

Analyzing Creep Mobility of Dzyaloshinskii Domain Walls with an Effective Elastic Band Model

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Chapter 1

The Dzyaloshinskii-Moriya Interaction and Spintronics Applications

In this introductory chapter, we will first provide some necessary background on the basic magnetic phenomena that will arise in our study of the creep motion of chiral domain walls before introducing and detailing the cause for this chirality, the Dzyaloshinskii-Moriya interaction, which is a quantum mechanical effect that has only recently been realized to have a significant impact on a number of very common magnetic thin film systems. Motivation for our study comes from the fact that while there are an array of prominent emerging applications for DMI, substantial difficulties and ambiguities have been encountered when trying to measure it.

1.1 Background on Magnetism and Magnetization Dynamics

1.1.1 Electron Spin

All electrons have both a fixed negative charge and a vectorial quantity called spin that corresponds to a fixed angular momentum and causes a magnetic dipole field to emanate from the electron. The alignment of large numbers of spins causes these dipole fields to sum, which serves as the primary origin of macroscopic ferromagnetic phenomena. Charges interact with each other through electrostatic forces and the manipulation of charge and its net flow, current, is the basis for electronics. Analogously, spins have a number of magnetostatic interactions and a net flow of spin, termed a spin current, can be manipulated to provide a basis for the emerging field of spintronics. Because of its scalar nature, producing a net change of charge in a volume requires electrons to be transported into or out of it. In contrast, a net change of spin can be accomplished simply through rotation of the electrons. Throughout the rest of this thesis, we will be concerned with the properties of ferromagnets, which at small length scales have a nonzero net spin angular momentum per unit volume termed the magnetization, $\mathbf{M}$.

1.1.2 Magnetostatic Energy Terms

Several different types of magnetic energy effects are important to determine the behavior of the magnetization in our materials of interest including exchange interactions, magnetocrystalline anisotropy, dipolar interactions, and Zeeman interactions. We outline them here in the framework of micromagnetics, where a magnetic material is described as a lattice of individual magnetic moments which are free to rotate but whose magnitudes per unit volume are fixed at the saturation magnetization M_s . While this is merely an approximation to a difficult and heavily quantum mechanical problem, it has proven extremely useful in describing even very complex magnetic behavior [5]. Exchange describes the tendency for nearby magnetic moments

in a ferromagnet to align parallel to each other and is based on the quantum mechanical Heisenberg exchange interaction which emerges from the Pauli exclusion principle. The exchange effect holds magnetic domains together. Magnetocrystalline anisotropy concerns the energy differences for moments lying along different directions in a crystal and originates from coupling between spin and orbital angular momenta known as the spin-orbit interaction. This anisotropy energy is dependent on the symmetry of the crystal structure as the magnetic energy must be the same along crystallographically equivalent directions. Structural asymmetry such as that between films of two different materials can also play a role and interfacial magnetocrystalline anisotropy terms can be produced where moments prefer to lie along or perpendicular to a boundary between layers. Dipolar, or demagnetizing, interactions refer to the non-quantum mechanical dipolar forces that moments exert on each other and can be considered as the interaction between magnetic “charges” at either end of a moment vector, with opposite charges attracting and like charges repelling. This dipolar interaction between moments decreases more slowly as a function of distance than the other interactions and in a rigorous sense introduces nonlocal terms to the magnetic energy. In many cases we can approximate the nonlocal form by replacing it with an effective term known as the shape anisotropy, which causes magnetization to tend to lie along the long axis of an asymmetric structure. In thin films this means magnetization will tend to be directed in the plane and in DWs it will be directed along the wall in the absence of other interactions. Finally, the Zeeman energy is just the contribution of externally applied fields and comes into play during the testing of materials and films in which we apply fields to switch or otherwise manipulate magnetization. For a thin perpendicular film in the xy plane that is taken to be uniformly magnetized through its thickness, all of these effects can be combined into a micromagnetic energy functional as follows:

$$E = \iint \left(A |\nabla \mathbf{m}|^2 - K m_z^2 + \frac{M_s^2}{2\mu_0} (N_x m_x^2 + N_y m_y^2) - \mu_0 M_s \mathbf{H} \bullet \mathbf{m} \right) dx dy \quad (1.1)$$

Here $\mathbf{m}$ is the dimensionless magnetization, A is the exchange stiffness, K is the net uniaxial anisotropy (magnetocrystalline minus shape), N_x and N_y are the shape factors in the x - and y -directions which account for the demagnetizing field in the case of a domain wall, and $\mathbf{H}$ is the applied field.

1.1.3 Landau-Lifshitz Gilbert Equation

The dynamics of this system are described by the LLG equation, where each moment undergoes damped precession (see Fig 1.1) around an effective field, given by the following equations of motion:

$$\frac{\partial \mathbf{m}}{\partial t} = -\frac{\gamma}{1+\alpha^2} \mathbf{m} \times \mathbf{H}_{eff} - \frac{\gamma\alpha}{(1+\alpha^2)M_s} \mathbf{m} \times \mathbf{m} \times \mathbf{H}_{eff} \quad (1.2)$$

$$\mathbf{H}_{eff} = \frac{1}{\mu_0 M_s} \frac{\partial E}{\partial \mathbf{m}} \quad (1.3)$$

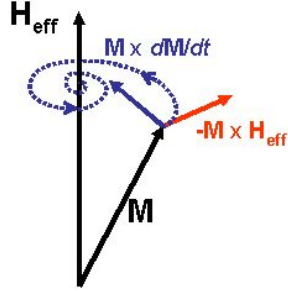


FIGURE 1.1: Diagram of the torques on each moment in the LLG equation.

In these equations, γ is the gyromagnetic ratio and α is the dimensionless damping constant, which describes a phenomenological relaxation toward equilibrium and is typically taken to depend only on the material.

1.1.4 Spin Transfer Torques

In addition to responding to applied fields, moments described in the LLG equation are also subject to manipulation by spin currents. Specifically, spin transfer torque results from relaxation of a spin current within a ferromagnet, which produces a torque on the permanent magnet's moment (see Fig 1.2). The STT effect is modeled in micromagnetics simulations by a modified LLG equation incorporating an additional two terms, as follows[1]:

$$\frac{d\mathbf{m}}{dt}_{stt} \propto T_{ad}\mathbf{m} \times (\mathbf{m} \times \mathbf{p}) + T_{fl}\mathbf{m} \times \mathbf{p} \quad (1.4)$$

Where T_{ad} is the anti-damping or adiabatic torque constant, T_{fl} is the field-like or non-adiabatic torque constant, and $\mathbf{p}$ is the moment vector of the injected spin current. STT can drive motion in magnetic domain walls by applying a torque to the internal magnetic moment of the wall, as described in section 1.3.2, and can be used to switch a magnetic tunnel junction purely with current, thereby allowing the write operation of an MRAM bit to be considerably simplified.

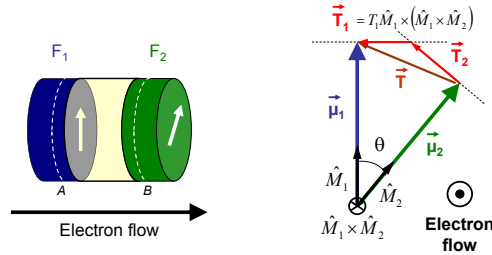


FIGURE 1.2: Schematic of spin transfer torque showing its relation to angular momentum conservation taken from [1].

1.2 The Dzyaloshinskii-Moriya Interaction

The origin of nearly all ferromagnetic and antiferromagnetic behavior is the result of the Heisenberg exchange interaction, a quantum mechanical effect that can be well described as a coupling between two spin angular momenta we will call S_1 and S_2 , taking the form

$$E_{ex} = -J \cdot \mathbf{S}_1 \bullet \mathbf{S}_2 \quad (1.5)$$

Here J is known as the exchange constant, which is positive for ferromagnetic coupling where it favors a parallel alignment of the spins and negative for antiferromagnetic coupling where an antiparallel alignment is favored. Using symmetry arguments, Dzyaloshinskii posited the existence of another type of exchange interaction which is antisymmetric with respect to exchange of its momenta, given by

$$E_{dmi} = -\mathbf{D} \bullet \mathbf{S}_1 \times \mathbf{S}_2 \quad (1.6)$$

Where $\mathbf{D}$ is known as the DMI vector. When the DMI vector is positive (negative), the interaction favors spins that are canted toward (away from) each other in a plane perpendicular to the vector. Another important feature of the interaction is that spins which are held parallel or antiparallel by the exchange interaction have zero DMI energy. The antisymmetric nature of DMI means that in collections of spins with high symmetry, many of the interactions will cancel out. Most notably, inversion symmetry restricts the net DMI vector to zero so bulk materials exhibiting DMI must have a crystal structure without an inversion center. This feature has kept the study of DMI contained to exotic materials until recently, due to the combination of improvements in film growth technology that allow interface effects to become important and

renewed interest in domain wall structure for spintronic applications[6]. To elaborate on the first point, we note that in a stack of two different thin films, inversion symmetry is broken even when both films have crystal structures with inversion centers. This property is known as structural inversion asymmetry (SIA) and is technically true for any asymmetric film stack, but in practice the magnetic films involved must be very thin in order to observe DMI due to the short range of exchange effects. Even in the case of ultrathin magnetic films, inside domains DMI is typically overcome by symmetric exchange and shows no effect on the equilibrium magnetization. However, at the boundary between domains of different magnetization, the domain wall (DW), moments must rotate in space from one easy axis direction to the other through the hard axis and are therefore inherently canted. DMI can lower or raise the energy of different canting orientations and determine the type of DW that is present at equilibrium.

1.2.1 Dzyaloshinskii Domain Walls in Perpendicular Thin Films

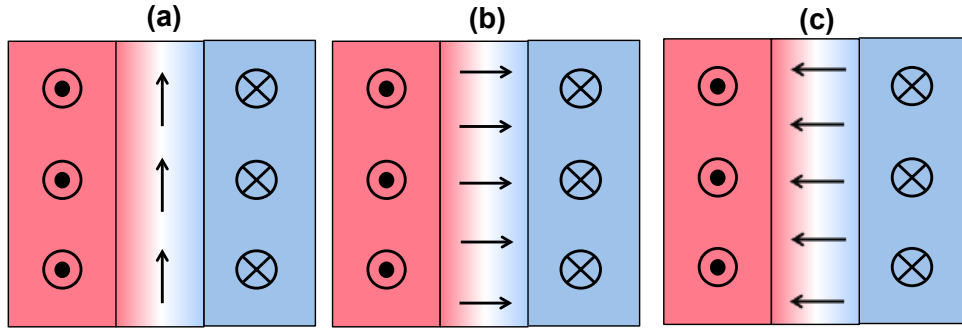


FIGURE 1.3: Top-down schematic of types of domain wall in a perpendicular thin film. (a) Bloch wall; (b) Right-handed Néel; (c) Left-handed Néel.

We now describe the three possible types of DWs in a thin film with a perpendicular easy axis (see Fig 1.3), with the first being the Bloch wall, where the magnetization rotates about an axis perpendicular to the DW. In unpatterned films without DMI, the Bloch wall is favored because it has a lower demagnetizing field. If such films are patterned into nanowires where

the length of the DW is smaller than its width, Néel walls have lower demagnetizing field and are the equilibrium structures. In these walls, the magnetization rotates about an axis parallel to the DW and can have different chirality, leading to right- and left-handed Néel walls. For a so-called up-down DW where the up domain is on the left, a right-handed Néel wall will rotate counterclockwise going from the up domain to the down domain. Owing to additional symmetry considerations laid out by Moriya[7] the net DMI vector must lie in the plane parallel to the DW. Positive (negative) DMI favors a right- (left-)handed Néel wall, disfavors the left (right), and has no energetic effects on Bloch walls. The stability of Néel walls predicted by DMI has been observed experimentally through a number of different methods [8–10] and from a practical perspective can allow for very fast DW motion from spin currents [11] described later, a property that is highly desirable to the nascent field of DW devices. Just as the regular exchange interaction has a micromagnetic form in the exchange stiffness term, DMI in thin films with SIA can be described micromagnetically as:

$$\iint D \left(m_z \frac{\partial m_x}{\partial x} - m_x \frac{\partial m_z}{\partial x} + m_z \frac{\partial m_y}{\partial y} - m_y \frac{\partial m_z}{\partial y} \right) \partial x \partial y \quad (1.7)$$

It has been demonstrated [12] that this micromagnetic term results in an effective in-plane field that exists only in the DW at steady state and points from the spin up domain to the spin down domain for positive D . For conditions that are outside of equilibrium, such as the switching of a magnetic tunnel junction (MTJ) free layer by thermal fluctuations, fields, or spin currents, DMI can have an impact on behavior as the magnetization becomes spatially nonuniform during the switching process.

1.3 Applications for Magnetic Materials with DMI

Now that the essential features of DMI have been detailed, we can explain why there is intense interest in the research community to understand and exploit it on several different fronts which are described below.

1.3.1 Domain Wall Motion by Spin Orbit Torques

The past decade of research has provided several key developments in the field of thin film magnetic phenomena; in addition to DMI, the importance of the spin hall effect (SHE) in driving DW motion has only recently been revealed [13]. The SHE is a case where the charge current through a normal metal generates a spin current in a direction transverse to the electron flow. The polarization direction of the spin current is perpendicular to both the charge and spin current directions. So for a charge current in the x -direction, a spin current with magnitude proportional to the material-dependent spin hall angle, θ_{SHE} , will be generated in the z -direction with spins polarized along the y -axis. This effect is due to spin orbit interactions in the normal metal and the spin torques generated by SHE are an example of spin orbit torques. For a magnetic nanowire that contains a DW and has a normal metal underlayer with nonzero θ_{SHE} , current passed through the nanowire will generate a flux of spins into the DW with polarization along the DW axis. For a Bloch wall, the spin current is polarized in the same direction as magnetization, so any torques are thermally activated and relatively weak. For a Néel wall, however, the spin current generates a non-adiabatic torque that has been shown to very efficiently drive the DW [14]. Combining DMI and SHE therefore enables fast current-driven motion of DWs.

1.3.2 Chiral Magnetic Textures

While we noted in section 1.2 that within domains DMI is typically overpowered by Heisenberg exchange, it will prove useful to explain what happens when it isn't. For very large values of D , the tendency for moments to take on a canted arrangement can overcome the tendency for them to align, leading to a nonuniform chiral magnetic ground state in the form of either a helical phase or what is called a skyrmion lattice [15, 16]. Even when DMI is not strong enough to fully stabilize them, skyrmions can exist as pointlike topological defects that superficially resemble a bubble domain but are much smaller [17]. The topological stability of the skyrmion makes them a topic of great interest to the magnetic physics community [18] and it has been suggested that strong emergent fields may arise from their interaction with currents that would allow for unprecedented current-driven mobility, though this has not yet been verified experimentally [19]. Skyrmions have very recently been created, imaged, and manipulated in metallic stacks at room temperature,[20] and engineering multilayers with large DMI is a crucial step in this process in order to increase skyrmion lifetimes.

1.3.3 Impact on MTJ Performance

An MTJ is a two terminal device that consists of two closely spaced ferromagnetic layers separated by an insulating tunnel barrier. The the phenomenon of tunneling magnetoresistance (TMR), the stack can be engineered so its resistance depends strongly on the relative polarization direction of the adjacent magnetic films, which in turn can be manipulated using STT. This strong, direct interaction between permanent magnetism and electron transport is critical for a number of present and potential applications in particular a type of high speed nonvolatile memory based on magnetoresistive effects, known as magnetoresistive random access memory (MRAM), is currently under development. Recent micromagnetic simulations [2] have shown

that DMI produces a decrease in the effective barrier for thermal switching of MTJs while simultaneously increasing the critical current required for STT switching, resulting in a substantially lower figure of merit for the an MRAM bit. During thermal switching, transition states with nonuniform magnetization can develop within the free layer of the MTJ and because DMI stabilizes some of these states it creates switching pathways with a lower thermal energy barrier (see Fig 1.4). Since STT acts as a uniform torque on the free layer, the lowered thermal switching barrier does not translate to lower critical current. To improve device behavior, we therefore wish to reduce DMI in the electrode as much as possible. In previous work [21, 22], we studied crystallization and interdiffusion processes in the MTJ electrodes and used these studies to aid the internal development of device stacks that could be employed to test switching efficiency. The goal of this project was to experimentally evaluate the impact of DMI on switching currents, and our attempts to develop electrodes with significant DMI will be discussed in chapter 5.

