The Electricity Market Game

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Abstract

This paper examines the effects of imperfect competition in unregulated electricity markets from a general equilibrium perspective, and demonstrates that horizontal market power can explain both the large peak-period price spikes observed recently in California and elsewhere, and the marked reduction in additions to capacity that have also occurred during the transition to competitive markets.

1 Introduction[1]

A primary economic rationale for restructuring of the electricity industry has been the promise of lower prices and more efficient power generation through market competition made possible by technological innovations that are allowing power generation to be separated from distribution. A key assumption behind this premise is that the technologies in question will result in competitive markets, rather than markets in which small numbers of firms exercise market power.

The recent events in the California electricity market – the large peak-load price spikes in the wholesale electricity markets, downstream bankruptcies of the regulated distribution utilities, rolling blackouts, and underinvestment in new generating capacity – have tarnished the promise of deregulation and raised the issue of imperfect competition in a particularly stark way. The analysis of these events by Borenstein, Bushnell, and Wolak [3] provides empirical support for the contention that the process of deregulation has not (at least as yet) been successful in creating the kind of competitive environment that will deliver the

[1]I am grateful to Yves Balasko, Alex Farrell, David Kelly, and to an anonymous referee and Associate Editor at the *Journal of Economic Theory* for many helpful comments and suggestions.

benefits that economists have been promising. While the Borenstein, Bushnell and Wolak study focuses on the California power markets, the problems of price spikes and under-investment in new generating capacity have also appeared in the Midwest and New England electricity markets as well, suggesting that horizontal market power may be a significant side-effect of deregulation in markets for electricity generally. Given the fact of imperfect competition in newly deregulated markets for electricity, then, one of the goals of this paper will be to develop a model of electricity pricing in imperfectly competitive markets and to explore the relationship between pricing during off-peak and peak periods of demand. A second goal of the paper will be to examine the incentives for expanding capacity that face producers in an imperfectly competitive market setting.

This is certainly not the first paper to examine issues of market power in newly deregulated markets for electricity (see, e.g. Gans, Price and Woods [4], Green and Newbery [5], Hogan [6], or Rudkevich, Duckworth and Rosen [9]), although it appears to be the first to examine the issue of horizontal market power in markets for electricity using the general equilibrium framework of the Shapley-Shubik market game. The development of the model closely follows that of Balasko [1] for the competitive model, which also examines the market for electricity from the general equilibrium perspective. A key feature of this framework is that of treating electricity delivered at different times as different commodities, thus permitting explicit consideration of demand differences occurring at different times of day or different months of the year. This approach is also a natural one given the fact that electricity must be generated as it is consumed. In addition, the general equilibrium approach allows one to model demand interactions across the different demand periods with respect to changes in electricity prices in each period, or with respect to changes in relative prices of non-electricity goods. These effects cannot be captured in a partial equilibrium setting, since demand in these models is specified exogenously. It is also well-known that in a general equilibrium setting, profit maximization is *not* the correct objective for an imperfectly competitive firm, since the production and pricing decisions of these firms affect relative prices, and hence the choice decisions of the firm's owners. Of course, capturing these effects in a partial equilibrium setting is impossible.[2] Finally, we note that Balasko's analysis of the competitive electricity market demonstrates the possibility of multiple equilibria, which may be a possible explanation for observed deviations of prices from marginal cost in real world electricity markets. We show here that imperfect competition in the same general equilibrium framework can also deliver these results under very simple assumptions about preferences.

[2]The general equilibrium approach provides an interesting parallel with several of the more sophisticated partial equilibrium approaches in the literature on imperfect competition in markets for electricity, which examine models of supply function equilibria. These equilibria have firms specifying quantities of power to provide in each of a sequence of periods, together with prices at which they are willing to supply these quantities. As our development of the general equilibrium model will show, the market game captures many of the features of the supply-function models. (See, e.g. Green and Newbery [5] or Rudkevich, Duckworth and Rosen [9] for details.)

The specific model we adopt is a version of the Shapley-Shubik market game. In Section 2, we lay out the general model, based on the formulation of Peck, Shell and Spear [8]. Since the complexity of the general model precludes the kinds of equilibrium calculations that bear on the policy issues of pricing and long-run investment in capacity, we turn in Section 3 to the study of an extended, but simplified, example of the model. The simplifications involve the specification of preferences and consideration of symmetric, deterministic equilibrium only. Finally, Section 4 discusses policy implications of the model.

2 The Electricity Market Game

The modeling strategy we adopt is based on the framework introduced by Balasko [1] which embeds the market for electricity in a simple general equilibrium setting in which all production and consumption activities take place at the same location. As in Balasko's paper, we assume that electricity can be produced and distributed over $T < \infty$ periods, so that electricity is a dated commodity. Furthermore, because of the unique physical properties of electric power, we assume that electricity available in one period cannot be stored for consumption in any later period.

In addition to electricity, we assume there is also a single consumption good (which we will frequently refer to as the numeraire good, following Balasko's terminology) which can either be consumed directly, or used in the production of electricity.

2.1 Electricity Production

Following Balasko, we assume that electricity producers have access to a constant returns to scale electricity generation technology. The installed capacity of a given power plant is denoted K. In the short-run, the installed capacity is fixed and constitutes a constraint on the producer's supply of power. In the long-run, capacity is variable and will be determined endogenously in the model. A producer's installed capacity can be increased by one unit (measured in kilowatts) by an investment of ρ units of the numeraire good. To supply one unit of electrical energy (measured in kilowatt hours), a producer must "burn" γ units of the numeraire.

With these definitions, we can characterize a typical producer's short- and long-run production sets as the collection of technically feasible activity vectors. An activity vector will be denoted by $(\mathbf{q}, \varsigma)$, where $\mathbf{q}' = \left[q^1, ..., q^T\right]$ is the vector of kilowatt hours of energy the producer supplies to the market, and λ is the amount of numeraire good used to produce this power. Adopting the usual general equilibrium convention on signing inputs and outputs, we assume that $\varsigma \leq 0$. The producer's short-run production set corresponding to installed

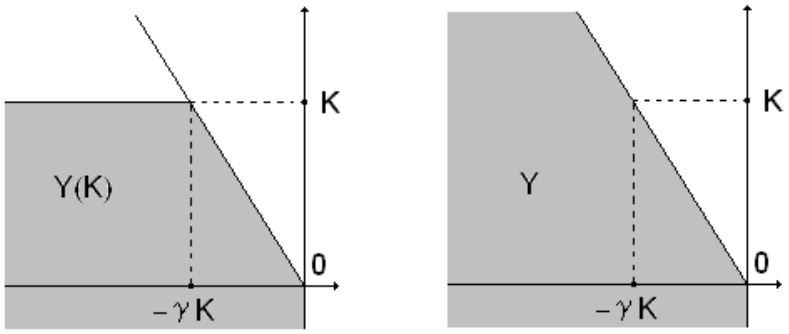


Figure 1: Short- and long-run production sets

capacity K is then given by

$$Y(K) = \left\{ (\mathbf{q}, \varsigma) \in \mathbb{R}^{T+1} \mid 0 \leq q^1, ..., q^T \leq K \text{ and } -\varsigma \geq \gamma \sum_{t=1}^{T} q^t \right\}.$$

The long-run production set is defined similarly, as

$$Y = \left\{ (\mathbf{q}, \mathbf{K}, \boldsymbol{\varsigma}) \in \mathbb{R}^{T+2} \mid 0 \leq q^1, ..., q^T \leq K \text{ and } -\varsigma \geq \rho K + \gamma \sum_{t=1}^{T} q^t \right\}.$$

Figure 1 illustrates the two production sets for the case of a single power output.

