinality of $\mathcal{R}^i$. Without explicitly writing the dependence on the message spaces, let $\mathcal{C}^i = \mathcal{C}^i(\mathcal{R}^i, \mathcal{M}^i)$ and let C^i be a generic element of $\mathcal{C}^i$. Let $C = \prod_i C^i$ and $\mathcal{C} = \prod_i \mathcal{C}^i$ be the collection of all menus. Let $\Gamma_{C,\mathcal{C}}$ be the game associated with menus $(C, \mathcal{C})$.

Definition 2 (**Equilibrium of Menu Games**). *A pure strategy equilibrium of a menu game*

is a collection of menus $\hat{\mathcal{C}}$ *and agents' choices* $\hat{C} \in \hat{\mathcal{C}}$ *and* $(\hat{b}_t^i, \hat{y}_t^i) \in C_t^i(\hat{b}^{i,t-1}, \hat{y}^{i,t-1}|\hat{C}^i) \quad \forall t \in \{1, ..., T\}, \quad \forall i \in \{1, ..., I\}$.[16]

1. *Agents' choice at time* t, $(\hat{b}_t, \hat{y}_t) : C_t(\hat{b}^{t-1}, \hat{y}^{t-1}|\hat{C}) \times \Theta^t \to C_t(\hat{b}^{t-1}, \hat{y}^{t-1}|\hat{C})$ *solves:*

$$U_t\left(b^{t-1}, y^{t-1}, \theta_t|\hat{C}\right) = \max_{(b_t, y_t) \in C_t\left(\hat{b}^{t-1}, \hat{y}^{t-1}|\hat{C}\right)} u\left(\sum_{i=1}^{I}(b_t^i + y_t^i)\right) - v\left(\frac{\sum_{i=1}^{I} y_t^i}{\theta_t}\right) + \\ +\beta \sum_{\theta_{t+1}} \pi(\theta_{t+1}) U_{t+1}\left(b^t, y^t, \theta_{t+1}|\hat{C}\right),$$

   subject to $\sum_{i=1}^{I}(b_t^i + y_t^i) \geq 0, \quad \sum_{i=1}^{I} y_t^i \geq 0 \quad \forall t.$

2. *Agents' choice at time* 0, $\hat{C} : \hat{\mathcal{C}} \to \hat{\mathcal{C}}$ *solves:*

$$\max_{C \in \hat{\mathcal{C}}} \sum_{\theta_1} \pi(\theta_1) U_1\left(b^0, y^0, \theta_1|C\right).$$

3. *For each* $i \in \{1, ..., I\}$, $\mathcal{C}^i$ *solves, taking as given* $\hat{\mathcal{C}}_{-i}$ *chosen by firms* $-i$ *and the agents' choice* $\hat{C}_{-i}$, $\{\hat{b}^t(\theta^t), \hat{y}^t(\theta^t)\}_{t=1}^T$*:*

$$V^i(\hat{\mathcal{C}}^i, \hat{\mathcal{C}}^{-i}) \equiv \min \sum_{t=0}^{T} \sum_{\theta^t} \pi(\theta^t) q^t \hat{b}_t^i(\theta^t)$$

   $\hat{b}_t^i(\theta^t) \in \hat{C}_t^i(\hat{b}^{i,t-1}, \hat{y}^{i,t-1}|\hat{C}^i)$, $\hat{C}_t^i(\hat{b}^{i,t-1}, \hat{y}^{i,t-1}|\hat{C}^i) \in \hat{\mathcal{C}}^i$
   $\hat{b}_t^{-i}(\theta^t) \in \hat{C}_t^{-i}(\hat{b}^{-i,t-1}, \hat{y}^{-i,t-1}|\hat{C}^{-i})$ *and* $\hat{C}_t^{-i}(\hat{b}^{-i,t-1}, \hat{y}^{-i,t-1}|\hat{C}^{-i}) \in \hat{\mathcal{C}}^{-i}$.

Denote the equilibrium allocation of a menu game by $(\hat{b}, \hat{y})$.

Note that a menu might contain more alternatives than the cardinality of the type space, implying that some alternatives are not chosen in equilibrium. Similarly, at time 0 a firm might offer more than one set of contracts, also implying that some contracts are offered and not chosen by agents in equilibrium. We denote a contract as **latent** if it is offered in equilibrium by a firm but is not chosen in equilibrium by any agent. As we show in

[16] We do not allow for random menus.

this paper, latent contracts have an important role in sustaining equilibrium allocations by preventing other firms from deviating to other contracts.

The following proposition shows that an equilibrium in a general communication system can be implemented as an equilibrium of a menu game. In this menu game, the collection of menus offered by each firm must be compatible with the general communication mechanism as defined above.

Proposition 1 (Delegation Principle)**.** *Let* (b^*, y^*) *be an equilibrium allocation of a general communication game* $\Gamma_{\mathcal{M},\mathcal{R}}$*. Then there exists* $(\hat{b}, \hat{y})$ *that is an equilibrium allocation of a menu game* $\Gamma_{C,\mathcal{C}}$ *and* $(b^*, y^*) = (\hat{b}, \hat{y})$*.*

Proof. In Appendix A. □

Proposition 1 states that for given message spaces $(\mathcal{M}, \mathcal{R})$, there exists a menu game that implements the same equilibrium allocation. It is important to note that message spaces restrict the menus that can be offered in a menu game. From the previous result, if firms are allowed to use unrestricted message spaces, the same equilibrium can be implemented if firms can offer unrestricted menus as stated in Corollary 1 in Martimort and Stole (2002). From now on, we focus on unrestricted menu games.

The presence of two rounds of communication (at time 0 and at every time t) allows to further simplify the unrestricted menu game. We show that any time t menu that contains latent points (allocations not chosen in equilibrium) can alternatively be replaced by a time t menu with the same number of elements as the type space and latent menus at time 0. This implies that, without loss of generality, we can restrict firms to offering time t menus that have the same cardinality of the type space, which we call minimal menus.

Definition 3 (**Minimal Menus**)**.** *A menu* $C^i \in \mathcal{C}^i$ *is minimal if for all* $C^{-i} \in \mathcal{C}^{-i}$ *and* $(b_t^i, y_t^i) \in C_t^i$*, for all* $C_t^i \in C^i$*, there exists* $\theta_t \in \Theta$*, such that* $(b_t^i, y_t^i) = (b^{*,i}(\theta^t), y^{*,i}(\theta^t))$*.*

Intuitively, a menu is minimal if all of its elements are chosen by some agent in equilibrium.

Proposition 2. *Let* $\mathcal{C} = \{\mathcal{C}^i, \mathcal{C}^{-i}\}$ *be an equilibrium of a menu game. There exists a payoff equivalent equilibrium* $\tilde{\mathcal{C}}$*, such that every* $\tilde{C}^i \in \tilde{\mathcal{C}}^i$ *is a minimal menu for all* i*.*

Proof. In Appendix A. □

3 Equilibrium Characterization

An important message of the previous section is that direct mechanisms might not be sufficient when characterizing the optimal contract. This means that firms might offer latent (off-equilibrium) contracts. In this section, the use of latent contracts plays an important role, in particular to show that an equilibrium exists. We show that equilibrium would fail to exist if firms were restricted to offer direct mechanisms.[17]

3.1 Characterization under Exclusive Contracts

Before characterizing the optimality conditions in our environment, we review two robust equilibrium conditions in an environment in which there is competition between insurance providers and *contracts are exclusive*. The seminal paper of Prescott and Townsend (1984) shows that in a general class of private information economy, the first welfare theorem holds. The decentralized economy is equivalent to a planning problem that maximizes the ex ante lifetime utility of the agents subject to feasibility and incentive compatibility constraints (in every period for every realization agents weakly prefer the allocation designed for them).

In an environment similar to ours, and in the presence of exclusive contracting, the equilibrium allocation has the following features:[18]

1. The marginal rate of substitution between consumption and leisure is equated to the

[17] Throughout the paper, an incumbent refers to a firm that offers a menu that contains transfers and/or output recommendations other than the null contract and some agent chooses some of these contracts in equilibrium. An entrant refers to an insurance provider that, at all times, every agent chooses the null contract from the menus offered by this firm. We assume the number of firms I is large enough so that an entrant always exists.

[18] For a review of the results of constrained efficient allocation in dynamic Mirrleesian environments, refer to Golosov, Tsyvinski, and Werning (2006).

marginal productivity only for the highest type, originally shown by Mirrlees (1971):

$$u'(c(\bar{\theta})) = \frac{1}{\bar{\theta}} v' \left(\frac{y(\bar{\theta})}{\bar{\theta}} \right), \tag{6}$$

$$u'(c(\theta)) > \frac{1}{\theta} v' \left(\frac{y(\theta)}{\theta} \right), \quad \forall\ \theta \neq \bar{\theta}, \theta \in \Theta, \tag{7}$$

where $\bar{\theta} \equiv \max_{\theta \in \Theta} \theta$. The intuition for this result is the following: in order to separate types, it is optimal to discourage less productive agents to work. This implies that all but the most productive agents work and consume less than they would in a competitive environment.

2. If preferences are separable in consumption and leisure, the marginal rate of substitution of consumption between any two periods differs from the intertemporal rate of transformation for all types (the standard Euler equation does not hold):

$$\frac{1}{u'(c(\theta^t))} = \frac{1}{\beta R} E \left[\frac{1}{u'(c(\theta^{t+1}))} | \theta^t \right], \quad \forall\ t, \theta^t. \tag{8}$$

This equation, derived originally by Rogerson (1985) and generalized in Golosov, Kocherlakota, and Tsyvinski (2003), implies that for all periods $u'(c(\theta^t)) < \beta R E \left[u'(c(\theta^{t+1})) | \theta^t \right]$. This means that it is optimal to make any type of agent saving constrained in order to encourage the truthful revelation of productivity in future periods.

3.2 Optimality Conditions under Non-exclusivity

We now derive the equilibrium conditions in the presence of non-exclusive contracting. This friction implies that the above equilibrium conditions cannot be implemented.

Under exclusivity, the optimal contract provides incentives to more skilled workers by discouraging less skilled agents to work (with respect to the full information allocation). The next lemma shows that this distortion cannot be implemented when contracts are not exclusive, since agents can work an extra amount to other firms.

Lemma 1. *In any equilibrium for every $\theta^t \in \Theta^t$, for all t the following holds:*

$$u'(b(\theta^t) + y(\theta^t)) \leq v'\left(\frac{y(\theta^t)}{\theta_t}\right)\frac{1}{\theta_t}, \tag{9}$$

where $b(\theta^t) = \sum_i b^i(\theta^t)$ and $y(\theta^t) = \sum_i y^i(\theta^t)$ and where $(b^i(\theta^t), y^i(\theta^t))$ are the contracts chosen by an agent with history θ^t from firm i at time t.

Proof. Suppose that for some history θ^t equation (9) does not hold:

$$u'(b(\theta^t) + y(\theta^t)) > v'\left(\frac{y(\theta^t)}{\theta_t}\right)\frac{1}{\theta_t}. \tag{10}$$

In this case, the agent would like to consume and work more than the equilibrium contract. An entrant can make strictly positive profits offering a supplemental contract with more consumption and output. Consider an entrant that offers the contract at time t, $C_t^E = \{(-\varepsilon, \delta^*(\varepsilon)), (0,0)\}$ where δ^* and ε are constructed as follows. Let $\delta^*(\varepsilon|\theta_t)$ be the solution of the following problem:

$$U(\varepsilon|\theta_t) \equiv \max_{\delta \geq 0} u(b(\theta^t) + y(\theta^t) + \delta - \varepsilon) - v\left(\frac{y(\theta^t) + \delta}{\theta_t}\right). \tag{11}$$

The first order condition for this problem is:

$$u'(b(\theta^t) + y(\theta^t) + \delta^*(\varepsilon|\theta_t) - \varepsilon) \leq v'\left(\frac{y(\theta^t) + \delta^*(\varepsilon|\theta_t)}{\theta_t}\right)\frac{1}{\theta_t}. \tag{12}$$

If $\varepsilon = 0$, the solution for the above problem is $\delta^*(0|\theta_t) > 0$ given that (53) holds. From the Theorem of the Maximum, the solution $\delta^*(\varepsilon)$ is continuous on ε. Fix $\epsilon_1 > 0$ such that $|\delta^*(0) - 0| > \epsilon_1$. There exists $\epsilon_2 > 0$ such that if $|\varepsilon - 0| < \epsilon_2$ then $|\delta^*(\varepsilon) - \delta^*(0)| < \epsilon_1$. Let ε be such that $0 < \varepsilon < \epsilon_2$.

