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# TOLLS: EFFICIENCY AND EQUITY ISSUES FOR INLAND WATERWAYS*

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Transportation requires large amounts of capital in fixed facilities such as roads, tracks, or canals. Inland waterway transport is unique among modes, since it neither owns its right-of-way nor pays taxes to support its construction and maintenance. Although user charges are widely employed to recover the costs of publicly provided investments, proper theoretical foundation has not been made for their application to inland waterway transport. In this paper we shall provide this foundation focusing on efficiency of allocation of resources, on equity of taxation, and on administration simplicity of each toll scheme.

§1 of this paper presents the criteria to be used in evaluating waterway user charges. §2 appraises possible alternative tolls and estimates the rates that would be needed to recover current expenditures. A combination of toll schemes (segment tolls, locking fees, and congestion tolls) are argued to be economically and politically feasible and to dominate fuel taxes and license fees. The Appendix presents an example of the benefits and costs of waterway expansion and a test of the predictive power of the queueing model.

## 1. Criteria for Evaluating User Charges

In this section we examine the basis for assessing user charges as well as three criteria by which alternative user charge methods can be evaluated.

### *Efficiency*

Efficiency requires that prices be set equal to long-run marginal costs for all modes. If price differs from marginal cost for one mode, four inefficiencies may result: (1) that mode may supply too much or too little service, (2) that mode will be advantaged or disadvantaged relative to other modes, (3) that mode may haul an inappropriate set of commodities, and (4) that mode may choose an inefficient mix of inputs. In a competitive economy, prices are driven to marginal cost. Market imperfections, inequitable taxes, or regulation cause distortions of the sort noted above. Unfortunately, it is not necessarily true that setting price equal to marginal cost for one mode will enhance efficiency if other modes are not doing so.[1]

### *Equity*

Equity requires that individuals pay in proportion to the services they receive. Inequity is sometimes a deliberate goal, as with some welfare programs or progressive taxation. For waterways, it is argued that virtually everyone consumes products transported on inland waterways, and so the benefits of lower prices are passed along to all consumers. Since all consumers benefit from cheap water transportation, all should be called upon to pay the taxes which finance it.

While plausible, the fact is that not all taxpayers benefit equally. Residents of the mountain states and those not buying the transported products benefit little but must still pay taxes. The inefficiency resulting from different pricing policies among the modes also causes inequity.

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[1] In technical terminology, there exists a second-best situation where all criteria for Pareto optimality cannot be obtained. See Otto Davis and Andrew Whinston [2]. However, it seems likely that, for shipment of bulk commodities, marginal cost pricing prevails.

*Administrative Simplicity*

The third goal for a user charge system is administrative simplicity. Administrative costs must be considered in determining whether user charges should be imposed, and if so, which system should be selected.

*Three Archetypical Waterways*

To clarify the arguments, consider three abstract situations; we neglect the private expenditures on each tow (fuel, etc.) and focus on social costs. (a) The first archetype is an existing waterway which requires no annual maintenance or operating expense and which has no congestion. (b) The second archetype is an existing waterway which does require annual expenditures for operations and maintenance, which may or may not experience congestion problems. (c) The third archetype is a proposed waterway or an existing waterway for which improvements are proposed.

If there is no congestion on the first waterway and no annual expenses, the marginal cost of providing access to a tow is zero. The Gulf of Mexico is such a waterway, and the cost of another ship using the Gulf, both short and long-run is zero. Any tax on users of this waterway will distort resources and work against both efficiency or equity. Past public investments are "sunk costs" and presumably should be forgotten.[2]

For the second waterway, assume there is no congestion. If the annual expenses are not related to the passage of a tow (i.e., maintenance is independent of usage), the marginal cost of a tow is zero. In this case the waterway is a pure public good, and equity and efficiency principles are in conflict: Any charge to waterway operators will curtail use and cause inefficient resource allocations; paying for the annual expense out of general tax revenues gives rise to inequity. The industry emphasizes intramodal efficiency and calls for "forever free" waterways. Economists and past administrations have emphasized equity and intermodal efficiency.