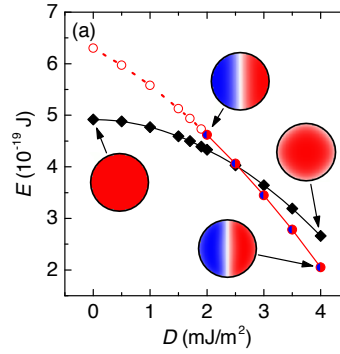


FIGURE 1.4: Plot showing the reduction in energy for a domain wall state in an MTJ electrode nanopillar with increasing DMI taken from [2].

1.4 Measuring DMI

The list of methods by which DMI has been observed in materials has grown rapidly throughout the past few years and includes a great deal of methods using exotic equipment and involved processing. Because we may wish to examine a large number of alloys for their DMI effects, we look for as high-throughput a method as possible.

1.4.1 Asymmetric Bubble Growth

One of the simplest techniques of measuring DMI to conduct, asymmetric bubble growth describes subjecting a small circular domain in a perpendicular thin film to an in-plane field while using an out-of-plane field to drive its growth. The left, right, and top/bottom sides of the domain are found to grow at different rates because the interaction of DMI and the applied in-plane field break the symmetry of the three different sides (see Fig 1.5), noting that the symmetry at the top and bottom are not broken so these are counted as a single side. This technique is appealing because it involves no patterning, as the starting domains can be preferentially nucleated at pre-made defect sites generated by ion beam or other damage [8]. It also uses MOKE to monitor domain growth, which we have easy access to. By far the largest drawback of this method is that the interpretation of asymmetric growth results are disputed and as of yet no single proposed mechanism appears to satisfactorily describe all observed experimental results.

Some of the first asymmetric bubble growth experiments were conducted on cobalt thin films held perpendicular by a platinum interface [3, 8, 23]. In these and other experiments, a second interface is present to make the stack either very asymmetric in order to produce large DMI or nearly symmetric for lower DMI. The effective DMI field will be parallel to the applied field for one side (left or right) of the wall and antiparallel for the opposite side (see Fig 1.6). In the following discussion we will refer to the wall where DMI and the applied field are parallel

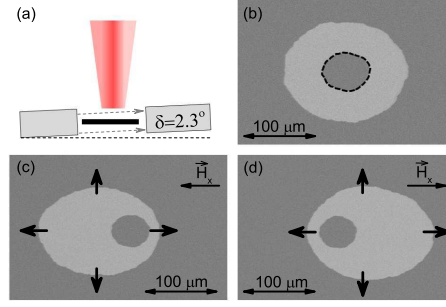


FIGURE 1.5: Bubbles grown in different in-plane fields taken from [3].

(antiparallel) as the low (high) energy wall. Typical results show that as the in-plane field is turned on, the low energy wall increases in velocity while the high energy wall decreases. At higher in-plane fields the high energy wall typically reaches a minimum velocity and then begins to increase in speed as the field is increased further. The stated explanation for this behavior was that the energy scale in a creep law fit, described in more detail in the following section, depends proportionally on wall energy. To a rough approximation the wall energy has a term proportional to $-(H_x - H_{dmi})^2$, so it was argued that the minimum velocity in the high energy wall corresponds to the point where H_x exactly cancels H_{dmi} and the wall energy attains a maximum. Several subsequent studies have raised issues with this description, possibly the most striking of which is a CoNi study conducted by Lau *et al.* [24] that shows the velocity of the high energy wall exceeding that of the low energy wall at sufficiently high field. This was attributed to the anisotropic nature of the wall energy driving toward an equilibrium in overall bubble shape, though no direct mechanism was provided. Attempts to investigate growth with anisotropic wall energy through modeling are described in the following two sections. Finally, we note that Jué *et al.* have shown that when their data is fit to a creep law, the difference in velocity doesn't have the characteristic of an energy scale at all and instead appears as a difference in damping between the two sides [25]. This behavior was attributed to an entirely new effect of SIA where the intrinsic damping depends on the chirality of magnetization. It

is clear that before results of these types of studies can be relied upon, a better theoretical groundwork must be developed which makes some progress in describing the varied behavior of experiments.

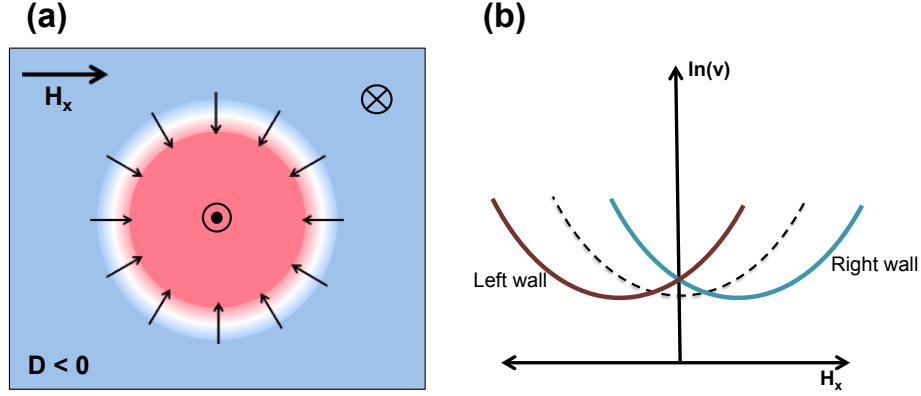


FIGURE 1.6: (a) Diagram showing bubble wall magnetization for $D < 0$. (b) Expected velocity behavior of the walls shown in (a) under the influence of an in-plane field according to the description of Hrabec et al.[3]. The dashed line indicates the expected behavior for $D = 0$.

1.5 Alternative Methods

Given the difficulty of quantitatively extracting DMI from asymmetric bubble growth experiments, it remains to be determined if results from the CoFeB/MgO system can be conclusive. Exploring other methods as potential supplements or alternatives to bubble growth could be critical to our ability to convincingly determine DMI for these films. Because of the increase in research attention to interfacial DMI, several techniques have recently been developed that may help with our analysis. We briefly detail these new methods, along with their strengths and weaknesses, below.

Current-Driven Motion - Spin transfer torque can be used to move a domain wall in much the same way as it is used to switch an MTJ, where electrons moving through the domain wall exert a torque in the domain they are entering, resulting in wall motion against the current

direction. The velocity of this wall motion has been shown to depend on an in-plane field in a manner similar to the idealized field-driven motion case for low in-plane fields [10]. The sample films are patterned as small wires and current pulses are applied over very short timescales, allowing larger driving forces and higher velocities than field-driven motion on blanket films but requiring more processing. This has the benefit of avoiding the creep regime, so effects of the pinning potential and thermal motion are reduced. Motion at velocities well above Walker breakdown, however, can also be very complex [26] and this method has not yet been subjected to the same scrutiny as bubble growth experiments.

Nucleation Field - Another method using patterned films, the nucleation field approach creates structures where, during switching, domains preferentially nucleate at triangular corners to produce a single, straight domain wall [9]. The critical field for nucleation depends on the energy of the wall and can be adjusted with an in-plane field, resulting in a shift in coercivity. The positive aspects of this method are that it deals directly with the wall energy and not the kinetics of wall motion; negatives include the fact that it was developed very recently and has few published results that conclusively indicate the change in coercivity is coming from DMI.

Maze Domain Collapse - In demagnetized films, domains become very small and the degree of uniformity of the magnetization decreases. These small domains form a maze-like pattern where walls are very closely spaced [27], and DMI stabilizes such nonuniform configurations, affecting both their structure and the perpendicular field at which they collapse. While this technique requires no patterning and can be carried out relatively easily, it does not take advantage of the symmetry of DMI during the measurement process and therefore other film properties such as the exchange strength and the magnetization can have qualitatively similar effects on maze domain collapse. In theory, DMI should have a quantitatively distinct impact on the structure of the maze domains, but in practice the size of domains is difficult to measure reliably in a demagnetized sample.

Brillouin Light Scattering - A more established method of measuring DMI, Brillouin light scattering involves optically exciting spin waves and determining their dispersion in the sample of interest [28]. Materials with DMI display a difference in the dispersion of spin waves depending on their wavevector. While this method is well represented in the literature and involves no patterning, it requires nonstandard equipment and expertise.

1.6 Thesis Outline

The primary focus for the rest of this thesis will be on investigating the anomalies that have been observed in asymmetric bubble growth experiments described in the previous section. Analytic results in creep theory have proven extremely successful in describing the dependence of the DW velocity behavior on the perpendicular driving field H_z , so we will look to them in greater depth in an attempt to understand the observed H_x dependences. We note that in the models presented so far to describe the asymmetry behavior, several aggressive assumptions have been made to simplify the problem but these may not be accurate. Our hypothesis is that an enhanced model of creep will more effectively represent the experimental behavior and therefore provide better insight into the underlying physical phenomena. This more complete model is developed in two stages - in chapter 3 we draw on analytic creep results which have been derived for single vortex pinning systems in superconductors to understand how the creep behavior depends in detail on all of the general elastic properties. These properties include the elastic modulus, interface width, pinning strength, effective mass, and friction. In chapter 4 we derive how these elastic properties can be expressed for the Dzyaloshinskii DW and explore their in-plane field dependence. Finally, in chapter 5 we combine the results of chapters 3 and 4 and apply the resulting models to experimental data in order to evaluate the efficacy of creep theory in explaining the trends in real materials.

Chapter 2

Experimental Techniques

2.1 Film Deposition and Characterization

Sputtering - This ubiquitous deposition method involves bombarding the material to be deposited (target) with an argon plasma, causing atoms to be ejected from the material's surface due to the impact of high energy ions. These ejected atoms can then accumulate on the surface of substrates placed in their path, producing a thin film. For depositing conductive layers, the plasma can be generated and sustained by applying a negative DC voltage to the target which also acts to pull positively charged argon ions in, supplying the energy required to sputter atoms and resulting in current flow through the target. This would result in charging for an insulating material such as MgO, so a large RF bias is applied to these targets while a smaller DC bias develops as a result - supplying sputtering energy but producing no net current flow. The relatively high deposition rates and uniformity of sputtering coupled with typically lower sensitivity to contaminants compared to techniques such as molecular beam epitaxy make it an ideal method for depositing films where crystalline quality is not critical, which is largely true for most films in an MTJ stack with the exception of MgO. Despite the strong dependence of

device behavior on MgO crystal quality, standard RF sputtering has been reported to produce MTJs of sufficiently high quality for our purposes [29], though we note that modifying the MgO sputtering process is a major area of industrial and academic research[30]. For all studies described in this document, films were deposited using either RF (for MgO) or DC (for all other materials) sputtering from one of two different deposition systems. Since the quality and properties of sputtered films, particularly that of MgO, depend strongly on equipment parameters including base pressure and geometry, our results vary depending on the equipment that was used. We refer to the first deposition system as the 5-target tool, as it is custom-built with five 5" targets directed at a rotating substrate table so deposition is in the face-to-face geometry. Base pressure on this piece of equipment is quite high at around 2×10^{-7} Torr, which we believe contributes to preventing deposition of high quality (001)-oriented MgO. In addition, the limit of five targets means a vacuum break is required during the deposition of a full stack SAF MTJ structure. The benefit of this system is that thickness gradients are very easy to deposit by controlling the substrate table rotation, so tuning thicknesses to optimize desired properties can be done in fewer deposition runs. The second system, which is newer, is an AJA tool with seven 2" targets and one 3" target (for MgO) in a confocal sputter-up arrangement. This tool typically reaches base pressures in the low 10^{-9} Torr range in order to deposit good MgO and has enough targets to run an entire device stack. Because of these improvements, all current development is done on the AJA system while most of the crystallization studies we described in chapter 1 were conducted before the AJA was installed using films from the 5-target tool.

Alternating Gradient Force Magnetometry (AGFM) - A recurring problem for thin film materials is that their absolute magnetic moment is very small owing to the low volume of magnetic material present, reducing the signal-to-noise ratio of moment measurements. AGFM has much greater sensitivity than VSM and can measure relative moments very accurately down to around a 10 nemu scale. This technique works by applying a gradient field to a sample that is free to

vibrate - thereby producing a force on the sample holder. Rapidly alternating this gradient field produces a vibration of the sample holder which is measured using a piezoelectric element. At most frequencies of the alternating gradient field, the diamagnetic response of a sample overwhelms the ferromagnetic signal so the frequency must be tuned to reduce the diamagnetic contribution. The accuracy of this tuning can affect the absolute strength of moment measurements, so saturation magnetization values obtained with AGFM must be used cautiously. Despite this drawback, the sensitivity of AGFM makes it an invaluable resource for measuring PMA in ultrathin films as coercivity and saturation field values can be obtained to high accuracy for samples with very low moment; we have used it extensively to optimize the PMA in MTJ stack structures.

Magneto-optical Kerr Effect (MOKE) Microscopy - The Kerr effect refers to changes in the refractive index and therefore reflective properties due to the magnetization of a material. This property manifests as a phase shift which can be imaged using phase contrast microscopy techniques, allowing the surface magnetization to be mapped directly. MOKE in combination with applied fields thus provides a method of imaging growing domains and providing experimental evidence of domain wall dynamics, which in turn are heavily influenced by DMI. The asymmetric bubble growth method described in chapter 1 and explored in detail in chapter 5 relies on these measurements, as do several of the potential alternative DMI measurement schemes. Because MOKE is an optical method it is limited in resolution to around $1\mu\text{m}$ and can therefore only study fairly large domains and switching behavior.

2.2 Micromagnetic Modeling

Micromagnetic modeling is a numerical technique for solving the magnetization dynamics using the LLG equation described in the previous chapter. In all micromagnetic results presented in

this thesis, we used Mumax³ version 3.8[31], which uses an explicit time Runge-Kutta method to solve the micromagnetic evolution equations. Calculations of the faceted DW profile and analysis of the static DW bending experiment were performed on a mesh of 1 x 1 x 2nm cells using the conjugate gradient energy minimization algorithm built into the software. Dynamic bending experiments used a maximum timestep of 10^{-14} seconds.

2.3 MTJ Development and Testing

The work we have carried out on MTJs is vertically integrated, covering everything from stack deposition to the electrical testing of fabricated devices. Here we outline our experimental methods, including a brief discussion of the theory of operation for each technique as well as elaboration on specific details related to our application of producing CoFeB/MgO/CoFeB MTJs.

2.3.1 Film Level Characterization

X-ray Diffraction (XRD) - Formation of the correct coherent interface required for high TMR in MTJs is dependent upon deposited MgO films having (001) crystallographic texture (this property is detailed in chapter 3). XRD provides a high-throughput method to determine the existence and quality of this desired texture, allowing for the quick evaluation of a large number of deposition conditions in terms of their impact on MgO structure. This technique works by irradiating a thin-film sample with a beam of monochromatic X-rays and measuring the intensity of diffraction as a function of the angle of the incident and diffracted beams with respect to the film surface. For structure identification, the angle of the incoming and outgoing beams are equal and are varied together in order to observe the change in diffracted intensity versus incident angle. If the film is crystallographically textured, this scan can produce peaks

in intensity which occur due to the constructive interference of diffraction from different Bragg planes parallel to the surface of the film [32]. For MgO, we look for the 002 peak that occurs when the beams are angled $\sim 21^\circ$ from the plane of the film when using Cu K_α X-rays. If this peak is present, we can determine the quality of the (001) texture by taking a rocking curve, which involves varying the tilt angle of the film while the incident and diffracted beams remain at the same position. Measuring the full width at half max (FWHM) for diffracted intensity versus the sample tilt angle indicates how the (001) plane normals of the MgO grains are distributed with respect to the film normal and serves as a figure of merit for film quality, with lower FWHM indicating stronger texture. XRD is a powerful tool as films can be tested with no additional processing, but we are typically limited to thicknesses of 5nm or greater for MgO in order to obtain reasonable signal. In our experiments, MgO films deposited using the 5-target system typically had very weak or nonexistent 002 peaks while those made using the AJA system produced 002 peaks with a minimum of 5.5° FWHM rocking curves. In principle, XRD can also be used to determine the presence and quality of (001)-oriented α -FeCo which crystallizes on the MgO surface after annealing of CoFeB, but the partially amorphous nature of this layer results in a much weaker signal and requires even higher thicknesses which diverge very significantly from device values. Ultrathin films like those found in devices are thus better served by studies using transmission electron microscopy.

