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"ESSAYS IN FINANCIAL ECONOMICS: CURRENCY RISK AND PRICING KERNEL VOLATILITY, CDS AND SOVEREIGN BOND MARKET LIQUIDITY, CDS AS SOVEREIGN DEBT COLLATERAL"

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ESSAYS IN FINANCIAL ECONOMICS:
CURRENCY RISK AND PRICING KERNEL VOLATILITY, CDS AND SOVEREIGN BOND MARKET LIQUIDITY, CDS AS SOVEREIGN DEBT COLLATERAL

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Dissertation Submitted in Partial Fulfillment of the Requirements for the Degree of

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Dedicated to my parents.

ABSTRACT

Essay 1: CDS and Sovereign Bond Market Liquidity

During the recent debt crisis in Europe, policy makers responded to the controversy surrounding CDS by implementing a series of policies that banned CDS trading. I use these bans as quasi-natural experiments to identify how derivative markets affect liquidity of the underlying cash market. I document that a temporary CDS ban increased bond market liquidity but a permanent ban instead decreased bond market liquidity. To explain these patterns, I build a dynamic search-theoretic model of over-the-counter bond and CDS markets that features an endogenous liquidity interaction between the two markets. My model shows that these opposing patterns are due to the fact that bond and CDS markets are substitute markets in the short run but are complementary markets in the long run. My results challenge existing theories of liquidity interaction among multiple markets and the common perception that the CDS market is a more liquid market than the bond market.

Essay 2: CDS as Sovereign Debt Collateral

A defining friction of sovereign debt is the lack of collateral that can back sovereign borrowing. This paper shows that credit default swaps (CDS) can serve as collateral and thereby support more sovereign borrowing. By giving more bargaining power to lenders in ex-post debt renegotiations, CDS becomes a commitment device for lenders to extract more repayment from the debtor country. This ex-post disciplining effect during debt renegotiations better aligns the sovereign's ex-ante incentives with that of the lender. CDS alleviates agency frictions that are present in any lending contracts but are particularly difficult to mitigate in sovereign debt context.

Essay 3: Currency Risk and Pricing Kernel Volatility[1]

A basic tenet of lognormal asset pricing models is that a risky currency is associated with *low* pricing kernel volatility. Empirical evidence indicates that a risky currency is associated with a relatively *high* interest rate. Taken together, these two statements associate high-interest-rate currencies with low pricing kernel volatility. We document evidence suggesting that the opposite is true, thus contradicting a fundamental empirical restriction of lognormal models. Our identification strategy revolves around using interest rate *volatility* differentials to make inferences about pricing kernel volatility differentials. In most lognormal models the two are monotonic functions of one another. A risky currency, therefore, is one with relatively low pricing kernel volatility *and* relatively low interest rate

[1] Joint work with Federico Gavazzoni and Chris Telmer

volatility. In the data, however, we see the opposite. High interest rates are associated with *high* interest rate volatility. This indicates that lognormal models of currency risk are inadequate and that future work should emphasize distributions in which higher moments play an important role. Our results apply to a fairly broad class of models, including Gaussian affine term structure models and many recent consumption-based models.

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ESSAY 1: CDS AND SOVEREIGN BOND MARKET LIQUIDITY

Abstract

During the recent debt crisis in Europe, policy makers responded to the controversy surrounding CDS by implementing a series of policies that banned CDS trading. I use these bans as quasi-natural experiments to identify how derivative markets affect liquidity of the underlying cash market. I document that a temporary CDS ban increased bond market liquidity but a permanent ban instead decreased bond market liquidity. To explain these patterns, I build a dynamic search-theoretic model of over-the-counter bond and CDS markets that features an endogenous liquidity interaction between the two markets. My model shows that these opposing patterns are due to the fact that bond and CDS markets are substitute markets in the short run but are complementary markets in the long run. My results challenge existing theories of liquidity interaction among multiple markets and the common perception that the CDS market is a more liquid market than the bond market.

1 INTRODUCTION

Are financial derivatives just redundant securities or do they affect the underlying asset, and in what ways? The recent crises in the US and in Europe and the policy debate surrounding these events illustrated our limited understanding of the recent financial innovations and derivatives such as credit derivatives and securitization. In this paper, I study both empirically and theoretically how derivatives affect price and liquidity of the underlying asset in a particular context: sovereign bond and credit default swap (CDS) markets.[2] The controversy surrounding CDS during the debt crisis in Europe culminated in a series of policies that banned "naked" purchases of CDS where investors buy CDS protection without actually owning the underlying government bonds. These policies serve as quasi-natural experiments that allow us to empirically identify the effect of naked CDS trading on the underlying bonds.

[2] A buyer of a CDS protection pays a periodic fee until either the contract matures or a default (or a similar event) occurs. In return, the protection seller transfers the purchased amount of insurance in the event of default. The contract specifies the reference entity, the contract maturity date, the insurance amount, and the events that constitute a credit event.

Using these bans and a diff-in-diff analysis, I document that permanent versus temporary CDS bans had completely opposite effects on bond market liquidity. When the EU voted in October 2011 to *permanently* ban naked CDS referencing EU countries, countries affected by the ban experienced a *decrease* in their bond market liquidity. When Germany *temporarily* banned naked CDS in May 2010, this pattern reversed: bond market liquidity temporarily *increased* instead.

To explain these opposing patterns and, consequently, shed light on how CDS markets affect the underlying bond market, I build a dynamic search-theoretic model of over-the-counter (OTC) bond and CDS markets. My model shows that, for traders who want a long exposure to credit risk, bond and CDS markets are substitute markets in the short run but are complementary markets in the long run. Depending on the nature of the ban, as a result, one effect dominates the other. When the CDS ban is temporary, long traders temporarily substitute out of the CDS into the bond market and bond liquidity temporarily increases. When the ban is permanent, however, as traders are forced to exit the CDS market, they pull out from the bond market also and bond liquidity decreases.

In the model, I capture the over-the-counter structure of bond and CDS markets using the search and bilateral bargaining mechanism of Duffie, Garleanu, and Pedersen (2005, 2007). A fraction of bond owners are hit by a liquidity shock that requires them to sell their bonds. Locating a buyer, however, involves search costs. When a seller finds a buyer, she takes into account the difficulty of locating a buyer again and resorts to selling her bond at a discounted price. Thus, as in the standard search framework, search costs create an illiquidity discount in bond prices.

I study how CDSs affect this illiquidity discount by modeling two novel features. The first is the presence of CDS markets. CDSs are derivative assets while bonds are fixed supply assets, and trading CDS contracts also involves search costs. Traders cannot directly short bonds but can buy (naked) CDSs to short credit risk. This assumption captures the fundamental difference between bond and CDS markets: it is cheaper to short credit risk using the CDS market than the bond market. The second novel feature is endogenous entry: the investors' entry rate into the underlying bond market endogenously adjusts to the introduction (and the elimination) of the CDS market.

In this environment, the complementarity effect works as follows. For traders looking to acquire a long position, selling CDSs and buying bonds are two different ways to be exposed to credit risk and they can search for a counterparty simultaneously in both the CDS and the bond market. This ability to simultaneously search in both markets reduces the expected search time of acquiring a long position in either market: long traders now have twice as many potential counterparties and, hence, a higher probability of finding a counterparty in either market. A marginal long trader – who would have been deterred by the search cost when there was just the bond market – is now willing to enter both the CDS *and* the bond market. As a result, the CDS market is complementary to the bond market: the existence of naked CDS buyers increases bond market liquidity by changing the ex-ante incentive of a marginal trader to enter and search for a long position in either market. Permanently banning naked CDS trading reverses this complementarity effect and eliminates the positive externality on bond market liquidity: long traders are forced to exit the CDS market but, by exiting the CDS market, they pull out from the bond market also.

When the CDS ban is temporary, the benefit of adjusting entry and exit into the bond and CDS markets (at the extensive margin) does not outweigh the cost of doing so. As a result, the aggregate number of traders across bond and CDS markets remains unchanged and there is only a movement at the intensive margin between bond and CDS markets. Long traders – who would have otherwise sold CDSs to the naked buyers – temporarily resort to trading in the bond market by buying bonds and thereby increase liquidity in the bond market.

The model mechanism critically relies on both endogenous entry and search frictions in the CDS market. Without search frictions in the CDS market, the CDS market is redundant: the existence of naked CDS buyers does not affect bond market liquidity. Thus, trading frictions in the CDS market create an interaction between bond and CDS markets that helps rationalize the empirical patterns. Also, in the data transaction costs in sovereign CDS markets are non-trivial: CDS bid-ask spreads are, on average, ten times larger than bond bid-ask spreads. The importance of trading frictions in the CDS market both in the model and in the data challenges the common perception and a common assumption in recent papers that the CDS market is a more liquid market. My results show that this is not the case.

The fact that bond and CDS markets can be complementary markets is a novel result in light of existing theoretical studies of the liquidity interaction between multiple asset markets. These studies highlight the migration (or equivalently, the substitution) effect. In these models, the aggregate number of traders across markets is kept fixed and, consequently, introducing additional markets necessarily results in a fragmentation and migration of traders across multiple markets. I show that an important interaction between multiple markets arises out of endogenizing traders' entry decision at the extensive margin (consequently, the aggregate number of traders across markets is endogenous) and this channel helps rationalize the observed empirical patterns.

This paper contributes to the existing literature by providing the first theoretical framework of over-the-counter trading in both the underlying and derivative markets. The framework features an interdependent endogenous bond and CDS market liquidity and can be used to analyze topics of growing interest such as the CDS-bond basis and the relative price discovery and to study how relative liquidity in bond and CDS markets affect them. Although I apply the model to sovereign bond and CDS markets, the model framework can applied to study a large class of assets and their derivatives that are traded over-the-counter: currency, commodities, asset-backed securities, and other fixed-income assets (e.g. corporate bond, interest rate). The controversy surrounding CDS during the debt crisis highlighted the lack of theoretical frameworks of OTC trading in both the underlying and derivative markets to help shape the debate.

The second main contribution of the paper is empirical. Empirically identifying how naked CDS trading affects the bond market is confounded by two issues. First, a direct measure of the amount of naked CDS purchases does not exist as we only observe the total amount of CDS purchased (the sum of naked and covered). The second issue is identifying causation as opposed to correlation. Using the CDS bans as quasi-natural experiments circumvents these issues. My diff-in-diff analysis exploits the realization of these bans, the timing of these bans, and the fact that some countries were affected while others were not. I also use daily data that I collected on over 3,200 individual bond issues across 66 government bond markets and CDS data for 66 countries including CDS spreads, liquidity, the amount of outstanding CDS, and the volume of CDS trade. Thus, the analysis is to my knowledge the most comprehensive study of sovereign bond and CDS markets.

This paper highlights a novel mechanism in which naked CDS buyers directly affect liquidity of the underlying bond market. The most commonly posited effect of CDS on the bond market is the "covered" CDS story: the ability to insure one's bond portfolio by buying CDS is likely to attract traders into the bond market and increase bond market liquidity. As for the effect of naked CDS trading, a common hypothesis is that it increases liquidity of the CDS market itself and, consequently, *indirectly* increases bond market liquidity by making CDS a cheaper hedging tool. These effects, however, cannot explain why permanent versus temporary CDS bans would affect bond market liquidity differently. This paper instead proposes a theory that rationalizes the opposite effects within the same theoretical environment.

Another effect that my mechanism is distinct from is the basis trade. In a basis trade, investors trade on an arbitrage opportunity arising from how credit risk is priced in bond and CDS markets versus the theoretical arbitrage relationship between the two securities. For example, if the CDS price is too low relative to bond spreads, then a basis trading strategy would involve buying bonds and buying CDS. Thus, the existence of the CDS market, by creating a potential arbitrage opportunity, may increase the amount of trade and liquidity in the bond market. But basis trades necessarily involve a long position in one market (e.g. buying bonds as in the example) but a short position in the other market (e.g. buying CDS). In contrast, in my mechanism, there is an increase in the volume of trade and liquidity in the bond market due to traders seeking a long position in *both* markets.

Finally, an important policy implication of my results is that permanently banning naked CDS trading adversely affected bond market liquidity, depressed bond prices, and thereby increased sovereign's borrowing cost exactly when governments were trying to avert a liquidity dry-up and credit risk spiral. This result is particularly important in the context of a sovereign debt crisis.

1.1 *Related Literature*

This paper belongs to the search literature of financial assets beginning with the seminal papers Duffie, Garleanu, and Pedersen (2005, 2007). My framework is closely related to the extensions of their environment to multiple assets by Vayanos and Wang (2007), Weill (2008) and, in particular, it is a variant of Vayanos and Weill (2008)'s framework that sheds light on the on-the-run phenomenon of Treasury bonds. I contribute to this literature, first, by modeling over-the-counter trading in derivatives in addition to trading in the underlying asset and, second, by endogenizing the entry decisions of agents into the market for the underlying asset in response to the introduction of the derivative market.

A related paper is Afonso (2011) who endogenizes the entry decisions of traders but in a single market setting. My model differs by featuring both multiple markets and endogenous entry and therefore sheds light on the rate of entry into one market as a result of introducing another market and on the mechanism through which traders migrate between different markets.

A search theoretic paper applied specifically to CDS markets is Atkeson, Eisfeldt, and Weill (2012) who in a static setting study how banks' CDS exposure arises endogenously depending on their size and their exposure to aggregate risk. In contrast, my paper focuses on naked CDS and studies in a dynamic setting the feedback from the CDS market into the bond market by allowing trade in both the bond and the CDS market as opposed to just the CDS market. Oehmke and Zawadowski (2013) explore how CDS affects bond prices in Amihud and Mendelson (1986) type framework with exogenous trading frictions. In contrast, my model features endogenous trading costs.

A related literature is equilibrium asset pricing models with exogenous trading frictions (see, for example, Amihud and Mendelson (1986), Acharya and Pedersen (2005)). My model features endogenous bond market liquidity and thereby allows for an endogenous interaction and a spillover between the underlying and the derivative markets.

A growing number of papers use reduced form approaches to price and quantify liquidity risk in bond and CDS markets. Longstaff, Mithal, and Neis (2005), Chen, Lesmond, and Wei (2007), Bao, Pan, and Wang (2011), for example, study liquidity of corporate bond markets and Beber, Brandt, and Kavajecz (2009) and Bai, Julliard, and Yuan (2012) of sovereign bond markets. Tang and Yan (2007), Chen, Fabozzi, and Sverdlove (2010), and Bongaerts, De Jong, and Driessen (2011) price liquidity risk in corporate CDS markets, and Beber, Brandt, and Kavajecz (2009), Bai, Julliard, and Yuan (2012) in sovereign CDS markets. These papers find a nontrivial magnitude of illiquidity in CDS markets. This paper complements this literature in two ways: first, it provides a theoretical framework to study bond and CDS liquidity, and, second, by using the CDS ban regulations, it documents novel empirical patterns in how bond and CDS market liquidity are interlinked.

Motivated by the theoretical arbitrage relation between how credit risk is priced through bond prices versus through CDS spreads, a growing number of papers study the joint dynamics of bond and CDS spreads, or equivalently the CDS-bond basis, as well as the relative price discovery mechanism in bond and CDS markets.[3] These papers' findings suggest that on average the arbitrage relation holds. But when it does not and the price of credit risk in these two markets deviates, where the price discovery takes place (determined by which of the two prices leads the other) is state dependent. In particular, one of the important determinants is the relative liquidity in these markets. I add to this literature by providing a tractable theoretical framework with endogenous liquidity interaction between the two markets and, hence, precise implications on liquidity and prices in both markets.