An entrant offering this contract makes strictly positive profits, proportional to ε, and the agent is strictly better off given that his utility is higher in some history with positive probability. This contract is always profitable for the entrant even if other type $\tilde{\theta}_t$ accepts the deviating contract. The only way to deter this deviation is to have some latent contract

that makes no agent willing to choose it. However, if such a contract existed, it would have been chosen in the original equilibrium, contradicting the fact that it is a latent contract. □

When contracts are exclusive, the provision of incentives imply that agents are savings constrained. The following lemma shows that this fails under non-exclusivity.

Lemma 2. *In any equilibrium for every $\theta^t \in \Theta^t$, for all t, the following holds:*

$$u'(c_t(\theta^t)) = \frac{\beta}{q} \sum_{\theta_{t+1}} u'(c_{t+1}(\theta^{t+1}))\pi(\theta_{t+1}), \tag{13}$$

where $c_t(\theta^t) = \sum_{i=1}^{I} (b_t^i(\theta^t) + y_t^i(\theta^t))$.

Proof. In appendix B. □

The intuition for the result is the following. If the equilibrium allocation does not satisfy the Euler equation, an entrant firm can offer a savings (borrowing) contract at time t with an implicit interest rate lower (higher) than the marginal rate of transformation. As long as this contract is accepted, the entrant makes strictly positive profits and such contract can be constructed in a way that provides higher utility to the agent.

In the next proposition, we show that in equilibrium the marginal rate of substitution (MRS) between consumption and leisure is equated to the marginal productivity for every history and also that the lifetime transfer received under any history is equal to zero, so that there is no cross-subsidization between types.

Proposition 3. *In any equilibrium the following two conditions hold:*

1. *Zero net present value of transfers:*

$$\sum_{t=1}^{T} \left(\frac{1}{q}\right)^{1-t} b_t(\theta^t) = 0 \quad \forall\ \theta^T \in \Theta^T. \tag{14}$$

2. *MRS equal to marginal productivity:*

$$u'(b(\theta^t) + y(\theta^t)) = v'\left(\frac{y(\theta^t)}{\theta_t}\right)\frac{1}{\theta_t} \quad \forall\ \theta^t, t. \tag{15}$$

Proof. In appendix B. □

So far we characterized three necessary properties of the equilibrium allocation: (13), (14), and (15). In subsection 3.3, we show that there is a unique allocation that satisfies these conditions, which we denote by $\{\hat{b}, \hat{y}\} = \{(b(\theta^t), y(\theta^t))_{t=1}^T \quad \theta^t \in \Theta^t\}$. The next proposition shows that an equilibrium exists by determining *strategies* of the firms (menus) that sustain this allocation as an equilibrium. A crucial element of the proof is that the equilibrium strategies must contain latent menus. These menus are similar to the ones derived in the characterization of equilibrium to show that any contract other than self-insurance is unprofitable.

Proposition 4. *Allocation $\{\hat{b}, \hat{y}\}$ is the unique equilibrium allocation of a menu game.*

Proof. We construct strategies of the firms and the agents that sustain allocation $\{\hat{b}, \hat{y}\}$ as an equilibrium. Let firm $i \in \{1, 2\}$ offer the following menus:

$$\hat{C}_1^i = \left\{ (b_1^i, y_1^i) : b_1^i \in \mathbb{R},\ y_1^i \in \mathbb{R}_+ |\ u'\left(b_1^i + y_1^i\right) = \frac{1}{\theta} v'\left(\frac{y_1^i}{\theta}\right)\ \forall \theta \in \Theta \right\},$$
$$\hat{C}_T^i(b^{i,T-1}, y^{i,T-1}) = \left\{ (b_T^i, y_T^i) : b_T^i = 0,\ y_T^i \in \mathbb{R}_+ |\ u'\left(-\frac{1}{q} b_{T-1}^i + y_T^i\right) = \frac{1}{\theta} v'\left(\frac{y_T^i}{\theta}\right)\ \forall \theta \in \Theta \right\},$$

and for periods $t = 2, \ldots, T-1$:

$$\hat{C}_t^i(b^{i,t-1}, y^{i,t-1}) = \left\{ (b_t^i, y_t^i) : b_t^i \in \mathbb{R},\ y_t^i \in \mathbb{R}_+ |\ u'\left(-\frac{1}{q} b_{t-1}^i + b_t^i + y_t^i\right) = \frac{1}{\theta} v'\left(\frac{y_t^i}{\theta}\right)\ \forall \theta \in \Theta \right\}.$$

These firms also offer the following latent menus:

Dynamic Contract: for all $t = 1, \ldots, T$

$$C_t^{i,D}(b^{i,t-1}, y^{i,t-1}) = \left\{ (b_t^i, y_t^i) : b_t^i \in \mathbb{R},\ y_t^i = 0 | b_t^i = -\frac{1}{q} b_{t-1}^i + x,\ x \in \mathbb{R} \right\}, \qquad b_T^i = b_0^i = 0$$

Static Contract:for all $t = 1, \ldots, T$

$$C_t^{i,S} = \{(0, \delta) : \delta \in \mathbb{R}_+\}$$

Remaining firms $i \in \{3, \ldots, I\}$ offer the null contract. Given these menus, the agents choose at time zero menu $\hat{C}^i$ from one of the two firms. We derive the agents' choices by backward induction. At time T, an agent with history (θ^{T-1}, θ_T) and past choices $(\tilde{b}(\theta^{T-1}), \tilde{y}(\theta^{T-1}))$ chooses from menu $C_T^i(\tilde{b}^i(\theta^{T-1}), \tilde{y}^i(\theta^{T-1}))$ the allocation $(\tilde{b}^i(\theta^T), \tilde{y}^i(\theta^T))$ such that $u'\left(-\frac{1}{q}\tilde{b}^i(\theta^{T-1}) + \tilde{y}^i(\theta^T)\right) = \frac{1}{\theta_T} v'\left(\frac{\tilde{y}^i(\theta^T)}{\theta_T}\right)$. For time $t \in \{1, ..., T-1\}$, an agent with history θ^t and past choices $(\tilde{b}^i(\theta^{t-1}), \tilde{y}^i(\theta^{t-1}))$ chooses from menu $C_t^i(\tilde{b}^i(\theta^{t-1}), \tilde{y}^i(\theta^{t-1}))$ allocation $(\tilde{b}^i(\theta^t), \tilde{y}^i(\theta^t))$ such that

$$u'\left(-\frac{1}{q}\tilde{b}^i(\theta^{t-1}) + \tilde{b}^i(\theta^t) + \tilde{y}^i(\theta^t)\right) = \frac{\beta}{q}\mathbb{E}_t\left[u'\left(-\frac{1}{q}\tilde{b}^i(\theta^t) + \tilde{b}^i(\theta^{t+1}) + \tilde{y}^i(\theta^{t+1})\right)\right], \quad (16)$$

$$u'\left(-\frac{1}{q}\tilde{b}^i(\theta^{t-1}) + \tilde{b}^i(\theta^t) + \tilde{y}^i(\theta^t)\right) = \frac{1}{\theta_t} v'\left(\frac{\tilde{y}^i(\theta^t))}{\theta_t}\right). \quad (17)$$

Given agents' choices, firm i's profit is $\sum_{t=1}^{T}\sum_{\theta^t} q^t \tilde{b}^i(\theta^t) = 0$.

We next show that such strategies constitute an equilibrium by showing that there are no profitable deviations by firms. In particular, the latent contracts $C^{i,S}$ and $C^{i,D}$ are sufficient to deter any potential deviations.[19]

As a first step, we show that is not profitable for any firm to offer a contract that specifies

[19]Note that only offering menus $\hat{C}^i$ is not an equilibrium since either an incumbent or an entrant will deviate, offering profitable welfare increasing menu, in the shape of a contingent contract. As an example, consider the following profitable deviation (motivated by Abraham and Pavoni (2005)). Let $\{\tilde{b}(b^{-1}), \tilde{y}(b^{-1})\}$ be the solution to the following problem:

$$\tilde{U}(b^{-1}) = \max_{b,y} \sum_{\theta} \pi(\theta)\left[u(b(\theta) + y(\theta)) - \frac{1}{\theta} v\left(\frac{y(\theta)}{\theta}\right)\right], \quad (18)$$

$$s.t. \quad u(b(\theta) + y(\theta)) - \frac{1}{\theta} v\left(\frac{y(\theta)}{\theta}\right) \geq u(b(\hat{\theta}) + y(\hat{\theta})) - \frac{1}{\theta} v\left(\frac{y(\hat{\theta})}{\theta}\right),$$

$$\sum_{\theta} \pi(\theta) b(\theta) = b^{-1}.$$

Note that $\tilde{U}(b^{-1})$ is strictly larger than the utility of autarky with b^{-1} additional (possibly negative) resources. The firm can deviate from the set of menus C^i defined above by substituting the $T-1$ menu with the original time T menu, and by replacing the time T menu with

$$\tilde{C}^T(b^{i,T-1}, y^{i,T-1}) = \left\{\{\tilde{b}(b^i_{T-1} - \varepsilon), \tilde{y}(b^i_{T-1} - \varepsilon)\} \middle| \text{ solves } (18) \text{and } \varepsilon > 0\right\},$$

For a sufficiently small ε, the agent prefers this contract to the original, and in addition, this deviation provides additional $\frac{\varepsilon}{q^{1-T}}$ profits.

only intertemporal transfers (without any output requirements). Suppose firm $j \neq 1, 2$ offers a menu C^j containing sequences of transfers $\{b_t\}_{t=1}^T$. For each feasible sequence in this menu,[20] define the net present values of a sequence by: $NPV(\{b_t\}_{t=1}^T) = \sum_{t=1}^T \frac{1}{q^{1-t}} b_t$. The menu C^j is chosen by agents and profitable only if it contains at least one feasible sequence with $NPV > 0$ and one with $NPV < 0$.[21] Denote by $\{\tilde{b}_t\}_{t=1}^T$ the feasible transfer with highest NPV. In the presence of menu C^D, all agents choose this sequence, implying that the entrant makes negative profits. Suppose not: there is an agent with history θ^T that chooses a sequence $\{b_t\}_{t=1}^T \neq \{\tilde{b}_t\}_{t=1}^T$. This agent is better-off by choosing the sequence $\{\tilde{b}_t\}_{t=1}^T$ and the following strategy in the menu C^D: $\delta_t = b_t - \tilde{b}_t$. This strategy enables him to replicate his original allocation and have extra resources, since the net present value of $\{\delta_t\}_{t=1}^T$ is negative:

$$\delta_0 = \sum_{t=1}^T \frac{1}{q^{1-t}} b_t - \sum_{t=1}^T \frac{1}{q^{1-t}} \tilde{b}_t < 0.$$

These additional resources can be used to increase consumption in any period, making the agent better-off.

The next step is to rule out contracts that offer jointly consumption transfers and output requirements. In appendix C we show that any contract that implies redistribution from unproductive to more productive agents reduces agents' welfare with respect to the equilibrium allocation. Thus, in the last period, a firm can either provide a contract with no redistribution or decrease welfare of the agents.[22]

For the dynamic case we focus on a two period example with two values of productivity shock, $\theta_H > \theta_L$. If the firm provides negative redistribution at time 1, given appendix C, the contract will not be chosen at time zero. The remaining alternative is to provide, at time 1, some redistribution from the productive to the unproductive agent. To do this, a firm must

[20] A sequence $\{b_t\}_{t=1}^T$ is feasible if $b_t \in C_t^j(b^{t-1}) \forall b_t, t$.

[21] If all sequences have NPV=0, the menu is not chosen, since the equilibrium allocation $\{\hat{b}, \hat{y}\}$ is the allocation that maximizes agents' welfare with no redistribution. Similarly, if all transfers are negative, the menu is also not chosen, while if all transfer have $NPV > 0$ the firm makes a loss.

[22] In a static environment this completes the proof since it rules out the existence of a contract that is, at the same time, profitable and preferred by the agents.

offer transfers with higher net present value together with higher output requirement. If not, the productive agent deviates, using both $C^{i,D}$ and $C^{i,S}$, replicating his original allocation and receiving transfers with higher NPV. Suppose now that at time 1, the θ_H agent receives transfers equal to $b_1 - \Delta$ while θ_L agent receives $b_1 + \Delta$ (with $\Delta > 0$). The best case for both agents is to receive transfers at time 2 that does not depend on the realization of the type in that period. Thus we can write transfers for the high type as $b_{2,H}$ and for the low type $b_{2,L}$. These transfers are such that $b_1 - \Delta + qb_{2,H} > b_1 + \Delta + qb_{2,L}$. This implies that a lower rate of return is charged to low productivity agents relative to high productivity agents. Since the low agent has lower consumption, this interest rate differential is welfare decreasing. Hence the benefits to the high agent are offset by the utility loss of the low agent. And, from an ex-ante perspective, the agent is better-off choosing the original equilibrium.