Transmission Electron Microscopy (TEM) - Direct imaging of the electrode and barrier structure of MTJ stacks can only be accomplished by high resolution TEM, which can provide crucial insight into the structure of the critical electrode-barrier interface and the crystallinity in both the MgO and CoFeB layers. TEM involves sending an incident beam of electrons through a thinned section of the film sample and analyzing the transmitted and diffracted beams to determine the sample's structure. Prior to this technique, the sample must be thinned to the range of ~ 10 -100nm before it becomes reasonably electron transparent. The fragility of very

thin sections and difficulty of sample prep means that only very small segments of the film cross section can be imaged and it can be difficult to interpret whether an observed difference between two stacks is due to changes in processing or just typical sampling variation. Still, TEM is critical to troubleshooting film development problems, we have used it to confirm (001) MgO and FeCo texture after annealing and analyzed physical changes due to different seedlayers.

Vibrating Sample Magnetometry (VSM) - For the crystallization studies described in chapter 1, we require a method to determine saturation magnetization to a high degree of accuracy. VSM is a standard technique in the study of magnetic material which uses induction to excite currents in pickup coils by vibrating a magnetized sample nearby; these currents are dependent on the strength of the sample magnetization and therefore allow it to be accurately measured. VSM equipment also contains an electromagnet so different fields can be applied to the sample and magnetic responses such as a standard hysteresis loop can be determined. Our particular tool is a Quantum Design PPMS with VSM Oven attachment, which has a vacuum-sealed sample chamber to run magnetic testing at $\sim 10^{-5}$ Torr and uses a helium-cooled superconducting magnet to apply fields up to 10 Tesla. While the samples we use for crystallization testing are thick enough that they can be saturated in-plane at fairly low fields, the vacuum chamber on the equipment is critical and allows for the use of special sample holder that contain a heating element and thermocouple so the sample can be annealed during measurement of its magnetization which allows changes that occur during crystallization to be measured in real time with fairly high throughput. The primary challenges for these measurements are the presence of contaminants, as the sample must be bonded to its holder with ceramic cement in order to ensure good thermal contact and the cement can pick up magnetic particles from the film during annealing, resulting in magnetic contamination if not all the cement is removed after measurement. Because our VSM has a characteristic noise floor of around $1\mu\text{emu}$ and the changes we are interested in correspond to 10-20% of the total sample moment, we are limited

to samples with total moment on the order of 20 μemu or above and therefore require at least 2-3nm of CoFeB for accurate results.

2.4 Patterning

In order to measure the TMR and switching behavior of MTJs, small pillar structures must be patterned out of the blanket film stacks so that confined currents can be passed through the junction. In broad terms, the patterning involves partially etching into our full stack structure to form nanopillars of between 50-150nm in diameter, then depositing leads to both the top and bottom of the pillar separately. This is accomplished in a sequence of six lithography levels, five of which are optical lithography and one of which is e-beam. A diagram of the film stack going through this process flow is provided in figure [2.1](#).

The detailed processing steps start with first depositing a carbon hardmask for the pillar etch and then sputtering thick tantalum marks for e-beam lithography to align the pillars to (b). After the nanopillar lithography, the hard mask undergoes a reactive ion etch (RIE) to expose the film surface and the pillars are etched in an argon ion mill with end-point detection that allows the etch to be stopped once the active MTJ layers are cleared (c). A SiN passivation layer is then deposited and planarized in the ion mill (d), followed by optical lithography to pattern the seed/bottom leads. Passivation over the bottom leads is etched by RIE, the remaining seedlayer is cleared in the ion mill, and an additional passivation layer is deposited to encapsulate the device (e). Small openings (vias) over the remaining seed material and over the nanopillar are patterned optically and etched in the RIE, deep enough to expose the hardmask remaining on the nanopillar but ideally not deep enough to expose the pillar sidewall (f). The remaining hardmask is then selectively etched in the RIE to expose the metal pillar surface (g), followed by another lithography and etch step which deepens vias over the remaining seed until the metal

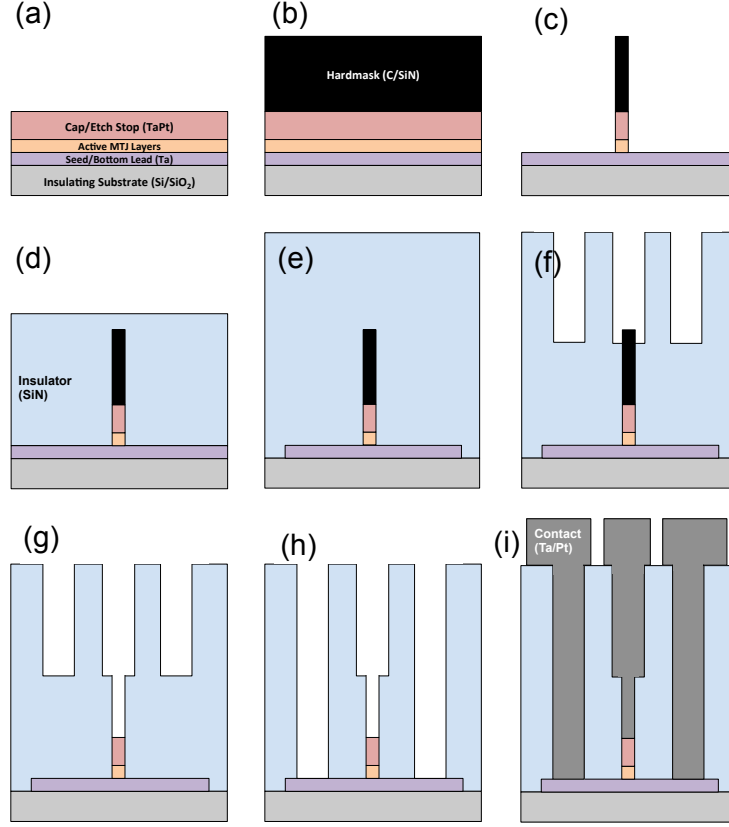


FIGURE 2.1: Standard process flow for e-beam nanopillars between 50-150nm in diameter, note the aspect ratio of the pillars and vias is heavily exaggerated. A description of each stage is provided in the text.

is exposed but leaves the pillar vias as they are (h). Finally, the surface is lightly etched and contacts are deposited which provide current paths through the device and through the bottom leads separately (i), which is required for the testing processes.

2.4.1 Device Testing

After patterning, TMR and RA product can be determined by electrical testing in an applied field. We note as a precaution that MTJs with low MgO thicknesses can be damaged by biases less than 1V, so electrostatic discharge (ESD) protection circuitry is required to ensure the junctions don't experience high transient currents while making contact to the probes. In

a typical scan, the applied field is started at a high enough value to saturate the device in the parallel state, then successive resistance measurements are taken at constant current as the applied perpendicular field is lowered. If the antiparallel state is stable, then at some critical field with opposite sign compared to the initial saturation field the low-coercivity layer will switch and the resistance will sharply increase. To determine TMR from the high and low resistance values, we also need to know how much of the total resistance is due to the junction. In our devices, the resistance of the bottom leads can become significant for low MgO thicknesses, so this background resistance must be measured by passing current through the bottom leads only and subtracted out. Once switching is observed, we can take a minor loop by starting in the antiparallel state and reducing the field until the low-coercivity layer switches back; the difference in the antiparallel to parallel (AP $\rightarrow$ P) and parallel to antiparallel (P $\rightarrow$ AP) switching field gives a measure of the coupling between the electrodes. Increasing the field from the antiparallel state will eventually result in the high-coercivity layer switching to the parallel state, providing information on the stability of the antiparallel state. Current switching with long pulses (100ms was used in the experiments shown in this document) is carried out on the same equipment, where resistance is measured as a function of the applied current after preparing the device in either the parallel or antiparallel state. Starting from the parallel state, increasing the current from the fixed layer to the free layer (so electrons are being transmitted from the free layer to the fixed layer) can result in a P $\rightarrow$ AP switch. Likewise if we start from the antiparallel state and increase current coming from the free layer, we can observe an AP $\rightarrow$ P switch. Because the device is a tunnel junction, the resistance will change depending on the applied voltage/obtained current and the raw current switching data will not be rectangular like the field switching data. This can be adjusted by taking the junction IV behavior into account. Real MRAM cells must be switched with much faster current pulses than 100ms and shorter pulses require a larger switching current[33]. Applying very short pulses requires a specialized

generator and waveguide leads to reduce self-induction, both of which will be used in future work.

Chapter 3

Review of Creep Theory for 1-D Elastic Manifolds

The theoretical study of elastic manifold dynamics dates back to the early 19th century work of Laplace in describing the capillary forces of fluids. In the intervening centuries, this line of reasoning has been formalized and expanded to applications in a wide variety of disciplines, providing insight into the behavior of a large number of physical objects, from soap bubbles to crystalline grain boundaries to polymers. In this chapter, we will review in some detail the branch of elastic manifold theory known as creep, which we are focusing on due to its relevance to the experimental studies of DMI outlined in Chapter 1. After briefly discussing how creep theory fits into the broader study of elastic manifolds, we will review the characteristic interface properties that affect mobility in the creep regime at low driving fields. Our analysis in this chapter draws heavily on creep theory that was developed in great detail to understand single vortex pinning in type II superconductors [34]. Magnetic DWs in thin films have been shown to be excellent model systems for creep theory [35–37] as their velocity can be directly determined

through magnetic domain imaging techniques such as MOKE and exhibits the expected critical behavior which we will derive later in this chapter.

3.1 Conditions for Creep

Generally, an elastic manifold is an extended object with a self-energy that increases along with its spatial extent, resulting in a tendency for a curved manifold to reduce its curvature in order to become more compact. The dynamics of an elastic manifold are broadly categorized by the dimensionality of the manifold, d , and the dimensionality of its motion, n . Such a case is referred to as a $d + n$ manifold. Our manifold of interest, a ferromagnetic domain wall in a thin film, is technically a three dimensional space with an additional degree of freedom corresponding to the internal magnetization direction in the plane of the film, but it can be treated effectively as a one dimensional object. In order for this to be valid, the DW width must be nearly constant, the film must be thin enough so that the magnetization is uniform along the film normal, and the internal degree of freedom must be a function of the DW orientation only. We will assume these approximations are valid and describe the requirements for these assumptions in section 3.3.1 and chapter 4. Because the domain wall is constrained to an effectively two dimensional film, we only have motion in one dimension and can describe the DW fully by two coordinates, z for the position along the axis of the DW orientation and u for the position along the axis of DW motion. Thus we have a 1+1 manifold, and the elastic energy can be expressed as

$$\mathcal{E}_{el} = \int \frac{\varepsilon t_f}{2} \left(\frac{\partial u}{\partial z} \right)^2 dz \quad (3.1)$$

Here ε is the stiffness (also known as elastic modulus) and t_f is the film thickness.

When elastic forces are coupled to any number of additional forces that tend to move the manifold or constraints that restrict its motion, a rich set of behaviors can result. A simple example is the aforementioned capillary theory developed by Laplace that is caused by the interplay between the driving force of pressure fields and the surface energy of a fluid interface. While several elementary cases like this have exact solutions, more complex scenarios are major areas of ongoing research in fields such as microstructural evolution, where the motion of elastic grain boundaries is constrained by requirements to preserve crystal topology [38]. The creep behavior we seek to explain results from a combination of elastic forces and three additional effects - pinning forces, a driving force, and thermal fluctuations.

$$F_{total} = F_{el} + F_{pin} + F_{drive} + F_{thermal} \quad (3.2)$$

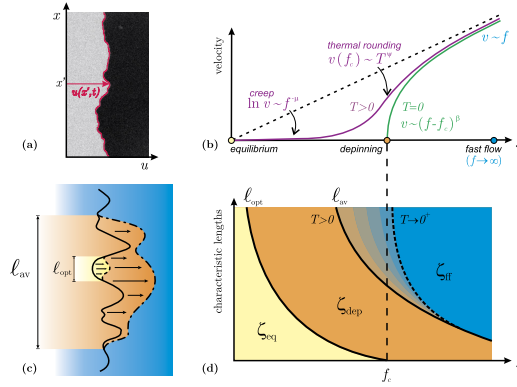


FIGURE 3.1: A schematic of different behaviors of the 1+1 manifold taken from [4].

Pinning forces are the result of static defects in the medium through which the manifold moves, in our case caused by material inhomogeneities in the magnetic thin film. Due to their static nature, these defects are also known as quenched disorder and will attract or repel the DW, resulting in an uneven potential energy landscape with barriers that must be surmounted in order for the DW to translate. Despite having a critical impact on a number of emerging

technological applications in magnetics such as racetrack memory, the pinning sites in many magnetic thin films are difficult to examine directly and remain poorly understood. As we elaborate on in the following section, several different forms for the pinning potential have been described theoretically and through analysis of experimental creep results we can narrow our focus down to the most likely candidates. The driving force we will focus on is a perpendicular magnetic field $f \propto H_z$, which produces a constant potential energy gradient along the direction of DW motion, causing the domain that is aligned parallel to the perpendicular field to expand. In addition to perpendicular fields, ferromagnetic DW creep can be driven by currents that produce spin torques on the DW by interacting with its internal degree of freedom. This type of driving results in deep changes to the dynamics and we will only briefly discuss current-driven motion as a possible avenue for future work in section 6.2.2. Before introducing thermal forces it will be helpful to outline the different behaviors that are possible from the interaction of elastic, pinning, and driving forces (Figure 3.1). Below a critical driving force $f = f_c$ there is no steady translation as the DW is able to reorient itself in the pinning potential so that elastic forces and pinning forces cancel the driving force and the DW is in a static equilibrium or pinned state. As the driving force is raised above f_c , the DW can be held in a pinned state only transiently as the combination of elastic forces and driving force will overcome the pinning forces. The DW therefore takes on a nonzero steady state velocity which exponentially increases as f is increased further and asymptotically approaches the velocity that would be obtained in the absence of any pinning potential, known as the flow regime.

Thermal fluctuations account for the random motion of the DW and are represented by a force term that takes the form of stochastic white noise, which is uncorrelated in both space and time. The inclusion of these forces allows the DW to sporadically overcome pinning sites even when $f < f_c$, resulting in a thermally-activated steady state velocity that depends exponentially on the driving force. This case defines the creep regime, and critical force-velocity relationships

can be expressed for limiting cases of f . In the low-driving limit, $f \rightarrow 0$, and the domain wall behavior is determined as a perturbation from the completely pinned state. In the depinning limit, $f \rightarrow f_c$, and the behavior is determined as a perturbation of the flow regime. Because the DW velocities in our experiments are several orders of magnitude below the flow velocity, we will evaluate only the low-driving limit where the velocity can be expressed as follows.

$$v_{creep} = v_0 \exp\left(-\frac{U}{kT}\right) \quad (3.3)$$

$$U \propto f^\mu$$

The remainder of this chapter will use scaling analysis to explore how the parameters of the creep law above arise from the competing forces outlined in equation 3.2. In order to put a form to these forces so the scaling behavior can be analyzed we will first need to make some broad assumptions about the nature and structure of the pinning.