In empirically analyzing naked CDS bans, this paper is related to Boehmer, Jones, and Zhang (2013) and Beber and Pagano (2013) who document that short-selling bans on stocks during the financial crisis adversely affected stock market liquidity. In contrast to these papers, I study how regulations that restricted trade in one market affected another related market and, thereby, make inferences on the underlying interaction between the related asset markets.

My work is also related to the literature that studies how CDS affects the issuer of the debt security on which these CDS contracts are written. Empirical studies include Ashcraft and Santos (2009) and Subrahmanyam, Tang, and Wang (2011) who study the effect on firms' cost of borrowing and credit risk, respectively.[4] Also Das, Kalimipalli, and Nayak (2013) document that corporate bond market liquidity did not improve with the inception of the CDS market, while Massa and Zhang (2012) and Shim and Zhu (2010) document that CDS markets increased corporate bond market liquidity. In contrast, my paper identifies the effect of naked CDS trading (as opposed to the CDS market in general) on bond market liquidity and focuses on sovereign bond and CDS markets.

On the theoretical front, Arping (2013) and Bolton and Oehmke (2011) formalize the tradeoffs associated with the empty creditor problem in the context of corporate debt and Sambalaibat (2012) in the context of

[3] Studies of the relative price discovery in corporate bond and CDS include Blanco, Brennan, and Marsh (2005) and in sovereign bond and CDS: Fontana and Scheicher (2010), Arce, Mayordomo, and Peña (2012), Ammer and Cai (2011), Calice, Chen, and Williams (2011), Delatte, Gex, and López-Villavicencio (2011). More specifically on the CDS-bond basis, see, for example, Blanco, Brennan, and Marsh (2005) and Bai and Collin-Dufresne (2011). See **?** for a survey of this literature.

[4] Ashcraft and Santos (2009) find CDS has beneficial effects on firms' cost of borrowing for safer firms but adverse effects for riskier firms as banks may lose the incentive to monitor firms. Subrahmanyam, Tang, and Wang (2011) find CDS increases firms' credit risk which they attribute to protected creditors' reluctance to restructure. Berndt and Gupta (2009) find that borrowers, whose loans have been sold off, underperform.

sovereign debt. Duffee and Zhou (2001) find that credit derivatives alleviate the lemons problem associated with banks having private information on their loans.[5] Thompson (2007) and Parlour and Winton (2009) study the tradeoffs that banks face in selling off versus insuring loans on their balance sheets. Thus, these papers have focused on issues surrounding *covered* CDS buyers who are *directly* exposed to the issuer's default risk. This paper instead focuses on how *naked* CDS buyers affect the issuer's cost of borrowing through their effect on bond liquidity and bond prices.[6]

This paper also contributes to the theoretical literature that studies the distribution of liquidity and trade across multiple markets. Examples that use information-based frameworks are Admati and Pfleiderer (1988), Pagano (1989), and Chowdhry and Nanda (1991), while search-theoretic ones are Vayanos and Wang (2007), Vayanos and Weill (2008), and Weill (2008). A typical result in these papers is that traders endogenously concentrate in one market and trade in the other market disappears. Multiple markets can co-exist under additional assumptions of heterogeneous agents and heterogenous markets so that there is a "clientele" effect.[7] The focus of these papers has been the endogenous cross-sectional distribution of liquidity and trade across markets and assets. This endogeneity, consequently, is on the intensive margin (i.e. the number of traders can vary in the cross-section but the aggregate number of traders is fixed), and hence, the results of these papers are effectively partial equilibrium effects. In my model, if the aggregate number of traders is kept fixed, then (similar to these papers) with the introduction of the CDS market, traders migrate from the bond market to the CDS market and bond market liquidity decreases. However, my model also shows that if the aggregate number of traders is endogenous to the introduction of an additional security (i.e. the endogeneity is on the extensive margin), then the result is the opposite: the number of traders and liquidity in the market for the underlying asset increases.

More broadly, this paper belongs to the literature on the impact of derivatives such as options and futures on the market for the underlying assets. A majority of this literature is empirical.[8] Theoretical frameworks that study the effect of derivatives on liquidity of the underlying asset market include Subrahmanyam (1991), Gorton and Pennacchi (1993), and John, Koticha, Subrahmanyam, and Narayanan (2003) and they also get

[5] Duffee and Zhou (2001) also show that that credit derivatives adversely affect the parallel loan sales market.

[6] Although I do not formally model the issuer's borrowing cost in the primary debt markets, He and Milbradt (2012) provide a formal treatment of the feedback loop between credit risk, the issuer's borrowing cost through the primary debt markets, and liquidity of the secondary bond markets.

[7] For example, Pagano (1989) shows that if markets differ in their fixed entry cost, then an equilibrium with multiple markets exists and has the following feature: the more liquid market has a larger fixed cost of entry and is also the market where only large traders (those needing a larger portfolio adjustment) are attracted to. This is because the larger market has a bigger absorbing capacity (i.e. minimal price impact) and the fixed entry cost can be spread over a large transaction size.

[8] See, for example, Chakravarty, Gulen, and Mayhew (2004) and the survey article, Mayhew (2000).

the "migration" result as the above multiple market information-based models.[9] I add to the literature by endogenizing entry. Also, these papers are based on Kyle (1985), Glosten and Milgrom (1985) type frameworks where illiquidity arises from asymmetric information. The stylized OTC search framework of my paper is better suited for sovereign bond markets for two reasons. First, trade in sovereign bond markets is fragmented across heterogenous bonds and, second, asymmetric information and insider trading are less severe with respect to governments than with respect to individual firms.

The paper is organized as follows. Section 2 presents the model environment while Section 3 derives the main theoretical results. Section 4 describes the data and gives the institutional details on bond and CDS markets. Section 5 documents the empirical patterns that motivate the model. Section 6.1 discusses how the model implications rationalize the observed empirical patterns and Section 7 concludes. All proofs are in the Appendix.

2 MODEL

Agents are heterogenous in their valuation of asset cash flows. Their valuations change randomly and thus generate trade in equilibrium. But finding a counterparty to trade with involves search costs that endogenously depend on the relative number of buyers and sellers. As a result, asset owners resort to selling their asset at a discount, and search costs create an illiquidity wedge in asset prices relative to the frictionless price.

In particular, time is continuous and goes from zero to infinity. Agents are risk neutral, infinitely lived, and discount the future at the constant rate $r > 0$. There is a bond with supply S that pays a coupon flow δ_b. In addition, agents can trade CDS in which a buyer of a CDS contract pays a premium flow p_c, and in return benefits from an expected insurance payment of δ_c. CDS allows both long and short positions to the underlying credit risk: a buyer of a CDS contract has a short exposure while a seller has a long exposure. I assume that bonds allow only a long exposure and that agents cannot short bonds directly. The bond coupon flow can be interpreted as an expected coupon flow: with intensity η the bond defaults but otherwise pays a dollar of coupon. Hence, $\delta_b = (1 - \eta)\$1$.

[9] Subrahmanyam (1991) and Gorton and Pennacchi (1993) using Kyle (1985) framework show that stock index futures market and security baskets, respectively, lower liquidity in the underlying stock market as some traders migrate to these derivative markets. John, Koticha, Subrahmanyam, and Narayanan (2003) also get similar results using Glosten and Milgrom (1985) framework to study the effect of options on stock market liquidity. Brennan and Cao (1996) and Cao (1999) using Hellwig (1980) environment show that options increase market depth of the underlying market. Other theoretical frameworks that study the effect of derivatives on aspects other than liquidity include Back (1993) and Biais and Hillion (1994). Back (1993) develops a framework based on Kyle (1985) to study the effect of options on price volatility. Biais and Hillion (1994) provide another information-based model of options and study their effect on price informativeness of the underlying asset.

Similarly, δ_c can be interpreted as an expected insurance payment: the CDS contract pays out a dollar if there is a default on the coupon payment, thus $\delta_c = \eta\$1$.

Agents' utility valuations of assets switch randomly between high, average, and low type where each type values the bond and CDS payoffs as given in Table 1. Specifically, let $\theta = 1$ denote a long position (exposed to risk) through the bond or CDS market, $\theta = 0$ no position, and $\theta = -1$ a short position (i.e. bought CDS). An agent with $\theta_b \in \{0, 1\}$ shares of the bond has a utility flow $\theta_b \left(\delta_b + x_t^b\right) - |\theta_b| y$. An agent with CDS position $\theta_c \in \{-1, 0, 1\}$ has a utility flow $-\theta_c \left(\delta_c + x_t^c\right) - |\theta_c| y$ where $x_t^b \in \{-x_b, 0, x_b\}$ and $x_t^c \in \{-x_{ch}, 0, x_{cl}\}$ are stochastic processes, and y is the cost of risk bearing that is positive for both long and short positions. An agent with $\{x_t^b = x_b, x_t^c = -x_{ch}\}$ is defined as a high type, with $\{x_t^b = 0, x_t^c = 0\}$ as an average, and with $\{x_t^b = -x_b, x_t^c = x_{cl}\}$ as a low type.

Parameters x_b, x_{ch}, x_{cl} can be interpreted as hedging benefits. High types, for example, may have an idiosyncratic endowment that is negatively correlated with the bond cash flow, while low types have an idiosyncratic endowment that is positively correlated with the bond. Thus, a low type agent would get an extra disutility of x_b from holding the bond ($\theta_b = 1$), while a high type would get an extra utility x_b. As CDS sellers ($\theta_c = 1$), a low type experiences a greater disutility paying out the insurance payment $(-\left(\delta_c + x_{cl}\right) - y)$ than a high type $(-\left(\delta_c - x_{ch}\right) - y)$. Conversely, as CDS buyers ($\theta_c = -1$), a low type benefits more from the insurance payment $(\left(\delta_c + x_{cl}\right) - y)$ than a high type $(\left(\delta_c - x_{ch}\right) - y)$. Appendix A.1 gives a simple example of how x_b, x_{ch}, x_{cl} can depend on the default intensity of the bond. Section A.2 in the appendix formally shows how, in an environment with risk averse agents, the hedging benefit is a function of the risk aversion parameter, the correlation between agents' idiosyncratic endowment and the bond cash flow, and the riskiness of the bond.

Table 1: Valuation of bond and CDS payments by high, average, and low type agents.

Agents are heterogenous in their valuation of bond and CDS cash flows. As shown in the "Bond Owner" column, high type agents derive a higher utility from a long exposure to the bond, while low type agents derive a disutility from a long exposure to the bond. Conversely, low type agents derive a higher utility from a short position (as shown in the "CDS Buyer" column), while high type agents derive a disutility from a short position. As a result, in equilibrium high type agents search for long positions while low type agents short credit risk. Average type agents are in between.

Types	Bond Owner ($\theta_b = 1$)	CDS Buyer ($\theta_c = -1$)
High	$\delta_b + x_b - y$	$\delta_c - x_{ch} - y$
Average	$\delta_b - y$	$\delta_c - y$
Low	$\delta_b - x_b - y$	$\delta_c + x_{cl} - y$

Assumption 1. $x_{ch} + x_{cl} > 2y > x_{ch}$

Assumption 1 ensures that low valuation agents want to short by buying CDS, while average types will not want to short. To see this, if a low type agent buys CDS from a high type, the buyer's flow surplus from the transaction is $(\delta_c + x_{cl}) - y - p_c$ while the seller's is $p_c - (\delta_c - x_{ch}) - y$. The total surplus is then $x_{ch} + x_{cl} - 2y$ which is positive from Assumption 1. If, instead, an average type buys CDS from a high type, the total surplus is $x_{ch} - 2y$ which is negative from Assumption 1.

There is an infinite mass of average valuation agents. A fixed flow $2F_h$ of average types switches to a high type, and a flow F_l switch to a low type. A high type agent enters to trade in the bond and the CDS market only if the expected value of trading as a high type (denoted by V_{hn}) is at least greater than the value of their outside option. I assume that half of the agents that switch to a high type do not have an outside option and hence always enter, while the other half has a positive opportunity cost of entering denoted by O_h.[10] Among these agents, let ρ be the fraction that enter:

$$\rho = \begin{cases} 1 & V_{hn}(\rho) > O_h \\ [0,1] \text{ if} & V_{hn}(\rho) = O_h \\ 0 & V_{hn}(\rho) < O_h. \end{cases} \tag{1}$$

Thus, the total flow of high types actually entering is $(1+\rho)F_h$.[11] Conversely, high types switch to an average type with Poisson intensity γ_d while low types switch to an average type with Poisson intensity γ_u. Thus, the steady state measure of high types is at least $\frac{F_h}{\gamma_d}$ while the steady state measure of low type agents is $\frac{F_l}{\gamma_u}$.

Assumption 2. $\frac{F_h}{\gamma_d} > S + \frac{F_l}{\gamma_u}$

Assumption 2 ensures that high types are the marginal investors in the bond.

2.1 *The Bond and the CDS Market*

Buyers and sellers in the bond market, whose measures are denoted by τ_{bb} and τ_{bs}, meet at a rate $\lambda_b \tau_{bb} \tau_{bs}$ where λ_b is the exogenous matching efficiency of the bond market. Given the total meeting rate, buyers find a seller with intensity $q_{bs} \equiv \lambda_b \tau_{bs}$, and sellers find a buyer with intensity $q_{bb} \equiv \lambda_b \tau_{bb}$. Once matched, a buyer and a seller Nash-bargain over price so that the buyer gets a fraction ϕ of the total gain from trade and the seller gets the remaining surplus. Analogously, in the CDS market CDS buyers find a seller with intensity $q_{cs} \equiv \lambda_c \tau_{cs}$, and sellers find a buyer with

[10] Afonso (2011) provides a more general setup in which there is a continuous distribution of agents with different outside values. My setup would be a special case of this.

[11] The assumption that a portion of high types are always entering is for simplicity and is a way to scale up the measure of high types in the economy so that even if $\rho = 0$, the steady state measure of high types is greater than the steady state measure of low types and the bond supply. This simplifies the derivation of existence and uniqueness of the steady state equilibrium without affecting the main channels of the model.

intensity $q_{cb} \equiv \lambda_c \tau_{cb}$ where τ_{cb} and τ_{cs} are the measures of CDS buyers and sellers, respectively.

2.2 *Agent Types and Transitions*

Table 2 shows the various types and their possible asset positions. μ_τ denotes the measure of an agent type $\tau \in \mathcal{T}$ and $\mathcal{T} \equiv \{hn, ln, hob, aob, hoc, aoc, lsc\}$ is the set of agent types. hn and ln are high and low non-owners, hob and aob are high and average bond owners, hoc and aoc are high and average types that have sold CDS, and lsc are low types who have bought CDS.

Table 2: Agent Types

An agent type is composed of their valuation type (high "h", average "a", low "l") and their asset position (θ_b, θ_c). Their asset position can be either a non-owner "n": $(\theta_b, \theta_c) = (0, 0)$, a bond owner "$ob$": $(\theta_b, \theta_c) = (1, 0)$, a CDS seller "$oc$": $(\theta_b, \theta_c) = (0, 1)$, or a CDS buyer "$sc$": $(\theta_b, \theta_c) = (0, -1)$.