Finally for $\{\hat{b}, \hat{y}\}$ to be sustained as an equilibrium allocation, at least two firms must offer the equilibrium and the latent contracts. If not, the unique firm active in equilibrium will re-optimize, and offer a contract that implies some redistribution (as the example in footnote 19) since no latent contract is preventing such deviation. □

Summarizing, the allocation $\{\hat{b}, \hat{y}\}$ can be sustained in equilibrium by at least two incumbents simultaneously offering the menu $\hat{C}^i$ and the latent contracts $C^{i,S}$, and $C^{i,D}$. This is necessary to prevent deviations by any firm to a more profitable and ex ante welfare improving contract that features redistribution. This result highlights the importance of allowing firms to offer latent contracts. If offering such contracts were not allowed, as in direct mechanisms, equilibrium would fail to exist in this environment.

3.3 Equivalence to Self-Insurance

In the previous propositions we showed that the equilibrium allocation satisfies a standard Euler equation, the marginal rate of substitution between consumption and leisure is equated to marginal productivity in every period, and the net present value of transfers received under any history is equal to zero (there is no redistribution). These equilibrium conditions are

the same optimality conditions in a decentralized economy in which agents can borrow and save at rate $R = 1/q$.

Let $\{c^*, y^*\} = \{c^*(\theta^t), y^*(\theta^t)\}_{t=1}^T$ be the solution to the following problem:

$$\max_{c,y\geq 0} \sum_{t=1}^{T} \sum_{\theta^t} \beta^{t-1} \pi\left(\theta^t\right) \left[u\left(c(\theta^t)\right) - v\left(\frac{y(\theta^t)}{\theta_t}\right)\right] \tag{19}$$

$$s.t. \qquad \sum_{t=1}^{T} \frac{c(\theta^t) - y(\theta^t)}{R^{1-t}} = 0, \quad \forall \theta^T,$$

where R is taken as given.

Proposition 5. *Let $\{\hat{b}, \hat{y}\} = \{\hat{b}(\theta^t), \hat{y}(\theta^t)\}_{t=1}^T$ be the equilibrium allocation of a menu game. Let the agents' consumption be $\hat{c}(\theta^t) = \hat{b}(\theta^t) + \hat{y}(\theta^t)$ for all θ^t and for all t. If $R = 1/q$, $c^*(\theta^t) = \hat{c}(\theta^t)$ and $y^*(\theta^t) = \hat{y}(\theta^t)$ for all θ^t and for all t.*

Proof. The first order conditions of (19) are:

$$u'(c(\theta^t)) = \beta R \sum_{\theta_{t+1}} u'(c(\theta^{t+1}))\pi(\theta_{t+1}), \tag{20}$$

$$u'(c(\theta^t)) = \frac{1}{\theta_t} v'\left(\frac{y(\theta^t)}{\theta^t}\right), \tag{21}$$

$$\sum_{t=1}^{T} \frac{c(\theta^t) - y(\theta^t)}{R^{1-t}} = 0, \quad \forall \theta^T. \tag{22}$$

A solution to (19) exists. Also, the maximization problem (19) has a strictly concave objective function and the constraint set is convex; hence, the first order conditions are necessary and sufficient for the optimum and the optimum is unique. □

The previous proposition summarizes how non-exclusivity and non-observability of contracts limit the ability to provide insurance and also the contracts that are offered in equilibrium. Our environment with firms interacting strategically and being allowed to offer any type of contracts, in equilibrium, is equivalent to an environment with competitive firms offering linear contracts with no redistribution. A immediate implication of the proposition

is that the equilibrium is unique in terms of allocation.

Corollary 1. *There is a unique equilibrium allocation of a menu game.*

4 Endogenous Insurance and Quantitative Analysis

In this section, we derive a simple testable model that endogenously generates heterogeneous insurance regimes. To do this, we relax the assumption on observability of the contracts. As in a costly state verification model, we give firms the option of paying a fixed cost, $\gamma \geq 0$, to monitor all the transactions an agent engages in.[23] We assume that agents are heterogeneous with respect to the probability distribution of the productivity shock. There are two groups of agents: the first group draws the productivity shock from a low mean distribution, while the second draws from a distribution with higher mean. We show that in this modified environment, different groups of agents will have access to different insurance possibilities. Using US survey data, we show that this extension can rationalize the coexistence of multiple insurance regimes observed in the data.

4.1 Monitoring Costs

At time 0 (and only at time 0), before offering a set of contracts to an agent of type $j \in \{1, 2\}$, each firm chooses between the following two options: pay a cost γ to observe all the contracts an agent engages in, and choose which contract to offer under full observability; or not pay the cost and offer the most profitable contract under non-exclusivity. Agents are heterogeneous with respect to the probability distribution of the productivity shock. A fraction of agents ("low mean agents") draws, at every time t, a shock $\theta_t \in \Theta$, distributed according to $\pi(\cdot)$ while the a fraction of agents ("high mean" agents) draws the productivity shock $\lambda\theta_t$, where $\theta_t \in \Theta$, distributed according to $\pi(\cdot)$ and $\lambda > 1$. Let $\bar{\theta}$ and $\lambda\bar{\theta}$ be the average productivity of, respectively, low and high mean agents. Whether an agent is a low or high mean is publicly

[23] Note that costly state verification models as in Townsend (1979) allow, upon paying the cost, the realization of uncertainty to be observable. Here we keep the realization of uncertainty private but allow the contracts an agent sign to be observable.

known by all the firms. For each group of agents, a firm decides whether to pay or not the monitoring cost and which contracts to offer in each case.

If a firm monitors an agent, the environment is equivalent to the one described in Prescott and Townsend (1984). We refer to optimal contract in this case as the "exclusive contract". If the monitoring cost is not paid, the environment is the one studied in previous sections of this paper. From Proposition 5, this environment is equivalent in terms of allocation to a self-insurance economy, in which agents can borrow and save at fixed rate R and are paid wages equal to marginal productivity. We refer to the optimal contract in this case as the "non-exclusive" contract.

To determine which contract each group of agent will have access to, for a given value of monitoring cost γ, firms compare the lifetime utility delivered under exclusive and non-exclusive contracts. This means that a firm finds profitable to pay the cost and offer the exclusive contract if agent's utility is higher in this case. If firms do not find it profitable to pay the cost, an agent will receive the lifetime level of utility associated with non-exclusive contracts. We show that, under a particular assumption on the utility function, there exists a level of the monitoring cost such that low mean agents have access to the non-exclusive contract, whereas high mean agents have access to the exclusive contracts. In appendix D, we show that if $\gamma = 0$, the exclusive contract is always preferred over the non-exclusive, since is cheaper to provide a given level of lifetime utility under exclusive contracts. For analytical convenience, we assume the following utility specification.

Assumption 1. $u(c) = \log c$; $v(l) = -a\log(1-l)$.

The zero profits level of lifetime utility ($\bar{w}^{NE}(\cdot)$) is defined as:

$$\begin{aligned} \bar{w}^{NE}\left(x\bar{\theta}\right) &= \max_{c,y} \sum_{t=1}^{T} \sum_{x\theta^t \in x\Theta} \beta^{t-1}\pi(\theta^t)\left[u\left(c(\theta^t)\right) - v\left(\frac{y(\theta^t)}{x\theta_t}\right)\right] \\ &\sum_{t} \left[\frac{c(\theta^t) - y(\theta^t)}{R^{1-t}}\right] = 0, \quad \forall \theta^T, \end{aligned} \tag{23}$$

where the argument $x = 1, \lambda$ refers to the agents' productivity distribution.

Similarly for the exclusive contracts, define $w^E(\cdot|\gamma)$ as follows. Note that for the exclusive contracts, the lifetime utility level that delivers zero profits also depends on the monitoring cost.

$$w^E\left(x\bar{\theta}|\gamma\right) = \max_{c,y} \sum_{t=1}^{T} \sum_{x\theta^t \in x\Theta} \beta^{t-1}\pi(\theta^t)\left[u\left(c(\theta^t)\right) - v\left(\frac{y(\theta^t)}{x\theta_t}\right)\right] \tag{24}$$

$$\sum_{\theta^t,t} \beta^{t-1}\pi(\theta^t)\left[u\left(c(\theta^t)\right) - v\left(\frac{y(\theta^t)}{x\theta_t}\right)\right] \geq \sum_{\theta^t,t} \beta^{t-1}\pi(\theta^t)\left[u\left(c(\tilde{\theta}^t)\right) - v\left(\frac{y(\tilde{\theta}^t)}{x\theta_t}\right)\right] \quad \forall\tilde{\theta}^t$$

$$\sum_{t}\left[\frac{c(\theta^t) - y(\theta^t)}{q^{t-1}}\right] = \gamma, \quad \forall\theta^T. \tag{25}$$

The following proposition states that there exists a value for the monitoring cost so that different agents have access to different insurance contracts.

Proposition 6. *There exists $\gamma^* > 0$ such that:*

$$\begin{aligned} \bar{w}^{NE}\left(\bar{\theta}\right) &= w^E\left(\bar{\theta}|\gamma^*\right) \\ \bar{w}^{NE}\left(\lambda\bar{\theta}\right) &< w^E\left(\lambda\bar{\theta}|\gamma^*\right) \end{aligned}$$

Proof. In appendix D. □

The steps to show the result are the following. We first show that, under non-exclusive contract, indirect lifetime utility of high mean agents is proportional to the lifetime utility of low mean agents (by a factor proportional to λ). The assumption on the utility function is crucial to show this result. Second, we show that, under exclusive contracts, the lifetime utility is scaled by a factor larger than λ. This implies that, for a given λ, there is a value of the monitoring cost so that the firms can promise a higher lifetime utility under the exclusive contract than under the non-exclusive. The same result can also be proved if $u(c,l) = \frac{\left(c^\alpha l^{1-\alpha}\right)^{1-\sigma}}{1-\sigma}$. The general CRRA case, with $u(c,l) = \frac{c^{1-\sigma}}{1-\sigma} + a\frac{l^{1-\sigma_l}}{1-\sigma_l}$, is verified numerically.[24]

[24] Another way to endogenously divide the population in two different insurance regimes is to assume agents are heterogenous with respect to the monitoring cost γ. In this case, we show that there exists a

4.2 Quantitative Implications

So far we showed that our extended model implies that different groups of agents have access to different insurance contracts. This implies that, along some dimensions, the allocations is characterized by a different set of equilibrium conditions. To test the implications of the model, we use US household survey data and divide the population by education attainment: those with less than a college degree and those who completed college or more. We estimate for each of the groups two implications of the model: an intertemporal optimality condition on consumption and an intratemporal condition on consumption and leisure. We consider the education level a proxy for a worker's average productivity and according to our model, agents with higher average productivity (college graduates) satisfy the optimality conditions of the exclusive contracts, while agents with lower average productivity (high school graduates) satisfy the optimality conditions of non-exclusive contracts.

Data

We use the Krueger and Perri Consumer Expenditure Survey (CEX) dataset for the period 1980 to 2003 and divide the population by the education level of the reference person. To abstract from college and retirement decisions, we restrict our sample to households with the reference person age is between 25 and 55.[25] We only consider reference person who worked more than 520 hours and less than 5096 hours per year and with positive labor income. We exclude households with wage less than half of the minimum wage in any given year and households who responded to all four interviews and with no missing consumption data. Table 8 (appendix E) describes the number of households in each stage of the sample selection. All the nominal data are deflated using the consumer price index calculated by the Bureau of Labor Statistics with base 1982-84=100.[26] In Table 9 (appendix E) we present

cutoff value γ^* such that if agents have cost γ, with $0 < \gamma \leq \gamma^*$, they have access to the exclusive contract and receive lifetime utility $w(\gamma)$. While agents with cost γ, with $\gamma > \gamma^*$, have access to the non-exclusive contract, receiving lifetime utility $\bar{w}^{NE}$.

[25] By stopping at age 55 we also minimize the disconnection between consumption expenditure and actual consumption (due to the progressive larger use of leisure in both preparation and shopping time) highlighted in Aguiar and Hurst (2005).

[26] For a more detailed description of the data and sample selection, refer to Ales and Maziero (2008).

some descriptive statistics of the sample considered. All the earnings variables and hours refer to the reference person, while the expenditure variables are total household expenditure per adult equivalent.[27] The consumption measure used includes the sum of expenditures on nondurable consumption goods, services, and small durable goods, plus the imputed services from housing and vehicles, as calculate by Krueger and Perri (2006).