2.2 The Market Game

The model is populated by two types of agents. *Producer agents* own power plants and can produce electricity. We assume there are P agents of this type and index them by $j = 1, ..., P$. Agents who cannot produce electricity will be called *standard agents*. These agents are endowed only with the numeraire good. We assume there are M agents of this type and index them by $h = 1, ..., M$. Since the demand for electricity occurs (as in the competitive model) over $T \geq 1$ periods, fully flexible pricing of demand in each period requires, in the market game setting, that transactions for power in each period and for the consumption good occur in $T+1$ "trading posts". The assumption of flexible pricing over all periods of demand is obviously unrealistic if we interpret the model as being literally one in which producers sell directly to consumers with prices established under long-term contracts that do not allow for peak-load pricing. Hence, if we view the periods in the model as different times of day, it probably doesn't have much to say about the real world, although as smart houses and factories evolve, this scenario becomes more realistic. On the other hand, if we view the

time periods as months, which correspond more closely to actual billing periods, the model becomes more germane, particularly in the kinds of deregulated environments that have evolved in the United States and Europe over the past 15 years. The flexible price assumption is also obviously at odds with the partially regulated structure of the California electricity markets, although even here, if we interpret consumers as utilities who know the preferences of their customers, and who may have to ration power during the peak periods defined by these preferences, the model does have some predictive power in terms of explaining the kinds of price spikes observed in the California market during the summer of 2000. It clearly has explanatory power for explaining these phenomena in the Midwest and New England power markets during the same time frame.

The formulation of the model below follows the Peck, Shell, and Spear [8] specification in which bids are made in some unit of account (inside money) rather than in terms of the numeraire good. This formulation avoids some well-known problems that can occur if the availability of the numeraire ends up constraining agents' access to credit in the market. We can still make direct price comparisons of the results for the imperfectly competitive market with those for competitive markets by renormalizing the prices appropriately.

2.2.1 Standard Agents

Standard agents are endowed only with the numeraire good. Agent $h's$ endowment is denoted by $\omega_h \gg 0$ for $h = 1, ..., M$. These agent's preferences are given by a standard utility function $u_h : \mathbb{R}^{T+1}_{++} \to \mathbb{R}$, $h = 1, ..., M$ which is assumed to be at least twice continuously differentiable, strictly increasing, strictly quasi-concave, and satisfies the Inada conditions

$$\lim_{x \to \partial \mathbb{R}^{T+1}_{+}} \|Du_h\| = +\infty$$

where $x \in \mathbb{R}^{T+1}$ denotes the agent's consumption bundle. In the market game, standard agents offer numeraire on the numeraire trading post in order to make bids on the electricity and numeraire trading posts. We denote agent h's bid on electricity trading post t by b^t_h, and let $\mathbf{b}'_h = \left[b^1_h, ..., b^T_h\right]$. Agent h's bid on the numeraire trading post is denoted by ξ_h. In order to avoid the indeterminacies identified by Peck, Shell and Spear [8] (and to focus attention on the role of producers in the imperfectly competitive environment), we impose a "sell-all" assumption on the standard consumers, so that each of them offers her full endowment on the trading post for the consumption good. Strategies for agents $h = 1, ..., M$ are then given by

$$S_h = \left\{ \left[\left(\mathbf{b}_h, \xi_h \right), \left(\mathbf{0}, \boldsymbol{\omega}_h \right) \right] \in \mathbb{R}^{2(T+1)}_{+} \right\}.$$

In keeping with the assumption that standard agents have no power production capabilities and offer all of their endowment of the consumption good on the market, h's quantity offer is just $\left(\mathbf{0}, \boldsymbol{\omega}_h \right)$.

Agents face budget constraints on what they may bid on each of the trading posts. For agents $h = 1, ..., M$, the budget constraint is

$$\sum_{t=1}^{T} b_h^t + \xi_h \leq \frac{B^0}{\Omega}\omega_h \tag{1}$$

where

$$B^0 = \sum_{k=1}^{M+P} \xi_k$$

and

$$\Omega = \sum_{h=1}^{M} \omega_h.$$

The constraint states that the amount that agent h can bid (in units of account) on each trading post must be less than or equal to the total amount of money available to the agent from the sale of her endowment. For standard agents, this is given by a share of the total bid on the numeraire trading post, with the share determined by the agent's offer of endowment (ω_h) relative to the total offer of the numeraire (Ω). Note that the total bid on the numeraire trading post derives both from the bids of standard agents and from those of electricity producers.

Since the aggregate bid for the numeraire includes agent h's bid, which also appears on the left-hand side of the constraint, the budget constraint can be simplified further by isolating all of agent h's bids on the left, yielding

$$\sum_{t=1}^{T} b_h^t + \frac{\Omega_{-h}}{\Omega}\xi_h \leq \frac{B^0_{-h}}{\Omega}\omega_h \tag{2}$$

where

$$\Omega_{-h} = \Omega - \omega_h$$

and

$$B^0_{-h} = B^0 - \xi_h.$$

2.2.2 Producers

Producers are endowed only with the technology to produce electricity, and make offers of power on each of the electricity trading posts in the amount $q_j^t \geq 0$ for $j = M + 1, ..., M + P$ and $t = 1, ...T$. Let $\mathbf{q}'_j = \left[q_j^1, ..., q_j^T\right]$. These agents make bids to purchase numeraire both for consumption and as inputs to production, as well as for electricity. We let $\mathbf{b}'_j = \left[b_j^1, ..., b_j^T\right]$ denote agent j's

bids for electricity, and ξ_j the bid for numeraire. Producer j's strategy set is then given by

$$S_j = \left\{ \left[\left(\mathbf{b}_j, \xi_j \right), \left(\mathbf{q}_j, \mathbf{0} \right) \right] \in \mathbb{R}_+^{2(T+1)} \right\}$$

for $j = M+1, ..., M+P$.

Producers face budget constraints on what they may bid on the electricity and numeraire trading posts. Agent j's budget constraint takes the form

$$\sum_{t=1}^{T} b_j^t + \xi_j \leq \sum_{t=1}^{T} \frac{B^t}{Q^t} q_j^t \tag{3}$$

for $j = M+1, ..., M+P$, where

$$B^t = \sum_{k=1}^{M+P} b_k^t$$

and

$$Q^t = \sum_{j=1}^{P} q_j^t.$$

As was the case for standard consumers, producer j's budget constraint will have his bids for electricity on both the left-hand and right-hand sides of the budget constraint, so that the constraint can be simplified by collecting the agent's own bids on the left-hand side. Doing this yields

$$\sum_{t=1}^{T} \frac{Q_{-j}^t}{Q^t} b_j^t + \xi_j \leq \sum_{t=1}^{T} \frac{B_{-j}^t}{Q^t} q_j^t \tag{4}$$

where

$$Q_{-j}^t = Q^t - q_j^t$$

and

$$B_{-j}^t = B^t - b_j^t.$$

2.2.3 Allocations

With the specifications of agents strategies given above, we now specify the allocations that agent's receive of electricity and the numeraire good. An agent's allocation of electricity in any period t will be denoted x_i^t where i denotes either a standard agent or a producer, and $t = 1, ..., T$. An agent's allocation of the numeraire good will be denoted x_i^0. With this notation, allocations are given as follows.