3.2 Types of Quenched Disorder

While the only necessary feature of quenched disorder is its ability to attract or repel a DW and therefore pin it, the nature of the pinning site-DW interaction and the spatial distribution of pinning sites can have a dramatic impact on the force-velocity behavior of the manifold and can even alter the scaling exponents and universality class. To begin characterizing the disorder, we will take the interaction between pinning sites and the DW to be short-ranged so that the pinning energy can be expressed in terms of an unchanging potential energy field. This is known as random bond or RB pinning and is capable of yielding a critical exponent μ that agrees with experimental studies of DW creep in metallic thin films [35]. In contrast, the pinning site-DW interaction could be long-ranged and produce what is called random force or

RF pinning. RF pinning yields larger values of the critical exponent which have been observed in creep experiments for ferromagnetic semiconductors [39]. Since we are interested in metallic films, we will assume RB pinning and can therefore express the pinning force as

$$F_{pin}(z) = -\frac{\partial V_{pin}(z, u)}{\partial u} \quad (3.4)$$

Where $V_{pin}(z, u)$ describes the pinning potential of a DW that goes through point (z, u) . In order to progress further, we must approximate the form of V_{pin} and therefore describe the distribution and geometry of pinning sites. As the pinning sites are taken to be defects, the value of V_{pin} is typically taken to be a random variable with no long range correlation. In one of the simplest cases that still produces rich behavior, we can take V_{pin} to be a random Gaussian field. This is a stochastic function described below that is parameterized by a pinning strength and correlation length, the latter of which corresponds to the size and spacing of pinning sites.

$$\begin{aligned} \langle V_{pin}(z, u) \rangle &= 0 \\ \langle V_{pin}(z, u) V_{pin}(z', u') \rangle &= \gamma e^{|u-u'|/R_c} \delta(z - z') \end{aligned} \quad (3.5)$$

Even with a fixed form of the pinning potential, the scaling behavior of the creep law will depend on the size of the spatial correlation distance R_c , producing either weak collective pinning or strong pinning as described in the following sections.

3.2.1 Weak Collective Pinning

The DW width λ presents a lower limit for the correlation length of the pinning potential. This is the case because V_{pin} describes the energy of the DW-pinning site interaction and not just

the pinning sites alone. So even if the pinning sites are spaced closer than the DW width as may be the case for atomistic defects, $V_{pin}(u, v)$ will include a sampling of the defects within a distance λ of the point (u, v) . In this scenario, called weak collective pinning, the DW is pinned not by individual defects but by the collective action of many pinning sites, with the net pinning force arising due to the DW favoring areas with high defect density. In weak collective pinning, the detailed geometry of defects becomes unimportant and the pinning potential can be parameterized by the properties of the isolated DW and a single pinning strength variable γ . Weak collective pinning considerably simplifies the analysis of creep behavior and is the most thoroughly understood type of pinning as a result.

3.2.2 Strong Pinning

Strong pinning results when the pinning site density decreases so that the DW is held in place by individual defects. The pinning strength can alternatively be described in terms of the properties of these individual defects using the average pinning force f_p and defect density n_p , giving

$$\gamma \propto f_p^2 n_p \lambda^2 \quad (3.6)$$

Analysis of this type of pinning is more difficult than weak collective pinning mainly because it requires more detailed information on the pinning sites. Despite this, some progress has been made in the past few decades to evaluate the scaling details for cases of strong pinning in vortex systems [40, 41]. While we will not explore the derivation here, this more recent work suggests that the creep law parameters will have different dependencies on interface properties than weak collective pinning even for point-like pinning defects. We will briefly present the corresponding scaling results for point-like strong pinning sites to illustrate different possibilities for explaining

the observed experimental data. Other types of strong pinning sites including extended defects have been explored for vortex systems but are beyond the scope of this thesis.

3.3 Evaluating the Energy Scale for the Creep Law

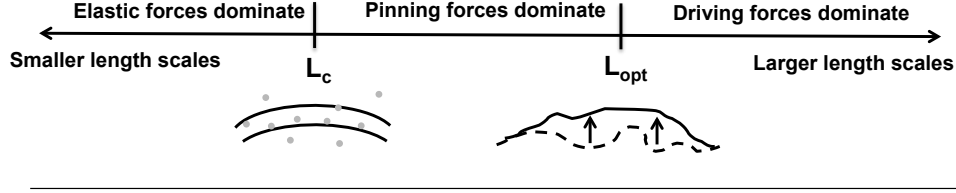


FIGURE 3.2: A diagram of how the different forces affecting the DW dominate at different length scales.

Before evaluating how the creep law energy scale U depends on DW properties, we will attempt to provide an interpretation of what this energy scale represents. In the presence of the random pinning potential and in the absence of a driving force (which we can neglect because we are interested in the low-driving limit), a static equilibrium is obtained where elastic forces and pinning forces balance and cause the DW to roughen on a characteristic length scale called the Larkin length or collective pinning length, denoted as L_c (Figure 3.2). We consider two points on this pinned DW which are a length $L \gg L_c$ apart and on the z -axis. Between these two points, the DW will tend to follow a weaving path through the pinning potential, but this path is not unique and there can be many alternatives which satisfy the force balance. These alternate paths are distinct metastable states which we can distinguish by their average distance from the z -axis in the direction of DW motion, $\bar{u}$, and the total energy (elastic and pinning), U_{meta} . Thermal forces can lead to transitions between metastable states that are called avalanches. The two values characterizing the metastable states have different power law dependencies on L that astoundingly can be determined exactly for 1+1 manifold dynamics using renormalization group theory [42]. When a small driving force is applied, the energy required to transition between metastable states is a sum of the difference in elastic and pinning energy given by

ΔU_{meta} and the energy change from the driving force which is proportional to $\Delta\bar{u}$. We can then use the known L dependencies of these terms to determine the L for which the energy barrier to transition between metastable states is a maximum, known as L_{opt} . This is the optimal or critical avalanche length, which is the rate limiting step in DW creep. The parameter U in the creep law is therefore the characteristic energy between metastable segments of length L_{opt} . In the following sections, we will derive expressions for L_c and L_{opt} in terms of the interface parameters, eventually allowing us to express U in terms of these parameters as well.

3.3.1 The Larkin Length

The characteristic length scale of DW curvature in a disordered medium is determined by the competition between elastic and pinning forces. Following Blatter [34], we can express this length scale by considering the energy of a segment of the DW of length L that has endpoints along the z -axis and curves outward from the axis a characteristic distance u . The sum of elastic and pinning energies is

$$\mathcal{E}_{el+pin} = \varepsilon t_f \frac{u^2}{L} - (\gamma \lambda t_f L)^{1/2} \quad (3.7)$$

The second term is the total pinning energy, which scales as the $\frac{1}{2}$ power of the volume of the DW because the pinning potential is random, so the average pinning energy decreases with the amount of space that is sampled, depending on volume to the $-\frac{1}{2}$ power. We consider a segment that is deformed from the z -axis into a new pinning potential, so that $u = \lambda$, and determine the length scale L_c that minimizes energy density.

$$\frac{\partial \mathcal{E}_{el+pin}/L}{\partial L} = -2\varepsilon t_f \lambda^2 L_c^{-3} + \frac{1}{2} (\gamma \lambda t_f)^{1/2} L_c^{-3/2} = 0 \quad (3.8)$$

$$L_c \propto \lambda \frac{\varepsilon^{2/3} t_f^{1/3}}{\gamma^{1/3}} \quad (3.9)$$

We can also determine the characteristic pinning energy, U_c of the segment simply by using $L = L_c$ in the second term on the right hand side of equation 3.7, obtaining

$$U_c \propto \lambda \varepsilon^{1/3} \gamma^{1/3} t_f^{2/3} \quad (3.10)$$

The critical force to depin the DW is just the force required to overcome this pinning energy

$$f_c = \frac{U_c}{\lambda L_c} = \frac{\gamma^{2/3} t_f^{1/3}}{\lambda \varepsilon^{1/3}} \quad (3.11)$$

These values are used in the next section to determine the properties of the DW at length scales larger than L_c . Before we move on to that analysis, we include a wrinkle to the preceding calculation that will arise in Chapter 4 when the stiffness ε of the DW is dispersive, meaning it depends on L . In this case, an additional term appears in equation 3.8. This added term forces us to replace ε in equations 3.9 and 3.10 with ε^* , given by

$$\varepsilon^* = \varepsilon(L_c) - \frac{L_c}{2} \frac{\partial \varepsilon}{\partial L}(L_c) \quad (3.12)$$

One problem with this expression is that it prevents us from solving explicitly for L_c as this term now appears on both sides of equation 3.9. This issue can be circumvented by numerically determining a self-consistent L_c .

An additional complication can arise if thermal fluctuations are relatively strong, which causes L_c to become a function of temperature. The large fluctuations cause the assumption of static

equilibrium to fail and tend to effectively smooth out the pinning potential. These become significant when thermal energy of the DW becomes comparable to the pinning energy at the nonthermal Larkin length, $L_c(T = 0)$. The temperature at which this occurs is known as the depinning temperature, T_{dp} . The expressions for T_{dp} and the temperature-dependent Larkin length, $L_c(T)$, are

$$T_{dp} = U_c/k \quad (3.13)$$

$$L_c(T) = L_c \left(1 + \left(\frac{T}{T_{dp}} \right)^5 \right) \quad (3.14)$$

Experimental analysis of the temperature dependence of DW velocity in Pt/Co thin films indicates a T_{dp} well over 1000° C [43], so we will ignore any temperature dependence of L_c as our experiments are conducted well below T_{dp} .

Finally, for strong pinning each segment is pinned by a single defect so the pinning energy is independent of the elastic modulus. The length scale for curvature is still determined by invoking mechanical equilibrium between the pinning forces and elastic forces, giving.

$$L_c^{sp} = \frac{\varepsilon^{1/2}}{\gamma^{1/4} n_p^{1/4}} \quad (3.15)$$

$$U_c^{sp} = \frac{\gamma^{3/4}}{\varepsilon^{1/2} n_p^{5/4} \lambda^2} \quad (3.16)$$

$$f_c^{sp} = \frac{\gamma^{3/4}}{n_p^{1/4} \varepsilon^{3/2}} \quad (3.17)$$

3.3.2 The Critical Avalanche Length

Seminal work by Huse, Henley, and Fisher [42] revealed key features of the statistical mechanics of a 1+1 elastic manifold in a random potential. The results, obtained using the method of images and later with renormalization group theory [44], are scaling expressions for the long range properties of the rough DW metastable states in terms of the properties at the Larkin length. As $L \rightarrow \infty$, the distance between metastable states was found to grow as

$$\Delta \bar{u}(L) = \lambda \left(\frac{L}{L_c} \right)^{2/3} \quad (3.18)$$

While the energy barrier between these states grows as

$$\Delta U_{meta}(L) = U_c \left(\frac{L}{L_c} \right)^{1/3} \quad (3.19)$$

Once a small field is applied, the energy barrier between metastable states can be expressed as

$$U(L) = \Delta U_{meta}(L) - fL\Delta \bar{u}(L) = U_c \left(\frac{L}{L_c} \right)^{1/3} - fL\lambda \left(\frac{L}{L_c} \right)^{2/3} \quad (3.20)$$

This is a critical nucleation problem, and we find the maximum energy barrier for an avalanche by taking the derivative with respect to L

$$\frac{\partial U}{\partial L} = \frac{1}{3}U_c L_{opt}^{-2/3} L_c^{-1/3} - \frac{5}{3}f\lambda L_{opt}^{2/3} L_c^{-2/3} = 0 \quad (3.21)$$

Solving for L_{opt} and using equation 3.11,

$$L_{opt} \propto L_c \left(\frac{f_c}{f} \right)^{3/4} \quad (3.22)$$

Inserting back into equation 3.21, we arrive at the critical energy barrier for creep motion.

$$U \propto U_c \left(\frac{f_c}{f} \right)^{1/4} \quad (3.23)$$

Which we can express in terms of interface properties using equations 3.10 and 3.11

$$U \propto \frac{\lambda^{3/4} \varepsilon^{1/4} \gamma^{1/2} t_f^{3/4}}{f^{1/4}} \quad (3.24)$$

Alternatively, strong pinning gives

$$U^{sp} \propto \frac{\gamma^{15/16}}{\varepsilon^{7/8} n_p^{21/16} \lambda^2 f^{1/4}} \quad (3.25)$$

The dependence of U on $f^{-1/4}$ is readily verifiable in the magnetic DW system by examining the velocity as a function of driving field H_z . This critical exponent has been observed to hold over many orders of magnitude of DW velocity in experiments conducted across a number of different material systems [36]. In the following chapter, we will attempt to describe how U depends on an in-plane magnetic field H_x , which manipulates the properties ε , λ , and γ .

3.4 Attempt Frequency and the Creep Law Prefactor

While the creep law prefactor v_0 has received considerably less attention in vortex theory than the Arrhenius term, it may also vary with the manifold properties. Since this problem has not

been directly addressed for vortex systems, the reasoning in this section is partially our own work and should be subjected to more scrutiny than the previous sections. We are particularly interested in the prefactor for our system because it has been suggested to contain information on a newly described magnetic effect known as chiral damping [25]. As with the energy scale, we begin with an interpretation of the meaning of v_0 , and will first reiterate that the exponential in equation 3.3 corresponds to the probability of successfully surmounting the energy barrier between metastable segments of length L_{opt} . The prefactor thus describes the velocity that the DW would have if every attempt at surmounting this energy barrier were successful, and we should be able to write v_0 as the product of the distance between the metastable states and the thermal attempt frequency.

$$v_0 \propto \Delta \bar{u}(L_{opt}) * \omega_{opt} \quad (3.26)$$

The first component has already been obtained in equations 3.18 and 3.22. The second is less trivial, like many of the other parameters we will assume a power law form at large length scales.

$$\omega(L) = \omega_c \left(\frac{L}{L_c} \right)^\zeta \quad (3.27)$$

Determining the exponent ζ is still in progress and requires an in-depth understanding of the renormalization group methods used to find the critical exponents discussed previously. The thermal frequency at the Larkin length, however, has close analogies to thermal time scales that have been evaluated for vortex creep [34]. In general, the oscillating DW segment will have both an effective mass due to its internal degree of freedom and friction due to Gilbert damping. The vast majority of studies involving DW motion focus on their translational behavior and can neglect the DW mass. We will show in section 4.2 that this is not necessarily valid for

DW bending modes and both mass-dominated and friction-dominated oscillations are possible for typical experimental conditions, which is relevant to our present discussion because the two limits have different characteristic frequencies.

$$\omega_c \propto \frac{1}{L_c} \left(\frac{\varepsilon}{m} \right)^{1/2}, \text{ mass-dominated oscillations} \quad (3.28)$$

$$\omega_c \propto \frac{\varepsilon}{\eta L_c^2}, \text{ damping-dominated oscillations} \quad (3.29)$$

In the above expressions, m is the DW effective mass and η is the friction which is proportional to the Gilbert damping parameter α . While in-plane field induced variation in the observed prefactor has been attributed to changes in α , we note here that v_0 will also depend on other interface properties which are known to have in-plane field dependencies which will need to be accounted for in order to deduce the correct form of α as a function of H_x . The dynamical interface properties m and η are evaluated in section 4.2 and their potential ability to describe experimental phenomena is explored briefly in Chapter 5.

Chapter 4

An Effective Elastic Band Model for Dzyaloshinskii Domain Walls

In the previous chapter, the governing behavior of a 1+1 elastic manifold was described in the low-field creep regime and expressed in terms of general pinned interface properties. This chapter will focus on evaluating these properties for a Dzyaloshinskii DW in terms of the parameters of the system, both material and experimental. We will develop an effective elastic band model for the DW through the derivation of approximate analytic forms for the key interface properties, followed by verification through comparison with micromagnetic results. We are particularly interested in the dependence of the properties on an applied in-plane magnetic field H_x , as the ability to alter the elastic properties with an externally applied field is a powerful tool for experimentally probing the creep law behavior. The results of such experiments are explored in the next chapter.