Intertemporal Optimality Conditions

The first implication we test is an intertemporal optimality condition regarding the evolution of consumption. We showed that if agents have access to exclusive contracts, the consumption allocation satisfies the following inverse Euler equation:

$$\frac{1}{u'\left(c_t\left(\theta^t\right)\right)}\frac{\beta}{q} = E_t\left[\frac{1}{u'\left(c_{t+1}\left(\theta^{t+1}\right)\right)}\right]. \tag{26}$$

On the other hand, if agents have access to non-exclusive contracts, the allocation must satisfy the following standard Euler equation:

$$u'\left(c_t\left(\theta^t\right)\right) = \frac{\beta}{q}E_t\left[u'\left(c_{t+1}\left(\theta^{t+1}\right)\right)\right]. \tag{27}$$

Assuming $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, these equations imply, respectively:

$$c_t(\theta^t)^{\sigma}\frac{\beta}{q} = E_t\left[c_{t+1}(\theta^{t+1})^{\sigma}\right], \tag{28}$$

$$c_t(\theta^t)^{-\sigma} = \frac{\beta}{q}E_t\left[c_{t+1}(\theta^{t+1})^{-\sigma}\right]. \tag{29}$$

These two equations can be nested in the following:

$$c_t(\theta^t)^b\left(\frac{\beta}{q}\right)^{\frac{b}{|b|}} = E_t\left[c_{t+1}(\theta^{t+1})^b\right]. \tag{30}$$

If the inverse Euler equation (28) holds, then $b > 0$, whereas if the standard Euler equation

[27] We use the Census definition of adult equivalent.

(29) holds, $b < 0$. Taking expectation of (30) at time t, we get

$$\sum_{\theta^{t+1}} \pi(\theta^{t+1}) \left[c_t(\theta^t)^b \left(\frac{\beta}{q}\right)^{\frac{b}{|b|}} - c_{t+1}(\theta^{t+1})^b \right] = 0, \quad \forall \theta^t. \tag{31}$$

For a given education group, we test whether the intertemporal consumption decision is compatible with exclusive or non-exclusive contracts by estimating the parameter b in (31). If, for an education group, the value of b is negative, the consumption of these agents is consistent with the predictions of (23). If the estimation of b has a positive value, it implies that agents' consumption satisfies the implications of (24). Our theory predicts that for more educated individuals the value of b is positive. The analysis here closely follows Ligon (1998) and Kocherlakota and Pistaferri (2008).[28]

Estimation Procedure and Results

A typical household is on the sample for a total period of four quarters. For the estimation, we construct sample averages as follows. Denote by $c_{i,t}$ the consumption for household i in the quarter that ends with month t, and let N_t be the number of observations available at time t. We de-seasonalize consumption with dummies corresponding to the month the household was interviewed. The sample analog of equation (31) is:

$$g(b) = \frac{1}{T} \sum_{t=3}^{T} \left[\left(\frac{\beta}{q}\right)^{\frac{b}{|b|}} \frac{1}{N_{t-3}} \sum_{i=1}^{N_{t-3}} c_{i,t-3}^b - \frac{1}{N_t} \sum_{i=1}^{N_t} c_{i,t}^b \right]. \tag{32}$$

As shown by Kocherlakota and Pistaferri (2008), this sample analog is still valid in the presence of multiplicative classical measurement error in the consumption data.

The main disadvantage of this sample analog is that, by taking means over the population, it does not take into account individual changes on consumption over time. An alternative

[28] In particular, Ligon (1998) tests whether the standard Euler or inverse Euler condition better describes the consumption behavior for three Indian villages. His results indicate that two out of three village provides evidence for the inverse Euler equation.

valid sample analog is the following:

$$\tilde{g}(b) = \frac{1}{T}\sum_{t=3}^{T}\left[\left(\frac{\beta}{q}\right)^{\frac{b}{|b|}}\frac{1}{N_t}\sum_{i=1}^{N_t}\left(\frac{c_{i,t-3}}{c_{i,t}}\right)^{b}\right] \tag{33}$$

where in this equation N_t is the total number of households with consumption data for time t and $t-3$. The estimation of this equation, in the presence of multiplicative classical measurement error in consumption, implies inconsistent estimation of the parameter β. A standard approach in the literature is to estimate the log-linearized version of this sample analog.[29] Simple algebra shows that the log-linearized versions of equations (26) and (27) result in the same log-linearized equation. This means that this procedure cannot be used to test whether the consumption of a group of household satisfy (26) or (27).

In table 1 we report the estimation of parameter b for the two education groups, assuming $\frac{\beta}{q} = 1$. We estimate the parameter b in (32) using non-linear generalized method of moments. We find that for college graduates $b = 0.855$, which is consistent with exclusive contracts. While for individuals with education less than college the estimation indicates $b = -1.128$, which corresponds to consumption evolving as predicted by non-exclusive contracts. Note that for agents with education less than college we reject that b is positive, while for college graduates we cannot reject a negative value for b.

As a robustness check, we perform the same estimation by dividing the population into four education groups: those with less than a high school education, those who completed high school, those with some college, and those who completed college or more. The results are reported in table 2 and are consistent with the previous one: for individuals with education less than college the estimated value of b is negative, while for college graduates, this value is positive.

Another robustness check performed is to estimate the moment condition for all the households that have answered at least one of the interviews, not only for the households who have answered the four interviews. The results are reported in table 3 and are consistent

[29] See Attanasio and Low (2004) and Ludvigson and Paxson (2001).

Table 1: Estimation results for risk aversion

	Education Group		
Consumption	**Less than College**	**College**	**All Sample**
Baseline de-seasonalized	-1.128	**0.855**	-1.030
	(0.459)	**(0.716)**	(0.728)
Baseline truncated*	-1.128	**0.864**	-1.030
	(0.458)	**(0.726)**	(0.728)

Estimation results for risk aversion from (31). A positive solution denotes the coefficient of risk aversion consistent with the household's decision under the constrained efficient contract, while a negative solution denotes the estimated risk aversion consistent with the group being under a borrowing and saving contract. *We drop households with consumption changes bigger than 5 times in absolute value.

Table 2: Estimation results for risk aversion: multiple education groups

	Education Group			
Consumption	**Less than HS**	**HS**	**Less than College**	**College**
Baseline de-seasonalized	-0.773	-1.346	-0.962	**0.855**
	(0.355)	(0.415)	(0.865)	**(0.716)**
Baseline truncated	-0.774	-1.348	-0.962	**0.864**
	(0.355)	(0.414)	(0.864)	**(0.726)**

Table 3: Estimation results for risk aversion

	Education Group		
Consumption	**Less than College**	**College**	**All Sample**
Baseline de-seasonalized	-0.970	**1.157**	-0.898
	(0.206)	**(0.467)**	(0.341)
Baseline truncated	-1.00	**1.155**	-0.904
	(0.210)	**(0.486)**	(0.360)

Estimation results for risk aversion from (31) with households who answered at least one interview.

Table 4: Estimation results for risk aversion

	Education Group		
Consumption	**Less than College**	**College**	**All Sample**
Baseline instrumented	-0.972	**1.167**	-0.890
	(0.213)	**(0.450)**	(0.365)

Estimation results for risk aversion from (31) using previous period interest rate as instrument.

with the results for the baseline sample. For both groups, the estimation of the coefficient of risk aversion is bigger than in the benchmark case and the standard errors are smaller. In this case, for both education groups, we can reject the value of b being the sign than the estimated.

We also perform our benchmark estimation by using the previous period interest rate as an instrument. The results are displayed in table 4 and are consistent with the benchmark results.

Intratemporal distortions

The second tested implication of the model regards the joint consumption and leisure decision

at a given time. Define, for an individual j, the intratemporal labor distortion as

$$\tau_{cl}^{j}(t) = \frac{1}{\theta_t^j} \frac{v'(\frac{y_t^j}{\theta_t^j})}{u'(c_t^j)} \tag{34}$$

As shown in Ales and Maziero (2008), if contracts are exclusive, an individual faces an *increasing* average value of τ_{cl} over the course of his working life. This result relies on the age dependent provision of incentives. As workers age (and the termination of the optimal contract gets closer) it is optimal to progressively provide more incentives using current promises of consumption and leisure (thus distorting more the static consumption-leisure condition) rather than promises of future consumption and leisure.

On the other hand, as proved in the previous section, if agents have access to non-exclusive contracts, $\tau_{cl}(t)$ is constant over age, since in this case the MRS is always equated to agent's marginal productivity. Hence evaluating how this distortion evolves over the working life provides another testable implication of the model.

Estimation Procedure and Results

Using the following utility function $u(c,l) = \frac{\left(c^{\alpha} l^{1-\alpha}\right)^{1-\sigma}}{1-\sigma}$, equation (34) is:

$$\tau_{cl}^{j}(t) = 1 - \frac{1-\alpha}{\alpha} \frac{1}{\theta_t^j} \frac{c_t^j}{L - l_t^j}, \tag{35}$$

The main advantage of using this utility is that the intratemporal distortion is not affected by the risk aversion parameter and α does not affect the behavior of $\tau_{cl}(t)$ over time.

To estimate the dependence of τ_{cl} on age, we regress its value on age. We run the regression on the standardized values of all variables.[30] We calculate the labor distortion as follows. We use as proxy for a worker's marginal productivity the imputed hourly wage, which is calculated dividing the total labor income by the total number of hours in a year For the measure of consumption, c_t^j we use total consumption expenditure, l_t^j is the yearly hours worked, and L is the feasible amount of yearly working hours, set at 5200. To abstract

[30] Precisely: $\tau_{cl}^{j}(age) = \bar{\tau}_{cl} + \delta * age + \varepsilon_{age}^{j}$.

Table 5: Intra temporal distortion: singles

	Education Group	
δ	**Less than College**	**College**
OLS	0.011 (0.008)	**0.016 (0.009)**
Robust Regression	-0.0018 (0.005)	**0.021(0.005)**
t-statistic	-0.36	3.93

from changes in family composition, we restrict the sample to individuals who are single.[31] The results of this estimation are displayed in table 5. To control for heteroscedasticity and outliers, we estimate δ using a robust regression and for completeness we also report the OLS estimation. The coefficient on age for the entire sample is positive, as highlighted in Ales and Maziero (2008). We find that a zero coefficient (implying independence over age) cannot be rejected for individuals with education less than college, whereas for college graduates the value of the coefficient is positive and significant, indicating that the labor distortion increases with age.

We also estimate (34) for a specification of the utility function that is separable on consumption and leisure. We assume $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ and $v(l) = \frac{l^{1-\sigma_L}}{1-\sigma_L}$ with $\sigma_L = 2$. Table 6 shows the results using for the coefficient of risk aversion the estimation of the Euler equation.[32] The result is the same as in the non-separable case: for less educated individuals, the coefficient on age is not significantly different than zero, while for college graduates this coefficient is positive. As a robustness check, we also calculate the labor distortion including married individuals in the sample. In this case, we consider as measure of consumption the total household consumption. For the labor variables, we assume that leisure for the husband and the wife are perfect substitutes and use total household earnings and hours to compute the distortion. Due to data limitation,[33] we restrict the sample to households with two or

[31] In our baseline sample the single individuals represent 18% of the population.

[32] We also estimate the equation for different values of risk version, within the range estimated in the literature, and the result is qualitatively unchanged.

[33] The CEX records hours and earnings for the reference person and the spouse.

Table 6: Intra temporal distortion: singles

	Education Group	
δ	**Less than College**	**College**
OLS	0.011 (0.008)	**0.016 (0.009)**
Robust Regression	-0.0078 (0.005)	**0.05(0.006)**
t-statistic	-1.44	7.28

Intra temporal distortion by age and education group for singles using estimates for risk aversion derived from the estimation of the Euler equation.

Table 7: Intra temporal distortion: couples

	Education Group	
δ	**Less than College**	**College**
OLS	-0.0007 (0.0003)	**0.0003 (0.0005)**
Robust Regression	-0.0004 (0.0002)	**0.0011 (0.0003)**
t-statistic	-1.76	3.59

Intra temporal distortion by age and education for household containing two adults.

less adults, the reference person and the spouse. In this case, the results are also consistent with our benchmark estimation, as reported in table 7.