If there is congestion on this second type of waterway, a nonpecuniary externality is present. Congestion will be discussed in detail below, but it should be noted that a congestion toll would serve both to raise revenue and to allocate resources better. Note that in either case each tow should be charged for any increased maintenance it imposes on the system.

The third archetypical waterway requires construction expenditures. Equity requires that users repay construction costs; efficiency requires that users pay the long-run marginal cost of use; there is no guarantee that equity and efficiency will result in the same toll.

*When Should Waterways Be Closed?*

Early in the 19th century, the large canal system in the northeastern U. S. became unprofitable as railroads were constructed; almost all canals were abandoned. Some modern waterways are so expensive to operate and maintain, given the volume of traffic, that they should be closed.

However, the decision to close a waterway is not simply a matter of comparing current costs to toll revenues. (1) Toll collections necessarily fall short of the total value of navigation benefits because of difficulties in collection and the inability to

[2] Such costs, however, may not be sunk in a general equilibrium context. Water carriers would have lower rates than if they had to pay for past public investments; this difference could result in competitive cargoes going by water rather than rail or could lead to cross-subsidization with other cargoes being charged higher rates to make up for losses (marginal cost equal to marginal revenue which is less than average cost) on water competitive cargoes. Secondary effects could extend to other competitive relationships with other modes, between commodities, locations, etc. If the essential characteristic of a sunk cost is that it not impinge on current resource allocation, such past waterway investments are not sunk at all.

extract all economic rent. (2) Waterways have long lives, and so one must look at the present discounted value of the stream of benefits and costs, not at just current values. (3) Waterways have other benefits, including recreation and flood control. Thus, a waterway should be closed if the present discounted value of all benefits, recreational as well as navigational, not just those that can be collected, is less than the present discounted value of all costs. Even though total benefits are difficult to estimate, a number of U. S. waterways are canditates for closure. (Table 1 shows the operating costs for each waterway segment.)

TABLE 1
*Segment Ton-Mile Taxes Required to Recover Operation and Maintenance Expenditures*

| Waterway Segment | Average Annual O & M Expenditures Fy 1970–74 (000 Dollars) | Actual Ton-Miles Cy 1972 (000) | O & M Cost per Ton-Mile (Mills) | Implied Toll per Mile for Loaded 1500-Ton Barge (Dollars) |
|---|---|---|---|---|
| Allegheny River | $ 990 | 80,025 | 12.4* | $ 18.60 |
| Apalachicola-Chattahoochee | 2,028 | 108,210 | 18.7* | 28.05 |
| Arkansas-Verdigris Waterway | 12,657 | 520,887 | 24.3* | 36.45 |
| Atlantic Intracoastal Wtrwy (Norfolk-Miami) | 3,265 | 691,882 | 4.72 | 7.08 |
| Columbia River (Pasco vicinity-The Dalles) | 17,225 | 2,688,815 | 6.4* | 9.60 |
| Cumberland River | 915 | 882,021 | 1.04 | 1.56 |
| Green and Barren Rivers | 852 | 1,467,228 | 0.53 | 0.87 |
| Gulf Intracoastal Waterway | 5,292 | 19,035,605 | 0.27 | 0.41 |
| Illinois Calsag Waterway (Indiana Harbor-Miss. River) | 4,723 | 8,350,420 | 0.57 | 0.86 |
| Kanawha River | 1,294 | 815,333 | 1.59 | 2.39 |
| Kentucky River | 1,084 | 39,748 | 27.3* | 40.95 |
| Lower Mississippi River (Cairo-Baton Rouge) | 7,953 | 60,563,824 | 0.13 | 0.20 |
| Missouri River | 12,363 | 1,280,385 | 9.7* | 14.55 |
| Mobile-Tombigbee-Warrior | 2,102 | 3,816,879 | 0.55 | 0.83 |
| Monongahela River | 2,271 | 1,528,000 | 1.49 | 2.24 |
| Ohio River | 12,984 | 32,055,618 | 0.41 | 0.62 |
| Ouachita-Black Rivers | 1,649 | 101,949 | 16.2* | 24.30 |
| Tennessee River | 2,171 | 3,755,867 | 0.58 | 0.87 |
| Upper Mississippi River (Minneapolis-Cairo) | 10,027 | 22,304,761 | 0.45 | 0.68 |
| Willamette River | 2,369 | 20,640 | 114.8* | 172.20 |
| Total | 104,214 | 160,108,097 | 0.651 | 0.98 |
| Total of high-cost segments | 50,365 | 4,840,659 | 10.4 | 15.6 |
| % High-cost segments of total | 48% | 3% | | |