4.1 Static Properties

The stiffness ε , DW width λ , and pinning strength γ can all be determined by the energetics of a static DW. In order to express these properties analytically, we require an approximate model for the DW energy. Using all the micromagnetic energy terms from section 1.1.2 and 1.2, we will consider a planar Dzyaloshinskii DW with orientation Θ and internal magnetization direction ϕ in the presence of an in-plane magnetic field. We take the direction of the applied field to define the x -axis while the effective field due to DMI is always oriented along the DW normal (Figure 4.1).

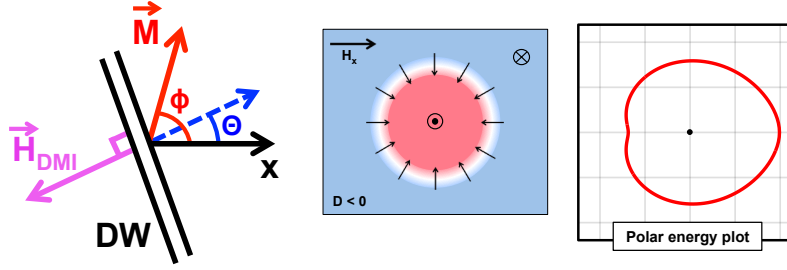


FIGURE 4.1: (Left) Top-down diagram of a Dzyaloshinskii DW clarifying the meaning of Θ and ϕ . (Middle) Bubble domain with effective fields due to DMI represented by the radial arrows. (Right) Polar energy plot corresponding to the bubble domain in an in-plane field.

As a first approximation, we will neglect any tilting of the magnetization in the domains due to the applied in-plane field and assume that the DW width is constant and equivalent to the width without an in-plane field or DMI, given by $\lambda_0 = \sqrt{A/K_{eff}}$. This allows us to integrate the micromagnetic energy functional and arrive at

$$\sigma(\Theta, \phi) = \sigma_0 - \pi D \cos(\phi - \Theta) - \pi \lambda_0 \mu_0 H_x M_s \cos(\phi) + \frac{\ln(2)}{\pi} t_f \mu_0 M_s^2 \cos^2(\phi - \Theta) \quad (4.1)$$

The first term is the Bloch wall energy $\sigma_0 = 4\sqrt{AK_{eff}}$, followed by a term due to DMI that favors a Néel wall and the Zeeman term that favors the internal magnetization aligning with

the applied field. The fourth term is the DW anisotropy energy rooted in the magnetostatic favorability of Bloch walls over Néel walls.[45] Minimizing DW energy with respect to internal magnetization results in the equilibrium values for a rigid wall as a function of orientation, $\sigma^{eq}(\Theta)$ and $\phi^{eq}(\Theta)$, which have been used in the calculation of DW tilting angles in nanowires and equilibrium droplet shapes via the Wulff construction.[46, 47] The combination of the in-plane field, which alters the wall energy depending on its absolute internal magnetization direction, and the other terms which couple the magnetization with orientation, produces an anisotropic wall energy as can be seen in the polar energy plot of a bubble domain. This anisotropic energy has an important impact on the stiffness of the DW, which we explore below.

4.1.1 Stiffness

As described in the previous chapter, an elastic manifold is characterized by its tendency to reduce curvature. This tendency is quantified by the elastic modulus, also known as stiffness or surface stiffness for a 1+1 manifold like the Dzyaloshinskii DWs we are interested in. If we take a planar DW segment with fixed endpoints and then force it to bend out of a straight line, both the energy $\mathcal{E}$ and arc length S of the DW segment will change. We can define the stiffness by this change as shown below.

$$\varepsilon = \frac{\Delta \mathcal{E}}{\Delta S} \quad (4.2)$$

This elastic energy scale has routinely been assumed to be $\varepsilon = \sigma$ for a magnetic domain wall[35, 37], which is strictly true only for the isotropic case where σ is not a function of the wall orientation. For an anisotropic interface, the energy of the curved wall will change not only because the length is increasing, but also because the orientations are changing. Here $\varepsilon = \tilde{\sigma}$ where $\tilde{\sigma}(\Theta) = \sigma(\Theta) + \sigma''(\Theta)$ depends not only on the energy of the local orientation but

also on the energies of orientations in close proximity to Θ . This form of the surface stiffness has previously found broad utility in describing the mobility of solid/liquid interfaces[48] and was first employed in creep theory to describe the movement of flux lines through pinning sites in anisotropic superconductors.[34, 49]

A stiffness value can be calculated from the equilibrium planar wall energy simply by

$$\tilde{\sigma}(\Theta, L \rightarrow \infty) = \sigma^{eq}(\Theta) + \frac{\partial^2 \sigma^{eq}}{\partial \Theta^2}(\Theta) \quad (4.3)$$

As noted above and discussed later in the text, this expression is valid only for long wavelength distortion and so provides insight into the long range stability of the planar domain wall. In Figure 4.2a, σ^{eq} and the long wavelength limit of $\tilde{\sigma}$ vs. $\mu_o H_x$ are compared for Dzyaloshinskii DWs with varying H_{DMI} , where positive H_x implies it is anti-parallel to H_{DMI} (henceforth referred to as the anti-parallel case). While σ^{eq} is symmetric about a maximum at H_{DMI} , $\tilde{\sigma}$ is highly asymmetric about a maximum centered at $H_x = 0$. Although σ^{eq} and $\tilde{\sigma}$ are qualitatively similar when the applied field is parallel to H_{DMI} , we note the striking result that as σ increases for anti-parallel H_x , $\tilde{\sigma}$ drops very rapidly at a field which is dependent on H_{DMI} . The calculation of $\tilde{\sigma}$ is complicated by the occurrence of negative values around H_{DMI} , as they suggest that a non-planar, faceted wall configuration is favored. The thermodynamic properties of faceted configurations have been explored in the study of crystal growth and are described geometrically by the pedal, $\Gamma(\Theta)$, of the equilibrium profile as determined via the Wulff construction on the polar energy plot, $\sigma(\Theta)$ (Figure 4.2b). The pedal gives both the driving force for faceting, $\Delta\sigma$, and the facet angles, $\Delta\Theta$, as indicated.[50][51] The faceting angle becomes non-zero where $\tilde{\sigma}$ is negative, and optimum orientations calculated from equation 4.18 show good agreement with numerical energy minimization results (Figure 4.2e).[31] Calculating stiffness from $\Gamma(\Theta)$, we find that the linear elastic response vanishes for cases where faceting is favored.

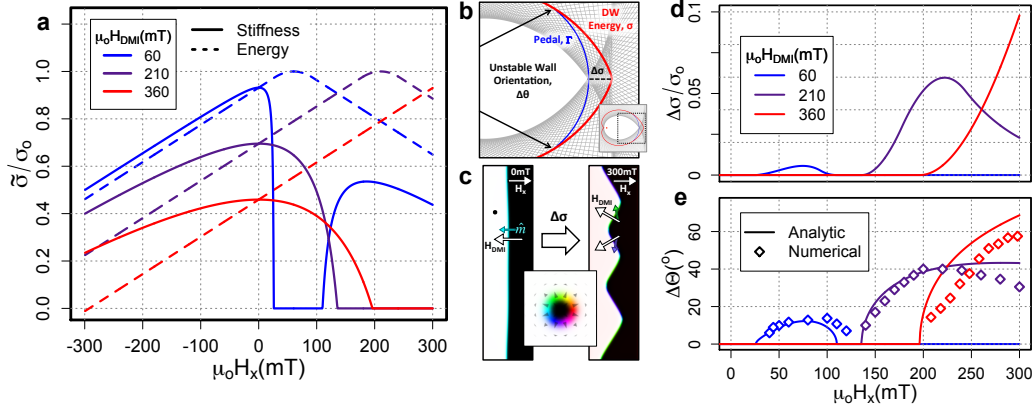


FIGURE 4.2: a) σ^{eq} and long wavelength $\tilde{\sigma}$ vs $\mu_0 H_x$ for varying $\mu_0 H_{DMI}$ in a Dzyaloshinskii DW with $\mu_0 H_k = 1$ T, $M_s = 600$ kA/m, and $t_f = 1.8$ nm. b) Wulff construction (grey), $\sigma^{eq}(\Theta)$ (red), and $\Gamma(\Theta)$ (blue) for a Dzyaloshinskii DW with $\mu_0 H_x = 300$ mT and $\mu_0 H_{DMI} = 360$ mT. The red point of the inset indicates the origin. c) Example calculation of wall faceting in Mumax³ for the same conditions as (b). d) Calculated driving force for faceting, $\Delta\sigma$. e) Calculated equilibrium facet orientation, $\Delta\Theta$, with superimposed numerical calculation results.

In real materials, a pinning potential can deform the domain wall at small length scales, in which case Heisenberg exchange along the wall will prevent the internal magnetization from assuming $\phi^{eq}(\Theta)$ everywhere and invalidate equation 4.3 (Figure 4.3). In order to express the stiffness in this case, the magnetization profile and energy for an infinitesimally curved domain wall can be calculated semi-analytically by considering perturbations of the 1-D expression in equation 4.18. We consider a narrow domain wall, neglecting non local terms in the demagnetizing energy. Augmenting the 1-D energy with an additional term for the exchange along the DW, the combined energy functional is

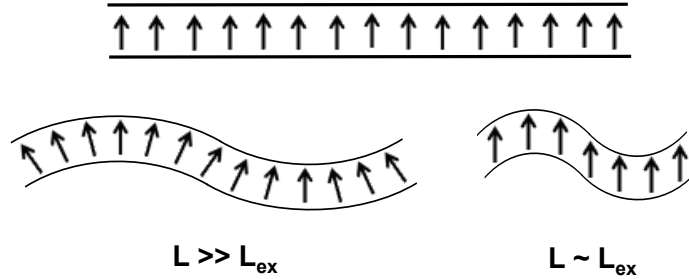


FIGURE 4.3: Diagram illustrating that when a Néel wall is deformed over large length scales, it can maintain a nearly Néel configuration everywhere. At smaller length scales, exchange forces the internal moments to align.

$$\mathcal{E} = \int \sigma(\Theta, \phi) + 2A\lambda \left(\frac{\partial \phi}{\partial s} \right)^2 ds \quad (4.4)$$

We expand the 1-D energy to second order about the orientation, Θ_o , and equilibrium magnetization, ϕ_o , for a straight DW segment

$$\begin{aligned} \sigma(\Theta, \phi) = \sigma + (\Theta - \Theta_o)\sigma_\Theta + (\phi - \phi_o)(\Theta - \Theta_o)\sigma_{\Theta\phi} \\ + \frac{1}{2}(\Theta - \Theta_o)^2\sigma_{\Theta\Theta} + \frac{1}{2}(\phi - \phi_o)^2\sigma_{\phi\phi} \end{aligned} \quad (4.5)$$

In the right hand side of the above equation, σ and its partial derivatives, indicated by subscripts, are all evaluated at (Θ_o, ϕ_o) . A stationary ϕ profile will satisfy the Euler-Lagrange equation

$$(\Theta - \Theta_o)\sigma_{\Theta\phi} + (\phi - \phi_o)\sigma_{\phi\phi} - 4A\lambda \frac{\partial^2 \phi}{\partial s^2} = 0 \quad (4.6)$$

A segment that is deformed into a circular arc of radius R has an orientation profile along the wall given by

$$\Theta(s) = \Theta_o - \frac{s}{R} \quad (4.7)$$

Which can be used to solve equation 4.6 for the magnetization profile, giving

$$\begin{aligned} \phi(s) = \phi_o + \frac{s}{R} \frac{\sigma_{\Theta\phi}}{\sigma_{\phi\phi}} + C_1 \sinh\left(\frac{s}{\Lambda}\right) + C_2 \cosh\left(\frac{s}{\Lambda}\right) \\ \Lambda = \lambda \sqrt{\frac{\sigma_o}{\sigma_{\phi\phi}}} \end{aligned} \quad (4.8)$$

Here Λ is the length scale for exchange along the domain wall, the vertical Bloch line width.

For a domain wall segment with fixed endpoints a length L apart, the bounds of integration for

large R are

$$s_{ep} = \pm R \arcsin \left(\frac{L}{2R} \right) \approx \pm \frac{L}{2} \left(1 + \frac{1}{6} \left(\frac{L}{2R} \right)^2 \right) \quad (4.9)$$

Combining the expressions for σ , Θ , ϕ , and s_{ep} , we evaluate $\mathcal{E}$ of the curved segment and will focus on the case where we do not fix the magnetization of the endpoints. Minimizing energy with respect to C_1 and C_2 we have

$$\begin{aligned} C_1 &= \frac{\sigma_{\Theta\phi}}{\sigma_{\phi\phi}} \frac{\Lambda}{2R} \operatorname{sech} \frac{L}{2\Lambda} \\ C_2 &= 0 \end{aligned} \quad (4.10)$$

The ground state energy of a curved domain wall can now be directly determined, from which we can extract the elastic response through the relation

$$\mathcal{E}(R) \approx L \left(\sigma + \frac{1}{6} \left(\frac{L}{2R} \right)^2 \tilde{\sigma} \right) \quad (4.11)$$

The result is a dispersive stiffness given by

$$\begin{aligned} \tilde{\sigma}(\Theta, L) &= \sigma + \sigma_{\Theta\Theta} - \frac{\sigma_{\Theta\phi}^2}{\sigma_{\phi\phi}} \zeta \left(\frac{L}{2\Lambda} \right) \\ \zeta(\ell) &= 1 - \frac{3}{\ell^3} (\ell - \tanh(\ell)) \end{aligned} \quad (4.12)$$

As $L \rightarrow \infty$, $\zeta \rightarrow 1$ and we recover an expression for $\tilde{\sigma}$ that is independent of symmetric exchange along the DW. This expression, which can be directly derived from equations 3 and 5, is the generalized stiffness for a surface with an orientational order parameter in local equilibrium that was first identified by Fournier to describe soft materials.^[52] Conversely, as $L \rightarrow 0$, $\zeta \rightarrow 0$ and the stiffness corresponds to the domain wall bending while maintaining a constant internal

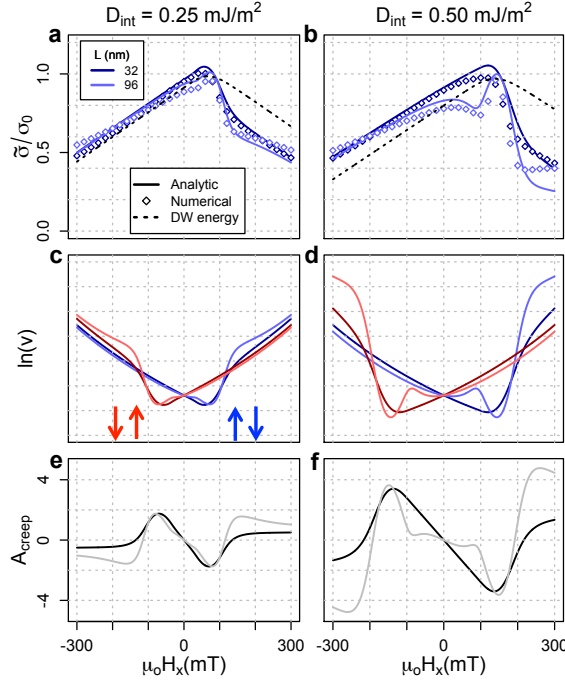


FIGURE 4.4: a-b) Normalized $\tilde{\sigma}$ vs $\mu_0 H_x$ for a) $D = 0.25 \text{ mJ/m}^2$ and b) $D = 0.5 \text{ mJ/m}^2$. Micromagnetic results were obtained from DW bending experiments outlined in the appendix to this work. c-d) Corresponding $\uparrow\downarrow$ and $\downarrow\uparrow$ DW velocity behavior for the length scales and D values in (a-b). e-f) Creep velocity asymmetry $\ln(v_{\uparrow\downarrow}/v_{\downarrow\uparrow})$. Dashed lines indicate the behavior of the DW energy.

magnetization direction. The third term in the expression for stiffness therefore corresponds to the energy decrease due to the DW moments relaxing to the ground state of the curved wall.