	(θ_b, θ_c)			
	$(0,0)$	$(1,0)$	$(0,1)$	$(0,-1)$
High	μ_{hn}	μ_{hob}	μ_{hoc}	
Average	∞	μ_{aob}	μ_{aoc}	
Low	μ_{ln}			μ_{lsc}

Figure 1 shows the transitions between types. High types want an exposure to the underlying credit risk by either buying a bond or selling CDS. If they switch to an average type, they will try to liquidate their existing long position by selling the bond or just exit the economy if they did not have any existing positions. Average types do not want neither a long nor a short exposure to risk so they just stay out of the markets. Low types want a short exposure which is possible by buying CDS.

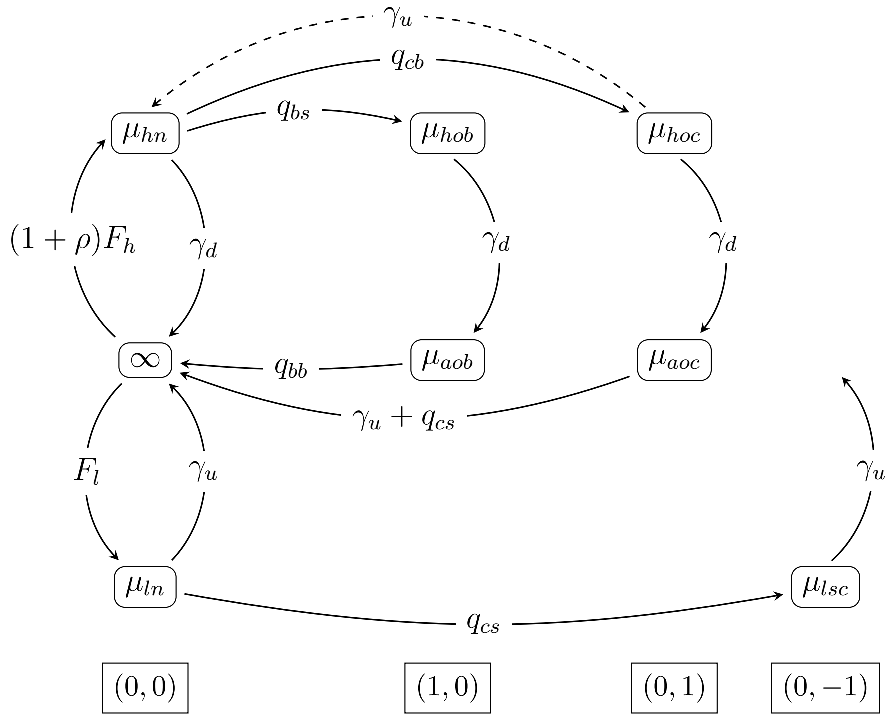
Since a high type non-owner (hn) wants a long exposure to credit risk, he will search to buy a bond or sell CDS and find counterparties with intensities q_{bs} and q_{cb}, respectively. Before he is even able to find a counterparty, he may switch to an average type and exit the economy. If he finds and trades with a bond-seller, he becomes a high type bond owner, hob. He is happy to hold that position until he is hit by a liquidity shock and becomes an average valuation, in which case he will become a bond seller (aob) to liquidate his bond position. Upon finding a bond buyer, he exits the market.

A high non-owner (hn) could also sell CDS (which occurs with intensity q_{cb}) and become a hoc type who has a long-exposure to credit risk. He is happy with this long exposure unless he switches to an average type and becomes one of aoc. As an average type, instead of remaining a CDS seller, he will try to unwind his position by searching for another CDS seller to take over his side of the trade at the original price. In practice, this is called *assignment* or *novation.*

Since a low non-owner (ln) wants to short credit risk, she searches to buy CDS and finds a counterparty with intensity q_{cs} and becomes a CDS holder, lsc. If she switches to an average type, she terminates her contract while her counterparty reverts back to an hn type and has to start over his search.

Figure 1: Transitions Between Agent Types

The figure shows the transitions between agent types. A flow of $(1+\rho)\,F_h$ agents enter the economy as high types and flow F_l as low types. High type agents are hit with a liquidity shock (and become an average valuation) with intensity γ_d. Conversely, low types switch to an average type with intensity γ_u. A trader seeking a long position (hn) finds a counterparty in the bond and the CDS market with probabilities q_{bs} and q_{cb}, respectively. A bond seller, aob, finds a buyer with probability q_{bb}. A trader seeking to establish a short position, ln, by buying CDS finds a counterparty with probability q_{cs}.



Given the search choices of agents, the measure of buyers and sellers in the bond and CDS markets are: $\tau_{bb} = \mu_{hn}$, $\tau_{bs} = \mu_{aob}$, $\tau_{cs} = \mu_{hn}$, $\tau_{cb} = \mu_{ln} + \mu_{aoc}$. Moreover, in the steady state, the measures of types are constant and the in-flow of agents has to equate the out-flow for each type as shown in Table 3.

Table 3: Flow-ins and outs

In the steady state equilibrium, the measure of agent types is constant: a flow of agents turning into a particular type (Flow-in) has to equal the flow of agents switching out of that type (Flow-out).

Type	Flow-in = Flow-out:
μ_{hn}	$(1+\rho)F_h + \gamma_u \mu_{hoc} = \gamma_d \mu_{hn} + (q_{bs} + q_{cb})\,\mu_{hn}$
μ_{ln}	$F_l = \gamma_u \mu_{ln} + q_{cs}\mu_{ln}$
μ_{hob}	$q_{bs}\mu_{hn} = \gamma_d \mu_{hob}$
μ_{aob}	$\gamma_d \mu_{hob} = q_{bb}\mu_{aob}$
μ_{hoc}	$q_{cb}\mu_{hn} = \gamma_d \mu_{hoc} + \gamma_u \mu_{hoc}$
μ_{aoc}	$\gamma_d \mu_{hoc} = \gamma_u \mu_{aoc} + q_{cs}\mu_{aoc}$
μ_{lsc}	$q_{cs}\mu_{ln} = \gamma_u \mu_{lsc}$

Bond market clearing imposes that the total of measure of bond owners has to equal the bond supply:

$$\mu_{hob} + \mu_{aob} = S. \tag{2}$$

CDS market clearing requires that the total number of CDS contracts sold has to equal the number of CDS contracts purchased:

$$\mu_{hoc} + \mu_{aoc} = \mu_{lsc}. \tag{3}$$

2.3 *Prices and Bargaining*

Prices of bonds and CDS arise from bilateral bargaining between buyers and sellers. Let V_τ denote the expected utility of type $\tau \in \mathcal{T}$. A bond buyer's marginal benefit of buying a bond is the increase in his expected utility $V_{hob} - V_{hn}$ and his marginal cost is the bond price p_b. Thus, he is willing to buy as long as the marginal benefit is greater than the marginal cost: $V_{hob} - V_{hn} \geq p_b$, and the smaller the price is, the larger is his surplus. Analogously, for a seller the marginal benefit of selling her bond is the bond price, p_b, and in return she is giving up the value of being a bond owner, V_{aob}, which is the marginal cost. Hence, she will sell as long as $p_b \geq V_{aob}$. Thus, the bond price has to lie in the interval: $V_{aob} \leq p_b \leq V_{hob} - V_{hn}$ and the length of this interval is the total surplus from trade. The buyer and the seller split the surplus proportional to their respective bargaining powers: ϕ and $1-\phi$. The greater the bargaining power of the buyer (i.e. higher ϕ), the lower the bond price:

$$p_b = \phi V_{aob} + (1-\phi)\,(V_{hob} - V_{hn}). \tag{4}$$

Analogously, a CDS seller and a CDS buyer Nash-bargain over the price such that the seller and the buyer get ϕ and $1-\phi$ fraction of the total surplus, respectively. The buyer's surplus is $V_{lsc} - V_{ln}$ and the seller's is $V_{hoc} - V_{hn}$. Thus, the CDS price is implicitly defined by:

$$V_{hoc} - V_{hn} = \phi \left(V_{lsc} - V_{ln} + V_{hoc} - V_{hn} \right). \tag{5}$$

A CDS seller who switches to an average, aoc, will choose to find another CDS seller to take over his side of the trade (at the original price) and exit with zero utility if $0 - V_{aoc} > 0$.

2.4 *Value Functions*

To determine the expected utilities of types, consider for example an hn type. In a small time interval $[t + dt]$, he could (a) switch to an average valuation (with probability $\gamma_d dt$ and get utility 0), (b) become a bond owner (with probability $q_{bs} dt$ and get $V_{hob} - p_b$), (c) become a CDS seller (with probability $q_{cb} dt$ and get utility V_{hoc}), or (d) remain an hn with probability:

$$\begin{aligned} V_{hn} = (1 - rdt) \Big(& \gamma_d dt(0) + q_{bs} dt(V_{hob} - p_b) + q_{cb} dt V_{hoc} \\ & + \left(1 - \gamma_d dt - q_{bs} dt - q_{cb} dt\right) V_{hn} \Big). \end{aligned}$$

After simplifying and taking the continuous time limit, we get:

$$rV_{hn} = \gamma_d(0 - V_{hn}) + q_{bs}(V_{hob} - p_b - V_{hn}) + q_{cb}(V_{hoc} - V_{hn}). \tag{6}$$

The flow value equations for the other types are analogously derived and are shown in Appendix (A).

2.5 *Equilibrium*

Definition 1. *A steady state equilibrium is given by types' measures $\{\mu_\tau\}_{\tau \in \mathcal{T}}$, prices $\{p_b, p_c\}$, entry decisions $\{\rho\}$, and value functions $\{V_\tau\}_{\tau \in \mathcal{T}}$ such that:*

1. *$\{\mu_\tau\}_{\tau \in \mathcal{T}}$ solve the steady state in-flow and out-flow equations in Table 3.*

2. *Market clearing conditions* (2) *and* (3) *hold.*

3. *Entry decisions, $\{\rho\}$, solve* (1)*.*

4. *Bond and CDS prices, $\{p_b, p_c\}$, solve* (4) *and* (5)*.*

5. *Agents' value functions, $\{V_\tau\}_{\tau \in \mathcal{T}}$, solve agents' optimization problem given by* (6)*, and* (A.14) – (A.19)*.*

The next proposition shows that a unique steady state equilibrium exists under the technical condition (7).

Proposition 1. *Suppose*

$$x_b - \frac{\left(x_{ch} + (x_{cl} - 2y)\left(\frac{q_{cs}+r+\gamma_u+\gamma_d}{q_{cs}+r+\gamma_u}\right)\right)}{\left(\frac{r+\gamma_d+\gamma_u+q_{cs}\phi_l+q_{cb}\phi_h}{q_{cb}\phi_h}\right)} > 0. \tag{7}$$

Then, for small search frictions there exists a unique equilibrium.

The proof is given in Appendix A. The proof of uniqueness involves the following steps. Given ρ, Appendix A shows that the set of equations that characterizes the dynamics of the population measures together with the market clearing conditions has a unique solution. Given this solution to the population measures, a linear system of equations characterizing the agents' value functions and prices uniquely determines $\{V_\tau\}_{\tau\in\mathcal{T}}$. Thus, for any $\rho \in [0, 1]$, V_{hn} is uniquely determined. The agent's entry decision can be either an interior solution or one of the two corner solutions ($\rho = 0$, $\rho = 1$). To show that the agents' entry decision has a unique solution, the appendix shows that if (7) holds, V_{hn} is a strictly decreasing function of ρ.

Existence can be established only in the frictionless limit ($\lambda_b \to \infty$, and $\lambda_c \to \infty$) and involves verifying that all the conjectured optimal trading strategies are indeed optimal. In particular, I first show that the total surplus from trading the bond is positive: $\omega_b = V_{hob} - V_{hn} - V_{aob} > 0$. By construction, this will ensure that a high type agent optimally chooses to buy a bond, while an average type will not want to be a bondholder and, if she had previously purchased a bond, will prefer to sell it. Second, Appendix A shows that the total surplus from trading CDS is positive $\omega_c = V_{hoc} - V_{hn} + V_{lsc} - V_{ln} > 0$. This will imply that high type agents will want to sell CDS while low type agents will want to buy CDS. Third, I verify that average type agents will prefer to stay out of the markets completely and not be a CDS buyer or a CDS seller: $0 - V_{aoc} > 0$. The latter ensures that agents who have previously sold CDS when they were high types will prefer to find another seller to take over his side of the trade (at the original CDS price) and exit with zero utility.

3 THEORETICAL RESULTS

To fix ideas, I will be interchangeably referring to buyers in the bond market (μ_{hn}) as liquidity providers. They also provide liquidity in the CDS market by selling CDS. Conversely, the liquidity demanders in the bond market are the bond sellers (μ_{aob}) and in the CDS market are the CDS buyers ($\mu_{ln}+\mu_{aoc}$). Note that the measure of these liquidity demanders and providers arise endogenously depending on the endogenous entry decision ρ, the efficiency of the matching functions λ_b and λ_c and the parameters that determine flows into the economy, $\{F_h, F_l\}$, and transitions between different valuations, $\{\gamma_d, \gamma_u\}$.

Proposition 2. *If the bond market is frictionless ($\lambda_b \to \infty$), the bond price is given by*

$$p_b = \frac{\delta_b + x_b - y}{r} \tag{8}$$

and the CDS market does not affect the bond market.

Proposition (2) shows that without search frictions in the bond market, the bond price is given by the present value of high types' valuation of the bond and, importantly, the CDS market does not affect the bond market. In the frictionless limit, a bond owner – who gets a liquidity shock and has a need to sell her bond – can do so instantaneously to another high type. As a result, bonds are held only by agents who derive a high utility from owning them and never by agents who have a low valuation. Consequently, the bond price is given by the valuation of high type agents since it is a weighted average of marginal valuations of the two types of bond owners.

Proposition 3. *The bond price is given by:*

$$p_b = \frac{\delta_b + x_b - y}{r} - \underbrace{\left[\gamma_d \frac{x_b}{rk} + \phi\left(q_{bs} + r\right)\frac{x_b}{rk}\right]}_{\textit{part of illiquidity discount}} - \underbrace{\frac{\left(q_{bb} + r\right)\left(1 - \phi\right)}{rk} q_{cb}\Delta_{hoc}}_{\textit{discount due to CDS}} \tag{9}$$

where

$$\Delta_{hoc} \equiv \frac{\phi(-\phi q_{bs} x_b + k x_c)}{\left[r + \gamma_d + \gamma_u + q_{cs}(1-\phi) + \phi q_{cb}\right] k - \phi q_{cb} q_{bs} \phi},$$

$$k \equiv r + \gamma_d + q_{bs}\phi + q_{bb}(1 - \phi).$$

Proposition (3) shows that with search frictions in the bond market the bond price is lower than the frictionless price in (8). To see why, a bond owner who gets a liquidity shock and has a need to sell her bond faces a difficulty of locating a counterparty. Due to this wait, she is stuck with a bond that she gets a disutility from. When she does find a buyer, she takes into account the difficulty of locating a buyer again and resorts to selling at a discounted price. Conversely, a potential bond buyer takes into account this trading friction in case he has a liquidity need in the future and has to reverse his trade and sell. If search costs are large, a potential buyer is only willing to buy at a low price, and a bond seller is also more willing to sell at a low price. Thus, search costs create an illiquidity discount in the bond price given by the difference between (9) and the frictionless price: the sum of the second and the third terms in (9). In particular, the third term is the additional discount in the bond price due bond buyers having the outside option of providing liquidity in the CDS market (by selling CDS). Thus, we are interested in the sum of the two terms which gives the total discount in the bond price created by search costs.