* High-Cost Segments.
Sources: U. S. Army [7].

## 2. Alternative User Charge Systems

Several user charge systems have been proposed for waterways. We describe each system and assess its ability to meet the criteria of efficiency, equity, and administrative simplicity. Tax levels will be estimated on the assumption that demand is completely inelastic. Taking account of demand elasticities and possible adjustments in the production function would increase these estimates 25–50 percent.

*The Fuel Tax*

Taxes levied on gasoline have paid for the interstate highway system and other highways. Many administrations have proposed a fuel tax for inland waterways ranging from \$0.02 to \$0.10 a gallon. At 1972 fuel consumption rates, a tax of 10.7¢ per gallon would be required to cover the \$112 million annual Federal operation and maintenance expenditures. To cover investment expenditures as well would require a tax of 31.8¢ per gallon.

The principal advantage of the fuel tax is administrative simplicity. It would be easy and cheap to collect these taxes. The tax would violate the efficiency criterion, however. Because it does not discriminate among different waterways, it would lead users of a few heavily-used waterways to subsidize the users of the little-used waterways. This distortion, however, is partly the result of keeping too many waterways open.[3] If the little-used waterways were closed, the inefficiencies caused by the fuel tax would be considerably reduced.

A large increase in the fuel price would also lead to reallocations within the industry, e.g., smaller towboats. The increase in the price of fuel, relative to other factor prices, would distort factor allocation. Finally, fuel taxes would not directly affect congestion. Indirectly, of course, they would reduce congestion in the aggregate sense since they would reduce total traffic.

*The Annual License Fee*

Another tax method used in paying for highways is the annual license fee. If the fee were determined by cargo capacity, jumbo barges would be charged \$5,600 annually to cover just operation and maintenance costs. If new construction costs were recovered, a fee of over \$16,000 per barge would be charged, a substantial proportion of the purchase price. Were the tax on horsepower, the annual fee would be \$372,000 to \$1,116,000 for a 6,000 horsepower towboat.

Administratively, the license tax is similar to the fuel tax since it is easy and cheap to administer. Equity and efficiency criteria are similar. The incidence of the tax is not related to the cost incidence of the waterways, so it could lead to cross-subsidization between waterways. Like the fuel tax, the license tax would motivate users to exert pressure to discontinue expenditures on uneconomic waterways.

The license tax would distort the efficiency of production. A tax on barge capacity would lead to fewer barges and thus a relative scarcity. A tax on horsepower would be similar to the fuel tax in leading to smaller vessels with higher productivity per horsepower.

The license tax would not directly reduce congestion. Like the fuel tax, it could indirectly reduce congestion by reducing traffic in the aggregate. If an equal license fee were imposed on all vessels, it could also reduce congestion indirectly by leading to fewer but larger vessels.

*The Segment Toll*

On turnpikes, users are charged for passing certain points (with the toll based on cargo capacity, value of cargo, or other criteria). Such a segment toll could offer the advantage of tying toll collection to expense by waterway. In Table 1 are data on the amount of cargo and the costs of operation and maintenance by waterway, along with estimated segment tolls. The tax per ton-mile varies from about 0.1 mill on the Lower Mississippi to 115 mills on the Willamette. The average segment toll of 0.65 mills per ton-mile would drop to 0.35 mills per ton-mile if some little-used segments were abandoned (representing 3% of traffic).