Stiffness behavior as a function of $\mu_0 H_x$ is explored in Figure 4.4 for a series of perturbation lengthscales and $\mu_0 H_{DMI}$ with Θ_o fixed at 8° to account for roughness as justified later. To better compare the trends in $\tilde{\sigma}$ with experiment, we have calculated a DW velocity using experimentally determined coefficients of the creep law for $H_x = 0$, $H_z = 4.25 \text{ mT}$ with the exponential factor scaling by $(\tilde{\sigma}/\tilde{\sigma}_{H_x=0})^{1/4}$, noting that the qualitative features are insensitive to the exact choice of coefficients and exponent. We see from either $\tilde{\sigma}$ or velocity that at very low length scales the effect of DMI is to both shift the curve horizontally, as described by Je *et al.*[8] for σ , and induce an asymmetric vertical shift about $\mu_0 H_{DMI}$ which is superficially similar to the chiral damping proposed by Jué *et al.*[25]. Unlike chiral damping, these two effects offset each

other at high fields so the stiffness converges for $\uparrow\downarrow$ and $\downarrow\uparrow$ walls. As the pinning length scale increases, sharp drops in stiffness develop at fields where the wall transitions from fully Néel to having some Bloch component. The most striking consequence is that the anti-parallel case can have multiple local extrema as well as a significant window where it is expected to have a greater velocity than its parallel counterpart before the two cases converge at much larger fields.

The behavior of the analytic model is confirmed by 2-D micromagnetic DW bending calculations conducted with mumax3 as described in chapter 2. The stiffness of a DW can be measured micromagnetically by determining how readily the domain wall deforms in spatially nonuniform perpendicular fields (Figure 4.5). Periodic boundary conditions are applied at the top and bottom of the simulation window and we know that in the upper half of the simulation, the elastic restoring force on the domain wall is balanced by the force of the Zeeman interaction from the perpendicular field.

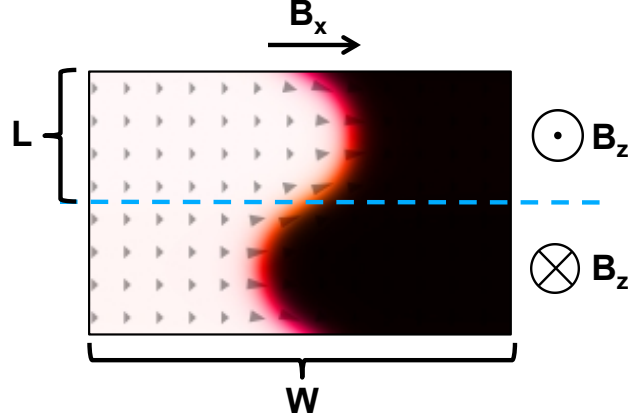


FIGURE 4.5: Snapshot of micromagnetic simulation grid showing an exaggerated domain wall bending experiment. Perpendicular fields of opposite signs are applied to the upper and lower halves of the grid in order to induce a sinusoidal deformation.

$$\tilde{\sigma} \frac{u}{L} = \frac{1}{4} B_z M_s L \quad (4.13)$$

Where $\tilde{\sigma}$ is the stiffness, u is the maximum domain wall displacement. The distance that the

DW bends is directly related to the change in average perpendicular magnetization of the top half, $\Delta \bar{m}_z^{top}$, that occurs when the z-fields are applied.

$$u = \frac{3}{4} W \Delta \bar{m}_z^{top} \quad (4.14)$$

Here W describes the size of the micromagnetic mesh in the direction perpendicular to the wall.

Solving for stiffness, we have

$$\tilde{\sigma} = \frac{L^2}{3W} \frac{B_z}{\Delta \bar{m}_z^{top}} \quad (4.15)$$

We note we can also get the change in total energy due to the applied fields from equation [4.13](#) and find the stiffness from the relation

$$\Delta \mathcal{E}_{tot} = \frac{1}{32} \frac{(B_z M_s L)^2}{\tilde{\sigma}} \quad (4.16)$$

In general, the analytic and micromagnetic methods agree quite well, with all the qualitative features matching up very well. There is some disagreement at relatively large in-plane fields, which could be due to the fact that DW width changes were ignored in the analytic method. The assumptions made to derive the analytic result also break down when L is on the order of λ . At these very short deformation length scales, we can no longer assume that the DW will maintain the same profile as the planar wall, and nonlocal effects for the dipolar interaction become more significant. For these situations the micromagnetic method is superior.

4.1.2 Domain Wall Width

The in-plane field dependence of the Dzyaloshinskii DW width has been addressed recently in the contexts of both the creep and flow regime[53, 54]. A key difficulty with this analysis is that there are several different methods of characterizing the DW width, each of which can yield distinct results. One technique is to assume that the profile of the polar angle of magnetization θ maintains the same form as a function of the distance along the DW normal, defined here by the u -axis. This amounts to assuming a magnetization profile given by

$$\theta(u) = 2 \arctan \left(e^{u/\lambda} \right) \quad (4.17)$$

Here λ is not necessarily equal to λ_0 when a field is applied. This prevents us from combining the exchange and anisotropy energy terms directly so that in place of σ_0 in equation 4.18, we get a sum of two terms with different λ dependencies, shown below.

$$\sigma(\Theta, \phi) = 2 \left(\frac{A}{\lambda} + K_{eff} \lambda \right) - \pi D \cos(\phi - \Theta) - \pi \lambda_0 \mu_0 H_x M_s \cos(\phi) + \frac{\ln(2)}{\pi} t_f \mu_0 M_s^2 \cos^2(\phi - \Theta) \quad (4.18)$$

If we minimize the energy with respect to λ , we get an approximation for the DW width as a function of in-plane field. Reintroducing this variable width to the above expression for energy, the resulting energy values show poor agreement with micromagnetic results even at moderate in-plane fields with very little tilting in the domain. We believe the problem with this calculation is that the DW profile at moderate fields is not simply a rescaled version of the zero-field DW profile, and that the in-plane field induces a significant change in the DW shape. To avoid this

problem, we will specifically define the geometric DW width, which is related to the gradient in m_z at the middle of the DW wall.

$$\lambda = \left(\frac{\partial m_z}{\partial u} \right)^{-1} \quad (4.19)$$

The various micromagnetic energy terms can be balanced at the DW center, where $m_z = 0$, to give

$$\lambda = \sqrt{\frac{1 + h_x \cos \phi_{eq}}{1 - h_x \cos \phi_{eq}}} \quad (4.20)$$

Where $h_x = \frac{\mu_0 M_s H_x}{2K_{eff}}$. The geometric DW width can also be directly calculated from micromagnetic results (Figure 4.6), with good agreement to the analytic result even at large values of the in-plane field.

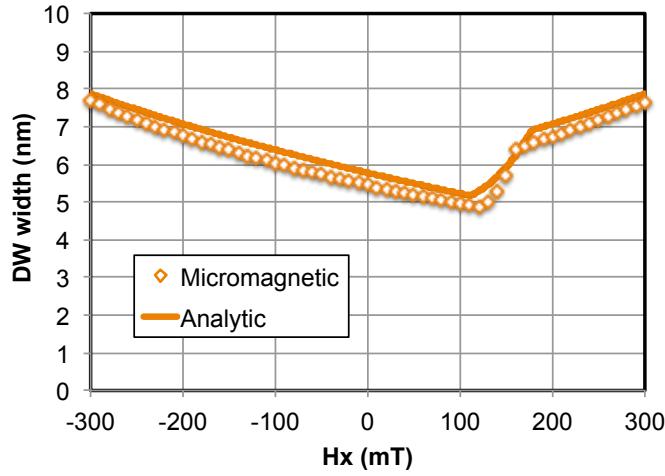


FIGURE 4.6: Comparison of analytic and micromagnetic results for the in-plane field dependence of the geometric DW width.

4.1.3 Pinning Strength

Spatial variation of the material parameters in thin perpendicular magnetic films creates an uneven energy landscape for a DW. Applying an in-plane field has no effect on the material parameters themselves, but it can alter the dependence of the DW energy on those material parameters and therefore change the pinning energy [55]. For instance, a domain wall segment that lies in an area of the film with average effective anisotropy K_{eff} will have a lower energy than a segment that lies in an area with a higher average effective anisotropy $K_{eff} + \delta K_{eff}$, so there will be an energy barrier that must be surmounted in order to move the DW from the low anisotropy area to the high anisotropy area. For a general material parameter X , we call the pinning by small spatial variations in the parameter δX pinning, and can quantify its strength simply by

$$\gamma_{\delta X} \propto \left| \frac{\partial \sigma}{\partial X} \right| \quad (4.21)$$

The pinning strength has been evaluated with micromagnetic simulations, and it is found that depending on the magnetic character of the dominant pinning mechanism, qualitatively different behavior can be observed. This could help explain the disparity in reported results between material systems. For instance, in δM_s and δK_u pinning the strength increases as the field increases, which would correspond to a reduction in velocity with application of an in-plane field that has been observed in some experiments [56] and stands in complete contrast to the simple model proposed in chapter 1. Being able to deduce the pinning mechanism from creep behavior would provide key insight into a technologically critical and difficult-to-study subject, but there are some important caveats to the prior analysis. First, there is no reason to believe that whatever physical defect causes the magnetic parameters to vary would only affect one of them, so the variation of material parameters are likely to be correlated. If that correlation

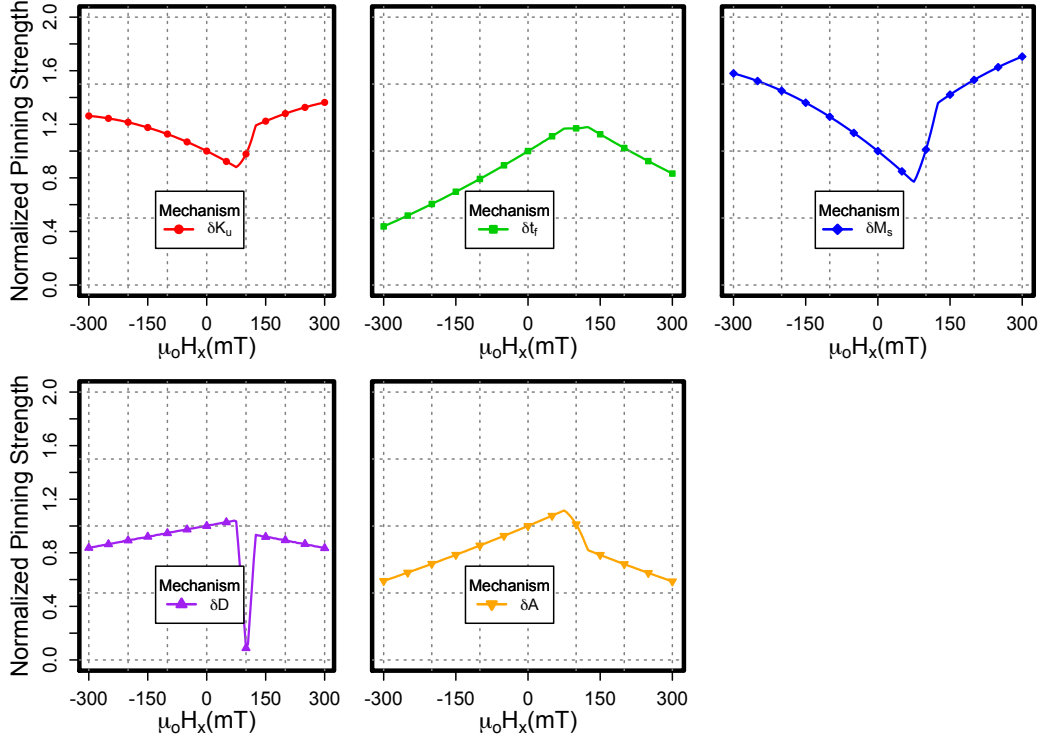


FIGURE 4.7: In-plane field dependence of pinning strength for a number of different mechanisms.

is known, the pinning strength can be determined with the above equation by expressing the various magnetic parameters in terms of an indirect parameter X .

Next, the pinning sites may not correspond to small changes in the material parameters, as there could be areas of the film where the parameters are very different than their average values. This would invalidate the perturbative approach and require more detailed micromagnetic analysis of the pinning site. Finally and perhaps most importantly, there does not have to be a single dominant pinning mechanism and/or the dominant mechanism can change as a function of the in-plane field.

The variation of the pinning strength in an in-plane field introduces a large number of unknowns into our analysis of DW velocity that we should not be able to ignore, given our determination that the magnitude of the pinning strength effect is significant for simple pinning models. If the

material parameters including DMI and other chiral effects were well known, however, it would be possible to extract the pinning strength as a function of in-plane field experimentally and use it to justify hypotheses regarding the identity of the pinning sites.

4.2 Dynamic Properties

The expansive field of ferromagnetic DW dynamics has remained productive throughout the past half century, punctuated by many major analytic achievements that have been supported experimentally, such as the description of Walker breakdown[57], spin transfer torque-driven DW motion [58], and more recently spin-orbit torque-driven DW motion [13]. One of the most common approximations made in the analytic description of DW motion is the rigid wall approximation [59] which allows the DW to be treated as a simple translating point object with an additional degree of freedom for the internal magnetization. This presents a problem for our purposes, because a completely rigid DW would not exhibit any creep phenomena and it is the deviations from rigidity, the bending modes [60], that will be our focus. Another feature of our analysis that distinguishes it from the majority of DW motion studies is that thermal forces are causing the motion, whereas thermal forces are frequently either ignored or expressed as a small perturbation in the presence of some other driving force [61]. For a general elastic band of length L with fixed endpoints, the governing equation for the motion is that of a damped harmonic oscillator.

$$m \frac{\partial^2 u}{\partial t^2} - \eta \frac{\partial u}{\partial t} - \frac{\varepsilon}{L} u + F_{therm} = 0 \quad (4.22)$$

Attempts to describe this motion analytically are detailed in the next section, but our primary approach is the ring-down method which will use micromagnetics. Ring-down analysis of an

oscillator involves establishing an excitation and then observing the decay of that excitation to establish the governing dynamics. In the bending experiments described in section 4.1.1, we applied a steady but spatially nonuniform perpendicular field to induce a sinusoidal distortion of the DW that will serve as our excitation, corresponding to a starting u value in 4.22. If we suddenly remove the perpendicular fields, the DW will relax exponentially to the planar state (Figure 4.8). If there is not too much friction, fourier analysis of the relaxation will reveal the characteristic frequency, and hence the effective mass, of the oscillator. The decay rate, when combined with the characteristic frequency information, yields the friction.

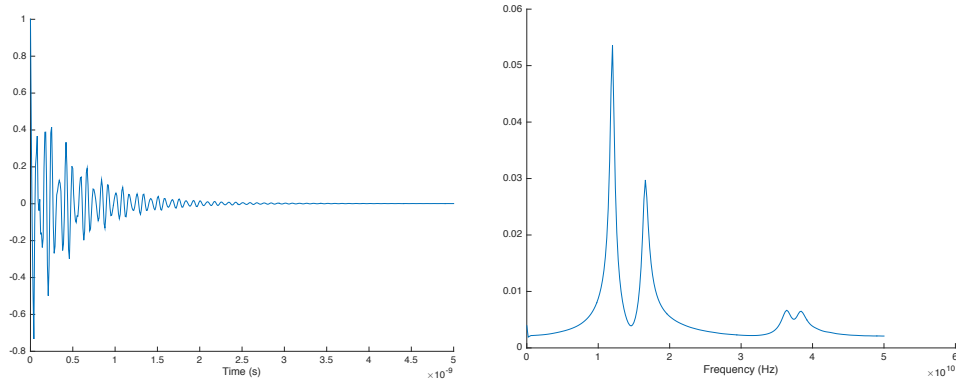


FIGURE 4.8: Results of a ring-down simulation showing an oscillating and decaying DW position (left) and its Fourier transform (right). The lowest frequency peak is believed to correspond to the simple bending mode while the identity of the second peak is currently unknown.