Definition 2. *The illiquidity discount, d, in the bond price is defined by the difference between the frictionless bond price* (8) *and the bond price with search frictions present in the bond market* (9)*:*

$$d \equiv \gamma_d \frac{x_b}{rk} + \phi\left(q_{bs}+r\right)\frac{x_b}{rk} + \frac{\left(q_{bb}+r\right)\left(1-\phi\right)}{rk} q_{cb}\Delta_{hoc}.$$

3.1 *The Effect of CDS on Bond Market Liquidity*

The next proposition gives the main theoretical result of the paper by analyzing how the introduction of the CDS market affects the bond illiquidity discount. It shows that the existence of naked CDS buyers increases bond market liquidity and, consequently, bond and CDS markets are complementary markets. In particular, when entry is endogenous and CDS search frictions are not too severe ($\lambda_c > \bar{\lambda}_c$), the existence of the CDS market lowers the illiquidity discount in the bond price. Figure 2 illustrates the result.

Proposition 4. *In the equilibrium of Proposition* (1)*, there exists* $\bar{\lambda}_c > 0$ *such that for all* $\lambda_c > \bar{\lambda}_c$*,*

$$d\left(\lambda_c\right) \leq d^{no\, cds}.$$

The proof is given in Appendix A and the mechanism consists of the following parts. First, for a given rate of entry, ρ, the introduction of the CDS market increases the value of entering the economy as a high type because now there are effectively twice as many counterparties (bond sellers and CDS buyers) that the high type agent can provide liquidity for. The probability of finding a counterparty in at least one of the two markets is, therefore, greater than if there was just the bond market. This is illustrated in Figure 3 by a vertically upward shift in $V_{hn}(\rho)$ to the solid red line. Attracted by the increase in the different ways of establishing a long position and supplying liquidity, high type agents enter at a higher rate.

Second, each additional entrant increases competition and lowers the profit for every other high type agent. This is illustrated by the downward slope of $V_{hn}\left(\rho\right)$ in Figure 3. They enter until the marginal entrant is indifferent between entering or not (where V_{hn} crosses the outside option, O_h). The above two mechanisms imply that the existence of the CDS market, in equilibrium, results in an increase in the number of high type agents and the aggregate supply of liquidity (denoted by the increase in ρ from ρ^{nocds} to $\rho^{cds}_{\lambda_c<\infty}$ in Figure 3).

Third, the increase in the rate of entry creates a positive externality in the bond market when the CDS market is subject to search frictions. Figure 4 illustrates the increase in the rate of entry of high types due to the introduction of the CDS market. If the CDS market is frictionless ($\lambda_c \to \infty$), the increase in the measure of all high type agents, $\left(\rho^{cds}-\rho^{nocds}\right)\frac{F_h}{\gamma_d}$, is exactly equal to the total demand for CDS (the measure of all low types, including those who have purchased CDS: $\frac{F_l}{\gamma_u} = \mu_{ln} + \mu_{lsc}$). This is because, upon entry, liquidity providers are able to locate and sell CDS immediately to the flow of CDS buyers, and, as a result, the increase in high types does not affect the bond market. But with search frictions, the increase in high type agents is strictly greater than the potential total demand for CDS ($\frac{F_l}{\gamma_u}$). When search frictions in the CDS market are high, a CDS seller can extract more rent from a CDS buyer. Thus, the increase in high type agents (the potential suppliers of CDS contracts) is greater than the potential demand for CDS.

As a result of this increase in high types, bond sellers free ride on the increased traffic of traders who are looking to establish a long position by either selling CDS or buying bonds. Figure 5 illustrates how the CDS market changes the bond market composition. Due to the increase in high types, there are more bond buyers, and consequently fewer bond sellers. Furthermore, as shown in Figure 6, the introduction of the CDS market also increases the volume of trade in the bond market.

3.1.1 *The Importance of CDS Search Frictions*

The positive externality created by the CDS market only exists when there are trading frictions in the CDS market. When the CDS market is frictionless, it has no effect on the bond price or the bond market composition: the number of bond buyers, sellers, or bond volume. As illustrated in Figure 5, when trading frictions in the CDS market decrease ($\lambda_c \to \infty$), the number of bond buyers and sellers (and hence bond volume) converges back to the benchmark environment without CDS. Proposition 5 formally shows that if the CDS market is frictionless, then it does not affect the illiquidity discount in the bond price.

Proposition 5. $\lim_{\lambda_c \to \infty} d(\lambda_c) = d^{\text{nocds}}$.

3.1.2 *The Importance of Endogenous Entry*

With exogenous entry, the above mechanism exactly reverses: the introduction of the parallel CDS market shrinks the size of the bond market as some agents who would have otherwise bought bonds migrate to the CDS market and sell CDS instead. Existing bond sellers effectively compete with CDS buyers for the same set of traders that can provide liquidity in either market. Due to fewer bond buyers, the congestion externality and search costs increase for bond sellers. Thus, with exogenous entry the effect of the CDS market is on the intensive margin: the total number of market participants is fixed, and there is only migration or substitution between the bond and the CDS market. With endogenous entry there is, on the extensive margin, a larger overall flow of traders into both bond and CDS markets. In particular, the increase in entry more than offsets the migration (i.e. the substitution) effect: it replaces the bond buyers that migrated to the CDS market and, due to search frictions in the CDS market, results in even greater number of potential bond buyers.

3.1.3 *Model Implication on a Permanent CDS Ban*

The above results showed that bond and CDS markets are complementary markets: the existence of naked CDS buyers increases bond market liquidity. These results imply that permanently banning naked CDS buyers will reverse this positive effect and lead to a decrease in bond market liquidity.

3.2 *A Temporary Naked CDS Ban*

So far I have compared the steady-state bond prices and bond market liquidity in settings with and without CDS markets when the aggregate number of market entrants can adjust towards these steady states. This analysis can speak to the effect of a permanent ban on naked CDS positions. In this section, I consider instead the immediate impact of a temporary ban on purchasing naked CDS.

I model a temporary naked CDS ban as a one-time unexpected drop in the number of naked CDS buyers. To focus on the immediate impact of the shock, I assume that the flow of entrants remains fixed in the short run as the economy rebounds back to the steady state equilibrium. Time can be relabeled so that $t = 0$ corresponds to the time at which this shock occurs. As the shock hits, the distribution of the measure of types switches to $\{\mu_\tau(0)\}_{\tau\in\mathcal{T}} = \{\bar{\mu}_\tau\}_{\tau\in\mathcal{T}}$. I define $\{\bar{\mu}_\tau\}_{\tau\in\mathcal{T}}$ such that all its elements are equal to the steady state measure of types except the measure of naked CDS buyers is instead zero: $\bar{\mu}_{ln} = 0$. The time-varying equilibrium measure of hn type agents from this shock back to the steady state is given by the solution to the following ODE:

$$\dot{\mu}_{hn}(t) = (1+\rho)F_h + \gamma_u\mu_{hoc}(t) - [\gamma_d\mu_{hn}(t) + (q_{bs}(t) + q_{cb}(t))\,\mu_{hn}(t)]$$

where the initial condition is given by $\{\mu_\tau(0)\}_{\tau\in\mathcal{T}} = \{\bar{\mu}_\tau\}_{\tau\in\mathcal{T}}$ and the entry rate ρ is kept fixed at the steady state level. The dynamics for the measures of other agents are analogously characterized in (A.45)–(A.51). Given this solution, agent hn's value function evolves according to:

$$\begin{aligned}\dot{V}_{hn}(t) = rV_{hn}(t) - [\gamma_d(0 - V_{hn}(t)) + q_{bs}(t)\,(V_{hob}(t) - V_{hn}(t) - p_b(t))\\ +q_{cb}(t)(V_{hoc}(t) - V_{hn}(t))]\end{aligned}$$

where

$$\begin{aligned}p_b(t) &= \phi V_{aob}(t) + (1-\phi)\,(V_{hob}(t) - V_{hn}(t)),\\ V_{hoc}(t) - V_{hn}(t) &= \phi\,(V_{lsc}(t) - V_{ln}(t) + V_{hoc}(t) - V_{hn}(t))\,.\end{aligned}$$

It is analogous for the other agents as shown in (A.52)–(A.58). Define $\Delta_{hob} \equiv V_{hob} - V_{hn}$, $\omega_b \equiv V_{hob} - V_{hn} - V_{aob}$, and $\omega_c \equiv V_{hoc} - V_{hn} + V_{lsc} - V_{ln}$. Then, we can rewrite all the ODEs for the value functions in terms of Δ_{hob}, ω_b and ω_c, for example:

$$\dot{V}_{hn}(t) = rV_{hn}(t) - [\gamma_d(0 - V_{hn}(t)) + q_{bs}(t)\phi\omega_b(t) + q_{cb}(t)\phi\omega_c(t)]$$

In turn, the solution for Δ_{hob}, ω_b and ω_c is given in Proposition 6.

Proposition 6. *Given the solution to the time-varying dynamics of agent measures, the dynamics for Δ_{hob} and V_{aob} are given by:*

$$\Delta_{hob} = \frac{\delta_b + x_b - y}{r} - \int_t^\infty e^{-r(s-t)} \left((\gamma_d + q_{bs}\phi)\, \omega_b + q_{cb}\phi\omega_c \right) ds,$$
$$V_{aob} = \frac{\delta_b - y}{r} + \int_t^\infty e^{-r(s-t)} q_{bb}\,(1-\phi)\,\omega_b ds,$$

where

$$\begin{bmatrix} \omega_b(t) \\ \omega_c(t) \end{bmatrix} = \int_t^\infty e^{-\int_t^s A(u)du} \begin{bmatrix} x_b \\ x_{cl} + x_{ch} - 2y \end{bmatrix} ds,$$

$$A(t) = \begin{bmatrix} r + \gamma_d + q_{bs}\phi + q_{bb}\,(1-\phi) & q_{cb}\phi \\ q_{bs}\phi & r + \gamma_d + \gamma_u + q_{cb}\phi + q_{cs}(1-\phi) \end{bmatrix}.$$

3.2.1 *Results*

Figures 7 and 8 plot the transition dynamics of types' measures and of the bond illiquidity discount from the CDS ban shock at $t = 0$ back to the steady state. The sudden drop in the number of naked CDS buyers frees up their counterparties who would have otherwise provided them liquidity by selling CDS. These liquidity providers temporarily substitute providing liquidity in the CDS market with providing liquidity in the bond market and trading as bond buyers. In turn, bonds sellers temporarily benefit from the ban as they now find bond buyers more quickly and hence face lower search costs. The temporary and sudden ban on naked CDS buyers, as a result, leads to an immediate increase in bond market liquidity. In the short term, for liquidity providers (i.e. the high type agents) bond and CDS markets are substitute markets. [12]

3.2.2 *The Implicit Cost of Entry*

The substitution effect arises because long traders resort to temporarily trading in the bond market instead of exiting entirely from both markets at the extensive margin. I arrive at this result by keeping the entry rate fixed which is a reduced form way to capture an adjustment cost of entry. Although I do not explicitly incorporate such adjustment cost of entry, eq. 10 illustrates one possible way of incorporating it. Now, in addition to comparing the value of entering $V_{hn}(\rho)$ with the outside investment opportunity O_h, high type agents have to take into account a cost of entry that varies with the entry rate:

[12] As the ban is lifted, the number of traders searching to buy CDS increases until the fraction of CDS buyers who finds a CDS seller equals the flow of new low type agents entering the economy.

$$\rho = \begin{cases} 1 & V_{hn}(\rho) - c(\rho) > O_h \\ [0,1] \text{ if } & V_{hn}(\rho) - c(\rho) = O_h \\ 0 & V_{hn}(\rho) - c(\rho) < O_h. \end{cases} \tag{10}$$

where $c'(\rho) > 0$ and $c''(\rho) > 0$. Figure 9 figuratively illustrates an example of a such cost function. The temporary CDS ban leads to a small decrease in the value of trading as a high type. When the scale of entry is already large, due to the convexity of $c(\rho)$, with a very small decrease in ρ, the cost decreases by a lot. As a result, the entry rate does not have to change much in response to a temporary ban. In contrast, with a permanent ban, the decrease in the value of trading as a high type is large. In addition, due to the convexity, as the entry rate ρ decreases, the resulting decrease in the cost of entry becomes less responsive. As a result, the entry rate has to decrease by a lot in response to a permanent ban. Alternatively, we can also back out how the short-run dynamics of the cost of entry has to look like from the dynamics of $V_{hn}(\rho^{ss})$ as shown in 10.

4 DATA AND MARKET DESCRIPTIONS

4.1 *Background on Sovereign Bond Market*

Government bonds trade in over-the-counter markets. A trader in the US, for example, shops for government bonds using phone calls, emails, messages and quotes through Bloomberg.[13] Locating a particular bond issue can be at times impossible. In European government bond markets, since their inception in 1988, MTS trading platforms have become an increasingly important trading venue. The MTS system is an inter-dealer trading platform that functions similar to electronic limit order markets and is not accessible to individuals. Despite its similarity to equity markets, trade is fragmented across heterogenous bonds and liquidity per bond is low.[14]

4.2 *Bond Market Data*

The bond price data comes from Thomson Reuters and consists of daily bid and ask price quotes for the period 2004-2012. Due to data access

[13] Trade in US Treasuries, however, is different than in other government bonds. See discussion in Vayanos and Weill (2008) and Fleming and Mizrach (2009) for institutional details specific to US Treasury markets.

[14] See Cheung, Rindi, and De Jong (2005) and Dufour and Skinner (2004) for more information on MTS trading platforms. Also, Pelizzon, Subrahmanyam, Tomio, and Uno (2013) analyze liquidity of Italian government bonds traded on MTS platforms. Although the Italian government bond market is one of the largest and the most liquid government bond markets, its liquidity, by daily trading volume and by the number of trades per bond, is comparable to the US municipal bond and the US corporate bond markets.

limitation, I use bonds that have not matured as of August 2012. To minimize differences across bonds, I use fixed coupon bonds. I exclude floating rate coupon bonds, perpetual bonds, index and inflation-linked bonds, and coupon strips. The final sample consists of 3,210 plain coupon bonds across 67 sovereigns. Thomson Reuters' bid and ask quotes are a composite of quotes collected from various sources including individual dealers, trade organizations such as the ICMA and IBoxx, and local market sources. Bond prices are quoted as a percent of the par (or face) value of the bond.[15] As each sovereign will have multiple bond issues that vary by maturity, currency, and coupon, prices are aggregated by taking the average of all bond issues. For robustness, I consider other ways of aggregating across bond issues including the average weighted by the bond issue size, specific maturities, and maturity buckets.

Table 8 shows the overall descriptive statistics and Table 10 at the country level. The average bond bid-ask spread across all bonds and countries was 0.95% of the mid price (or 95 basis points). From the country-level Table 10, we see a lot of cross-country difference in the average bond bid-ask spread and that bid-ask spreads widen with credit risk: Greece has the highest average bid-ask spread of 3.51% (351 bps) while the U.S. has the lowest at 0.04% (4 bps).