For $h = 1, ..., M$ and $t = 1, ..., T$

$$x_h^t = \frac{b_h^t}{B^t} Q^t \tag{5}$$

and

$$x_h^0 = \frac{\xi_h}{B^0} \Omega. \tag{6}$$

For $j = M + 1, ..., M + P$ allocations are given by

$$x_j^t = \frac{b_j^t}{B^t} Q^t \tag{7}$$

for $t = 1, ..., T$, and

$$x_j^0 = \frac{\xi_j}{B^0} \Omega - \gamma \boldsymbol{\iota} \cdot \mathbf{q}_j \tag{8}$$

where $\boldsymbol{\iota}$ denotes a sum vector. Note that for producer agents, the sell-all requirement implies that they cannot directly consume their own output of electricity, but rather, must purchase it on the open market. Similarly, they cannot contract to sell electricity and then make good on the contract by purchasing electricity in the spot market. As we noted above, the sell-all requirement is imposed to restrict the kind of indeterminacy found by Peck, Shell and Spear, although the effects of such indeterminacies (when they exist) is clearly an interesting issue for future consideration. Similarly, relaxing the assumptions that producers cannot import power from other markets also poses interesting questions for future research.

The allocations rules are quite intuititive, stating that each agent's allocation of a commodity is determined by giving the agent the fraction of the total offer of the good on the trading post, with the share determined by the agent's bid on the trading post as a fraction of the total bid. These rules can also be interpreted as giving the agent her bid divided by the price of the good determined on the trading post (which is given by the ratio of total bid to total quantity offered). These specifications of the allocations are standard for $h = 1, ..., M$. For producer agents, the allocation rules incorporate the constraints imposed by production. Agent j's allocation rule for electricity reflects that fact that he need not offer the full short-run capacity on the market at any point in time, although the amount he does offer must be less than capacity. The specification of j's allocation of the consumption good reflects the fact that agent j produces electricity, and hence must allocate his purchases of the consumption good between his own consumption and the input requirements for producing the output vector $\mathbf{q}_j$. Finally, it is easy to verify that summing allocations over all the agents uses exactly the quantities of all goods offered on the markets, so that all markets clear.

2.2.4 Best Responses

Both types of agent in the model choose their bid and offer strategies as best responses to the bids and offers of other agents, that is, so as to maximize utility subject to the budget constraints, taking other agents' actions as given.

For agents $h = 1, ..., M$, their optimization problems are

$$\max_{(\mathbf{b}_h, \xi_h)} \; u_h \left(\frac{b_h^1}{B^1} Q^1, ..., \frac{b_h^T}{B^T} Q^T, \frac{\xi_h}{B^0} \Omega \right)$$

subject to

$$\sum_{t=1}^{T} b_h^t + \frac{\Omega_{-h}}{\Omega} \xi_h \leq \frac{B_{-h}^0}{\Omega} \omega_h.$$

For producer agents $j = M+1, ..., M+P$ the optimization problem is

$$\max_{(\mathbf{b}_j, \xi_j, \mathbf{q}_j)} \; u_j \left(\frac{b_j^1}{B^1} Q^1, ..., \frac{b_j^T}{B^T} Q^T, \frac{\xi_j}{B^0} \Omega - \gamma \boldsymbol{\iota} \cdot \mathbf{q}_j \right)$$

subject to

$$\sum_{t=1}^{T} \frac{Q_{-j}^t}{Q^t} b_j^t + \xi_j \leq \sum_{t=1}^{T} \frac{B_{-j}^t}{Q^t} q_j^t$$

and

$$\mathbf{q}_j \leq K \boldsymbol{\iota}$$

First-order conditions for agents $h = 1, ..., M$ are

$$u_{ht} \frac{B_{-h}^t Q^t}{\left(B^t\right)^2} - \lambda = 0 \text{ for } t = 1, ..., T \tag{9}$$

$$u_{h0} \frac{B_{-h}^0 \Omega}{\left(B^0\right)^2} - \lambda \frac{\Omega_{-h}}{\Omega} = 0. \tag{10}$$

Those for agents $j = M+1, ..., M+P$ are

$$u_{jt} \frac{B_{-j}^t Q^t}{\left(B^t\right)^2} - \lambda \frac{Q_{-j}^t}{Q^t} = 0 \text{ for } t = 1, ..., T \tag{11}$$

$$u_{j0} \frac{B_{-j}^0 \Omega}{\left(B^0\right)^2} - \lambda = 0 \tag{12}$$

$$u_{jt}\frac{b_j^t}{B^t} - \gamma u_{j0} + \lambda\frac{B^t Q_{-j}^t}{\left(Q^t\right)^2} - \mu^t = 0 \tag{13}$$

$$\boldsymbol{\mu}\cdot\left[K\boldsymbol{\iota} - \mathbf{q}_j\right] = 0 \tag{14}$$

where $\boldsymbol{\mu}' = \left[\mu^1, ..., \mu^T\right]$. This last equation is just the complementary slackness condition from the Kuhn-Tucker conditions stating that if some producer's capacity constraint in period t is not binding, then the multiplier μ^t associated with the constraint must be zero.

Finally, we adopt the standard definition of the Nash equilibrium as any collection of bids and offers for all agents each of which is a best response to the bids and offers of other agents.

The model as specified here is significantly more complex than the standard market game, because of the presence of production. Hence, at an abstract level, issues of existence and uniqueness of equilibria arise. While we will not pursue these issues in full generality here, we do note that establishing existence for the case where the marginal cost of producing power is small should be reasonably straightforward. The argument is as follows.

In the absence of capacity constraints, if $\gamma = 0$, then electricity producers will find it optimal to produce as much as they possibly can, since the marginal utility of producing an extra kilowatt hour is

$$u_{jt}\frac{b_j^t}{B^t} + \lambda\frac{B^t Q_{-j}^t}{\left(Q^t\right)^2}$$

which is always positive as long as the aggregate bid for time t electricity is positive. Assuming, then, that $B^t > 0$, when the capacity constraints are present, they will bind when $\gamma = 0$, so that the last set of first-order conditions above becomes

$$u_{jt}\frac{b_j^t}{B^t} + \lambda\frac{B^t}{K} = \mu^t.$$

Note next that when all producers supply K to the market with $\gamma = 0$ the model reduces to a sell-all market game in which electricity producers are endowed with K for each of the t periods. Hence, we now have an almost standard market game model in the sense that it corresponds to the standard market game specification (with the sell-all assumption), except for the assumption that some agents have zero endowment of some goods.

This assumption poses a potential problem since the argument Peck, Shell, and Spear used to show that aggregate bids cannot be zero in equilibrium breaks down here. This argument was based on using the fact that when all agents have strictly positive endowments of all goods, the set of individually rational, feasible allocations is compact and bounded from below. This fact can then be used to show that any equilibrium must have all agents making strictly positive bids. Without some condition which will guarantee that equilibrium bids are

strictly positive, it is possible to have equilibria in which because no other agents make positive bids, agent h's budget constraint then forces her to post a zero bid.