4.2.1 Inertia

The DW effective mass identified by Döring [62] arises from the internal degree of freedom ϕ of the moving DW taking on a different direction than for the static DW ϕ_{eq} . This occurs because the driving field results in a precessional torque on the internal magnetization which must be balanced by the DW anisotropy which corresponds to $\sigma_{\phi\phi}$ using the notation of section 4.1.1. The effective mass for a rigid, translating DW is known as the Döring mass and is given by

$$m_D \propto \frac{M_s^2 (1 + \alpha^2)}{\sigma_{\phi\phi} \gamma_{gm}^2} \quad (4.23)$$

Where α is the Gilbert damping constant and γ_{gm} is the gyromagnetic ratio. For a nonuniformly translating DW, ϕ will vary along the length of the segment and we must consider exchange along the DW, just as we did in section 4.1.1 for the stiffness. The governing equations for this type of behavior have been described in the context of bubble dynamics without DMI by Slonczewski [63].

$$\begin{aligned} \sigma_{\phi\phi} \left(\phi - \phi_{eq} - \Lambda^2 \frac{\partial^2 \phi}{\partial s^2} \right) &= \frac{M_s}{\gamma_{gm}} (1 + \alpha^2) \frac{\partial u}{\partial t} \\ \frac{\partial \phi}{\partial t} &= -\alpha \frac{\partial u}{\partial t} \end{aligned} \quad (4.24)$$

Just as we determined stiffness by introducing a parabolic distortion of the position of the DW and determining the resulting potential energy, here we introduce a parabolic distortion of the velocity of the DW and determine the resulting kinetic energy to extract the effective mass. The result is a dispersive effective mass given by

$$\begin{aligned} m(L) &= m_D \zeta \left(\frac{L}{2\Lambda} \right) \\ \zeta(\ell) &= \frac{8}{15} + 4\ell^{-5} \left(\ell - \frac{1}{3}\ell^3 - \tanh \ell \right) \end{aligned} \quad (4.25)$$

Combining the mass with the stiffness as described in section 3.4 gives a characteristic frequency that can be compared to the results from the ring down method (Figure 4.9). While the agreement is good for lower length scales, at larger scales the deviations are very significant.

This problem was similarly observed for the stiffness and may be improved by including variable DW width.

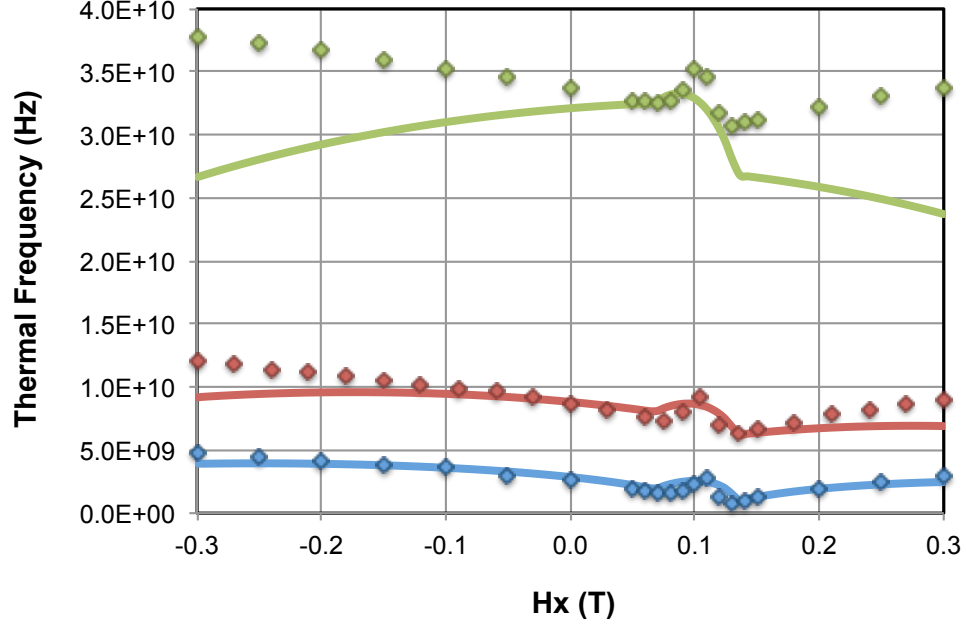


FIGURE 4.9: Comparison of analytic and micromagnetic determination for the thermal elastic mode frequency for $L=16\text{nm}$ (blue), 32nm (red), and 64nm (yellow-green).

4.2.2 Friction

In depth analytic and micromagnetic measurements of the friction will be left as a possible topic for future work, but we can draw some conclusions from a cursory analysis of the micromagnetic ring down results. We have found that the damping ratio of short wavelength DW oscillations is very small up until a Gilbert damping value around 0.1, at which point the characteristic frequency could be dominated by friction. This indicates that for smaller values of α , we would expect the frequency to be mass-dominated. This has some implications for the chiral damping effect that we will explore in part in the next chapter.

Chapter 5

Comparison of Creep Models to Experiment

Recent studies on asymmetric field-driven growth of magnetic bubble domains in perpendicular thin films exhibiting an interfacial DMI have provided a wealth of experimental evidence to validate models of creep phenomena, as we have seen in the previous chapter that key properties of the DW can be altered with the application of an external in-plane magnetic field. While asymmetric growth behavior has been attributed to the highly anisotropic DW energy which results from the combination of DMI and the in-plane field, many experimental results remain anomalous. In this chapter we seek to understand how readily the asymmetric DW mobility studies in several material systems can be described in terms of a more in-depth application of creep theory which was developed in the previous two chapters.

5.1 Data Processing and Fitting Techniques

Asymmetric bubble domain expansion experiments yield information on the creep velocity for all DW orientations, but geometric effects may skew the results as the orientation of a point on the bubble can change over time [24]. These type of considerations have been well studied in growing crystals [64] and can be modeled if the energy of the roughened DW is known, but we will ignore them here as they complicate our analysis. As the creep models that were derived in chapter 3 are based on a macroscopically planar DW, and because a Dzyaloshinskii DW is most sensitive to an in-plane field applied perpendicular to the DW, the only data that we will attempt to fit is for the left and right sides of the bubble domain where the field is parallel or antiparallel to the effective field from DMI. Several studies have also been conducted on nearly planar DWs by nucleating and expanding a rectangular domain and they indicate that the mobility is not strongly affected by the geometry, at least for the parallel and antiparallel cases. If we assume the bubble domain is an up domain, then the left (right) side is a $\downarrow\uparrow$ ($\uparrow\downarrow$) DW and we can denote its velocity by $v_{\downarrow\uparrow}$ ($v_{\uparrow\downarrow}$). Measuring the velocities for both sides at both positive and negative H_x is technically redundant because by symmetry arguments $v_{\uparrow\downarrow}(H_x) = v_{\downarrow\uparrow}(-H_x)$, but this redundancy serves as a useful check on the sample alignment because the equation does not hold for a tilted sample where H_x and $-H_x$ will have opposite components normal to the tilted film and therefore change the driving force. The velocity data can be transformed in order to separate out and more easily evaluate the chiral magnetic effects by defining antisymmetric and symmetric components

$$A_{creep}(H_x) = \ln \left(\frac{v_{\uparrow\downarrow}(H_x)}{v_{\downarrow\uparrow}(H_x)} \right) \quad (5.1)$$

$$S_{creep}(H_x) = \ln \left(\frac{v_{\uparrow\downarrow}(H_x)v_{\downarrow\uparrow}(H_x)}{v_{\uparrow\downarrow}(0)v_{\downarrow\uparrow}(0)} \right) \quad (5.2)$$

Since chiral magnetic effects produce asymmetry in the creep velocity behavior, A_{creep} will contain much of the information on DMI. The H_x field dependence of the velocity has dominated our discussion up to this point, but there is also considerable utility in determining the H_z field dependence of the velocities. Most importantly, the critical exponent can be verified by examining the linear correlation between $\ln(v)$ and $H_z^{-1/4}$. This relation holding lends some support to the concept that creep theory is an accurate description of the governing behavior and may also be able to describe the H_x dependence. Finally, analyzing the H_z dependence for different H_x values can in theory separate out the H_x dependence of the prefactor and the energy scale of the creep law. This information is extremely useful for attributing results to specific quantities as certain properties appear only in the energy scale, but these measurements are very difficult to get precise results on in practice. This difficulty arises from the fact that creep velocities are prone to noise as there is no guarantee that particularly large pinning sites are evenly distributed and the changes in H_x dependence with H_z can be subtle.

Before moving on to the specific models used to fit the experimental data, we briefly outline the fitting procedure. All of the models under consideration contain multiple parameters that are not readily measurable and can therefore be tuned to produce a better fit. In order to save time and eliminate bias, we have used automated fitting routines to select parameter values, minimizing the least squared error in A_{creep} and S_{creep} using a steepest descent method with a line search. To limit the possibility of overfitting, some parameters such as L_c and Θ_0 are restricted to a physically reasonable value or range and are verified to have little effect on estimates of the primary parameters of interest such as D .

5.1.1 Chiral Effects arising from DMI

From chapter 2, creep behavior can broadly be described by the equation

$$v_{creep} = v_0 \exp -U^* \quad (5.3)$$

Where we have nondimensionalized the characteristic energy barrier by normalizing to the thermal energy. Several studies [3, 8] have suggested that the in-plane field dependence of the creep velocity is well described using an energy barrier given by

$$U^*(H_x) = U^*(0) \left(\frac{\sigma(H_x)}{\sigma(0)} \right)^{1/4}, \text{ “energy model”} \quad (5.4)$$

This is derived by assuming that the elastic modulus ε is equal to the DW energy, which we have shown not to be the case in section 4.1.1. Using the actual elastic modulus gives

$$U^*(H_x) = U^*(0) \left(\frac{\tilde{\sigma}(H_x)}{\tilde{\sigma}(0)} \right)^{1/4}, \text{ “stiffness model”} \quad (5.5)$$

We will compare these models based on the experimental results in section 5.2, but it is important to note that both models ignore the in-plane field dependence of the DW width and the pinning strength, as well as any in-plane field dependencies present in the prefactor. These are not valid assumptions because if creep theory were strictly true, we would expect all of these terms to be significant. Unfortunately, for a model to be fully consistent with creep theory it needs to describe the pinning mechanism, which is a nontrivial challenge to say the least. Based on the fact that the chiral energy model is able to describe some experiments fairly well, we will take the stiffness model simply as a starting point to investigate more complete explanations for the various observed velocities.

5.1.2 Chiral Damping and Anomalous Chiral Energy

A chirality-dependent damping was suggested by Jué [25] based on velocity asymmetry that appeared to come from the creep law prefactor without any appreciable energy scale asymmetry that would be expected from DMI according to either of the models of the previous section.

$$v_0(H_x) = v_0 (1 + \alpha_{cd} \cos(\phi_{eq}(H_x))) , \text{ chiral damping} \quad (5.6)$$

Here α_{cd} is a parameter between -1 and 1 that characterizes the strength of the chiral damping. From first principles, it has been shown that chiral damping can arise from spin-orbit coupling and SIA [65], but it is not clear if the observed magnitude of the effect is in accord with these mechanisms. Other studies have also found chirality-dependent asymmetry that appears to be independent of DMI [66], but indicate that this may originate in the energy scale. We will refer to this as anomalous chiral energy (ACE) because it does not seem to depend directly on DMI and represent it with a model generated by multiplying a term of the same form as the chiral damping equation with the energy scale models of the previous section.

$$U_{ace}^*(H_x) = U^*(H_x) (1 + \epsilon_{ace} \cos(\phi_{eq}(H_x))) , \text{ anomalous chiral energy} \quad (5.7)$$

5.2 Material Systems

As outlined in section 1.3, there are several different technological applications for which the analysis of DMI is relevant and each uses distinct materials. For DW motion applications, CoNi multilayers are desirable for their relatively low damping and therefore large maximum velocities. For p-MTJ electrodes, the most commonly used material is CoFeB with an MgO

interface for PMA and TMR. We have conducted asymmetric bubble expansion experiments on stacks from both of these systems and interpret the results below, making use of the models presented in the previous section.

5.2.1 CoNi Multilayers

One of the most well-studied sources of PMA and DMI is Co grown on a (111) Pt surface, and we will use this structure as a standard before moving on to less well-characterized materials. The first sample we will investigate is a $1.8nm$ thick $[Co/Ni]_{2.5}$ stack grown on a (111) Pt seedlayer with a TaN capping layer. In Figure 5.1, we observe a distinct asymmetry in the Co/Ni system characterized by a rapid increase in velocity for large positive H_x and a more gradual monotonic increase for negative H_x noting that there is little difference between the trends for bubble domains or planar DWs. The large observed asymmetry about the minimum in velocity immediately suggests that the energy model alone will not be able to describe the behavior. A series of Kerr images are included to highlight the evolution of the wall morphology with increasing H_x . Although there are likely to be changes to the morphology on a length scale not resolvable by Kerr microscopy, it does appear that the wall profile becomes more irregular for increasing H_x that could be due to a reduction of wall stiffness.

For this sample, the stiffness model is able to provide a significantly improved fit to the data compared to the energy model. The parameters $L_c = 50nm$ and $\Theta_0 = 8^\circ$ have been selected to improve fit but their values are also fairly consistent with prior work estimating the Larkin length in Pt/Co systems[35] and MOKE images of the roughness, respectively. We also note the model is fairly insensitive to them. Measurements at different driving fields confirm the expected critical exponent and allow us to fix U_0^* and v_0 . This unfortunately reduces the fit so that there is a clear symmetric shift about the minimum. Thus while the stiffness model seems to be qualitatively describing the behavior, we already have an indication that our it may be

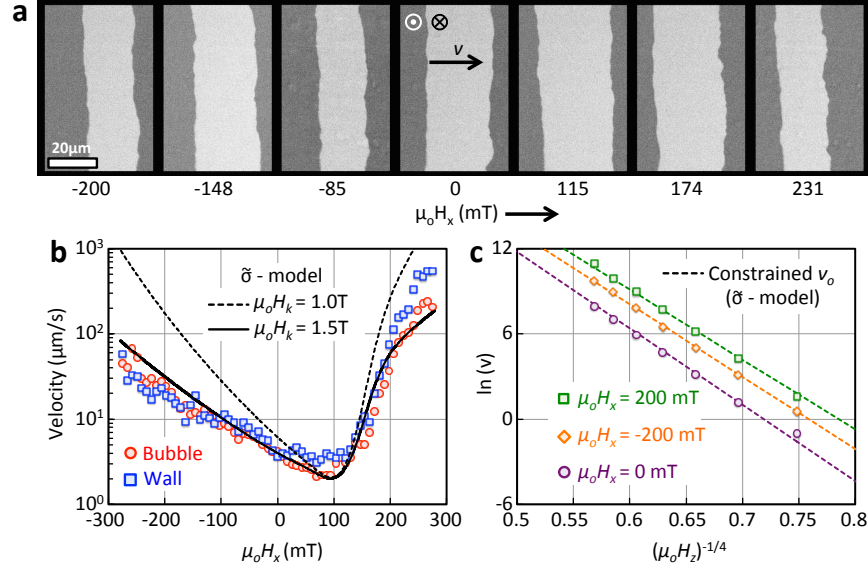


FIGURE 5.1: a) Series of subtractive Kerr images of a DW in the Co/Ni multi-layer thin film with indicated $\mu_0 H_x$ where the bounds of grey region represent the displacement of an $\uparrow\downarrow$ wall. b) Experimental DW velocity vs. $\mu_0 H_x$ for $\mu_0 H_z = 4.25 \text{ mT}$ with best fit from the dispersive stiffness model. The fitting parameters use values of v_0 and U_0^* taken from experimental results at $H_x = 0$. c) DW velocity vs $\mu_0 H_z$ for a series of $\mu_0 H_x$. Dashed lines correspond to creep parameters used in our stiffness model where v_0 is constrained (no H_x dependence).

too simple. The experimentally obtained H_k for this sample is $1T$, but we have included a plot including $H_k = 1.5T$ to show that the fit is can be made nearly perfect if a symmetric shift is included.