Segment tolls would be relatively easy to collect since tows could be identified as they transited locks or other points on a waterway. Alternatively, tows must keep logs

[3] Moreover, insofar as reduced expenditures would be reflected in reduced fuel tax rates, users would be motivated to scrutinize proposed new investments in waterway projects and exert pressure to avoid funding uneconomic projects.

of their movements, and so it would be easy to charge them per mile of travel in each segment. Waterways are currently defined in terms of expenditures on maintenance, navigation, and construction; these waterway definitions could serve as the toll segments. Segment tolls would achieve one aspect of efficiency that neither of the previous two tolls could achieve; they would equate marginal costs and revenues by waterway. Also, they would not distort resource allocation within the industry. However, they would do little to curtail congestion. While the effect of any charge would be to curtail traffic, segment tolls would be lightest on the most heavily-used (congested) waterways and heaviest on the lightly-used (uncongested) ones.

*Lockage Fees*

Waterways might be financed by charging a fee to transit a lock. The lockage fees would be on the order of \$100 to \$1,000 to recover operation and maintenance costs of the system and about three times as high to cover construction costs as well.

The effects of lockage fees on resource allocation would depend on how they were applied. Clearly, there are some costs directly associated with a lockage; thus, lockage fees could be set at the marginal cost of one lockage. This method would be efficient if combined with a segment toll to recover costs not associated with a lockage. It makes little sense to use only lockage fees, since not all waterways have locks (e.g., the lower Mississippi).

Lockage fees might also be set to approximate congestion tolls, as discussed in the next section, along with efficiency implications. Lockage fees are administratively easy to collect but would lead to distortions if used as the sole basis for recovering costs.

*Congestion Tolls*

A tow frequently finds that it must wait for others to be served before transiting a lock. As more tows attempt to use the waterway, the average waiting time at locks increases and raises costs. A modern towboat costs about \$2.5 million with nearly as much capital in the barges and cargo. The cost of keeping such a towboat waiting along with its flotilla of barges will be in excess of \$200 per hour.[4]

When congestion exists, each tow imposes higher costs on all other tows. Waiting can be reduced by charging a congestion toll equal to the entire additional cost each tow imposes on the system.

For a waterway with locks, congestion is determined by the capacity of the locks; i.e., tows will be queued at the locks waiting for service as more tows attempt to use the waterway. We assume that the number of tows arriving at any lock during an hour is a random variable, as is the number of tows that can be served in an hour. For mathematical simplicity, we assume that both of these stochastic distributions are Poisson (the assumption is tested in the Appendix). Let $K$ be the number of tows demanding service during a year:[5] $\lambda = K/8760$ is the number demanding service during an hour. Let $1/\mu$ be the expected time (hours) taken by the lock to serve a tow. Then from queueing theory, we know that the expected waiting time in queue, $T_{LQ}$, and total locking time, $T_L$, are

$$T_{LQ} = \lambda/\mu(\mu - \lambda), \quad T_L = 1/\mu + T_{LQ} = 1/(\mu - \lambda). \tag{1}$$

Some illustrative calculations for a wide range of $K$ are shown in Table 2.[6] Note that $\rho$ is the percentage of time the lock is being utilized (assuming that the locking

[4] Labor expenses would be more than \$100 per hour for the crew while the cost of delay for towboat, barges, and equipment would be almost \$100 per hour.

[5] The arrival rate is assumed to be uniform over time of day, day of week, and season. While the size of tows may vary and while ice sometimes stops traffic, this assumption is a reasonable approximation.

[6] L. Lave and J. DeSalvo [4].