4.3 *Background on the CDS Market*

As discussed before, credit default swaps are over-the-counter derivative contracts that resemble insurance protection against a default or a similar event (referred to as a "credit event") on bonds of a specific firm or government (the "reference entity"). A buyer of CDS protection pays a periodic fee (equivalently, the CDS price, premium, or spread) until either the contract matures or a credit event occurs. In return, the buyer gets paid by the seller the protection amount that was purchased (called "notional") in the event of default or a similar event on *any* one of the bonds covered by the contract of the reference entity. Thus, CDS contracts are written on the level of firms and governments and not at an individual bond level.[16] The standard notional amounts are in the range of $10-20 million.[17] Prices of CDS contracts are paid quarterly and are quoted as annualized percentages of the contract notional.[18] The contract specifies the reference entity, the contract maturity, the notional amount, the set

[15] For example, if the bond price is 95, the bond is trading at 95 cents on the dollar.

[16] This means, for example, if you are a holder of bond "A" of Greek government and Greece defaults on another bond "B" and both bonds are covered by the contract, you will be still be paid out even if your bond "A" has not been defaulted on.

[17] This is comparable to the most common transaction sizes of 5, 10, 25 million euros in, for example, the MTS Global Market (see Cheung, Rindi, and De Jong (2005)).

[18] For example, if the price of a CDS contract with $10 million notional is 200 basis points, the protection buyer pays $0.2 million annually in quarterly installments of $0.05 million. The price of a CDS contract can be thought of as, in its simplest form, the probability of default times one minus the recovery rate. For example, if a one year CDS contract is trading at 200 basis points, and the recovery rate was zero, then the implied probability of default is 2%.

of bonds of the reference entity covered by the contract, and the default events that constitute a credit event. The standard credit events for sovereign CDS are Failure to Pay and Debt Restructuring.[19]

Credit events are determined by the International Swaps and Derivatives Association (ISDA), the governing body for the CDS market.[20] Protection buyers get paid the difference between the notional and the recovery value (effectively, the price of defaulted bonds) that is determined through a special post-credit-event auction. For example, if an investor bought CDS with a notional of $10 million and the recovery rate is 25%, she receives $7.5 million in cash. ISDA finalizes the actual list of eligible bonds that can be submitted into the auction, and oversees the auction. At the end of the auction, all bonds submitted into the auction are bought and sold at the same final bond price and this final price is the price or the recovery rate that settles *all* CDS contracts on that reference entity. In addition to receiving the cash portion, CDS buyers have the option of requesting a "physical settlement" of contracts by selling the bond during the auction.

4.4 *CDS Data*

I now describe the CDS data used in the paper. CDS price data comprises of daily bid and ask price quotes from CMA for the five year maturity contracts over the period 2004-2012. Following market standards, they are reported in basis points. Table 8 summarizes the CMA CDS price data for all sovereigns and Table 10 at the country level. CDS notional data comes the Depository Trust and Clearing Corporation (DTCC) which provides post trade electronic confirmation service to CDS market participants. According to the DTCC, at least more than 90% of all worldwide trades in the CDS market gets recorded in their information warehouse. The DTCC provides historical data on both the volume of trade and the outstanding amount of protection. The volume data is the total notional of all trades on an average day per quarter for each sovereign over the period 2010 Q2 - 2012 Q2. The outstanding data consists of the outstanding gross notional, net notional, and the number of contracts for each sovereign over the period October 31, 2008 - July 28, 2012 at a weekly frequency.

[19] For corporate CDS, bankruptcy is an additional standard credit event. There are three kinds of restructuring that vary by how restrictively they limit the set of eligible bonds: Modified Restructuring (MR), Modified Modified Restructuring (MMR), and Complete (or "old", "full") Restructuring (CR). MR is the most restrictive limiting eligible bonds to have maturity of up to 30 month after the declaration of a credit event, then MMR with 60 month maturity, and CR is the least restrictive with the standard 30-year maturity limit on bonds. CDS on North American reference entities usually feature MR (except CDS on high credit risk firms tend to completely exclude any debt restructuring as a credit event), while CDS on European firms feature the less restrictive MMR. Debt restructuring on CDS on sovereigns, on the other hand, most commonly specify CR.

[20] Credit events are decided by the "determination committee" of ISDA which consists of 10 big dealer banks (e.g Bank of America, Barclays, BNP Paribas, Citibank, Credit Suisse, Deutsche Bank, Goldman Sachs, JPMorgan, Morgan Stanley, and UBS) and five buy side firms that tend to be hedge funds.

Table 9 summarizes the DTCC data for all sovereigns and Table 11 at the country level. My analysis focuses on the outstanding CDS net notional as the amount of CDS purchased.

5 EMPIRICAL RESULTS

In this section, I document the following patterns regarding sovereign bond and CDS markets:

1. A greater amount of CDS purchased is associated with more liquid bond market. Specifically, the amount of CDS net notional outstanding is positively and significantly correlated with bond market liquidity even after controlling for credit risk, and debt outstanding. Consistent with this pattern, when the EU voted in October 2011 to *permanently* ban naked CDS on governments of the EU countries, countries affected by the ban experienced a decrease in their bond market liquidity.

2. On May 18 2010, Germany temporarily banned naked buying of CDS on governments of the Eurozone and the ban was effective overnight. Immediately following the ban, the bond market liquidity increased for the countries affected by the ban. Thus, the pattern documented previously (that bond liquidity increases with more CDS positions) reversed during the initial period of the ban.

5.1 *Pattern 1: The Time-Series Pattern between CDS and Bond Market Liquidity*

5.1.1 *Contemporaneous Regressions*

I first document that the amount of CDS net notional outstanding is positively and significantly correlated with bond market liquidity (narrower bond bid-ask spreads) controlling for credit risk and debt outstanding.

An empirical measure that captures the amount of naked CDS positions in isolation does not exist. Therefore, I use CDS net notional outstanding as a measure proportional to the overall amount of naked positions. Figure 15 plots the time series pattern for Italy of CDS net notional and the proportional bond bid-ask spread over 2008-2012. Overall, a greater amount of CDS purchased is associated with narrower bond bid-ask spreads, i.e. greater liquidity. However, both bond market liquidity and the amount of CDS purchased are correlated with credit risk and the size of the bond market. Table 13 shows for the sample of European Union countries the results of regressing the bond bid-ask spread on CDS net notional while controlling for credit risk (proxied by CDS price) and the gross government debt outstanding. The coefficient estimate of CDS net notional (-0.261, se: 0.0396) says that if CDS net notional increases by one billion US dollar, the proportional bond bid-ask spread decreases by 0.261 (% of mid price), or by about one fourth of the average proportional bid-ask spread (the average was about 1.11% of the mid price as shown in Table 12 of descriptive statistics).[21] An alternative specification of CDS net notional as the log of the ratio of CDS net notional to gross debt outstanding gives qualitatively the same result.

5.1.2 *Vector Autoregressions*

As the regressions in Table 13 of contemporaneous variables do not necessarily imply causation, in this section I carry out vector autoregressions to explore the direction of causation. To summarize, VAR and VECM results suggest that it is not just bond market liquidity driving CDS net notional and that CDS net notional also affects bond market liquidity.

First, I test whether there is a cointegration relationship between CDS price, CDS net notional and the bond bid-ask spread. If there is no evidence of cointegration among the variables, I carry out the simpler VAR-in-differences. For 14 out of 23 European Union countries, there is no evidence of cointegration while for the other 9 there is. For the former 14, Table 14 shows the results of the VAR-in-differences: it shows the p-values of the Granger-causality tests. For 5 out of these 14, CDS net notional Granger-causes bond bid-ask spreads, while for 2 out of 14, bond liquidity Granger-causes CDS net notional. Thus, there is a stronger evidence that CDS net notional affects bond market liquidity.

For the other 9 countries for which the variables are cointegrated, I use vector error-correction models (VECM). The idea behind VECM is that a long term equilibrium relationship exists between cointegrated variables:

$$x_t - \alpha_0 - \alpha_1 \mu_t - \alpha_2 d_t = 0$$

where x_t is a measure of credit risk, μ_t is CDS net notional, and d_t is the proportional bond bid-ask spread. Changes in the variables can be characterized as adjustments to deviations from this long term equilibrium plus responses to the lagged changes:

$$\Delta x_t = \lambda_x (x_{t-1} - \alpha_0 - \alpha_1 \mu_{t-1} - \alpha_2 d_{t-1}) + \sum_{j=1}^{p-1} \beta_{1j} \Delta x_{t-j} + \sum_{j=1}^{p-1} \delta_{1j} \Delta \mu_{t-j} + \sum_{j=1}^{p-1} \gamma_{1j} \Delta d_{t-j},$$

$$\Delta \mu_t = \lambda_{notl} (x_{t-1} - \alpha_0 - \alpha_1 \mu_{t-1} - \alpha_2 d_{t-1}) + \sum_{j=1}^{p-1} \beta_{2j} \Delta x_{t-j} + \sum_{j=1}^{p-1} \delta_{2j} \Delta \mu_{t-j} + \sum_{j=1}^{p-1} \gamma_{2j} \Delta d_{t-j},$$

$$\Delta d_t = \lambda_{bond} (x_{t-1} - \alpha_0 - \alpha_1 \mu_{t-1} - \alpha_2 d_{t-1}) + \sum_{j=1}^{p-1} \beta_{3j} \Delta x_{t-j} + \sum_{j=1}^{p-1} \delta_{3j} \Delta \mu_{t-j} + \sum_{j=1}^{p-1} \gamma_{3j} \Delta d_{t-j}.$$

Gonzalo and Granger (1995) proposed a notion similar to Granger causality in the VECM framework. The variable that adjusts less to the

[21] Throughout, the panel regression analyses with time series data adjust for the fact that errors are correlated within country by including country fixed effects. In addition, I capture any non-fixed effect by modeling AR(1) correlation structure in disturbances to allow for the correlation between residuals to decay with time (as is the case with time series data). Standard errors also allow heteroskedasticity and correlation of disturbances across countries (e.g. Eurozone countries are more correlated with each other than with a non-Eurozone country).

deviation from the long term equilibrium would be considered to be the more important driver of the long run equilibrium (i.e. it Granger-causes the other variables in the long run). Conversely, the variable that adjusts the most (indicated by a significant adjustment coefficient) has a more transitory as opposed to a permanent effect on the other variables. Table 15 shows the adjustment coefficients and t-statistics for CDS net notional and bond bid ask spreads. The t-statistics are generally bigger for the bond bid-ask spread than for CDS net notional. This suggests that CDS net notional affects bond market liquidity.

Next, I conjecture that changes in the amount of naked CDS positions affect CDS market liquidity and explore whether CDS liquidity Granger-causes bond market liquidity.[22] Table 16 reports the results of the Granger-causality tests. For 18 out 24 European Union countries, the CDS bid-ask spread Granger-causes the bond bid-ask spread, whereas for only 10 out of 24 countries the bond bid-ask spread Granger-causes CDS bid-ask spread.

The above VAR and VECM results suggest that it is not just bond market liquidity driving CDS net notional and that CDS net notional also affects bond market liquidity.

5.2 *Pattern 1 using The Permanent CDS Ban*

In this subsection, I examine how bond market liquidity changed following the EU's decision to ban naked CDS. The above results relied on the argument that changes in CDS net notional, keeping credit risk and debt outstanding fixed, is attributable to changes in naked CDS positions. The ban decision targeting naked CDS in particular helps to identify the potential effect of naked positions on bond market liquidity.

5.2.1 *The Description of the Ban*

Throughout 2011, market participants faced uncertainty over whether the EU would adopt measures to ban naked CDS. The uncertainty was finally resolved on October 18, 2011 when, after months of negotiations, the European Parliament and the EU states passed a law to permanently ban naked CDS.[23] The legislation applied to all CDS transactions referencing governments of the EU regardless of the geographic location of the transaction or the legal jurisdiction of the financial institution involved in the transaction.[24]

[22] This set of regressions explores whether one market affects the other more and does not focus on the sign of the correlation. The sign of the correlation is sensitive to whether I use absolute CDS bid-ask spread or relative bid-ask spread (i.e. normalized by the mid price).

[23] For the draft of the law (number: 16338/11 EF 152 ECOFIN 739 CODEC 1873) that was agreed upon by the European Parliament and the Council of the European Union, see `http://register.consilium.europa.eu/pdf/en/11/st16/st16338.en11.pdf`.

[24] It also applied to CDS referencing three other European Economic Area countries: Iceland, Norway, Liechtenstein. But I will simply refer to the countries affected by the ban as the EU although I am including these other three in the analysis.

The final draft of the law was published March 2012 (Regulation EU No 236/2012).[25] Although the legislation was to be in effect beginning November 1, 2012, the March 2012 regulation stated that traders who enter new contracts after March 2012 would have to unwind them by November 2012. Contracts entered into before March 2012 could remain in place even beyond November 2012. Figure 11 compares the total CDS purchased referencing EU governments versus countries not affected by the ban. We see that the total amount of CDS purchased on EU sovereigns started to dramatically decrease starting around the time the law was passed and has been declining ever since. This decrease did not occur for countries not affected by the ban. Thus, anticipating the difficulty of renewing their contracts beyond March 2012, traders started to decrease their activity already beginning fall of 2011.

I describe now the main restrictions that the legislation imposed. Market participants were generally confused about how to actually interpret and satisfy each of these restrictions. The legislation considered a CDS purchase to be covered if it was hedging a portfolio of assets the value of which had a historical correlation of at least 70% with the government bond price over 12 months (or more) prior the CDS purchase. If a CDS purchase could not satisfy this at the time of the purchase, it would be considered naked and hence prohibited. The underlying portfolio could consist of, for example, long positions with respect to private entities within the reference entity country or even long positions through CDS itself. The correlation requirement would be automatically satisfied if the underlying position being hedged was governments bonds (at all federal and local levels of the government), the liability of state enterprises, or the liability of enterprises that are guaranteed by the sovereign. The legislation exempted market making activities.

After the purchase, traders did not have to maintain the correlation throughout the CDS contract since the price of the underlying portfolio can vary. But the size of underlying positions had to remain "proportional" to the amount of CDS purchased. In other words, a trader could not buy bonds with an intent to sell them back once she purchases CDS.[26] In

[25] Additional details emerged later with supplemental regulations EU No 826/2012 (29 June 2012), EU No 827/2012 (29 June 2012), and EU No 918/2012 (5 July 2012). For these drafts, see `http://ec.europa.eu/internal_market/securities/short_selling/index_en.htm`.

[26] In addition, the regulation had various disclosure requirements of short positions through equity, sovereign bond and CDS markets. It also restricted short selling of equity and attempted to restrict naked short selling of governments bonds. Naked short selling is the sale of a security without having pre-borrowed. By definition, naked short selling is limited and temporary since the short seller has to borrow or buy the security to deliver it within the sale settlement period (usually 3 days or less). Otherwise, it results in a delivery failure. According to Comotto (2010), naked short selling of government bonds occurs rarely. When they do occur, it is intraday (for few hours and the short sell is covered), or occur because of operational errors. The regulation required that in order to short sell government bonds, a trader had to either have "located" the bond or have pre-borrowed it. The pre-borrowing arrangement prior to selling is the regular (covered) short selling and entails a contractual repo claim to a bond. But the locate requirement is a soft constraint and does not involve a contractual claim as it can be easily satisfied by email or phone. This regulation,

terms of how the regulation was enforced, institutions were supposed to be able to provide such evidence of hedging if they were asked.