One possible way around this is to modify the individual rationality condition from one based on guaranteeing agents at least the same utility as they receive at their endowment (which may be $-\infty$ in our framework) to one that guarantees that each agent is able to purchase at least her endowment. This assumption is certainly consistent with the sell-all specification since without it, agents could end up being forced to put all their endowment on the relevant trading posts, and receive nothing in return. With this assumption, an aggregate bid $B^0 = 0$ for the consumption good is not consistent with equilibrium. Given the Inada conditions on preferences, this will then imply that all standard agents make positive bids for electricity, so that for all t, $B^t > 0$ in equilibrium. Hence, we are guaranteed an interior equilibrium. Application of the kinds of regularity techniques developed in Peck-Shell-Spear can then be used to show that equilibrium will continue to exist if the marginal cost γ is greater than zero, but sufficiently close to zero.

Since our fundamental interest in this paper is to compare competitive and imperfectly competitive outcomes in the electricity market, we forego a more detailed analysis of the issues of existence and uniqueness of equilibria in the electricity market game in favor of an in-depth examination of a more tractable example of the model.

3 An Extended Example: The Log-linear Economy

In order to get concrete results, we will simplify the model laid out above by focusing on a particular specification of preferences, and by confining our attention to the symmetric Nash equilibria of the model. The fundamental simplification we make is to assume that standard agents, who are endowed with the numeraire good, derive no utility from consuming the numeraire. All these agents care about is electricity consumption. Producer agents, on the other hand, own the means for producing electricity, but care only about the consumption of the numeraire good. This specification of the model greatly simplifies the market interactions in the model, since producer agents are the only ones bidding on the trading post for the numeraire good, while only standard agents bid on the electricity markets.

For the example, all agents have log-linear preferences. Standard agents' utility functions are given by

$$u_h(\mathbf{x}) = \sum_{t=1}^{T} \alpha_t \ln\left(x^t\right)$$

while those of producer agents are given by

$$u_j\left(x^o\right) = \ln\left(x^0\right).$$

Note that we are assuming that all agents of the same type have the same utility function. This assumption allows us to restrict attention to symmetric equilibria in the model, where agents of the same type make the same bids and offers. Finally, we also assume that all standard agents have the same endowment $\omega_h = \omega$ and that all producer agents have access to the same technology and face the same short-run capacity constraints.

With this specification of preferences, the marginal utilities that enter into the first-order conditions for the optimal bids and offers are given by

$$u_{ht} = \frac{\alpha_t}{x^t} = \frac{\alpha_t B^t}{b^t Q^t} \text{ for } t = 1, ..., T$$

and

$$u_{j0} = \frac{1}{x^0} = \frac{1}{\xi \frac{\Omega}{B^0} - \gamma \boldsymbol{\iota} \cdot \mathbf{q}}.$$

3.1 Best Response Functions and Symmetric Equilibrium

We use the first-order conditions calculated in the previous section to determine the best response functions of each of the agents for the example economy.

3.1.1 Standard Agents

For standard agents, the general first-order conditions are

$$u_{ht} \frac{B^t_{-h} Q^t}{\left(B^t\right)^2} - \lambda = 0 \text{ for } t = 1, ..., T.$$

Substituting u_{ht} from the expression above then yields

$$\frac{\alpha_t B^t_{-h}}{b^t B^t} = \lambda$$

or

$$b^t = \frac{\alpha_t B^t_{-h}}{\lambda B^t}.$$

Substituting into the budget constraint

$$\sum_{t=1}^{T} b^t_h = \frac{B^0_{-h}}{\Omega} \omega$$

yields

$$\frac{1}{\lambda}\sum_{t=1}^{T}\frac{\alpha_t B_{-h}^t}{B^t} = \frac{B_{-h}^0}{\Omega}\omega.$$

Solving for λ gives

$$\hat{\lambda} = \frac{\Omega}{\omega B_{-h}^0}\sum_{t=1}^{T}\frac{\alpha_t B_{-h}^t}{B^t}.$$

Hence, we obtain the standard agent's optimal bid as

$$\hat{b}^t = \frac{\alpha_t B_{-h}^t}{B^t}\frac{B_{-h}^0\omega}{\Omega}\left[\sum_{t=1}^{T}\frac{\alpha_t B_{-h}^t}{B^t}\right]^{-1}.$$

We now impose the requirement that bids be symmetric (offers for the standard agents are already symmetric, given the assumption that agents have the same endowment and the requirement that they offer it all on the market). With M standard agents and P producer agents, we have

$$\begin{aligned} B^t &= Mb^t \\ B_{-h}^t &= (M-1)\,b^t \\ B_{-h}^0 &= P\xi \\ \Omega &= M\omega \end{aligned}$$

Making these substitutions on the right-hand side above, we get

$$\begin{aligned} \hat{b}^t &= \frac{(M-1)}{M}\alpha_t \cdot \frac{P\xi}{M}\left[\sum_{t=1}^{T}\frac{\alpha_t\,(M-1)}{M}\right]^{-1} \\ &= \frac{P}{M}\frac{\alpha_t}{\sum_t \alpha_t}\xi. \end{aligned}$$

3.1.2 Producers

For producer agents, the general first-order conditions for the choice of $\mathbf{q}$ and x^0 are

$$u_{j0}\frac{B_{-j}^0\Omega}{\left(B^0\right)^2} - \lambda = 0$$

$$\lambda\frac{B^t Q_{-j}^t}{\left(Q^t\right)^2} - \gamma u_{j0} - \mu^t = 0$$

$$\boldsymbol{\mu}\cdot\left[K\boldsymbol{\iota} - \mathbf{q}_j\right] = 0.$$

Substituting for u_{j0} in the first-expression, we have

$$\left[\frac{1}{\xi\frac{\Omega}{B^0}-\gamma\boldsymbol{\iota}\cdot\mathbf{q}}\right]\frac{B^0_{-j}\Omega}{\left(B^0\right)^2}=\lambda$$

or

$$\left[\frac{1}{\xi\Omega-B^0\gamma\boldsymbol{\iota}\cdot\mathbf{q}}\right]\frac{B^0_{-j}\Omega}{B^0}=\lambda.$$

For the time being, we focus on the case where the capacity constraint is *not* binding, so that $\mu^t=0$. In this case, substituting for u_{j0} in the second first-order condition yields

$$\lambda\frac{B^tQ^t_{-j}}{\left(Q^t\right)^2}-\frac{\gamma B^0}{\xi\Omega-B^0\gamma\boldsymbol{\iota}\cdot\mathbf{q}}=0.$$

Substituting for λ from the first equation gives

$$\left[\frac{1}{\xi\Omega-B^0\gamma\boldsymbol{\iota}\cdot\mathbf{q}}\right]\frac{B^0_{-j}\Omega}{B^0}\cdot\frac{B^tQ^t_{-j}}{\left(Q^t\right)^2}=\frac{\gamma B^0}{\xi\Omega-B^0\gamma\boldsymbol{\iota}\cdot\mathbf{q}}$$

or

$$\frac{B^0_{-j}\Omega}{B^0}\cdot\frac{B^tQ^t_{-j}}{\left(Q^t\right)^2}=\gamma B^0$$

which implies that

$$\frac{B^tQ^t_{-j}}{\left(Q^t\right)^2}=\frac{\gamma\left(B^0\right)^2}{B^0_{-j}\Omega}.$$

Now, imposing the assumption that we are at a symmetric equilibrium, this expression becomes

$$\frac{M\hat{b}^t\left(P-1\right)}{P^2q^t}=\frac{\gamma P^2\xi}{\left(P-1\right)M\omega}.$$

Hence,

$$\hat{q}^t=\frac{\left(P-1\right)^2M^2\omega}{\gamma\xi P^4}\hat{b}^t.$$

Substituting for $\hat{b}^t$ from the expression derived above, we get

$$\hat{q}^t=\frac{\left(P-1\right)^2M\omega}{\gamma P^3}\frac{\alpha_t}{\sum_t\alpha_t}.$$

Finally, we may normalize the bid $\xi = 1$ since the budget constraint for a producer agent

$$\xi = \sum_{t=1}^{T} \frac{B_{-j}^t}{Q^t} q^t$$

when evaluated using the bids and offers obtained above yields an identity. This also makes sense from an economic perspective, since bids are denominated in units of account, and until we specify how much of this unit of account is available, price (or bid) levels are undetermined.