TABLE 2
*Congestion Toll Calculations*

| $K$ | $\rho = \frac{100\lambda^*}{\mu}$ | $T_L$ | $T$ | $\frac{\partial T}{\partial K}$ | $\theta$ | $\frac{-\partial T}{\partial \mu}$ |
|---|---|---|---|---|---|---|
| 1 . . . | 0.01 | 1.00 | 1 | 1.00 | 0.00 | 1 |
| 100 . . . | 1.14 | 1.01 | 101 | 1.02 | 0.01 | 102 |
| 1,000 . . . | 11.40 | 1.13 | 1,129 | 1.28 | 0.15 | 1,280 |
| 2,000 . . . | 22.90 | 1.30 | 2,592 | 1.69 | 0.39 | 3,380 |
| 2,190 . . . | 25.00 | 1.33 | 2,920 | 1.77 | 0.44 | 3,876 |
| 3,000 . . . | 34.30 | 1.52 | 4,563 | 2.31 | 0.79 | 6,930 |
| 4,000 . . . | 46.70 | 1.84 | 7,361 | 3.39 | 1.55 | 13,560 |
| 5,000 . . . | 57.10 | 2.33 | 11,648 | 5.43 | 3.10 | 27,150 |
| 6,000 . . . | 68.50 | 3.18 | 19,043 | 10.11 | 6.93 | 60,660 |
| 7,000 . . . | 80.00 | 4.98 | 34,841 | 24.80 | 19.82 | 173,600 |
| 8,000 . . . | 91.40 | 11.52 | 92,210 | 132.71 | 121.19 | 1,061,680 |
| 8,750 . . . | 99.90 | 876.00 | 766,500 | 767,376 | 766,500 | 6,714,548,000 |

Notes:

$\mu$: average number of tows serviced per hour by the lock (to illustrate the calculation, $\mu$ is assumed equal to unity).

$K$: number of tows serviced per year; $\lambda^*$ is the number of tows serviced per hour.

$\rho$: utilization rate of lock (the lock is operating $\rho$ percent of the time); $\rho/100 = K/(8{,}760\mu) = \lambda^*/\mu$.

$T_L$: average locking time (including waiting time) in hours. $T_L = 8{,}760/(8{,}760\mu - K) = 1/(\mu - \lambda^*)$.

$T$: total locking time per year for all tows in hours. $T = KT_L = 8{,}760K/(8{,}760\mu - K) = 8{,}760\lambda^*/(\mu - \lambda^*) = K/(\mu - \lambda^*)$.

$\partial T/\partial K$: marginal locking time for another tow: the change in total locking time due to the addition of a tow. $\partial T/\partial K = 8{,}760/(8{,}760\mu - K)^2\mu = T_L^2\mu$.

$\partial\gamma/\partial\mu$: marginal locking time for improved lock: the change in total locking time due to an increase in the service rate of the lock. $\partial\gamma/\partial\mu: -8{,}760/(8{,}760\mu - K)^2K = T_L^2K$.

$\theta(\partial T/\partial K) - T_L\theta V$ is the optimal toll (where $V$ is the hourly cost of keeping a tow waiting). This toll equates private cost and social cost.

rate, on the average, is one tow per hour). Also shown is $T = KT_L$, the total locking time for all tows during the year; it rises more than proportionately as tows are added. The increase in locking time due to an extra tow $\partial T/\partial K$ is shown in Table 2.

This $\partial T/\partial K$ is the total cost that an extra tow on the waterway would be imposing on the system. The cost is stated in terms of extra hours that all tows are delayed.

The price that each tow currently pays for using the waterway is its average locking time, $T_L$. Note the $T_L$ rises rapidly, but not as rapidly as $\partial T/\partial K$. The cost that a tow should perceive in its decision to use a waterway is $V(\partial T/\partial K)$, where $V$ is the cost of an hour's delay; the cost it actually perceives is $T_L$. Under the assumption that all tows have a uniform value per hour of $V$ and that they are served on a first come, first served basis, the difference between these two is the congestion toll needed to optimize resource allocation. This is also shown in Table 2.