5.2.2 *The Effect of the Permanent Ban on Bond Market Liquidity*

Now consider what happened to bond liquidity. Figure 12 shows that following the ban bond market liquidity decreased for countries that were affected by the ban. I formally show this with a difference-in-difference analysis. I set the ban period, denoted by T_b, to be a four month period starting the week after October 18th through the end February 2012. I explore the following panel regression using four-months of data before and after the ban of all countries:

$$d_{it} = c + \gamma_i + \lambda_t + X'_{it}\beta + \delta D_{i \in EU, t \in T_b} + \epsilon_{it} \tag{11}$$

where d_{it} is the bond bid-ask spread (% of the mid price) of country i at time t, c is a constant, γ_i and λ_t are the coefficients on country and time fixed effects, respectively, and X_{it} is a set of controls. $D_{i \in euro, t \in T_b}$ is a dummy variable that equals one for the country-date observations for which the ban was in place. Thus, the control group is countries outside the EU (hence not affected by the ban) and the treatment group is the EU countries. The coefficient of interest is δ: it measures the effect of the CDS ban on liquidity of the European Union government bond markets.

Table 17 shows the regression results of (11) controlling for debt outstanding and CDS price as a measure of credit risk. The coefficient estimate of the *EU CDS Ban* (δ) in Column (1) is positive and statistically significant (0.271, st. err: 0.105) and shows that the ban is associated with 27% increase in the proportional bid-ask spread.[27] The average bid-ask spread for the EU countries was about 1% of the mid price or \$1 of round trip transaction cost for every \$100 of transaction. Relative to this average, the round trip transaction cost increased 27% from \$1 to \$1.27. Columns (1) and (2) show that including CDS price as a measure of credit risk qualitatively does not change the results.

5.2.3 *Alternative Specifications*

To allow for the possibility that bond bid-ask spreads for different countries followed different trends, Columns (3)-(6) allow for country specific trends. Column (7) includes instead a group specific trend: treated and control countries, as a group, followed different trends. We see that the observed decrease in bond liquidity is robust to including country or group trends. As a robustness, Column (5) excludes Greece as a potential outlier, and Column (6) restricts the control group countries to just OECD countries. Both give qualitatively the same result.

as a result, did not affect short selling of government bonds but mainly targeted the CDS market.

[27] See Footnote 21 for discussion on standard errors. In addition, Table 23 shows the regression results of collapsing the time series data into two pre and post ban periods and finds qualitatively the same result using the OECD sample.

We observe the total amount of CDS purchased and not covered and naked purchases separately. Since the ban targeted naked CDS trading in particular, the decrease in the total amount of CDS purchased during this period should capture more a decrease in naked rather than covered CDS purchases, and hence, should capture the amount of naked CDS positions outstanding prior to the ban.[28] Thus, if banning naked CDS positions caused bond liquidity to decrease, a greater decrease in CDS net notional should be associated with a greater increase in the bond bid-ask spread. To check this hypothesis, Column (4) adds an interaction term between the *EU CDS Ban* dummy variable with the change in net notional following the ban, *EU CDS Ban*$*\Delta Notl$. The positive and the statistically significant coefficient of the interaction term shows that among countries subject to the ban those that had potentially more naked CDS positions outstanding prior to the ban experienced even a greater widening of the bond bid-ask spread.

To check the hypothesis that the change in net notional following the ban is correlated with more naked CDS positions prior to the ban, Table 22 shows the correlation between the change in net notional during this period and the past level of CDS net notional controlling for debt outstanding and credit risk. We see a statistically significant positive correlation for the EU countries but not for the non-EU countries.

5.2.4 *Possible Endogeneity of Regulations*

Short selling bans are usually imposed in times when regulators are concerned with stability and liquidity in financial markets. If the regulation was passed in anticipation of a decrease in liquidity, then the subsequent observed decrease in bond liquidity following the ban may not be due to the ban (while the ban itself was ineffective in improving market conditions). However, this argument does not explain why liquidity increased following the temporary German ban.

Nevertheless, since the ban targeted particularly the naked CDS buyers, it still allows us to approximate the amount of naked CDS positions that had existed before the ban by using the decrease in the total CDS purchased following the ban. I check whether the cross-country variation in the drop has an explanatory power for the bond liquidity level *before* the ban for countries subject to the ban. Table 18 shows the estimates of the following regression:

$$\begin{aligned} d_{it-1} = c + \gamma_i + \lambda_{t-1} + X'_{it-1}\beta_1 + \beta_2\Delta_{i(t-1,t)}Notl \\ +\delta D_{i\in EU,t\in T_b} * \Delta_{i(t-1,t)}Notl + \epsilon_{it-1}. \end{aligned} \tag{12}$$

[28]The legislation applied to new CDS contracts and not existing positions. So the decrease in CDS net notional captures the maturation of CDS contracts that otherwise would have been renewed had there not been the ban.

We are interested in δ.[29] We see from the estimate of β_2 that normally future changes in CDS net notional are not correlated with the past level of bond liquidity. But the decrease in net notional that countries subject to the ban experienced, specifically during the ban, is associated with a higher level of bond liquidity (tighter bid-ask spreads) prior to the ban. Thus, a potentially greater amount of naked CDS positions outstanding prior to the ban is correlated with a higher pre-ban level of bond liquidity controlling for the pre-ban levels of credit risk and debt outstanding.

5.3 *Pattern 2: The Temporary CDS Ban*

5.3.1 *The Description of the Ban*

On Tuesday May 18th 2010, Germany prohibited naked purchase of CDS referencing Eurozone governments.[30] As recent as month prior to the ban Germany's rhetoric had been that there is no need to ban naked CDS trading. The regulation was unexpected by market participants and was implemented within the same day that the media first reported it. News about the ban first appeared around 1pm on Tuesday May 18 2010 through Reuters. But the official details of the legislation did not emerge until late in the evening around 9:30pm. The regulation was effective from midnight the same day (within two and half hours from the release of the official statement) and was to be in effect through March 31, 2011. However, later on July 27, 2010 the regulation was made permanent.

The regulation also banned naked short selling of 10 leading German financial stocks and naked short selling of eurozone governments bonds that were allowed to be listed on Germany's domestic stock exchange. The naked bond short selling restriction, as a result, applied to only a few German and Austrian bonds.

The May 18th 2010 regulation did not specify the territorial scope of the regulation. So it is not clear whether market participants interpreted the regulation to apply to all transactions regardless of the geographic location and the institution. According to Allen & Overy LLP and ISDA's conversations with BaFin (Germany's financial regulatory body), BaFin confirmed that the regulation applied to transactions where at least one of the counterparties is located in Germany. It would not, for example, apply to a transaction between the New York branch and London Branch of Deutsche Bank.

5.3.2 *Results*

In this section, I explore how this regulation impacted bond market liquidity. Figure 13 plots the cross country average of the bond bid-ask spread. The dashed line shows the average for the EU countries not

[29] Since we are looking at the past level of bond liquidity, the interpretation of $\Delta Notl$ in the current setup is different from the previous set-up.

[30] For the draft of the regulation, see `http://www.bafin.de/SharedDocs/Aufsichtsrecht/EN/Verfuegung/vf_100518_kreditderivate_en.html`.

affected by the ban (i.e. naked CDS referencing these countries could still be purchased), while the solid line plots the average for the EU countries affected by the ban (i.e. the Eurozone countries). Two vertical lines are drawn for the week before the ban and the week of the ban. We can see that for the countries affected by the ban, there was a large and sudden narrowing of the bond bid-ask spread while this did not occur for the countries not affected by the ban. Figure 14 in the Appendix demonstrates the time series of CDS net notional around the ban.

To test this pattern formally, I carry out an exercise analogous to the EU ban. I set the initial period of the ban, denoted by T_b, to be a month long period starting the week after the ban inception.[31] I explore the following regression using four months of data before and after the ban using the sample of EU countries:

$$d_{it} = c + \gamma_i + \lambda_t + X_{it}^{'}\beta + \delta D_{i \in euro, t \in T_b} + \epsilon_{it} \tag{13}$$

where d_{it} is the bond bid-ask spread (% of mid), c is a constant, γ_i and λ_t are the coefficients on country and time fixed effects, respectively, and X_{it} is a set of controls. $D_{i \in euro, t \in T_b}$ is a dummy variable that equals one for the country-date observations for which the ban was in place. The control group is the non-Eurozone countries within the EU (hence not affected by the ban) and the treatment group is the Eurozone countries. Thus, the difference δ is the effect of the CDS ban on liquidity of the Eurozone government bond markets.

Table 24 shows the regression results of (13). The coefficient estimate of the *CDS Ban* (δ), as shown in columns (1) and (2), is negative and statistically significant. During the initial period of the ban, countries subject to the CDS ban experienced larger decrease in the bond bid-ask spread relative to the countries not subject to the ban. Comparing Column (1) and column (2) shows that including CDS price does not make a difference. As Columns (3) and (6) show, controlling for country or group specific trends, respectively, does not change the results. The observed decrease in the bid-ask spread is also to robust to excluding Greece as a potential outlier (Column (5)).

Since the ban targeted naked CDS in particular, the decrease in CDS net notional following the ban should be associated more with the amount of naked CDS positions that had existed before the ban than the amount of covered CDS.[32] Thus, if banning naked CDS positions caused bond liquidity to increase, a larger drop in CDS net notional should be associated with a larger decrease in the bond bid-ask spread. Column (4) checks this hypothesis as a further robustness. It includes an

[31] Although the temporary ban was initially effective through March 2011, I restrict to a narrower ban period to capture the immediate impact of the ban. Given that the ban applied to institutions within Germany only, with more time, trades are likely to have shifted to other European countries outside Germany.

[32] Similar to the EU ban, the legislation banned new CDS contracts and would not have applied to existing CDS contracts. Thus, any decrease in net notional is capturing more the maturation of CDS contracts that would have otherwise been renewed had there not been the ban.

interaction of *CDS Ban* dummy with the change in net notional following the ban ($CDS\ Ban * \Delta Notl$). The negative and statistically significant slope coefficient of the interaction term suggests that among countries subject to the ban, those that had potentially more naked CDS positions prior to the ban experienced an even greater decrease in the bid-ask spread.

These results suggest that, first, naked CDS positions in particular have an effect on bond market liquidity, and second, the positive correlation between bond liquidity and the amount of CDS positions that we saw in the previous section reversed during the initial period of the ban.

6 DISCUSSION

6.1 *Model Implications and the Empirical Patterns*

I discuss now how the model mechanism rationalizes these contradictory empirical patterns. The model suggests that, in the long term, bond and CDS markets are complementary markets. Thus, when the CDS market is shut down permanently, it adversely affects liquidity of the bond market which is consistent with the observed decrease in bond market liquidity after the permanent ban. Specifically, if the number of naked CDS buyers permanently decreases for an exogenous reason (as happened with the permanent EU ban), long traders – who would have been counterparties to naked CDS buyers – are forced to exit the CDS market. But by exiting the CDS market, they exit the bond market also. As a result, bond liquidity and bond prices decrease. Importantly, bond and CDS markets are complementary markets only in the presence of search frictions in the CDS market. Hence, trading frictions in the CDS market create an interaction between bond and CDS markets that helps explain the empirical patterns.

In the short term, on the other hand, bond and CDS markets are substitute markets. As a result, when the CDS market is shut down temporarily, the immediate effect is an increase in bond market liquidity which is consistent with the observed increase in bond liquidity following the temporary German ban. This is because the migration effect (specifically, its reverse) dominates: long traders do not exit at the extensive margin but instead resort to temporarily trading in the bond market. Bond sellers temporarily benefit from a greater number of bond buyers who would have otherwise sold CDS.

6.2 *The Distinction from Other Mechanisms*

The mechanism of this paper is separate from other effects that CDS markets might have on bond market liquidity. The most commonly thought of effect is the "covered" CDS story: the ability to insure one's bond portfolio by buying CDS is likely to attract traders into the bond market and increase bond market liquidity. The most commonly thought of effect of naked CDS trading is that it may increase liquidity of the CDS market itself and, consequently, may *indirectly* increase bond market liquidity by making CDS a cheaper hedging tool. These effects, however, cannot explain why permanent versus temporary CDS bans would affect bond market liquidity differently. This paper instead proposes a theory that rationalizes the opposite effects of different bans within the same theoretical environment.

Another effect that my mechanism is distinct from is the basis trade. In a basis trade, investors trade on an arbitrage opportunity arising from how credit risk is priced in bond and CDS markets versus the theoretical arbitrage relationship between the two securities. For example, if the CDS price is too low relative to bond spreads, then a basis trading strategy would involve buying bonds and buying CDS. Thus, the existence of the CDS market, by creating a potential arbitrage opportunity, may increase the amount of trade and liquidity in the bond market. But basis trades necessarily involve a long position in one market (e.g. buying bonds as in the example) but a short position in the other market (e.g. buying CDS). In contrast, in my mechanism, there is an increase in the volume of trade and liquidity in the bond market due to traders seeking a long position in *both* markets.

6.3 *Search vs. Asymmetric Information*

Another plausible effect of the CDS market is that as an instrument to trade on negative news, shorting credit risk through the CDS market may aggravate adverse selection problems in the bond market and may amplify a potential "run" on sovereign bond markets. This, in turn, may lead to a further liquidity dry-up in the bond market. Although plausible, this mechanism on its own cannot explain why different bans would affect bond liquidity differently.

In the above scenario, potential bond investors as a group may have asymmetric information from what the sovereign knows about itself. It is also possible that illiquidity arises from asymmetric information amongst traders as in Kyle (1985) and Glosten and Milgrom (1985) type frameworks. The search framework is better suited for sovereign bond markets for two reasons. First, trade in sovereign bond markets are fragmented across heterogenous bonds. Second, asymmetric information and insider trading is less severe with respect to governments than with respect to individual firms.

7 CONCLUSION

This paper studies, both empirically and theoretically, the interaction between bond and CDS markets, and, in particular, how naked CDS trading affects liquidity of the underlying bond market. To identify how naked CDS trading affects bond market liquidity, I use two naked CDS bans implemented in Europe as quasi-natural experiments and analyze how they affected sovereign bond market liquidity. I document that the 2011 permanent EU ban adversely affected bond market liquidity but the 2010 temporary German instead increased bond market liquidity.

To reconcile these contradictory patterns, I build a search theoretic framework with interdependent bond and CDS markets liquidity. I show that, in the long term, bond and CDS markets are complementary markets. The introduction of the CDS market creates a positive externality in the bond market and increases bond market liquidity by attracting traders into both the CDS and the bond market. This result implies that permanently banning the CDS market will adversely affect bond market liquidity: by pulling out from the CDS market, traders pull out from the bond market also. But, in the short term, there is a substitutability between these two markets so that when the CDS market is banned only temporarily, instead of pulling out from both markets, traders temporarily migrate to the bond market.

My paper shows that different CDS bans can have different effects on liquidity of the underlying bond market. But the main policy implication of the paper is that permanently banning naked CDS trading will, in the long term, adversely affect bond market liquidity and hence increase sovereigns' cost of borrowing.