3.2 Off-peak Electricity Prices

Having determined the optimal bids and offers for the example, we can now compute the price of electricity (in units of account) in any off-peak period as

$$\hat{p}^t = \frac{B^t}{Q^t} = \frac{\left[P \frac{\alpha_t}{\sum_t \alpha_t}\right]}{\frac{(P-1)^2 M\omega}{\gamma P^2} \frac{\alpha_t}{\sum_t \alpha_t}} = \frac{\gamma P^3}{M\left(P-1\right)^2 \omega}.$$

In order to make comparisons between this result and the corresponding pricing results for the competitive market (see, e.g. Balasko [1]), we need to convert this price (which is in terms of units of account) into one which is in terms of the numeraire good. To do this, we divide $\hat{p}^t$ by the price of the numeraire good, which is $\hat{p}^0 = P/M\omega$. This yields

$$\hat{\pi}_t = \frac{\hat{p}^t}{\hat{p}^0} = \frac{\gamma P^3}{M\left(P-1\right)^2 \omega} \frac{M\omega}{P} = \gamma \left[\frac{P}{P-1}\right]^2.$$

There are several things to note about this result. First, it represents a mark-up of marginal cost that depends on the number of producers – but not consumers – in the market. As the number of producers gets large, the mark-up factor approaches one, consistent with the standard result that in perfectly competitive environments, off-peak prices equal marginal costs. Secondly, from the pricing expression, it is easy to calculate the mark-up over marginal cost due to imperfect competition in the electricity market:

$$\begin{aligned} \hat{m}_t &= \frac{\hat{\pi}_t}{\gamma} - 1 \\ &= \left[\frac{P}{P-1}\right]^2 - 1 \\ &= \frac{2P-1}{\left(P-1\right)^2}. \end{aligned}$$

For this example, if we take $P = 7$ (as in California), then $\hat{m}_t = 13/36 \doteq 0.36$. This number is about twice the annual mark-up estimated by Borenstein, Bushnell and Wolak [3] of 18.3% for the California market in 1998. As these authors

note, however, the number of producers operating in the California market changes between peak and off-peak periods, with power imports from other states accounting for the supply differences. While the mark-up percentage for the California market is fairly large, the corresponding numbers for other regions of the country are more reasonable. In the New England pool, for example, there are 29 generators, implying a mark-up of only 7.2% in that market. Similarly, in the Missouri-Kansas region, with 22 generators, the implied mark-up is 9.6%. These numbers are significantly smaller than the 10% to 25% deadweight losses due to regulatory inefficiency[3] estimated by Maloney, McCormick and Sauer [7]. We also note that if there are 40 power providers in our model, the predicted mark-up over marginal cost is about 5.2% which is certainly in the ballpark for the Justice Department's benchmark for determining whether a market is competitive. We conclude from this, then, that while deregulated markets will almost never be perfectly competitive, reasonable amounts of competition will lead to efficiency improvements over the regulated environment, at least for off-peak periods.

3.3 Peak-period Electricity Prices

The results obtained above are applicable to off-peak periods, that is, periods in which offers are such that the capacity constraints are not binding. Hence, we turn next to the case where some capacity constraint binds. For this section, it will be convenient to let $\delta_t = \alpha_t / \sum \alpha_t$. Then

$$\hat{q}^t = \frac{(P-1)^2 M\omega}{\gamma P^3}\delta_t.$$

Now, suppose $\hat{q}^t \geq K$ so that the capacity constraint is binding. Then the first-order conditions for a producer agent become

$$\left[\frac{1}{\xi\frac{\Omega}{B^0} - \gamma\boldsymbol{\iota}\cdot\mathbf{q}}\right]\frac{B^0_{-j}\Omega}{(B^0)^2} = \lambda$$

and

$$\lambda\frac{B^t Q^t_{-j}}{(Q^t)^2} - \frac{\gamma B^0}{\xi\Omega - B^0\gamma\boldsymbol{\iota}\cdot\mathbf{q}} - \mu^t = 0.$$

Substituting for λ from the first equation in the second yields

$$\left[\frac{1}{\xi\frac{\Omega}{B^0} - \gamma\boldsymbol{\iota}\cdot\mathbf{q}}\right]\frac{B^0_{-j}\Omega}{(B^0)^2}\frac{B^t Q^t_{-j}}{(Q^t)^2} - \frac{\gamma B^0}{\xi\Omega - B^0\gamma\boldsymbol{\iota}\cdot\mathbf{q}} - \mu^t = 0$$

[3]The inefficiencies identified by Maloney, McCormick and Sauer stem from the fact that regulated off-peak prices are generally higher than the marginal cost of generating electricity, because of guaranteed capital recovery allowances and related administrative overhead. See Balasko [1] for a comparison of off-peak and peak-period pricing relative to capital recovery in a competitive market.

or

$$\frac{B^t Q^t_{-j}}{\left(Q^t\right)^2} = \frac{\gamma\left(B^0\right)^2}{B^0_{-j}\Omega} + \frac{B^0}{B^0_{-j}\Omega}\left[\xi\Omega - B^0\gamma\boldsymbol{\iota}\cdot\mathbf{q}\right]\mu^t.$$

At the symmetric equilibrium, we have

$$\frac{M\left(P-1\right)}{P^2 q^t}\hat{b}^t = \frac{\xi\gamma P^2}{\left(P-1\right)M\omega} + \frac{\xi P}{\left(P-1\right)M\omega}\left[M\omega - P\gamma\boldsymbol{\iota}\cdot\mathbf{q}\right]\mu^t.$$

Substituting for $\hat{b}^t$ and solving this for q^t yields

$$q^t = \frac{\left(P-1\right)^2 M\omega\delta^t}{\gamma P^3 + P^2\left(M\omega - P\gamma\boldsymbol{\iota}\cdot\mathbf{q}\right)\mu^t}.$$

We note in passing that when the capacity constraint does not bind, so that $\mu^t = 0$, this reduces to the expression derived above for q^t. Now, since $q^t = K$ when the capacity constraint is binding, we may use the expression above to solve for the multiplier μ^t as

$$\hat{\mu}^t = \frac{\left(P-1\right)^2 M\omega\delta^t - \gamma K P^3}{KP^2\left(M\omega - P\gamma\boldsymbol{\iota}\cdot\mathbf{q}\right)}.$$