All these calculations and derivations are carried out under the assumption that all tows have a uniform value per hour of $V$ and that they are served on a first come, first served basis. If either of these assumptions is violated, the congestion toll is no longer optimal, and one of the more complex schemes outlined in the next section is required.

The next logical question concerns improvement of the lock: At what level of utilization or at what amount of toll collection should the lock be improved so that it can serve more tows? To calculate this, we need to know $\partial T/\partial\mu$, the rate at which waiting time changes as the service rate is changed; this is calculated in Table 3. Calculus is not a good tool for this problem since changes in the service rate can probably only occur in discrete steps, whereas calculus assumes infinitesimal changes.

TABLE 3

*The Benefit of Lock Expansion*

| $K$ | $T_L(K)$ | $T_L^*(K)$ | $B = VK\Delta T_L$ |
|---|---|---|---|
| 3,000 . . . | 1.521 | 0.363 | $ 347,200 |
| 4,000 . . . | 1.840 | 0.415 | 568,100 |
| 5,000 . . . | 2.327 | 0.690 | 819,800 |

Notes:

$K$: number of tows serviced per year; $T_L(K)$: total locking time (waiting plus service) per tow; $T_L^*(K)$: total locking time per tow, assuming the service rate ($\mu$) has doubled.

$B$: the estimated benefit of lock expansion (assuming demand is fixed).

Instead, the benefit should be calculated as the savings in total locking time, $B = (T_L(K) - T_L^*(K))KV$, where $T_L^*$ is total locking time after the improvement.

*Congestion Toll Procedures*

If tows have different costs associated with waiting, "first come, first served" can be improved upon.[7] Assuming that all bidders act independently, auctioning places in the queue would result in both the greatest toll revenue and the best resource allocation. Marginal tows could wait and pay little while priority cargo could avoid delay. If all tow-masters bid the same amount per hour to save an hour of waiting time, the procedure would reduce to the lockage fee described above.[8]

A second procedure would charge each tow $200 for each hour of delay caused each tow in back of it (unless it vacated its position). No bidding would be required, and so it would be administratively simpler; it would be less efficient however, since the tow with the greatest cost for delay would not be locked through first. Since both these procedures involve discretion, they involve greater administrative costs and open the possibility of collusion.

A third procedure would charge $200 per hour for delaying other tows without provision for vacating one's position; this is the congestion toll shown in the table. A final scheme would charge tows based on the average wait during a period (such as daylight hours during winter). The third and fourth procedures are average tolls or lockage fees that would eliminate uncertainty about the amount of the toll and be simple administratively, but they would not enable marginal towboats to pass free or allow priority tows to go first.

*Equity and Congestion Tolls*

The congestion toll is designed to reduce the resource costs of congestion; it need not serve the same function as a user charge, since it might produce either more or less revenue than public expenditures for operation and maintenance costs of the waterways and construction costs. As shown in considering a fuel tax, equity does not necessarily solve the congestion or other efficiency problems.

## Conclusion

No one type of toll avoids all problems of efficiency, equity, and administrative simplicity. Efficiency requires that a tow be charged (a) its effect on annual

[7] For other treatments, see Kleinrock [3], Naor [5], and Yechiali [8], [9].

[8] If all towmasters bid $200 per hour to save delay, they would be charged $200 multiplied by the actual number of hours they delayed other tows. The queueing formula used to derive the optimal toll is based on calculating the expected wait for each tow. We anticipate that the actual waiting times would average out to the expected waiting time.

maintenance, (b) the marginal cost of going through a lock, and (c) the cost of the additional waiting it imposes on other users because of congestion. Administrative simplicity demands that these charges be easily computed and assessed. Equity requires that the maintenance and additional investment costs of a waterway segment be paid by its users. Yet charging tows more than the sum of the three charges noted above violates the efficiency criterion.

If efficiency is the prime concern, a segment toll plus locking fee and congestion toll —suitably averaged to attain administrative simplicity—are best. If equity and simplicity are the prime concerns, the segment toll might be set to bring total receipts up to the maintenance and investment cost associated with each segment.