5 Conclusion

In this paper, we study a decentralized environment when firms compete for the provision of insurance. We focus on how the presence of non-exclusive trades endogenously limits the contracts offered, and consequently the amount of insurance implemented. We consider an environment in which consumers are privately informed about their skill shocks that evolve over time and can sign non-observable contracts with insurance providers. Our main results

are that competition reduces the amount of insurance provided, the equilibrium is equivalent to a self-insurance economy, and only linear contracts are offered. Also, in equilibrium there is no redistribution.

To derive testable implications of the model, we extend the model and relax the assumption on the observability of contracts: firms can pay a cost to observe all the contracts an agent signs. Assuming agents are heterogeneous with respect to this cost, we find that agents with lower monitoring costs have access to the constrained efficient contract, while agents with higher monitoring costs have access to contracts that implement the self-insurance allocation. This implies that the first group of agents attains a higher level of lifetime utility. Considering education as a proxy for lifetime utility, we test the different intertemporal and intratemporal implications of this model using US data. We find that agents with a high level of education satisfy the optimality conditions of the constrained efficient model while the consumption and hours of agents with less education evolve according to the borrowing-savings economy.

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A Proofs of Section 2

Proof of Proposition 1

Proof. The proof is by construction. Starting from the equilibrium strategies of a general communication game, we construct strategies for a menu game and show that these strategies constitute an equilibrium.

Define as in (2) and (5) respectively the menus and the collection of menus that are compatible with message spaces $(\mathcal{M}, \mathcal{R})$. Define the strategy of firm i in this menu game as:

$$\hat{\mathcal{C}}^i = \{C^i \subseteq C^i\left(G^i\right) | G^i = \mathbf{Im}(\phi^{i,*})\}. \tag{36}$$

The collection of menus $\hat{\mathcal{C}}^i$ contains all the subsets of the allocation space that are consistent with the collection of allocation functions in the original equilibrium. Agents' strategies are defined as follows.

$$\hat{C}^i = \{\hat{C}^i_t \in \hat{\mathcal{C}}^i : \hat{C}^i_t = \mathbf{Im}(g^{i,*}_t | m^{i,t-1,*}) \text{ and } g^{i,*}_t = \phi^{i,*}(r^{i,*})\}$$

$$(\hat{b}^i(\theta^t), \hat{y}^i(\theta^t)) = g^{i,*}_t(m^{i,t,*}(\theta^t)).$$

Note that by construction $\hat{C}^i_t \in \hat{\mathcal{C}}^i$ and $(\hat{b}^i(\theta^t), \hat{y}^i(\theta^t)) \in \hat{C}^i_t$, $\forall \theta^t$, $\forall t$. The menu $\hat{C}^i_t$ is the subset of allocation space, $\mathbb{R}^2$, that corresponds to the allocation function chosen by the agent in the original equilibrium. Also $(\hat{b}^i, \hat{y}^i)$ corresponds to allocation determined by the allocation function given the equilibrium message sent by each type θ^t. If agents and firms follow these strategies, the equilibrium allocation in the menu game is the same as in the original equilibrium.

First let's show that the agents' strategies are an equilibrium. Suppose that at some time t, for some firm i $\exists$ $(b^i_t, y^i_t) \in \hat{C}^i_t$ such that:

$$u\left(\sum_{i=1}^{I}(b^i_t + y^i_t)\right) - v\left(\frac{\sum_{i=1}^{I} y^i_t}{\theta_t}\right) + \beta \sum_{\theta_{t+1}} \pi(\theta_{t+1}) U_{t+1}\left(\hat{b}^{t-1}, b_t, \hat{y}^{t-1}, y_t, \theta_{t+1} | \hat{C}\right) >$$
$$u\left(\sum_{i=1}^{I}(\hat{b}^i_t + \hat{y}^i_t)\right) - v\left(\frac{\sum_{i=1}^{I} \hat{y}^i_t}{\theta_t}\right) + \beta \sum_{\theta_{t+1}} \pi(\theta_{t+1}) U_{t+1}\left(\hat{b}^t, \hat{y}^t, \theta_{t+1} | \hat{C}\right).$$

Since $(b^i_t, y^i_t) \in \hat{C}^i_t$, there exists $m^i_t \in \mathcal{M}^i_t$ such that $(b^i_t, y^i_t) = g^{i,*}_t(m^{i,t})$. Replacing in the agents' payoff:

$$u\left(\sum_{i=1}^{I}\left(b^i(m^{i,t}) + y(m^{i,t})\right)\right) - v\left(\frac{\sum_{i=1}^{I} y(m^{i,t})}{\theta_t}\right) + \beta \sum_{\theta_{t+1}} \pi(\theta_{t+1}) U_{t+1}\left(m^t, \theta_{t+1} | g^*\right) >$$
$$u\left(\sum_{i=1}^{I}\left(b^i(m^{i,t,*}) + y(m^{i,t,*})\right)\right) - v\left(\frac{\sum_{i=1}^{I} y(m^{i,t,*})}{\theta_t}\right) + \beta \sum_{\theta_{t+1}} \pi(\theta_{t+1}) U_{t+1}\left(m^{t,*}, \theta_{t+1} | g^*\right).$$

But this contradicts $m^{i,*}$ being an equilibrium in the original game. Now suppose $\hat{C}^i$ is not an equilibrium for some i. There exists some $C^i \in \hat{\mathcal{C}}^i$ such that:

$$U(C^i, \hat{C}_{-i}) > U(\hat{C}).$$

Since $C^i \in \hat{\mathcal{C}}^i$, $\exists\ r^i \in \mathcal{R}^i$ such that $C^i = \mathbf{Im}(g^i)$ and $g^i = \phi^{i,*}(r^i)$. Replacing in the agents' payoff:

$$U\left(g^i, g^*_{-i},\right) > U\left(g^{i,*}, g^*_{-i}\right).$$

But this contradicts $r^{i,*}$ being an equilibrium in the original game.

Finally, we check that firms' strategies constitute an equilibrium. Suppose $\exists\ \mathcal{C}^i \in \mathcal{C}^i(\mathcal{R}^i, \mathcal{M}^i)$ such that $V^i(\mathcal{C}^i, \hat{\mathcal{C}}^{-i}) > V^i(\hat{\mathcal{C}}^i, \hat{\mathcal{C}}^{-i})$.

Since $\mathcal{C}^i \in \mathcal{C}^i(\mathcal{R}^i, \mathcal{M}^i)$, there exists ϕ^i such that $g^i = \phi^i(r^{i,*})$. Replacing in the firm's payoff in the original game $V^i(\phi^i, \phi^*_{-i}) > V^i(\phi^{i,*}, \phi^*_{-i})$. But this contradicts $\phi^{i,*}$ being an equilibrium in the original game. □

Proof of Proposition 2

Proof. We show the equivalence by construction. For a given firm i, by assumption there exists at least one $C^i \in \mathcal{C}^i$ which is not minimal. As notation let $C^i \times C^{-i} = C^1 \times C^i ... \times C^N$, and let $U(C^i, C^{-i})$ be the lifetime utility of a sequence of menus C as defined in equilibrium. Define the set

$$P\left(C^i\right) = \{C^{-i} \in \mathcal{C}^i \mid U(C^i, C^{-i}) \geq U(\tilde{C}^i, C^{-i}) \quad \forall \tilde{C}^i \in \mathcal{C}^i\}. \tag{37}$$

The set $P\left(C^i\right)$ contains all the menus C^{-i} offered by other firms $-i$ that resulted in C^i being chosen from firm i. Note that if C^i is the unique element of $\mathcal{C}^i$ the set $P\left(C^i\right) = \mathcal{C}^i$.
For each $C^{-i} \in P\left(C^i\right)$, construct the following sequence of menus:

$$\tilde{C}^i_t\left(C^{-i}|C^i_t\right) \equiv \left\{\left(b^{i,*}_t\left(\theta^t, C^i, C^{-i}\right), y^{i,*}_t\left(\theta^t, C^i, C^{-i}\right)\right) \in C^t_i, \quad \forall\ \theta_t \in \Theta\right\}, \quad \forall\ C^{-i}_t \in C^i, \forall t. \tag{38}$$

Each set $\tilde{C}^i_t\left(C^{-i}|C^i_t\right)$ contains the actual equilibrium choices of each type of agent and is a minimal menu. Let $\tilde{C}^i(C^{-i}) \equiv \{\tilde{C}^i_t\left(C^{-i}|C^i_t\right) \forall\ C^{-i}_t \in C^i, \forall t\}$. Finally let $\tilde{C}^i = \{\tilde{C}^i\left(C^{-i}\right)\ \forall\ C^{-i} \in P\left(C^i\right)\}$. We now replace the menu $C^i \in \mathcal{C}^i$ by $\tilde{C}^i$ and show that the equilibrium is the same. Let $\tilde{\mathcal{C}}^i = \{(\mathcal{C}^i \backslash C^i), \tilde{C}^i\}$. We prove the statement in two steps. We first show that each element of $\tilde{C}^i$ is chosen by the agent if and only if C^i was chosen in the original equilibrium. We then show that $\tilde{\mathcal{C}} = \{\tilde{\mathcal{C}}^i, \mathcal{C}^{-i}\}$ is an equilibrium of the menu game by showing that none of the firms $-i$ deviates to any $\tilde{\mathcal{C}}^{-i}$.

To show the first step, given that C^i was chosen in the original equilibrium $U(C^i, C^{-i}) \geq U(\hat{C}^i, C^{-i}) \quad \forall \hat{C}^i \in \mathcal{C}^i$. By construction, we have that $U(\tilde{C}^i(C^{-i}), C^{-i}) \geq U(C^i, C^{-i})$ and $U(\tilde{C}^i(C^{-i}), C^{-i}) \geq U(\hat{C}^i, C^{-i})$, for all $\hat{C}^i \in \tilde{C}^i$, so that $U(\tilde{C}^i(C^{-i}), C^{-i})) \geq U(C'^i, C^{-i})$ for all $C'^i \in \tilde{\mathcal{C}}^i$. To prove the reverse, suppose $\hat{C}^i \in \tilde{C}^i$ is chosen by the agent. By the definition of $\tilde{C}^i$, if $(b^i_t, y^i_t) \in \hat{C}^i_t \in \hat{C}^i$ then $(b^i_t, y^i_t) \in C^i_t \in C^i$ so $U(C^i, C^{-i}) \geq U(\hat{C}^i, C^{-i})$ and $U(C^i, C^{-i}) \geq U(C^{i'}, C^{-i})$ for all $C^{i'} \in \tilde{C}^i$. Given that $\hat{C}^i$ is chosen then $U(\hat{C}^i, C^{-i}) \geq U(C^{i'}, C^{-i})$ for all

$C^{i'} \in \mathcal{C}^i \backslash \tilde{C}^i$. Combining these inequalities, we get that $U(C^i, C^{-i}) \geq U(C^{i'}, C^{-i})$ for all $C^{i'} \in \mathcal{C}^i$, implying that C^i is chosen in the original equilibrium.

Suppose there exists a collection of menus $\tilde{\mathcal{C}}^{-i}$ so that $V^{-i}(\tilde{\mathcal{C}}^{-i}, \tilde{\mathcal{C}}^i) > V^{-i}(\mathcal{C}^{-i}, \tilde{\mathcal{C}}^i)$. Let C^* denote the equilibrium choice of the agent, such that $U\left(C^{i,*}, C^{-i,*}\right) \geq U(\hat{C}^i, \hat{C}^{-i})$, for all $(\hat{C}^i, \hat{C}^{-i}) \in \tilde{\mathcal{C}}^i \times \tilde{\mathcal{C}}^{-i}$. The first case is if $\tilde{\mathcal{C}}^{-i} \cap P\left(C^i\right) = 0$. If $C^{-i,*} \in \tilde{\mathcal{C}}^{-i} \cap \mathcal{C}^{-i}$, we immediately reach a contradiction since C^* was also chosen in the previous equilibrium so that profits must be equal. If $C^{-i,*} \notin \tilde{\mathcal{C}}^{-i} \cap \mathcal{C}^{-i}$, then we reach a contradiction with $\mathcal{C}$ being an equilibrium, since firm $-i$ would have deviated from offering the menu $\mathcal{C}^{-i} \backslash P\left(C^i\right) \cup C^{-i,*}$ and would make strictly greater profits.

The second case is if $\tilde{\mathcal{C}}^{-i} \cap P\left(C^i\right) \neq 0$. In this case, if $C^{-i,*} \in P\left(C^i\right)$ we immediately reach a contradiction since the agent chooses the same menu in both equilibrium so profits are the same. If $C^{-i,*} \notin P\left(C^i\right)$ then we contradict $\mathcal{C}$ being an equilibrium, since firm $-i$ would deviate from offering $\mathcal{C}^{-i} \backslash P\left(C^i\right) \cup C^{-i,*}$.