Note that to fully determine $\hat{\mu}^t$ we need to calculate $\boldsymbol{\iota}\cdot\mathbf{q}$. For this, let $\mathbb{T} = \{1,...,T\}$, $\mathbb{H} = \{t|q^t = K\}$ and $\mathbb{L} = \mathbb{T}\backslash\mathbb{H}$. Then

$$\begin{aligned}
\boldsymbol{\iota}\cdot\mathbf{q} &= \sum_{t\in\mathbb{L}} q^t + \sum_{t\in\mathbb{H}} q^t \\
&= \frac{\left(P-1\right)^2 M\omega}{\gamma P^3}\sum_{t\in\mathbb{L}}\delta^t + \sum_{t\in\mathbb{H}} K \\
&= \frac{\left(P-1\right)^2 M\omega}{\gamma P^3}\sum_{t\in\mathbb{L}}\delta^t + \left(\#H\right)K.
\end{aligned}$$

With $q^t = K$, it follows that the peak-period price is

$$\hat{p}^t_H = \frac{B^t}{Q^t} = \frac{P\delta^t}{PK} = \frac{\delta^t}{K}.$$

Renormalizing this to put it in terms of the numeraire, we get

$$\hat{\pi}^t_H = \frac{M\omega\delta^t}{KP}.$$

Note that the period t budget share δ^t such that K is the optimal quantity for an electricity producer to offer is given by

$$\begin{aligned}
\tilde{q}^t &= \frac{\left(P-1\right)^2 M\omega}{\gamma P^3}\delta_t = K \\
&\Rightarrow \quad \tilde{\delta}_t = \frac{\gamma K P^3}{\left(P-1\right)^2 M\omega}.
\end{aligned}$$

At this budget share, the peak-load price is

$$\begin{aligned}\hat{\pi}_H^t &= \frac{M\omega\tilde{\delta}_t}{KP} = \frac{\gamma KP^3 M\omega}{KP\left(P-1\right)^2 M\omega} \\ &= \gamma\left(\frac{P}{P-1}\right)^2\end{aligned}$$

in agreement with the off-peak price at this demand level.

One thing to note about the peak price is that for a fixed number of producer agents P, the peak price increases linearly with the number of electricity consumers, so that the larger the total demand, the larger the peak-load price. This is in contrast with the fact that off-peak prices depend only on the marginal cost of producing power and on the number of producer agents in the market. The validity of this statement obviously depends on holding *per capita* endowments of agents constant as we change the actual number of agents on the demand side of the market, but this seems a reasonably way of asking how peak period prices related to the total demand in the market.

The fact that the peak-period price of electricity in units of account depends only on the capacity and the budget share going to power in period t is something of an artifact of the strong assumptions on preferences in the example. It seems likely that when both producers and standard agents are bidding for the numeraire good, the number of standard consumers will likely enter into the determination of this price. In any case, the example does deliver the clear message that the relative price of electricity during peak periods can be arbitrarily large – even orders of magnitude larger than the off-peak relative price. Thus, the model is consistent with the appearance of large price spikes during peak periods of the kind observed on the California wholesale power markets recently. Finally, we also note that if we let the number of producers and standard agents go to infinity at the same rate, we obtain the competitive equilibrium relative price for the associated competitive log-linear economy

$$\pi_{CH}^t = \frac{\omega\delta^t}{K} \tag{15}$$

which will be smaller than that of the imperfectly competitive model whenever there are more electricity consumers than producers. Balasko [1] notes that in a competitive environment, peak-period equilibrium prices necessarily exceed marginal cost, and need not be unique. Hence, the observation that peak prices exceed marginal cost, or that they differ in similar circumstances is not incompatible with the market being competitive. It is an open question, however, whether reasonable specifications of competitive markets will deliver the kinds of spikes in relative prices that emerge from the imperfectly competitive model. Some answer to this question would appear to be important, considering the views in the media that the price spikes constitute *prima facia* evidence of "market manipulation". If we take "market manipulation" to mean illegal collusive behavior by firms in the market, then the results presented here are

a counter-example to the popular conception, because the imperfectly competitive environment does deliver the observed peak-period price spikes, but they occur as *non-cooperative* outcomes of the market game, which in no way involve collusion to monopolize the market. They are simply a consequence of the optimizing behavior of the small number of firms in the market, and hence, an attribute of the market itself.

3.4 Long-run Analysis

According the publications of the California Public Utilities Commission, electricity demand in California is growing at rates that significantly exceed the growth in generation capacity of electricity suppliers. Indeed, additions to generation capacity during the 1998-99 period plunged by over 90% from their 1996-98 levels. While California's problems have been the most visible, the United States as a whole has suffered from a failure of new generating capacity to keep up with the growth in demand for electricity. Hence, in this section, we develop some results on industry incentives to expand capacity over the long-run when markets are imperfectly competitive.

There are two ways to approach this question. The first involves comparing the equilibrium quantity offers of producers with those that would be offered in a competitive setting. While this comparison is straightforward to make in the model, the interpretation of the result is sensitive to the way in which we evaluate the limits as the number of producers and consumers gets large. To see this, note that the competitive quantity offer of a producer is given by

$$\hat{q}^t_{comp} = \frac{\omega}{\gamma}\delta^t \tag{16}$$

as long as we evaluate the limit by letting M and P approach infinity at the same rate. The ratio of the competitive offer to the imperfectly competitive offer is then given by

$$\frac{\hat{q}^t_{comp}}{\hat{q}^t} = \frac{P^3}{M\left(P-1\right)^2}. \tag{17}$$

If we examine this ratio along a sequence of M's and P's at which $M = P$, we will conclude that imperfectly competitive producers always offer less to the market than would their competitive market counterparts. On the other hand, for any fixed P, by taking M sufficiently large, we can make the ratio above less than 1, in which case, if there is adequate capacity for producers to produce the associated output, we are forced to conclude that the imperfectly competitive firms produce *more* than their competitive counterparts. Because of this ambiguity in interpreting the relationship between the imperfectly competitive and competitive quantity offers, we turn to an examination of the incentives producers have at any given symmertic equilibrium to either expand or contract generating capacity over the long-run in the model. We note that this provides only a partial answer to the question of long-run incentives facing the industry,

since it does not address the question of whether an individual producer has incentives to deviatiate unilaterally (by expanding or contracting capacity) given that other producers keep their capacity fixed. We will show, however, that the industry as a whole not only has no incentive to expand capacity, they have positive incentives to reduce capacity. Hence, in the long-run, the possibility exists that firms in the industry may collude and withhold capacity from the market. Alternatively, if we view the decision on whether to expand capacity in a repeated game setting, one of the possible non-cooperative outcomes (assuming sufficient patience on the part of the firms) is for capacity to remain stagnant relative to demand, or even to contract.