Key model ingredients that help reconcile the observed patterns are, first, search frictions in the CDS market. The complementariness of bond and CDS markets arises only in the presence of search frictions in the CDS market. The CDS market is otherwise redundant and does not affect bond market liquidity. The second key model ingredient is endogenous entry. The fact that bond and CDS markets can be complementary markets is a novel result in light of existing theoretical studies of the liquidity interaction between multiple asset markets. These studies highlight the migration (or equivalently, the substitution) effect. In these models, the aggregate number of traders across markets is kept fixed and, consequently, introducing additional markets necessarily results in a fragmentation and migration of traders across multiple markets. My results show that an important interaction between multiple markets arises out of endogenizing the aggregate number traders across markets by endogenizing traders' entry decision at the extensive margin.

Agents' flow value equations are analogously derived to (6):

$$
\begin{aligned}
rV_{ln} &= \gamma_u(0 - V_{ln}) + q_{cs}(V_{lsc} - V_{ln}) && \text{(A.14)}\\
rV_{hob} &= \delta_b + x_b - y + \gamma_d(V_{aob} - V_{hob}) && \text{(A.15)}\\
rV_{aob} &= \delta_b - y + q_{bb}(0 - V_{aob} + p_b) && \text{(A.16)}\\
rV_{hoc} &= p_c - (\delta_c - x_{ch}) - y + \gamma_d(V_{aoc} - V_{hoc}) + \gamma_u(V_{hn} - V_{hoc}) && \text{(A.17)}\\
rV_{aoc} &= p_c - \delta_c - y + q_{cs}(0 - V_{aoc}) + \gamma_u(0 - V_{aoc}) && \text{(A.18)}\\
rV_{lsc} &= -p_c + (\delta_c + x_{cl}) - y + \gamma_u\,(0 - V_{lsc}) && \text{(A.19)}
\end{aligned}
$$

Proof of Proposition 1. The proof of uniqueness is shown in Lemma 1 and the proof of existence is shown in Lemma 2. □

Lemma 1. *Suppose* (7) *holds, then the steady state equilibrium is unique.*

Proof. First fix ρ, then using the in-flow out-flow equations and the market clearing conditions (2) (3), $\mu_{ln}, \mu_{hob}, \mu_{aob}, \mu_{hoc}, \mu_{aoc}, \mu_{lsc}$ can be solved as a function of μ_{hn}:

$$
\begin{aligned}
\mu_{ln} &= \frac{F_l}{\gamma_u + \lambda_c \mu_{hn}} && \text{(A.20)}\\
\mu_{hob} &= \frac{S\lambda_b\mu_{hn}}{\lambda_b\mu_{hn} + \gamma_d} && \text{(A.21)}\\
\mu_{aob} &= S - \frac{S\lambda_b\mu_{hn}}{\lambda_b\mu_{hn} + \gamma_d} && \text{(A.22)}\\
\mu_{hoc} &= \frac{\lambda_c F_l \mu_{hn}}{\gamma_u\,(\lambda_c\mu_{hn} + \gamma_d + \gamma_u)} && \text{(A.23)}\\
\mu_{aoc} &= \frac{\gamma_d F_l \lambda_c \mu_{hn}}{\gamma_u\,(\lambda_c\mu_{hn} + \gamma_u)\,(\lambda_c\mu_{hn} + \gamma_d + \gamma_u)} && \text{(A.24)}\\
\mu_{lsc} &= \frac{\lambda_c F_l \mu_{hn}}{\gamma_u\,(\lambda_c\mu_{hn} + \gamma_u)} && \text{(A.25)}
\end{aligned}
$$

And μ_{hn} itself is a solution to:

$$
(1+\rho)F_h - \gamma_d\mu_{hn}\left(\frac{S\lambda_b}{\lambda_b\mu_{hn} + \gamma_d} + \frac{\lambda_c F_l}{\gamma_u\,(\lambda_c\mu_{hn} + \gamma_d + \gamma_u)} + 1\right) = 0 \qquad \text{(A.26)}
$$

The LHS of (A.26) is positive at $\mu_{hn} = 0$, decreasing in μ_{hn}, and is negative for large μ_{hn}, hence (A.26) has a unique positive solution. Thus, (A.26) uniquely determines μ_{hn} and has a positive solution, while other μ's are uniquely determined by (A.20)-(A.24). Next, once μ's are solved, the value functions and prices are uniquely determined by a linear system of equations: (6),(A.14) -(A.19), and (4)-(5).

We are left with the endogenous entry decisions:

$$
\rho = \begin{cases} 1 & V_{hn}(\rho) > O_h \\ [0,1] \text{ if} & V_{hn}(\rho) = O_h \\ 0 & V_{hn}(\rho) < O_h \end{cases} \qquad \text{(A.27)}
$$

There are three cases: two corner solutions $\rho = 0$, and $\rho = 1$, and an interior solution. Next, I show that V_{hn} is strictly decreasing in ρ, which will imply that under each case the equilibrium is unique. The derivation in the proof of existence shows that:

$$
V_{hn} = \frac{q_{bs}x_b\phi + \Delta_{hoc}q_{cb}\,(r + \gamma_d + q_{bb}(1-\phi))}{(r+\gamma_d)\,k}
$$

where

$$\Delta_{hoc} = \frac{x_{ch} + (q_{cs} + r + \gamma_u + \gamma_d)\frac{x_{cl}-2y}{r+\gamma_u+q_{cs}} - \frac{1}{k}q_{bs}\phi x_b}{\frac{(1-\phi)q_{cs}+r+\gamma_u+\gamma_d}{\phi} + \frac{1}{k}q_{cb}\left(r + \gamma_d + (1-\phi)\, q_{bb}\right)}$$

None of the μ's other than μ_{hn} directly depend on ρ but depend only indirectly through μ_{hn}, thus we write:

$$\begin{aligned}
\frac{\partial V_{hn}(\rho)}{\partial \rho} &= \frac{\partial \mu_{hn}}{\partial \rho}\left(\frac{\partial V_{hn}}{\partial q_{bs}}\frac{\partial q_{bs}}{\partial \mu_{hn}} + \frac{\partial V_{hn}}{\partial q_{bb}}\frac{\partial q_{bb}}{\partial \mu_{hn}} + \frac{\partial V_{hn}}{\partial q_{cb}}\frac{\partial q_{cb}}{\partial \mu_{hn}} + \frac{\partial V_{hn}}{\partial q_{cs}}\frac{\partial q_{cs}}{\partial \mu_{hn}}\right)\\
&= \frac{\partial \mu_{hn}}{\partial \rho}\left(\frac{\partial V_{hn}}{\partial q_{bs}}\frac{\partial(\lambda_b\mu_{aob})}{\partial \mu_{hn}} + \frac{\partial V_{hn}}{\partial q_{bb}}\frac{\partial(\lambda_b\mu_{hn})}{\partial \mu_{hn}} + \frac{\partial V_{hn}}{\partial q_{cb}}\frac{\partial \lambda_c(\mu_{aoc}+\mu_{ln})}{\partial \mu_{hn}} + \frac{\partial V_{hn}}{\partial q_{cs}}\lambda_c\right)\\
&= \frac{\partial \mu_{hn}}{\partial \rho}\left(\frac{\partial V_{hn}}{\partial q_{bs}}\frac{\partial(\lambda_b\mu_{aob})}{\partial \mu_{hn}} + \frac{\partial V_{hn}}{\partial q_{bb}}\frac{\partial(\lambda_b\mu_{hn})}{\partial \mu_{hn}} + \frac{\partial V_{hn}}{\partial q_{cb}}\frac{\partial \lambda_c(\mu_{aoc}+\mu_{ln})}{\partial \mu_{hn}} + \frac{\partial V_{hn}}{\partial q_{cs}}\lambda_c\right)
\end{aligned} \tag{A.28}$$

Next, I derive $\frac{\partial V_{hn}}{\partial q_{bs}}$, $\frac{\partial V_{hn}}{\partial q_{bb}}$, $\frac{\partial V_{hn}}{\partial q_{cb}}$, and $\frac{\partial V_{hn}}{\partial q_{cs}}$.

$$\begin{aligned}
\frac{\partial V_{hn}}{\partial q_{bb}} &= -\frac{q_{bs}\ \phi_h}{(r+\gamma_d)\,k^2}\phi_l B\\
\frac{\partial V_{hn}}{\partial q_{bs}} &= \frac{\phi_h\,(r+\gamma_d+q_{bb}\phi_l)}{(r+\gamma_d)\,k^2}B\\
\frac{\partial V_{hn}}{\partial q_{cs}} &= \frac{q_{cb}\,(r+\gamma_d+q_{bb}\phi_l)}{k\,(r+\gamma_d)\,C}\left(-\frac{\phi_l A}{\phi_h C} - \frac{(x_{\text{cl}}-2y)\,\gamma_d}{(q_{cs}+r+\gamma_u)^2}\right)\\
\frac{\partial V_{hn}}{\partial q_{cb}} &= \frac{A\,(r+\gamma_d+q_{bb}\phi_l)}{k\,(r+\gamma_d)\,C\phi_h}\left(\frac{r+\gamma_d+\gamma_u+q_{cs}\phi_l}{C}\right)
\end{aligned}$$

where

$$\begin{aligned}
B &\equiv x_b + \frac{q_{cb}}{C}\left(\frac{(r+\gamma_d+q_{bb}\phi_l)}{k}q_{cb}\frac{A}{C} - A - \frac{(r+\gamma_d+q_{bb}\phi_l)}{k}x_b\right)\\
A &\equiv x_{\text{ch}} + \frac{(x_{\text{cl}}-2y)\,(q_{cs}+r+\gamma_d+\gamma_u)}{q_{cs}+r+\gamma_u} - \frac{q_{bs}x_b\phi_h}{r+\gamma_d+q_{bs}\phi_h+q_{bb}\phi_l}\\
C &\equiv \frac{r+\gamma_d+\gamma_u+q_{cs}\phi_l}{\phi_h} + \frac{q_{cb}\,(r+\gamma_d+q_{bb}\phi_l)}{k}
\end{aligned}$$

From here, $\frac{\partial V_{hn}}{\partial q_{cb}} > 0$ while $\frac{\partial(\lambda_c(\mu_{aoc}+\mu_{\text{ln}}))}{\partial \mu_{hn}} < 0$ implying that the third term in (A.28) is negative. Since $\frac{\partial V_{hn}}{\partial q_{cs}} < 0$, the fourth term (A.28) is also negative. But the sign of both $\frac{\partial V_{hn}}{\partial q_{bs}}$ and $\frac{\partial V_{hn}}{\partial q_{bb}}$ depend on the sign of B. Thus, consider B:

$$\begin{aligned}
B &= x_b + \frac{q_{cb}}{C}\frac{(r+\gamma_d+q_{bb}\phi_l)}{k}q_{cb}\frac{A}{C} - \frac{q_{cb}}{C}A - \frac{q_{cb}}{C}\frac{(r+\gamma_d+q_{bb}\phi_l)}{k}x_b\\
&= x_b\left(1 - \frac{q_{cb}}{C}\frac{(r+\gamma_d+q_{bb}\phi_l)}{k}\right) - \left(1 - \frac{q_{cb}}{C}\frac{(r+\gamma_d+q_{bb}\phi_l)}{k}\right)\frac{q_{cb}}{C}A\\
&= \left(1 - \frac{q_{cb}}{C}\frac{(r+\gamma_d+q_{bb}\phi_l)}{k}\right)\left(x_b - q_{cb}\frac{A}{C}\right)
\end{aligned}$$

First, $0 < \frac{q_{bb}\phi_l+\gamma_d+r}{k} < 1$ and $0 < \frac{q_{cb}}{C} < 1$. To see the latter, let $\phi_l = \phi_h$, then $C > q_{cs}$. From Assumption 2:

$$\mu_{hn} + \mu_{hoc} + \mu_{hoc} \geq \frac{F_h}{\gamma_d} > S + \frac{F_l}{\gamma_u}$$

But using the CDS market clearing condition, we have $\frac{F_l}{\gamma_u} = \mu_{\text{ln}} + \mu_{\text{lsc}} = \mu_{\text{ln}} + (\mu_{\text{hoc}} + \mu_{aoc})$. Thus,

$$\mu_{hn} + \mu_{\text{hoc}} + \mu_{\text{hob}} > S + \mu_{\text{ln}} + (\mu_{\text{hoc}} + \mu_{aoc})$$

Cancel μ_{hoc},

$$\mu_{hn} + \mu_{\text{hob}} > S + \mu_{\text{ln}} + \mu_{aoc}$$

$$\mu_{hn} > (S - \mu_{\text{hob}}) + \mu_{\text{ln}} + \mu_{aoc} > \mu_{\text{ln}} + \mu_{aoc}$$

Hence, $q_{cs} > q_{cb}$ and $C > q_{cs} > q_{cb}$. Thus, the term in the first bracket of B is positive. Now consider the term in the second bracket of B, $x_b - q_{cb}\frac{A}{C} = x_b - q_{cb}\Delta_{\text{hoc}}$:

$$\begin{aligned} x_b - q_{cb}\Delta_{\text{hoc}} &= x_b - q_{cb}\frac{x_{\text{ch}} + \frac{(x_{\text{cl}}-2y)(q_{cs}+r+\gamma_d+\gamma_u)}{q_{cs}+r+\gamma_u} - \frac{q_{bs}\phi_h}{k}x_b}{\frac{r+\gamma_d+\gamma_u+q_{cs}\phi_l}{\phi_h} + \frac{q_{cb}(r+\gamma_d+q_{bb}\phi_l)}{k}} \\ &= x_b - \frac{x_{\text{ch}} + (x_{\text{cl}} - 2y)\left(1 + \frac{\gamma_d}{q_{cs}+r+\gamma_u}\right) - \frac{q_{bs}\phi_h}{k}x_b}{\frac{r+\gamma_d+\gamma_u+q_{cs}\phi_l}{q_{cb}\phi_h} + \frac{k-q_{bs}\phi_h}{k}} \\ &= \frac{\left(\frac{r+\gamma_d+\gamma_u+q_{cs}\phi_l+q_{cb}\phi_h}{q_{cb}\phi_h}\right)x_b - \left(x_{\text{ch}} + (x_{\text{cl}} - 2y)\left(\frac{q_{cs}+r+\gamma_u+\gamma_d}{q_{cs}+r+\gamma_u}\right)\right)}{\frac{r+\gamma_d+\gamma_u+q_{cs}\phi_l}{q_{cb}\phi_h} + \frac{k-q_{bs}\phi_h}{k}} \end{aligned}$$

The sign of the expression depends on the numerator:

$$\left(\frac{r + \gamma_d + \gamma_u + q_{cs}\phi_l + q_{cb}\phi_h}{q_{cb}\phi_h}\right)x_b - \left(x_{\text{ch}} + (x_{\text{cl}} - 2y)\left(\frac{q_{cs} + r + \gamma_u + \gamma_d}{q_{cs} + r + \gamma_u}\right)\right)$$

This expression is positive from (7). Thus, $\frac{\partial V_{hn}}{\partial q_{bs}} > 0$ and together with $\frac{\partial \mu_{aob}}{\partial \mu_{hn}} < 0$ implies that the first term of (A.28) is negative. Also, since $\frac{\partial V_{hn}}{\partial q_{bb}} < 0$, the second term of (A.28) is also negative.