Repeating this procedure for every non-minimal menu C^i in the original $\mathcal{C}^i$, we construct $\tilde{\mathcal{C}}$ where every menu is minimal. □

B Proofs of Section 3

B.1 Proof of Lemma 2

Proof. Suppose that for some history $\hat{\theta}^t$ equation (13) does not hold.
Case 1:

$$u'(c_t(\hat{\theta}^t)) < \frac{\beta}{q} \sum_{\theta_{t+1}} u'(c_{t+1}(\hat{\theta}^t, \theta_{t+1}))\pi(\theta_{t+1}). \tag{39}$$

In this case, the agent is savings constrained. An entrant can make strictly positive profits offering a savings contract at a rate lower than $1/q$, contradicting the original allocation being an equilibrium. The first step is to construct the contract to be offered by a firm. Let $\delta^*(\varepsilon)$ be the solution of the following problem:

$$U(\varepsilon) \equiv \max_{\delta \geq 0} u(c_t(\hat{\theta}^t) - \delta) + \beta E_t u\left(c_{t+1}(\hat{\theta}^t, \theta_{t+1}) + \delta\left(\frac{1}{q} - \varepsilon\right)\right). \tag{40}$$

The first order condition for this problem is:

$$u'(c_t(\hat{\theta}^t) - \delta) \geq \beta(\frac{1}{q} - \varepsilon) E_t u'\left(c_{t+1}(\hat{\theta}^t, \theta_{t+1}) + \delta\left(\frac{1}{q} - \varepsilon\right)\right). \tag{41}$$

If $\varepsilon = 0$, the solution for the above problem is $\delta^*(0) > 0$ given that (39) holds. From the Theorem of the Maximum, the solution $\delta^*(\varepsilon)$ is continuous on ε. Fix $\epsilon_1 > 0$ such that $|\delta^*(0) - 0| > \epsilon_1$. $\exists\ \epsilon_2 > 0$ such that if $|\varepsilon - 0| < \epsilon_2$ then $|\delta^*(\varepsilon) - \delta^*(0)| < \epsilon_1$. Let ε be such that $0 < \varepsilon < \epsilon_2$. Consider an entrant that offers the contract $C_t = \{(\delta^*(\varepsilon), 0), (0, 0)\}$ and $C_{t+1} = \{(-\delta^*(\varepsilon)(\frac{1}{q} - \varepsilon), 0), (0, 0)\})$ and the contract $(0, 0)$ for all other periods. This firm

is making strictly positive profits, proportional to $\delta^*(\varepsilon)\varepsilon$, and the agent is strictly better off keeping the original equilibrium together with this contract since increases the his utility in a history with positive probability and keeps the same utility in all other histories.

Hence, under the original equilibrium, a firm can offer a contract that makes strictly positive profits. This contradicts the allocation being an equilibrium.

The other case can be proved using a similar argument. □

B.2 Proof of Lemma 3

Proof. For a history θ^{t-1}, define the net present value of transfers received from time t onwards by:

$$A_t(\theta^{t-1}, \theta_{t-1}^T) = \sum_{n=t}^{T} \left(\frac{1}{q}\right)^{t-n} b_n(\theta^{t-1}, \theta_{t-1}^n), \tag{42}$$

where $\theta_{t-1}^n = (\theta_t, \theta_{t+1}, \ldots, \theta_n)$ is the sequence of shocks following history θ^{t-1} from time t to n and $b_n(\theta^{t-1}, \theta_{t-1}^n)$ is the equilibrium transfer chosen at time n by an agent with history θ^n. We show, using a backward induction argument, that for all t, $A_s(\theta^{s-1}, \theta_{s-1}^T)$ is independent of θ_{s-1}^T for all $s \geq t$. This implies that $A_1(\theta^T)$ is the same for all $\theta^T \in \Theta^T$. If $A_1(\theta^T) > 0$, firms make strictly negative profits in equilibrium and would be better off offering a null contract. If $A_1(\theta^T) < 0$, an entrant can offer the same sequence of transfers giving an additional transfer $\varepsilon > 0$ in the terminal period. Since the sequence of transfers is not contingent and is profitable for all types, there is no latent contract that makes it unprofitable.

1. **Equations (14) and (15) hold for $t = T$.**

We first show that at time T, transfers are independent of realization of time T shock and then show that for time T equation (15) holds.

Equation (14) holds at $t = T$:

Suppose that (14) does not hold and let $b(\theta^T) = \min_{b \in C(b^{T-1}, y^{T-1})} b$ and $b(\theta^{T-1}, \hat{\theta}_T)$ the second smallest b. Denote by $\hat{\theta}^T = (\theta^{T-1}, \hat{\theta}_T)$. The contradiction argument relies on the incumbent firm deviating to an allocation that delivers higher profits. First note that it must be true that $y(\hat{\theta}^T) + b(\hat{\theta}^T) > y(\theta^T) + b(\theta^T)$. If not, given that $b(\hat{\theta}^T) > b(\theta^T)$ then $y(\hat{\theta}^T) < y(\theta^T)$, an entrant firm can offer the following contract $\tilde{C}_T = \{(-\varepsilon, y(\theta^T) - y(\hat{\theta}^T)); (0, 0)\}$, for some ε small enough. An agent with type θ^T is better off by choosing allocation $(b(\hat{\theta}^T), y(\hat{\theta}^T))$ in menu C_T together with $(-\varepsilon, y(\theta^T) - y(\hat{\theta}^T))$ in menu $\tilde{C}_T$. With these choices, his utility is:

$$u\left(b(\hat{\theta}^T) - \varepsilon + y(\theta^T)\right) - v\left(\frac{y(\theta^T)}{\theta_T}\right) > u\left(b(\theta^T) + y(\theta^T)\right) - v\left(\frac{y(\theta^T)}{\theta_T}\right)$$

where the inequality holds as long as $b(\hat{\theta}^T) - \varepsilon > b(\theta^T)$. No latent contracts can prevent this deviation, since it is profitable for the entrant as long as some agent accepts it.[34]

The equilibrium allocation, being optimal for the agent, must satisfy the following:

$$u(b(\hat{\theta}^T) + y(\hat{\theta}^T)) - v\left(\frac{y(\hat{\theta}^T)}{\hat{\theta}_T}\right) \geq u(b(\theta^T) + y(\theta^T)) - v\left(\frac{y(\theta^T)}{\hat{\theta}_T}\right), \tag{43}$$

$$u(b(\theta^T) + y(\theta^T)) - v\left(\frac{y(\theta^T)}{\theta_T}\right) \geq u(b(\hat{\theta}^T) + y(\hat{\theta}^T)) - v\left(\frac{y(\hat{\theta}^T)}{\theta_T}\right). \tag{44}$$

Case 1 If (43) holds with equality, an agent of type $\hat{\theta}_T$ is indifferent between his equilibrium choice and the choice of agent θ_T. However, the insurance providers receive strictly higher profits from the allocation θ_T, since by assumption $b(\hat{\theta}^T) > b(\theta^T)$. This incumbent can deviate to an alternative menu that differs from the original by offering at time T only the allocation chosen by agent θ_T. No latent contract can induce lower profits to deter this deviation, since now the deviating incumbent offers a subset of the allocations that were available in the original equilibrium. The argument also holds if the equilibrium allocation is divided between multiple insurance providers.

Case 2 Suppose that (43) holds with strict inequality. Following the argument in the previous case, for any type $\bar{\theta}_T$ such that $b\left(\bar{\theta}_T\right) > b_T(\theta^T)$, it must be true that:

$$u(b(\bar{\theta}^T) + y(\bar{\theta}^T)) - v\left(\frac{y(\bar{\theta}^T)}{\bar{\theta}_T}\right) > u(b(\theta^T) + y(\theta^T)) - v\left(\frac{y(\theta^T)}{\bar{\theta}_T}\right). \tag{45}$$

Otherwise, the incumbent firm will offer only the contract containing $b_T(\theta^T)$.

Consider the following deviation by an incumbent firm $\tilde{b}(\hat{\theta}^T) = b(\hat{\theta}^T) - \varepsilon$ and $\tilde{b}(\theta^T) = b(\theta^T) + \varepsilon - \delta$ for $\varepsilon, \delta > 0$ and $\varepsilon > \delta$ (to be defined explicitly below) and keeping unchanged all the other allocations.[35] This deviation reduces the spread of transfers and increases incumbent's profit by a factor proportional to δ.

To show that such deviation is profitable, thus reaching a contradiction, we show that there is no latent contract $\alpha \equiv (\alpha_b, \alpha_y)$ that can induce a reduction in the profits of this firm. Suppose such contract exists. One possibility is to induce θ_T agents, when faced with the deviating allocation $\tilde{b}$, to choose $\tilde{b}(\hat{\theta}^T)$. This would imply a reduction of profits, since $\tilde{b}(\hat{\theta}^T) > \tilde{b}(\theta^T)$. Such latent contract has to satisfy:

$$u(\tilde{b}(\hat{\theta}^T) + y(\hat{\theta}^T) + \alpha_b + \alpha_y) - v\left(\frac{y(\hat{\theta}^T) + \alpha_y}{\theta_T}\right) > u(\tilde{b}(\theta^T) + y(\theta^T)) - v\left(\frac{y(\theta^T)}{\theta_T}\right). \tag{46}$$

[34] Note that this case arises in the solution of the constrained efficient allocation: high skilled agents work more and make positive transfers to less skilled agents. The deviation $\tilde{C}_T$ makes this allocation unprofitable in our environment, since it induces skilled agents to choose the allocation designed for low skilled agents and working an additional amount with entrant.

[35] If there are multiple θ with values equal to $b(\hat{\theta}^T)$ or $b(\theta^T)$, the same deviation applies to all such transfers.

Since α is not chosen in the original equilibrium, it must also be true that

$$u(b(\theta^T)+y(\theta^T))-v\left(\frac{y(\theta^T)}{\theta_T}\right)\geq u(b(\hat{\theta}^T)+y(\hat{\theta}^T)+\alpha_b+\alpha_y)-v\left(\frac{y(\hat{\theta}^T)+\alpha_y}{\theta_T}\right). \tag{47}$$

However, $u(\tilde{b}(\theta^T)+y(\theta^T))>u(b(\theta^T)+y(\theta^T))$ and $u(b(\hat{\theta}^T)+y(\hat{\theta}^T)+\alpha_b+\alpha_y)>u(\tilde{b}(\hat{\theta}^T)+y(\hat{\theta}^T)+\alpha_t+\alpha_y)$, which combined with (47) implies

$$u(\tilde{b}(\theta^T)+y(\theta^T))-v\left(\frac{y(\theta^T)}{\theta_T}\right)>u(\tilde{b}(\hat{\theta}^T)+y(\hat{\theta}^T)+\alpha_b+\alpha_y)-v\left(\frac{y(\hat{\theta}^T)+\alpha_y}{\theta_T}\right), \tag{48}$$

contradicting (46). As before, consider any other type $\bar{\theta}_T\neq\theta_T$ with $b\left(\bar{\theta}_T\right)>b\left(\theta_T\right)$:

$$u(\tilde{b}(\theta^T)+y(\theta^T)+\alpha_b+\alpha_y)-v\left(\frac{y(\theta^T)+\alpha_y}{\bar{\theta}_T}\right)>u(b(\bar{\theta}^T)+y(\bar{\theta}^T))-v\left(\frac{y(\bar{\theta}^T)}{\bar{\theta}_T}\right). \tag{49}$$

Since a latent contract is not chosen in the original equilibrium, it must also be true that

$$u(b(\bar{\theta}^T)+y(\bar{\theta}^T))-v\left(\frac{y(\bar{\theta}^T)}{\bar{\theta}_T}\right)\geq u(b(\theta^T)+y(\theta^T)+\alpha_t+\alpha_y)-v\left(\frac{y(\theta^T)+\alpha_y}{\bar{\theta}_T}\right). \tag{50}$$

The previous equation must hold with equality, otherwise in the original equilibrium the deviating firm would not offer contract $b\left(\bar{\theta}_T\right)$. Let

$$\begin{aligned}\Delta(\bar{\theta}) \;=\; \min_{\alpha\in C_T^{-i}}\Bigg\{&u(b(\bar{\theta}^T)+y(\bar{\theta}^T))-v\left(\frac{y(\bar{\theta}^T)}{\bar{\theta}_T}\right)+\\&-u(b(\theta^T)+y(\theta^T)+\alpha_b+\alpha_y)+v\left(\frac{y(\theta^T)+\alpha_y}{\bar{\theta}_T}\right)\Bigg\}.\end{aligned} \tag{51}$$

This gives the minimum utility gain agent $\bar{\theta}^T$ receives from choosing allocation $(b(\bar{\theta}^T),y(\bar{\theta}^T))$ instead of $(b(\theta^T),y(\theta^T))$ combined with any other latent contract α. Since (50) holds with strict inequality, $\Delta\left(\bar{\theta}\right)$ is strictly positive for each $\bar{\theta}$. Let $\bar{\alpha}\equiv\arg\min\Delta(\bar{\theta})$. There exists $\varepsilon\left(\bar{\theta}\right)>0$ such that

$$\begin{aligned}&u(b(\bar{\theta}^T)+y(\bar{\theta}^T))-v\left(\frac{y(\bar{\theta}^T)}{\bar{\theta}^T}\right)\geq\\&u(b(\theta^T)+y(\theta^T)+\bar{\alpha}_t+\bar{\alpha}_y+\varepsilon\left(\bar{\theta}\right))-v\left(\frac{y(\theta^T)+\bar{\alpha}_y}{\bar{\theta}^T}\right)>\\&u(b(\theta^T)+y(\theta^T)+\bar{\alpha}_b+\bar{\alpha}_y+\varepsilon\left(\bar{\theta}\right)-\delta)-v\left(\frac{y(\theta^T)+\bar{\alpha}_y}{\bar{\theta}^T}\right).\end{aligned} \tag{52}$$

Let $\varepsilon = \min_{\bar{\theta} \neq \theta} \varepsilon\left(\bar{\theta}\right)$. Under this choice of ε, the above equation contradicts (50). Equation (52) also implies that for all $\bar{\theta} \neq \theta$, choice following the deviation is the same as in the original equilibrium.