To characterize the incentives facing producers at the symmetric equilibrium, we examine producer consumption, since (given the monotonicity of the utility function) the incentive to expand or contract capacity turns on how this affects consumption. For this analysis, we order the peak-demand budget shares δ^t from lowest to highest and let

$$\mathbb{H}_\tau = \left\{ t \in \mathbb{H} \mid \delta^t \leq \delta^\tau \right\}.$$

When the capacity constraint is binding at some level K_0, a producer agent's consumption is given by

$$\hat{x}^0_{\mathbb{H}} = \frac{M\omega}{P}\left[1 - \left(\frac{P-1}{P}\right)^2 \sum_{t \in \mathbb{L}} \delta^t\right] - \gamma\left(\#\mathbb{H}\right) K_0.$$

It should be clear from this expression that if capacity expands only marginally (so that no peak period is converted into an off-peak period), then this capacity expansion only reduces consumption for producers. Hence, we assume that capacity expansion actually covers at least one peak period of demand. So, if the producer expands capacity to $K_1 = K_0 + \Delta K$ to cover the first τ peak-

periods of demand, her consumption is

$$\begin{aligned}
\hat{x}^0_{\mathbb{H}_\tau} &= \frac{M\omega}{P}\left[1-\left(\frac{P-1}{P}\right)^2\left(\sum_{t\in\mathbb{L}}\delta^t+\sum_{t\in\mathbb{H}_\tau}\delta^t\right)\right]-\gamma\left(\#\mathbb{H}-\tau\right)K_1-\rho\Delta K \\
&= \frac{M\omega}{P}\left[1-\left(\frac{P-1}{P}\right)^2\sum_{t\in\mathbb{L}}\delta^t\right]-\gamma\left(\#\mathbb{H}\right)\left(K_1\right)- \\
&\quad -\frac{M\omega}{P}\left(\frac{P-1}{P}\right)^2\sum_{t\in\mathbb{H}_\tau}\delta^t+\gamma\tau K_1-\rho\Delta K \\
&= \frac{M\omega}{P}\left[1-\left(\frac{P-1}{P}\right)^2\sum_{t\in\mathbb{L}}\delta^t\right]-\gamma\left(\#\mathbb{H}\right)K_0- \\
&\quad -\frac{M\omega}{P}\left(\frac{P-1}{P}\right)^2\sum_{t\in\mathbb{H}_\tau}\delta^t+\gamma\tau K_1-\left[\gamma\left(\#\mathbb{H}\right)+\rho\right]\Delta K \\
&= \hat{x}^0_{\mathbb{H}}-\frac{M\omega}{P}\left(\frac{P-1}{P}\right)^2\sum_{t\in\mathbb{H}_\tau}\delta^t+\gamma\tau K_1-\left[\gamma\left(\#\mathbb{H}\right)+\rho\right]\Delta K \\
&= \hat{x}^0_{\mathbb{H}}-\gamma\sum_{t\in\mathbb{H}_\tau}\hat{q}^t+\gamma\tau K_1-\left[\gamma\left(\#\mathbb{H}\right)+\rho\right]\Delta K.
\end{aligned}$$

The gain to expanding capacity is then

$$\hat{x}^0_{\mathbb{H}_\tau}-\hat{x}^0_{\mathbb{H}}=\gamma\tau K_1-\left[\gamma\left(\#\mathbb{H}\right)+\rho\right]\Delta K-\gamma\sum_{t\in\mathbb{H}_\tau}\hat{q}^t.$$

This will be non-negative as long as

$$\gamma\tau K_1\geq\left[\gamma\left(\#\mathbb{H}\right)+\rho\right]\Delta K+\gamma\sum_{t\in\mathbb{H}_\tau}\hat{q}^t$$

or

$$\gamma\tau K_1-\gamma\sum_{t\in\mathbb{H}_\tau}\hat{q}^t\geq\left[\gamma\left(\#\mathbb{H}\right)+\rho\right]\Delta K.$$

Since the capacity K_1 covers the demands in the sub-periods H_τ, it follows that $K_1=\sup\left[\hat{q}^t\right]_{t\in H_\tau}$. Let t^* be the period in which the supremum is attained, so that $K_1=\hat{q}^{t^*}$. The condition above can then be written as

$$\hat{q}^{t^*}-\frac{1}{\tau}\sum_{t\in\mathbb{H}_\tau}\hat{q}^t\geq\frac{\left[\gamma\left(\#\mathbb{H}\right)+\rho\right]}{\gamma\tau}\Delta K.$$

Now, let

$$\bar{q}_\tau=\frac{1}{\tau}\sum_{t\in\mathbb{H}_\tau}\hat{q}^t.$$

Then, since $\Delta K = \hat{q}^{t^*} - K_0$, we have

$$\hat{q}^{t^*} - \bar{q}_\tau \geq \frac{[\gamma(\#\mathbb{H}) + \rho]}{\gamma\tau}\hat{q}^{t^*} - \frac{[\gamma(\#\mathbb{H}) + \rho]}{\gamma\tau}K_0$$

or

$$\left[\frac{\gamma(\#\mathbb{H}) + \rho}{\gamma\tau} - 1\right]\hat{q}^{t^*} + \bar{q}_\tau \leq \frac{[\gamma(\#\mathbb{H}) + \rho]}{\gamma\tau}K_0$$

or

$$K_0 \geq \left[1 - \frac{\gamma\tau}{\gamma(\#\mathbb{H}) + \rho}\right]\hat{q}^{t^*} + \frac{\gamma\tau}{\gamma(\#\mathbb{H}) + \rho}\bar{q}_\tau.$$

Since the right-hand-side is a convex combination of things strictly larger than K_0, it follows that no producer will wish to expand capacity. In fact, as the following calculation shows, the industry will generally have incentives to *reduce* capacity if it can.

As before, a producer's consumption given some capacity K_0 is

$$\hat{x}^0_{\mathbb{H}} = \frac{M\omega}{P}\left[1 - \left(\frac{P-1}{P}\right)^2 \sum_{t\in\mathbb{L}} \delta^t\right] - \gamma(\#\mathbb{H})K_0.$$

If the producer reduces capacity just sufficiently to make the highest off-peak demand a peak demand, then

$$K_1 = \sup_{t\in\mathbb{L}} \left\{\hat{q}^t\right\} = \hat{q}^{t^*}.$$

The producer's consumption after the capacity reduction will be

$$\hat{x}^0_{\mathbb{H}^+} = \frac{M\omega}{P}\left[1 - \left(\frac{P-1}{P}\right)^2 \sum_{t\in\mathbb{L}^-} \delta^t\right] - \gamma(\#\mathbb{H} - 1)K_1.$$

Here, we use $\mathbb{H}^+$ to denote the expanded set of peak periods, and $\mathbb{L}^-$ to denote the reduced set of off-peak periods. Note also that we are assuming that there is no after-market for capacity, so that if a producer reduces capacity by one kilowatt-hour, she does not recover the investment cost ρ. Now, rewrite this expression as

$$\begin{aligned}
\hat{x}^0_{\mathbb{H}^+} &= \frac{M\omega}{P}\left[1 - \left(\frac{P-1}{P}\right)^2 \sum_{t\in\mathbb{L}^-} \delta^t - \left(\frac{P-1}{P}\right)^2 \left[\delta^{t^*} - \delta^{t^*}\right]\right] - \\
&\quad -\gamma(\#\mathbb{H} - 1)[K_0 - \Delta K] \\
&= \frac{M\omega}{P}\left[1 - \left(\frac{P-1}{P}\right)^2 \sum_{t\in\mathbb{L}} \delta^t + \left(\frac{P-1}{P}\right)^2 \delta^{t^*}\right] - \\
&\quad -\gamma(\#\mathbb{H})K_0 + \gamma K_0 + \gamma(\#\mathbb{H})\Delta K - \gamma\Delta K \\
&= \hat{x}^0_{\mathbb{H}} + \frac{M\omega}{P}\left(\frac{P-1}{P}\right)^2 \delta^{t^*} + \gamma K_0 + \gamma[\#\mathbb{H} - 1]\left[K_0 - \hat{q}^{t^*}\right] \\
&= \hat{x}^0_{\mathbb{H}} + \gamma\hat{q}^{t^*} + \gamma[\#\mathbb{H}]K_0 - \gamma[\#\mathbb{H} - 1]\hat{q}^{t^*} \\
&= \hat{x}^0_{\mathbb{H}} + \gamma[\#\mathbb{H}]K_0 - \gamma[\#\mathbb{H} - 2]\hat{q}^{t^*}.
\end{aligned}$$