Ir is another species that has been reported to give interfacial DMI when placed adjacent to Co [67], though not as strong as Pt and the sign of D for Ir seeds is widely given to be opposite that of Pt. This difference in sign has previously been exploited to additively combine the effects by using Pt as an underlayer and Ir as a capping layer or vice versa. We have attempted to use alloys of Pt and Ir as a seed layer to tune DMI over a large range by altering the alloy composition. The full stack is Ta/Pt/PtIr/[Co/Ni]_{2.5}/TaN and the PtIr alloys were deposited using alternating sub-monolayer deposition. Representative velocity behavior is shown in Figure 5.2 and it is immediately apparent that the velocity behavior for Ir-rich seeds is not well described by either the energy or stiffness models alone. The velocity shows a very strong asymmetry but the

shift in the minimum velocity is quite small. Since the DMI-based effects we have considered both shift the minimum and cause the asymmetry, we will have to include additional chirality-dependent terms from section 5.1.2 if we wish to reproduce the data. Either of the expressions from section 5.1.1 can be combined with either of the expressions from section 5.1.2 to give four possible models. Of these, only the combination of the energy model and chiral damping is unable to fit all of the data satisfactorily so we will neglect it. The other three models are able to capture the observed velocity variations remarkably well, so we will have to differentiate between them by analyzing the fitted values (Figure 5.3).

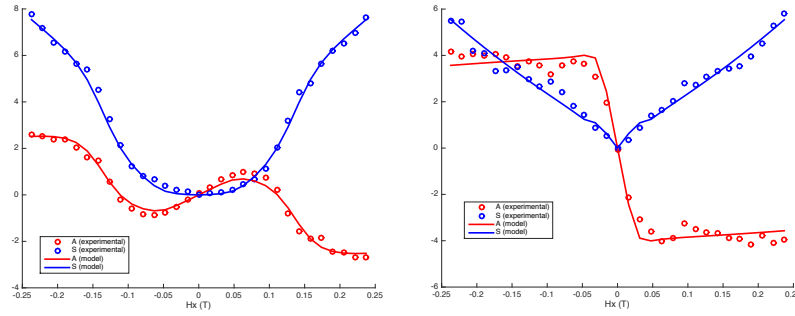


FIGURE 5.2: Typical model fit for asymmetric bubble growth in CoNi multilayers grown on Pt-rich (left) and Ir-rich (right) seed layers. The stiff + ACE model is shown but other models with low fitting error produce similar results.

All three models agree that as the Pt concentration is increased, DMI increases in magnitude. The energy model, however, suggests that D for the Pt-rich seed is substantially larger than the value obtained from the stiffness models. An independent measurement of DMI could therefore establish a preference for one over another. The two ACE models both show linear trends in the weight of the chiral term ϵ_{ace} , but with opposite slopes. The stiffness + ACE model shows nearly no ACE for a Pt seed, while the energy + ACE model has maximum anomalous effect for the Pt seed. The chiral damping model has a fairly small value for α_{cd} for the Pt seed and a very large value for the Ir seed. It is interesting to note that the original evidence for chiral damping involved an increase in velocity of around a factor of 2, while the estimate here for the Ir seed would correspond to a factor of 50. This is not necessarily impossible because the

regular Gilbert damping is much lower for these CoNi multilayers than for the Co single layer in Jué *et al*, but it would indicate that whatever is causing the damping to be chiral is by far the dominant dissipation mechanism in the material. In contrast, the same velocity asymmetry would be result from a relatively modest 10% change in the energy barrier. Finally, the models show some differences in the values of U_0^* that are produced. These can ostensibly be selected for by looking at the H_z dependence, but this may be demanding too much quantitative accuracy from the relatively simple approximations of creep law behavior that have been used. We will save further interpretation of these results for the concluding commentary in chapter 6.

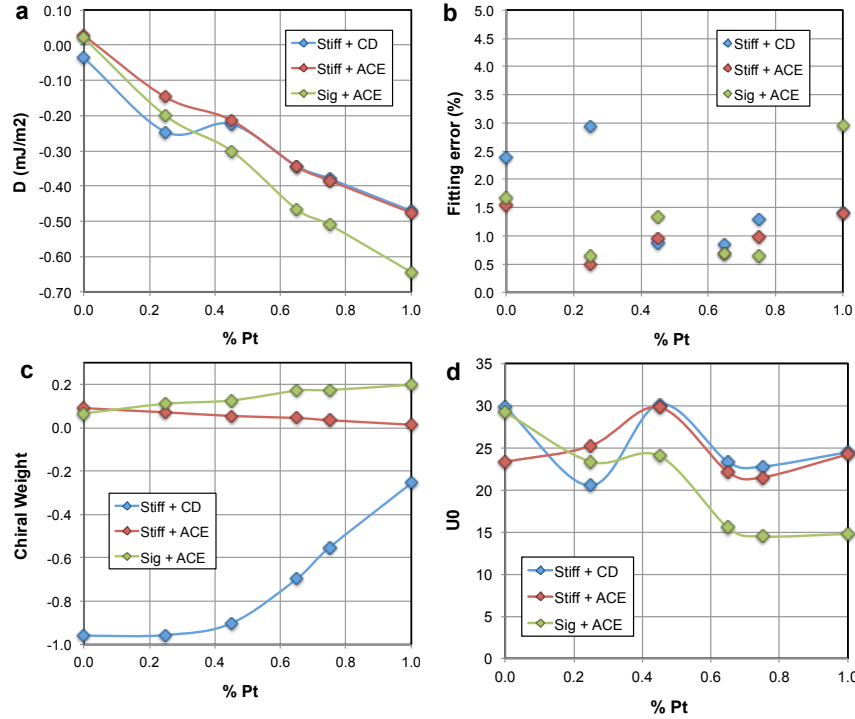


FIGURE 5.3: a) Plot of the DMI values as a function of seedlayer Pt content extracted from the model fits for three different models. In all cases L_c is taken to be $50nm$ and Θ_0 is taken to be 8° b) Plot of fitting error defined as the squared error divided by the total sum of squared experimental A_{creep} and S_{creep} . c) Plot of the fitted value for the weight of the chiral term, α_{cd} for chiral damping models and ϵ_{ace} for anomalous chiral energy models. d) Plot of U_0 fitting values.

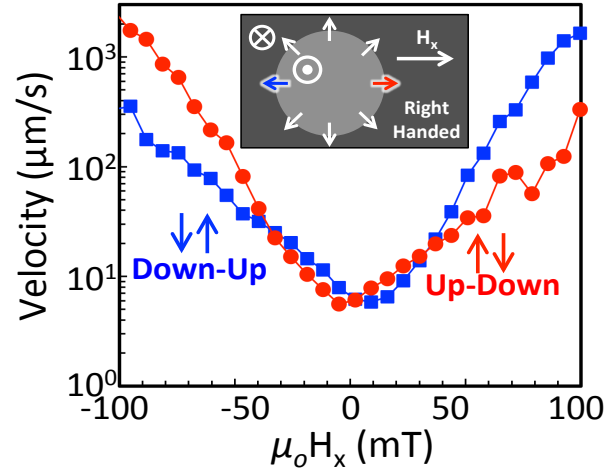


FIGURE 5.4: Velocity asymmetry in TaMo/CoFeB/MgO bubble studies.

5.2.2 CoFeB Films

In the CoNi material system, the seed and capping layers produced both PMA and DMI for the magnetic film. For CoFeB electrodes, PMA is established by the MgO interface which has been indicated to possess very little DMI due to weak spin orbit coupling. The other material adjacent to CoFeB is typically a heavy refractory metal with larger spin orbit coupling and therefore has a potential to induce chiral magnetic effects. While the adjacent heavy metal layer is not believed to be the origin of the PMA, it can have a poorly understood impact on the anisotropy and can determine whether or not the CoFeB can be crystallized with a perpendicular easy axis, thereby restricting our choices. Perpendicular samples were successfully developed with TaHf alloy seed layer, but these exhibited very little asymmetry in the velocity behavior. Alloys of Ta and Mo as seeds produced some behavior similar to the Ir-rich seeds of the previous section that could be described by anomalous chiral energy, but not readily explained in terms of DMI (Figure 5.4). Thin seed layers of W have been reported to have a significant DMI, but our perpendicular samples to date have all exhibited highly dendritic grain growth (Figure 5.5). While we are troubleshooting this problem, recent reports [68] of bubble growth studies in these systems seem to suggest that a stiffness model could describe at least some of the behavior.

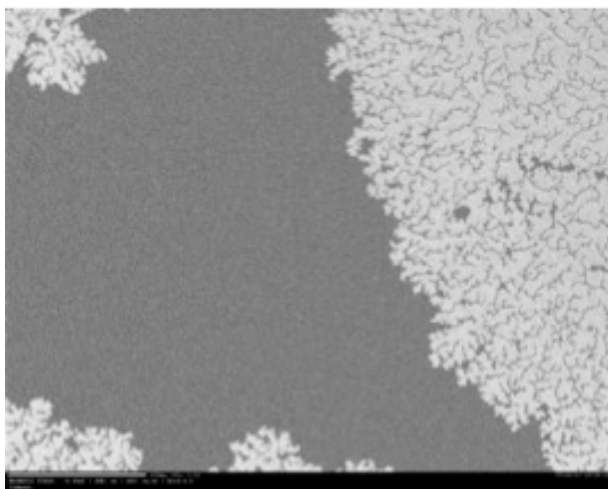


FIGURE 5.5: MOKE image of dendritic domains in a W/CoFeB/MgO stack.

Chapter 6

Conclusions and Future Work

6.1 Conclusions

In this thesis, we have examined the theory of creep phenomena for chiral magnetic domain walls in an attempt to provide insight into asymmetric bubble domain expansion experiments and improve their ability to quantify DMI. Our analysis of creep theory for elastic manifolds led us to develop an effective elastic band model for a Dzyaloshinskii DW in an in-plane field using analytic techniques [69], which allows key DW properties relevant to creep behavior to be evaluated very quickly. Micromagnetic methods were developed to measure the DW elastic properties and verify the analytic expressions. This work highlights and enhances parallels between Dzyaloshinskii DWs and other interfaces including soft materials with internal orientation and vortices in anisotropic superconductors, the latter of which are particularly notable because of the vast amount of research that has been directed toward understanding their pinning. DW pinning is critically important to future technological applications that require low critical driving currents and our work here makes steps to improve our understanding of a regime where the pinning dominates.

In chapter 5, we applied the creep models to actual experimental results for Co/Ni multilayer systems on PtIr alloy seed layers, identifying three candidate models that are able to precisely describe the observed asymmetric bubble growth behavior. While further experiments are necessary to determine model accuracy and identify the best choice, the stiffness + anomalous chiral energy scheme is particularly appealing because it is able to describe the behavior of a multilayer with a Pt seed layer almost purely in terms of DMI and a multilayer with an Ir seed layer almost purely in terms of the anomalous chiral energy. This suggests a picture where Ir affects the film in a fundamentally different way than Pt, though the mechanism by which Ir would be able to introduce a DW chirality dependence for the creep law energy barrier without actually altering the average energy of the chiral DW remains unknown.

6.2 Future Directions

Throughout the thesis, several short open topics were identified that could add to the main thrust of this work and we will briefly list them here. First, the constant DW width approximation, which was used in several of the analytic expressions for the elastic band model, should be replaced by the actual in-plane field dependent DW width in order to attain better agreement with micromagnetic results. The intrinsic parts of the elastic band model could also be completed by deriving an analytic expression for the DW friction. More complete models of the creep law energy scale can be applied to the experimental results in attempts to determine the in-plane field dependence of the pinning strength. On the experimental front, W/CoFeB/MgO stacks with reasonable domain structure should be tested to see if they conform to any of the fitting models. Finally, we introduce two additional thrusts of more intermediate scope below.

6.2.1 Langevin Modeling

By far the most common method of numerically modeling magnetic phenomena is micromagnetics, but the low-field creep regime is particularly difficult to simulate with this technique. The main reason is because the DW motion in this regime is highly stochastic, so the DW spends most of the time pinned and only moves forward in short unpredictable bursts. Therefore in order to measure an accurate average steady state velocity, the simulated time must be very large, likely on the order of milliseconds. For a large 2-D micromagnetic simulation, this becomes quite computationally expensive. Many simulated DW motion problems have been made more computationally accessible by running a 1-D simulation where the only magnetic variation is in the direction of DW motion, but this is equivalent to the rigid wall approximation and as we described in chapter 3, rigid DWs will not display creep behavior. Langevin modeling uses a governing equation similar to the effective elastic band model developed in this thesis and reduces the dimensionality of the simulation to a 1-D model along the length of the manifold. This type of modeling has been shown to give $\mu = -\frac{1}{4}$ if the thermal forces are strong enough [70] and has found some success in describing features of vortex line creep experiments [71]. We have already adapted such a model for use with the anisotropic surface tension presented by DMI and an in-plane field. The equation of motion for each point on this wall is:

$$\frac{du(z_i)}{dt} = \frac{1}{\gamma} \left(-\frac{d\sigma}{du(z_i)} + F_p + F_t + F \right) \quad (6.1)$$

Where $u(z_i)$ is the distance the wall has traveled at a point z_i along its length, F_p is the pinning force, F_t is the thermal force, $-\frac{d\sigma}{du(z_i)}$ is the effective force stemming from the change in wall surface energy (including changes in length and normal angle) due to the motion at point z_i , F is the driving force originating from the H_z field, and γ is a friction coefficient that is proportional

to the Gilbert damping term α . The pinning force is generated as the gradient of a gaussian random field representing the pinning potential and the thermal force is generated as a random stochastic value with variance proportional to γkT . The equation of motion is solved using a second order stochastic Runge-Kutta algorithm. As the code has not yet been optimized for speed, we are limited to the study of velocities on the order of $\sim 1\text{m/s}$ or higher. A snapshot of the simulation is shown in Figure 6.1. The walls move in avalanches as expected and we are able to recreate the desired $\mu = -\frac{1}{4}$, but the slope of the Langevin creep law fit diverges significantly from experiment. Parallelizing the code and getting it to run on a GPU would be critical to making this outcompete micromagnetic modeling, which already has readily available GPU compatible software.

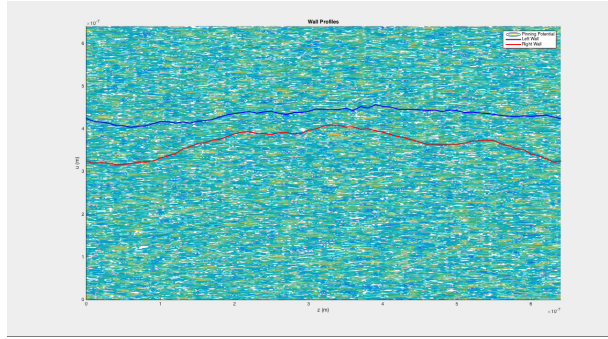


FIGURE 6.1: A snapshot of the Langevin model simulation, with DWs moving upward.

6.2.2 Extension to Current-Driven Creep

It has been shown analytically that current-driven creep caused by adiabatic spin transfer torque has a different critical exponent, $\mu = -\frac{1}{2}$, than field-driven creep where $\mu = -\frac{1}{4}$ [72]. This work also showed that the critical exponent for non-adiabatic spin transfer torque is $\mu = -\frac{1}{4}$ by expressing it as a long-ranged driving force. Recent work in transport theory describing DW motion in the flow regime has been able to provide an origin for the non-adiabatic torque by giving the adiabatic torque a finite correlation length along the DW [73]. By incorporating this finite-range torque into our model, which already describes the response of the DW to

a nonuniform force with different transverse lengths, we may be able to explain intermediate critical exponents, which have been observed in recent current-driven creep experiments [\[74\]](#).

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