The last step in the proof requires checking that the time $T-1$ incentive constraints hold. This is necessary in order to leave the decision of the agents unchanged at time $T-1$. Note that for a given $\varepsilon > 0$, there exists $\delta^* > 0$ that makes the utility, calculated in time $T-1$, of the modified contract the same as in the original contract. To see this, note that if $\delta = \varepsilon$ the change in utility of the agent is negative following the proposed deviation, while if $\delta = 0$ the utility change is positive, since the agent now faces a reduction in the spread of consumption at time T because $y(\hat{\theta}^T) + b(\hat{\theta}^T) > y(\theta^T) + b(\theta^T)$. This implies that there exists an intermediate value of δ^* such that $\varepsilon > \delta^* > 0$ so that the change is zero. Hence, the time $T-1$ decision will be unchanged if $\delta = \delta^*$.

Equation (15) holds at time $t = T$:

Lemma 1 implies that there is only one case left to consider. Suppose that for some $\theta^T = (\theta^{T-1}, \theta_T)$

$$u'(b(\theta^{T-1}) + y(\theta^T)) < v'\left(\frac{y\left(\theta^T\right)}{\theta_T}\right)\frac{1}{\theta_T}. \tag{53}$$

In this case, the agent would like to consume and work less than the equilibrium contract. A deviation that reduces the total output and consumption by agent θ^T cannot be provided by an entrant, since a worker cannot deliver negative hours. However, an incumbent firm will find it optimal to deviate from the equilibrium contract, offering an allocation with lower consumption and lower output requirement and making strictly positive profits. Formally, it offers the original contract at all time $t < T$ and at time T, a menu that contains a null contract, the modified allocation chosen by θ^T and the original allocation chosen by the remaining types:

$$C_T\left(b(\theta^{T-1}), y(\theta^{T-1})\right) =$$
$$\left\{\left(b(\theta^T) + y(\theta^T) + \delta^*(\varepsilon|\theta_T) - \varepsilon, y(\theta^T) + \delta^*(\varepsilon|\theta_T)\right); (0,0); \left(y(\hat{\theta}^T) + b(\hat{\theta}^T), y(\hat{\theta}^T)\right)\ \hat{\theta}^T \neq \theta^T\right\}$$

where δ^* and ε are constructed in a similar fashion to the proof of Lemma 1, with the constraint $\delta \leq 0$.
With this deviation, the incumbent makes strictly positive profits, proportional to ε, and there exists ε so that agents' utility is unchanged following this deviation. This guarantees that no deviation at time $T-1$ takes place. This contract is always profitable for the incumbent even if another type $\tilde{\theta}_T$ accepts it. If an agent with type $\tilde{\theta}^T$ is able to choose the pair $(b(\theta^T) + y(\theta^T) + \delta^*(\varepsilon|\theta_T) - \varepsilon, y(\theta^T) + \delta^*(\varepsilon|\theta_T))$ at time T, it implies that he must also have chosen the allocation sequence $\{(b(\theta^n) + y(\theta^n), y(\theta^n))\}_{n=1}^{T-1}$ in previous periods. From the previous step in the proposition, transfers from any history are independent of time T; i.e., this agent will receive transfers with the same net present value as in the original choice.

Hence, the deviation is profitable.

2. **Equations (14) and (15) hold for $t < T$.**

As an inductive assumption, suppose (14) holds for $t+1$. We now show it holds for period t. Rewrite the net present value of transfers as:

$$A_t(\theta^{t-1}, \theta_{t-1}^T) = \sum_{n=t}^{T} \left(\frac{1}{q}\right)^{t-n} b_n(\theta^{t-1}, \theta_{t-1}^n) =$$
$$b_t(\theta^{t-1}, \theta_t) + q \sum_{n=t+1}^{T} \left(\frac{1}{q}\right)^{t+1-n} b_n(\theta^{t-1}, \theta_{t-1}^n) = b_t(\theta^{t-1}, \theta_t) + qA_{t+1}(\theta^t, \theta_t^T).$$

By way of contradiction, there exist θ_t and $\hat{\theta}_t$ following history θ^{t-1} such that

$$b_t(\theta^{t-1}, \theta_t) + qA_{t+1}(\theta^{t-1}, \theta_t) < b_t(\theta^{t-1}, \hat{\theta}_t) + qA_{t+1}(\theta^{t-1}, \hat{\theta}_t). \tag{54}$$

By the inductive assumption $b_t(\theta^{t-1}, \theta_t) < b_t(\theta^{t-1}, \hat{\theta}_t)$. As in the proof for time T, the contradiction argument relies on deviations by entrants to guarantee that (15) holds and on deviations by entrant and incumbent firms to imply that the net present value of transfers is zero.

Under the inductive assumption, the agent faces no distortion on both his intratemporal margin and intertemporal margin (recall Lemma 2) from time $t+1$ onward. This implies that the equilibrium allocation from time $t+1$ onwards is equivalent to a self-insurance economy (this will be formally proved in Proposition 5). Let $S_{t+1}(x)$ be the utility the agent receives from entering time $t+1$ with a level x of net present value of assets. The value function S is monotonically increasing in the level of assets. Given this, the agents' equilibrium choices at time t satisfy the following:

$$u(b(\hat{\theta}^t) + y(\hat{\theta}^t)) - v\left(\frac{y(\hat{\theta}^t)}{\hat{\theta}_t}\right) + \beta S_{t+1}\left(qA_{t+1}(\hat{\theta}^t) - b(\hat{\theta}^t)\right) \geq$$
$$u(b(\theta^t) + y(\theta^t)) - v\left(\frac{y(\theta^t)}{\hat{\theta}_t}\right) + \beta S_{t+1}\left(qA_{t+1}(\theta^t) - b(\theta^t)\right), \tag{55}$$

and

$$u(b(\theta^t) + y(\theta^t)) - v\left(\frac{y(\theta^t)}{\theta_t}\right) + \beta S_{t+1}\left(qA_{t+1}(\theta^t) - b(\theta^t)\right) \geq$$
$$u(b(\hat{\theta}^t) + y(\hat{\theta}^t)) - v\left(\frac{y(\hat{\theta}^t)}{\theta_t}\right) + \beta S_{t+1}\left(qA_{t+1}(\hat{\theta}^t) - b(\hat{\theta}^t)\right). \tag{56}$$

If $y(\theta^t) \geq y(\hat{\theta}^t)$, an entrant can offer the following menu that enables the agent to work additional hours and move resources between time t and time $t+1$:

$$\begin{aligned}\tilde{C}_t &= \left\{\left(b(\theta^t) - b(\hat{\theta}^t), y(\theta^t) - y(\hat{\theta}^t)\right); (0,0)\right\},\\ \tilde{C}_{t+1} &= \left\{\left(-\frac{1}{q}[b(\theta^t) - b(\hat{\theta}^t)] - \varepsilon, 0\right)\right\}.\end{aligned}$$

This menu generates strictly positive profits to the entrant, proportional to ε. If this menu is offered, agent θ^t will deviate, accepting the allocation for $\hat{\theta}^t$ together with the allocation specified in the entrant's menu. This is due to the fact that the agent can now replicate his original time t level of output and have access to a strictly higher net present value of transfers at a cost equal to ε.

Suppose now that $y(\theta^t) < y(\hat{\theta}^t)$. The first case we consider is when consumption at time t is higher for the agent with a higher net present value of transfer, $y(\theta^t) + b(\theta^t) < y(\hat{\theta}^t) + b(\hat{\theta}^t)$. As in the argument for period T, inequality (55) cannot hold with equality. This enables us to reduce the time t spread of consumption between histories θ^t and $\hat{\theta}^t$. Following the same steps of time T, a contradiction can be reached.

The final case is $y(\theta^t) < y(\hat{\theta}^t)$ and $y(\theta^t) + b(\theta^t) \geq y(\hat{\theta}^t) + b(\hat{\theta}^t)$. This case violates the inter-temporal Euler equation for at least one of the two types, thus contradicting Lemma 2. To see this, suppose that the Euler equation (13) holds for agent θ^t. We have

$$\begin{aligned}&u'(y(\theta^t) + b(\theta^t)) = \frac{\beta}{q}\sum_{\theta_{t+1}} \pi(\theta_{t+1}) u'(c(\theta^{t+1}))\\ \Rightarrow\ &u'(y(\hat{\theta}^t) + b(\hat{\theta}^t)) \geq \frac{\beta}{q}\sum_{\theta_{t+1}} \pi(\theta_{t+1}) u'(c(\theta^{t+1}))\\ \Rightarrow\ &u'(y(\hat{\theta}^t) + b(\hat{\theta}^t)) > \frac{\beta}{q}\sum_{\theta_{t+1}} \pi(\theta_{t+1}) u'(c(\hat{\theta}^t, \theta_{t+1})),\end{aligned}$$

where the last implication follows from the fact that an agent with higher transfer will have higher consumption at time $t+1$, thus a lower expected marginal utility of consumption.

To conclude, given that it was shown that the net present value of transfers is independent of the time t choice, we can follow the same steps as in time T to show that equation (53) holds for time t. □

C No profitable deviation with redistribution.

We show that there is no profitable deviation at time T that implies some redistribution between agents.

We first show that any deviation, if chosen by agents, is such that transfers $\{b(\theta^{T-1},\theta_i)\}_{\theta_i\in\Theta}$ satisfy the following ordering: for all i,j if $\theta_i>\theta_j$ then $b(\theta^{T-1},\theta_i)>b(\theta^{T-1},\theta_j)$. Suppose not, so there exists $\theta_i>\theta_j$ with $b(\theta^{T-1},\theta_i)<b(\theta^{T-1},\theta_j)$. Let $\{\hat{b}_T(\theta^{T-1}),\hat{y}(\theta^T)\}$ be the allocation chosen from the contract $\hat{C}$ at time T.[36] The agents' choices must satisfy the following, for all $\theta,\hat{\theta}$:

$$u\left(\hat{b}_T(\theta^{T-1})+b(\theta^T)+\hat{y}(\theta^T)+y(\theta^T)\right)-v\left(\frac{\hat{y}(\theta^T)+y(\theta^T)}{\theta}\right)\geq$$
$$u\left(\hat{b}_T(\theta^{T-1})+b(\theta^{T-1},\hat{\theta})+\hat{y}(\theta^{T-1},\hat{\theta})+y(\theta^{T-1},\hat{\theta})\right)-v\left(\frac{\hat{y}(\theta^{T-1},\hat{\theta})+y(\theta^{T-1},\hat{\theta})}{\theta}\right).$$

Using this equation for θ_i and θ_j and from convexity of v, we have that $\hat{y}(\theta^T)+y(\theta^T)>\hat{y}(\theta^{T-1},\hat{\theta})+y(\theta^{T-1},\hat{\theta})$. Agent θ_i is better-off with the following strategy: choosing the pairs $(\hat{b}_T(\theta^{T-1}),\hat{y}(\theta^{T-1},\hat{\theta}))$ and $(b(\theta^{T-1},\hat{\theta}),y(\theta^{T-1},\hat{\theta}))$ and from menu C_T^S choosing $\delta_i=\hat{y}(\theta^T)+y(\theta^T)-(\hat{y}(\theta^{T-1},\hat{\theta})+y(\theta^{T-1},\hat{\theta}))$. This allows him to have the same output requirements as in the original choice but higher consumption transfers.