The other proposed user-charge plans (fuel taxes or license fees) are dominated by these proposals. While they might offer slight advantages in terms of administrative simplicity, these advantages are small compared to the substantial distortions in equity and efficiency.

## Appendix

*An Application: The Illinois Waterway*

An example might serve to clarify this analysis. A study was undertaken to determine whether lock capacity on the Illinois waterway should be expanded [6]. A second lock, twice the size of the existing one, was proposed to be installed parallel to the old lock at each of the seven existing structures. It was assumed that the installation of this second lock would double the service rate. It is possible that the service rate might be more or less than doubled since the narrowness of the channel means that both locks would experience restricted operations (physical lock capacity actually triples under this plan). The study determined an annual cost for the lock expansion which averaged $656,000 for each of the seven locks on the Illinois waterway.

Suppose there were a single lock on the waterway and that the cost of keeping a tow waiting were $100 per hour. Then the lock should be improved if at least 6,500 tow-hours could be saved per year. Such a savings would be obtained if $K = 5{,}000$. For this number of trips, the average locking time $T_L$ would be 2.33 hours at the old service rate of $\mu = 1$ per hour. At the new service rate of two tows per hour, $T_L$ would be 0.69 hours. The improvement would result in a saving of 8,200 tow-hours per year as calculated:[9] $B = (2.33 - 0.69)5{,}000\,V = 8{,}200\,V = \$820{,}000$. Since the saving, $820, 000, exceeds the cost, it would pay to expand the lock capacity when the utilization rate is approximately 57%. The calculation is shown in Table 3.

That the predictions of queueing theory correspond to reality can be seen from Table 4, taken from Lave and DeSalvo [4].

The table shows that an increase in $K$, the number of tows demanding service, increases both waiting time and the amount of time the lock is utilized.[10]

[9] In this calculation, it is assumed that demand is fixed since no estimate of the price elasticity of demand was available. Probably the best estimate of when to expand capacity in this example would be at about 50% of capacity. An independent check of the reasonableness of this figure is provided by the analysis of Bottoms [1]. Bottoms assumes that half the barges will be empty ($p = \frac{1}{2}$) and concludes that the "practical" capacity of a waterway is about 25% of the tonnage that could be moved if all barges were full and there were always a tow waiting to be served. Using the same $p$, our conclusions are identical.

[10] This research was supported by a contract between the Department of Transportation and Charles River Associates. The views are solely those of the authors and do not necessarily represent the views of the Department of Transportation. They are indebted to Robert Gallamore, Robert Burns and Gerald Kraft for helpful comments. Any remaining errors are those of the authors.

TABLE 4
*Test of Locking Model*

| Lock | $K$ | $\lambda$ | $\mu$ | Predicted $T_{Lq}$ | Observed $T_{Lq}$ | Relative Error |
|---|---|---|---|---|---|---|
| Starved Rock | 6243 | 0.71 | 1.13 | 1.50 | 1.38 | 0.09 |
| Marseilles | 5953 | 0.68 | 1.01 | 2.04 | 2.02 | 0.01 |
| Dresden Island | 6245 | 0.71 | 1.39 | 0.75 | 1.09 | 0.31 |
| Brandon Road | 6491 | 0.74 | 1.19 | 1.38 | 1.39 | 0.01 |
| Lockport | 6424 | 0.73 | 1.62 | 0.50 | 0.55 | 0.09 |

Notes:
$\mu$ is the service rate of the lock. These service rates were calculated for 1949 and 1950. The rates did not change between these years, and there is little reason to expect them to change over time. Predicted $T_{Lq} = \lambda/\mu(\mu - \lambda)$. Observed $T_{Lq}$ is derived from the log of one towboat. The observations are for the first months in 1967 and represent the average of about 35 transits of each lock. Relative error is derived as the difference between predicted and observed waiting time divided by the observed waiting time.

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