It now follows that

$$\hat{x}^0_{\mathbb{H}^+} - \hat{x}^0_{\mathbb{H}} = \gamma\left[\#\mathbb{H}\right] K_0 - \gamma\left[\#\mathbb{H} - 2\right] \hat{q}^{t^*}.$$

Hence, the gain to consumption will be non-negative as long as

$$\gamma\left[\#\mathbb{H}\right] K_0 - \gamma\left[\#\mathbb{H} - 2\right] \hat{q}^{t^*} \geq 0$$

or

$$K_0 \geq \frac{\#\mathbb{H} - 2}{\#\mathbb{H}} \hat{q}^{t^*}$$

which is always true. Hence, the industry always has an incentive in the long-run to reduce capacity.

The analysis presented here obviously depends heavily on the assumption that the technology exhibits constant returns to scale, and the results on long-run incentives to expand capacity will change if such expansion exhibits any kind of increasing returns effect. On the other hand, the constant returns to scale assumption does appear to be consistent with the emergence of relative low cost simple and combined cycle gas turbine generation technologies.

Given the constant returns assumption, one question we can ask is how competition affects the incentive to expand capacity. While the current model doesn't give a complete answer to this question, it can provide some partial answers. In particular, if we keep the level of demand fixed by fixing the number of electricity consumers M, then as the number of producers increases, the outputs q^t of each producer decrease. If the number of producers is large enough, then, the capacity constraints in the model will not bind, and each producer's consumption becomes independent of her investment in generating capacity. Hence, to the extent that entry into the generation market provide sufficient capacity to cover all demand periods, the incentive to reduce capacity goes away. Of course, the question the current model cannot answer is where new entrants to the generating sector come from and in response to what incentives. Thus, one subject of future research will be to model the entry decision by allowing standard agents to pay a certain cost in exchange for transforming themselves from electricity consumers to electricity producers.

3.5 Robustness Issues

An obvious question about the specification of the model outlined here is whether the results it yields are robust to perturbations in the specification of the model. The simplifying assumption that producer agents don't themselves directly consume electricity is useful in that it provides an explicit separation of the supply and resulting pricing decisions of producers from their interests as consumers. This does not seem unreasonable, since, in a real world setting, the supply and pricing decisions would be corporate, and while corporate employees might not be happy (as consumers) with the resulting outcomes, this would not be likely to have much effect on the corporate entity. On the other hand, the fact that

only producers purchase the numeraire good makes it impossible to examine the limiting case of a monopoly producer, since the model has a singularity at $P = 1$. The additive separability in the specification of preferences seems to be fairly standard in the empirical literature which tries to estimate time-of-day electricity demands from micro data, and seems reasonably innocuous as an assumption about short-run demand. It is also straightforward to show that the specific log-linear specification can be relaxed to allow for additively-separable CRR preferences of the form

$$u_h = \sum_{t=1}^{T} \alpha_t \frac{[x_h^t]^{1-\eta}}{1-\eta}$$

without changing the basic results derived above, except for a re-weighting of the budget shares δ^t. Similarly, because producers only consume the numeraire good, their optimal bids and offers are independent of the actual specification of the utility function (since monotonic transformations yield the same demands). Hence, in the context of the extended example, we can view producers are caring only about profit maximization, although this will not be robust in more general environments. There are obvious directions in which these specifications can be relaxed, for example by eliminating the homotheticity assumption and allowing the exponents in the CRR specification above to differ over time periods, or by permitting producers to consume electricity, but each of these generalizations comes at the price of reduced tractability of the model.

4 Conclusions and Policy Implications

The general equilibrium analysis of horizontal market power in the market for electricity via the market game model has a number of important implications for the on-going process of deregulating electricity markets.

The model examined here suggests that limited competition among generators during off-peak periods will lead to prices which exceed marginal cost, in line with a number of other studies of horizontal market power in electricity markets based on partial equilibrium frameworks. The extent of the mark-up depends on the number of generating firms in the market; when there are 20 producers in the model, the mark-up is around 11%. With 50 firms, this drops to below 5%.

A more interesting feature of the model is its ability to match the peak-period price-spike phenomenon observed in a number of wholesale electricity markets. These spikes can be quite large if the number of peak-period electricity consumers is large relative to the number of producers. As in the case of off-peak prices, having more producers reduces the price spike effect, although some spiking of prices will always be present (even in perfectly competitive environments) during peak periods since these prices are the only mechanism by which demand is rationed during these periods.

Finally, the model also demonstrates a clear incentive for producers to reduce capacity when possible, which may in part explain the extreme drop in

additions to new capacity in California and elsewhere in the period following implementation of deregulation.

From a policy perspective, horizontal market power in markets for electricity poses a challenge to the process of deregulation, and raises the question of whether deregulation is proceeding too quickly or even whether it should be rolled back. If the deregulation process results in a shake-out in which generating capacity ends up concentrated in the hands of a few large firms, then recent events in California may prove to be the norm rather than the exception. This scenario could occur, for example, if policymakers end up encouraging reliance on large fixed cost generating technologies, such as coal-fired or nuclear power plants. A similar result could obtain if the natural gas industry becomes significantly concentrated and ends up increasing the price of natural gas for combined cycle gas turbine generators, making the older but larger plants more economical. An obvious question here is whether increased diligence by antitrust authorities can reduce these problems. While attention to such obvious abuses as taking generators offline for unneeded maintenance is warranted, the fact remains that most of the undesired effects in the model (such as peak-period price spikes or the incentive to reduce capacity) are simply the result of agent's looking after their own interests in an imperfectly competitive, but non-cooperative environment.

Perhaps the clearest policy implication to come out of the model is the need to encourage competition as the electricity industry restructures itself. Competition has beneficial effects both on pricing and on the long-run decision to invest in new capacity. Fostering competition can be accomplished during restructuring by requiring not only divestiture of generation assets by utilities during the phase when generation is separated from distribution, but also by maximizing the number of new generating firms created in the process, subject, of course, to the technical constraints imposed by the number and nature of existing generating plants. While it isn't clear whether a broader divestiture could have been organized in the California market, there was at least one instance in which generating assets from different utilities (Pacific Gas and Electric, and Southern California Edison) were sold to the same company (NRG Energy; see the California Energy Commission's web site http://www.energy.ca.gov/electricity/divestiture.html). There have also been indications in the business press that continued regulatory impediments to new power plant construction (based on community siting and environmental concerns) significantly raised the cost of entry into the California generation industry. Even so, the fact that other regions of the country which impose less regulatory burden on new power plant construction have experienced similar reductions in new investment in generating capacity suggests that policymakers need to pay increased attention to fostering competition as part of the deregulatory process.[4]

[4]The issue of stranded costs obviously also has important implications for competitiveness in the electricity market, since the deadweight cost increases associated with compensating utilities for prior investments which are no longer economically viable also adds to the cost of entry for new (and more efficient) generators.

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