We now show that any negative intratemporal transfers (transferring from less to more productive agents) induce a utility level lower than under self-insurance. Let $N=|\Theta|$ be the number of possible shock realizations. From the previous result, we focus on the case with transfers $\{b(\theta^{T-1},\theta_i)\}_{\theta_i\in\Theta}$ ordered so that for all i,j if $\theta_i>\theta_j$ then $b(\theta^{T-1},\theta_i)>b(\theta^{T-1},\theta_j)$. Define the time T utility of an agent θ with level of transfers equal to b, that can optimally chose the amount to work by the following:

$$W(b,\theta)=\max_{y\geq 0}u(b+y)-v\left(\frac{y}{\theta}\right). \tag{57}$$

Denote by $y^*(b,\theta)$ the solution of problem (57) characterized by:

$$u'(b+y^*(b,\theta))=\frac{1}{\theta}v'\left(\frac{y^*(b,\theta)}{\theta}\right). \tag{58}$$

Note that, for a given b, y^* is increasing in θ, since v is convex. The envelope condition for (57) implies:

$$\frac{\partial W(b,\theta)}{\partial b}=u'(b+y^*(b,\theta))>0. \tag{59}$$

Given the definition of W, the time T utility under menu $\hat{C}$ can be written as:

$$\hat{u}_T=\sum_{i=1}^{N}\pi(\theta_i)W(\hat{b}_T(\theta^{T-1}),\theta_i). \tag{60}$$

[36] From proposition (3), transfers in contract $\hat{C}$ do not depend on time T realization of the shock.

Let $\bar{W}$ be the time T level of utility derived following a deviation by an entrant receiving transfers b and let $W^N(b)$ be the following:

$$W^N(b) = \sum_{i=1}^{N} \pi(\theta_i) W(b_i + \hat{b}_T(\theta^{T-1}), \theta_i), \tag{61}$$

where each individual W is as in (57), from here onwards abusing notation we set $W(b_i, \theta_i) \equiv W(b_i + \hat{b}_T(\theta^{T-1}), \theta_i)$. We consider the most favorable case for the consumer and assume that the deviation incurs zero profits, so that $\sum_{i=1}^{N} \pi_i b_i = 0$. We will show that $\bar{W}$, the utility under the deviation is such that $\bar{W} < \hat{u}_T$. As a first step we show the following

$$\sum_{i=1}^{N} \pi_i b_i W'(0, \theta_i) < 0, \tag{62}$$

this can be shown by multiplying and dividing the above by $W'(0, \underline{\theta})$ where $\underline{\theta}$ is the smallest θ_i. This implies that the sign of (62) is determined by the sign of the following

$$\sum_{i=1}^{N} \pi_i b_i W'(0, \theta_i) = W'(0, \underline{\theta}) \sum_{i=1}^{N} \pi_i b_i \frac{W'(0, \theta_i)}{W'(0, \underline{\theta})},$$

which is negative given the zero profit assumption and the fact that $W'(0, \theta_i)$ is decreasing in θ_i. Define a scale parameter $g \in [0, 1]$ for all the transfers, and define the following function of the scale parameter

$$G(g) = \sum_{i=1}^{N} \pi_i W(g \cdot b_i, \theta_i), \tag{63}$$

note that $G(0) = \hat{u}_T$ and by definition of W, $\bar{W} \leq G(1)$; we will show that G is monotonically decreasing in g. We have that

$$\frac{\partial G'(g)}{\partial g} = \sum_{i=1}^{N} \pi_i b_i W'(g \cdot b_i, \theta_i), \tag{64}$$

where $W'(g \cdot b_i, \theta_i) = u'(g \cdot b_i + y^*(g \cdot b_i, \theta_i))$. As in the previous case, we also have that

$$\begin{aligned} u'(g \cdot b_i + y^*(g \cdot b_i, \theta_i)) &< u'(y^*(0, \theta_i)), \qquad if\ b_i > 0, \\ u'(g \cdot b_i + y^*(g \cdot b_i, \theta_i)) &> u'(y^*(0, \theta_i)), \qquad if\ b_i < 0. \end{aligned}$$

This then implies that $G'(g) < G'(0)$ for all $g > 0$, from (62) we have that $G'(0) < 0$ so that $G(1) < G(0) = \bar{u}_T$.

D Proofs of Section 4

Optimality of Exclusive Contracts under Zero Costs.

Lemma 3. *For all feasible utility levels w, $V(w) > \Pi(w)$.*[37]

Proof. Let $\{c^{NE}, y^{NE}\}$ and $\{c^{E}, y^{E}\}$ be the solution of (23) and (24), respectively. Since $\{c^{NE}, y^{NE}\}$ is in the constraint set of (24), $V(w) \geq \Pi(w)$ for all w. Suppose there exists w such that $V(w) = \Pi(w)$. This implies that $\{c^{NE}, y^{NE}\}$ is one of the solutions of (24) for this w. Let $\theta_l = \min_\theta \Theta$. A necessary first order condition for a solution of (24) is for all feasible w:

$$u'(c(\theta^{t-1}, \theta_l)) > \frac{1}{\theta_l} v' \left(\frac{y(\theta^{t-1}, \theta_l)}{\theta_l} \right), \quad \forall\ \theta^{t-1}. \tag{65}$$

However, since $\{c^{NE}, y^{NE}\}$ is a solution of (23), it must satisfy the following necessary first order condition:

$$u'(c(\theta^{t-1}, \theta_l)) = \frac{1}{\theta_l} v' \left(\frac{y(\theta^{t-1}, \theta_l)}{\theta_l} \right), \quad \forall\ \theta^{t-1}. \tag{66}$$

This contradicts $\{c^{NE}, y^{NE}\}$ being a solution of (24).[38] So no such w exists. □

Proof of Proposition 6

To prove the proposition, we first show the following two lemmas. For notation, let $U(c, y, \Theta)$ the life-time utility of any allocation $\{c(\theta^t), y(\theta^t)\}$ when shocks are in Θ.

Lemma 4. $w^{NE}\left(\lambda\bar{\theta}\right) = \frac{1-\beta^T}{1-\beta}\log\lambda + w^{L}\left(\bar{\theta}\right)$.

Proof. Let $\{c(\theta^t), y(\theta^t)\}$ be the solution of (23) for low mean agents. To prove the claim, we show that $\{\lambda c(\theta^t), \lambda y(\theta^t)\}$ solves the above problem for high mean agents. Suppose not, then there exists an allocation $\{\hat{c}(\lambda\theta^t), \hat{y}(\lambda\theta^t)\}$ that delivers higher utility $U(\hat{c}, \hat{y}, \lambda\Theta)$. Consider the allocation $\left\{\frac{\hat{c}(\theta^t)}{\lambda}, \frac{\hat{y}(\theta^t)}{\lambda}\right\}$. This allocation is in the constraint set of problem (23) and delivers utility

$$U\left(\frac{\hat{c}}{\lambda}, \frac{\hat{y}}{\lambda}, \Theta\right) = U(\hat{c}, \hat{y}, \lambda\Theta) - \frac{1-\beta^T}{1-\beta}\log\lambda. \tag{67}$$

By the contradicting assumption, $U(\hat{c}, \hat{y}, \lambda\Theta) > U(\lambda c, \lambda y, \lambda\Theta) = U(c, y, \Theta) + \frac{1-\beta^T}{1-\beta}\log\lambda$, which implies $U(\hat{c}, \hat{y}, \lambda\Theta) - \frac{1-\beta^T}{1-\beta}\log\lambda > U(c, y, \Theta)$. Using (67), we get $U\left(\frac{\hat{c}}{\lambda}, \frac{\hat{y}}{\lambda}, \Theta\right) > U(c, y, \Theta)$, contradicting allocation $\{c(\theta^t), y(\theta^t)\}$ solving (23) for low mean agents. □

Lemma 5. $w^{E}\left(\lambda\bar{\theta}\right) > w^{E}\left(\bar{\theta}\right) + \frac{1-\beta^T}{1-\beta}\log\lambda$.

[37] The set of feasible initial utility levels is the open interval $\left(\frac{1-\beta^{T+1}}{1-\beta}\underline{U}, \frac{1-\beta^{T+1}}{1-\beta}\bar{U}\right)$, where $\underline{U} = \inf_{c,l\geq 0} u(c) - v(l)$ and $\bar{U} = \sup_{c,l\geq 0} u(c) - v(l)$.

[38] Note that (65) holds with equality only for the highest realization of utility.

Proof. Let $\{c(\theta^t), y(\theta^t)\}$ be the solution of (24) for low mean agents. Consider the relaxed problem with the surplus constraint (25) holding as a weak inequality. Note that the allocation $\{\lambda c(\lambda\theta^t), \lambda y(\lambda\theta^t)\}$ is in the constraint set of this relaxed problem when agents are high mean. Also, this constraint must hold with equality (otherwise the extra surplus can be distributed in an incentive compatible way, increasing agent's utility). This implies that the allocation that solves the problem must deliver strictly higher utility. This implies $w^E\left(\lambda\bar{\theta}\right) > w^E\left(\bar{\theta}\right) + \frac{1-\beta^T}{1-\beta}\log\lambda$. □

Proof of Proposition 6

Proof. Let $\gamma^* = V\left(w^{NE}\left(\bar{\theta}\right)\right)$, where the function V is the solution of the following problem:

$$V(w_0) = \max_{c,y} \sum_{\theta^t,t} q^t \pi(\theta^t)[y(\theta^t) - c(\theta^t)] \tag{68}$$

$$\sum_{\theta^t,t} \beta^{t-1}\pi(\theta^t)\left[u(c(\theta^t)) - v\left(\frac{y(\theta^t)}{\theta_t}\right)\right] = w_0$$

$$\sum_{\theta^t,t} \beta^{t-1}\pi(\theta^t)\left[u(c(\theta^t)) - v\left(\frac{y(\theta^t)}{\theta_t}\right)\right] \geq \sum_{\theta^t,t} \beta^{t-1}\pi(\theta^t)\left[u(c(\tilde{\theta}^t)) - v\left(\frac{y(\tilde{\theta}^t)}{\theta_t}\right)\right] \quad \forall\tilde{\theta}^t$$

When writing problems (68), we abuse notation by denoting by θ the agents' labor productivity for both groups of agents.

This implies $w^E\left(\bar{\theta}|\gamma^*\right) = w^{NE}\left(\bar{\theta}\right)$. Also, $w^E\left(\bar{\theta}|\gamma^*\right) + \frac{1-\beta^T}{1-\beta}\log\lambda = w^{NE}\left(\bar{\theta}\right) + \frac{1-\beta^T}{1-\beta}\log\lambda$. Using the previous lemmas,

$$w^E\left(\lambda\bar{\theta}|\gamma^*\right) > w^E\left(\bar{\theta}|\gamma^*\right) + \frac{1-\beta^T}{1-\beta}\log\lambda = w^{NE}\left(\bar{\theta}\right) + \frac{1-\beta^T}{1-\beta}\log\lambda = w^{NE}\left(\lambda\bar{\theta}\right).$$

It is possible to break the indifference of the firms with respect to low mean agents by considering the monitoring cost $\gamma = \gamma^* + \varepsilon$ for some ε small enough. The result holds in this case, since $w^{NE}\left(\bar{\theta}\right) > w^E\left(\bar{\theta}|\gamma^*\right)$ and $w^E\left(\bar{\theta}|\gamma^*\right)$ is continuous on γ, so we can replicate the same steps. □

E Data

Table 8: Sample selection for CEX data

	CEX
Baseline sample	69,816
Hours restriction	46,559
Earnings <=0	46,002
Labor income <=0	45,745
Minimum wage restriction	43,802
Age >=25 and <= 55	36,871
Final sample	36,871

Numbers indicate total observations remaining at each stage of the sample selection.

Table 9: Summary statistics for the CEX sample used.

	CEX (80-04)
Age	39.17 (8.74)
Education	
High school dropout	6.99
High school graduate	29.26
College	60.46
Race	
White	86.95
Black	9
Family composition	3.07 (1.58)
Average earnings ($)	30,340 (20,406)
Average annual consumption ($)	13,542 (6,842)
Food ($)	3,791 (1,965)
Rent ($)	262 (487)
Hours	2.123 (567)

Note - All dollar amounts in 1983 dollars.