Finally from (A.26) and using the Implicit Function Theorem,

$$\frac{\partial \mu_{hn}}{\partial \rho} = \frac{F_h}{\gamma_d\left(\frac{s\lambda_b\gamma_d}{(\lambda_b\mu_{hn}+\gamma_d)^2} + \frac{\lambda_c f_l(\gamma_d+\gamma_u)}{\gamma_u(\lambda_c\mu_{hn}+\gamma_d+\gamma_u)^2} + 1\right)}$$

Thus, $\frac{\partial \mu_{hn}}{\partial \rho} > 0$, and, consequently, $\frac{\partial V_{hn}(\rho)}{\partial \rho} < 0$. □

Lemma 2. *Existence*

Proof. To show existence we verify that the conjectured optimal trading strategies are in fact optimal. In particular, first, we show that the total surplus from trading the bond is positive: $\omega_b = V_{hob} - V_{hn} - V_{aob} > 0$. By construction, this will ensure that individual surpluses to the buyer and the seller of the bond are positive: a high type agent optimally chooses to buy the bond, and an average type agent prefers to sell her bond. Second, we show that the total surplus from trading CDS is positive $\omega_c = V_{hoc} - V_{hn} + V_{lsc} - V_{ln} > 0$. This will imply that the high type agents will want to sell CDS while low type agents want to buy CDS. Third, we verify that the average type agents will prefer quit being a CDS seller: $0 - V_{aoc} > 0$. Thus, agents who have previously sold CDS when they were high types will prefer to find another seller to take over his side of the trade and exit the market with zero utility. I proceed by first deriving ω_b, ω_c, V_{aoc}.

Subtracting rV_{ln} (A.14) from rV_{lsc} (A.19) and defining $\Delta_{lsc} \equiv V_{lsc} - V_{ln}$, we get:

$$\Delta_{lsc} = \frac{\delta_c + x_{cl} - y - p_c}{r + \gamma_u + q_{cs}}$$

From (5):

$$\Delta_{hoc} = \frac{\phi}{1-\phi}\Delta_{lsc} = \frac{\phi}{1-\phi}\frac{\delta_c + x_{cl} - y - p_c}{r + \gamma_u + q_{cs}}$$

Also from the value function of V_{aoc},

$$V_{aoc} = \frac{p_c - (y + \delta_c)}{r + \gamma_u + q_{cds}} \tag{A.29}$$

Using (A.17) and substituting in the expression for V_{aoc}:

$$rV_{hoc} = p_c - (\delta_c - x_{ch}) - y + \gamma_d(\frac{p_c - (y + \delta_c)}{r + \gamma_u + q_{cds}} - V_{hoc}) - \gamma_u \Delta_{hoc}$$

Add $\gamma_d V_{hoc}$ to both sides:

$$(r + \gamma_d)V_{hoc} = p_c - (\delta_c - x_{ch}) - y + \frac{p_c - (y + \delta_c)}{r + \gamma_u + q_{cds}} - \gamma_u \Delta_{hoc}$$

Subtract $(r + \gamma_d)V_{hn}$ from both sides:

$$(r + \gamma_d + \gamma_u)\Delta_{hoc} = p_c - (\delta_c - x_{ch}) - y + \frac{p_c - (y + \delta_c)}{r + \gamma_u + q_{cds}} - (r + \gamma_d)V_{hn}$$

Thus, we have three equations and three unknowns, Δ_{hoc}, p_c, V_{hn}:

$$\Delta_{hoc} = \frac{\phi}{1-\phi}\frac{\delta_c + x_{cl} - y - p_c}{r + \gamma_u + q_{cs}}$$

$$(r + \gamma_d + \gamma_u)\Delta_{hoc} = p_c - (\delta_c - x_{ch}) - y + \frac{p_c - (y + \delta_c)}{r + \gamma_u + q_{cds}} - (r + \gamma_d)V_{hn}$$

$$V_{hn} = \frac{q_{bs} x_b \phi + \Delta_{hoc} q_{cb} \left(r + \gamma_d + q_{bb}(1-\phi)\right)}{(r + \gamma_d)\, k} \tag{A.30}$$

where the latter comes from the solution to the equations for V_{hob}, V_{aob}, and V_{hn}. The solution for Δ_{hoc} is given by:

$$\Delta_{hoc} = \frac{x_{ch} + (q_{cs} + r + \gamma_u + \gamma_d)\,\frac{x_{cl} - 2y}{r + \gamma_u + q_{cs}} - \frac{1}{k} q_{bs} \phi x_b}{\frac{(1-\phi)q_{cs} + r + \gamma_u + \gamma_d}{\phi} + \frac{1}{k} q_{cb} \left(r + \gamma_d + (1-\phi)\, q_{bb}\right)} \tag{A.31}$$

From here:

$$p_c = \delta_c + x_{cl} - y - \frac{1-\phi}{\phi}(r + \gamma_u + q_{cs})\Delta_{hoc} \tag{A.32}$$

$$\omega_c = \frac{1}{\phi}\Delta_{hoc}$$

Using the solution to the equations for V_{hob}, V_{aob}, and V_{hn}:

$$\omega_b = \frac{x_b - q_{cb}\Delta_{hoc}}{r + \gamma_d + \phi q_{bs} + (1-\phi) q_{bb}} \tag{A.33}$$

To consider small search frictions, define $\epsilon \equiv \frac{1}{\lambda_b}$ and $n \equiv \frac{\lambda_c}{\lambda_b}$. We show existence for $\epsilon = 0$. Then by continuity, existence is established in the neighborhood of $\epsilon \equiv 0$ or for small search frictions. With the change of variables, (A.26) becomes:

$$(1 + \rho)F_h - \gamma_d \mu_{hn} \left(\frac{S}{\mu_{hn} + \epsilon \gamma_d} + \frac{n F_l}{\gamma_u \left(n \mu_{hn} + \epsilon(\gamma_d + \gamma_u)\right)} + 1 \right) = 0 \tag{A.34}$$

From (A.34), for any $\rho \in [0,1]$, μ_{hn} asymptotically converges to $\mu_{hn} = \frac{(1+\rho)F_h}{\gamma_d} - (S + \frac{F_l}{\gamma_u})$ therefore $0 < \lim_{\lambda_b,\lambda_c \to \infty} \mu_{hn} < \infty$ and $\lim_{\lambda_b,\lambda_c \to \infty} q_{bb} = \infty$. This also implies from (A.22) that $\lim_{\lambda_b,\lambda_c \to \infty} \mu_{aob} = 0$ and q_{bs} converges to a finite number. Analogously, $\lim_{\lambda_b,\lambda_c \to \infty} q_{cs} = \infty$ and from (A.20) and (A.24): $0 < \lim_{\lambda_b,\lambda_c \to \infty} q_{cb} < \infty$.

To show $\omega_c > 0$ using these limits, consider the numerator of Δ_{hoc}:

$$x_{ch} + (q_{cs} + r + \gamma_u + \gamma_d) \frac{x_{cl} - 2y}{r + \gamma_u + q_{cs}} - \frac{1}{k} q_{bs} \phi x_b$$

Using the above limits of q_{cs}, q_{bs}, and q_{bb}, it converges to $x_{\mathrm{ch}} + x_{\mathrm{cl}} - 2y$ which is positive by Assumption 1.

From (A.29), in order for $V_{aoc} < 0$, the CDS price has to be less than $p_c < \delta_c + y$. From (A.31) and (A.32):

$$p_c = (\delta_c + x_{\mathrm{cl}}) - y - \frac{(1-\phi_h)(q_{cs} + r + \gamma_u)\left(\left(x_{\mathrm{ch}} + \frac{(x_{\mathrm{cl}}-2y)(q_{cs}+\gamma_d+r+\gamma_u)}{q_{cs}+r+\gamma_u}\right) - \frac{x_b q_{bs} \phi_h}{k}\right)}{\phi_h \left(\frac{q_{cb}(q_{bb}\phi_l + \gamma_d + r)}{k} + \frac{(1-\phi)q_{cs} + \gamma_d + r + \gamma_u}{\phi_h}\right)}$$

This converges to $\delta_c + y - x_{ch}$, which is less than $\delta_c + y$. Thus, $V_{aoc} < 0$. Average types will not want to buy CDS because the flow utility would be $\delta_c - y - p_c$. Given that $p_c \to \delta_c + y - x_{ch}$, this converges to $x_{ch} - 2y$ which is negative by Assumption (1). To show $\omega_b > 0$, consider the numerator of (A.33): $x_b - q_{cb}\Delta_{hoc}$. Since $0 < \lim q_{cb} < \infty$ and Δ_{hoc} converges to zero, $x_b - q_{cb}\Delta_{hoc}$ converges to $x_b > 0$. The above results show existence for $\epsilon = 0$. By continuity, existence is also established near $\epsilon = 0$.

□

Proof of Proposition 2. The bond price is $p_b = \phi(V_{hob} - V_{hn}) + (1-\phi)V_{aob}$. Solving V_{hob} and V_{aob}:

$$V_{hob} = \frac{\delta_b + x_b - y}{r} - \frac{\gamma_d (x_b + q_{bb}(1-\phi)V_{hn})}{r(r + \gamma_d + q_{bb}(1-\phi))} \tag{A.35}$$

$$V_{aob} = \frac{\delta_b + x_b - y}{r} - \frac{(r+\gamma_d)(x_b + q_{bb}(1-\phi)V_{hn})}{r(r + \gamma_d + q_{bb}(1-\phi))} \tag{A.36}$$

where from the earlier derivation:

$$V_{hn} = \frac{q_{bs} x_b \phi + \Delta_{hoc} q_{cb} (r + \gamma_d + q_{bb}(1-\phi))}{(r+\gamma_d) k} \tag{A.37}$$

Thus, we derive the limits of q's, and Δ_{hoc} as $\lambda_b \to \infty$ for an arbitrary λ_c. With the change of variable, $\epsilon \equiv \frac{1}{\lambda_b}$, (A.26) becomes:

$$(1+\rho)F_h - \gamma_d \mu_{hn} \left(\frac{S}{\mu_{hn} + \epsilon\gamma_d} + \frac{\lambda_c F_l}{\gamma_u (\lambda_c \mu_{hn} + \gamma_d + \gamma_u)} + 1\right) = 0$$

For $\epsilon = 0$,

$$\frac{(1+\rho)F_h}{\gamma_d} - S - \mu_{hn} \left(\frac{\lambda_c F_l}{\gamma_u (\lambda_c \mu_{hn} + \gamma_d + \gamma_u)} + 1\right) = 0 \tag{A.38}$$

For any $\rho \in [0,1]$, the LHS of (A.38) is positive at $\mu_{hn} = 0$, decreasing in μ_{hn}, and is negative for large μ_{hn}, Hence, (A.38) has a positive finite solution, $0 < \lim_{\lambda_b \to \infty} \mu_{hn} < \infty$, and this implies $\lim_{\lambda_b \to \infty} q_{bb} = \infty$, and $k \to \infty$. This also implies from (A.22) that $\lim_{\lambda_b \to \infty} \mu_{aob} = 0$ and q_{bs} converges to a finite number. Analogously, $\lim_{\lambda_b \to \infty} q_{cs} = \infty$ and from (A.20) and (A.24): $0 < \lim_{\lambda_b \to \infty} q_{cb} < \infty$.

Then as discussed above, the numerator of Δ_{hoc} converges to a finite number, while the denominator converges to ∞, thus, $\Delta_{hoc} \to 0$. So $V_{hn} \to 0$, hence $V_{hob} \to \frac{\delta_b + x_b - y}{r}$, $V_{aob} \to \frac{\delta_b + x_b - y}{r}$ and $p_b \to \frac{\delta_b + x_b - y}{r}$. □

Note that since $V_{hn} \to 0$, $\rho \to 0$. This is why the assumption that there is some proportion of high types who do not have an outside opportunity and always enter simplifies some of the proofs. Otherwise, as high types enter at a smaller and smaller rate, the steady state measure of high types can become smaller than $S + \frac{F_l}{\gamma_u}$. As a result, the marginal investor of the bond is not necessarily the high type and the frictionless price is not given by the valuation of the high types.

Proof of Proposition 3. Combining (A.35)-(A.37) we get the bond price. □

Proof of Proposition 4. Consider the interior solution $V_{hn}(\rho^{cds}) = O_h$. Since the bond price is $p_b = \phi(V_{hob} - V_{hn}) + (1-\phi)V_{aob}$, for an interior solution ($V_{hn}^{nocds} = V_{hn}^{cds} = O_h$) it is sufficient to show that $V_{hob}(q_{bb}^{cds}) > V_{hob}(q_{bb}^{nocds})$ and $V_{aob}(q_{bb}^{cds}) > V_{aob}(q_{bb}^{nocds})$. From (A.35) and (A.36), the derivative with respect to q_{bb}:

$$\frac{\partial V_{hob}}{\partial q_{bb}} = -\frac{\gamma_d \left(\left(r + \gamma_d \right) V_{hn} - x_b \right) (1-\phi)}{r \left(r + \gamma_d + q_{bb}(1-\phi) \right)^2}$$

$$\frac{\partial V_{aob}}{\partial q_{bb}} = -\frac{\left(r + \gamma_d \right) \left(\left(r + \gamma_d \right) V_{hn} - x_b \right) (1-\phi)}{r \left(r + \gamma_d + q_{bb}(1-\phi) \right)^2}$$

Thus, the condition for both V_{hob} and V_{aob} to be increasing in q_{bb} at $q_{bb} = q_{bb}^{nocds}$ is: $(r + \gamma_d)V_{hn} - x_b < 0$ evaluated at $q_{bb} = q_{bb}^{nocds}$.

Without CDS, the solution for V_{hn} is

$$V_{hn}^{nocds} = \frac{q_{bs} x_b \phi}{(r+\gamma_d)(r+\gamma_d + q_{bs}\phi + q_{bb}(1-\phi)} \tag{A.39}$$

Rearranging we get:

$$\left(r + \gamma_d \right) V_{hn} = \frac{q_{bs}\phi}{(r + \gamma_d + q_{bs}\phi + q_{bb}(1-\phi)} x_b < x_b$$

Next, we show that $q_{bb} = \lambda_b \mu_{hn}$ increases with CDS. Consider the solution for V_{hn}:

$$V_{hn}^{cds} = \frac{x_b q_{bs}^{cds} \phi_h}{k^{cds} \left(\gamma_d + r \right)} + \frac{q_{cb} \Delta_{\text{hoc}} \left(q_{bb}\phi_l + \gamma_d + r \right)}{k^{cds} \left(\gamma_d + r \right)} \tag{A.40}$$

Compare this with (A.39). The fact that $V_{hn}^{nocds} = V_{hn}^{cds} = O_h$ and that the second term of (A.40) is asymptotically positive implies that:

$$\frac{x_b q_{bs}^{cds} \phi_h}{k^{cds} \left(\gamma_d + r \right)} < \frac{x_b q_{bs} \phi_h}{k \left(\gamma_d + r \right)}$$

The term $\frac{x_b q_{bs} \phi_h}{k(\gamma_d + r)}$ is strictly decreasing in μ_{hn}. Thus, it has to be the case that $\mu_{hn}^{cds} > \mu_{hn}^{nocds}$.

Now consider the corner solution $\rho = 1$. This will be the case when $V_{hn}(\rho = 1) > O_h$. Keeping ρ fixed, when CDS matching efficiency λ_c decreases, V_{hn} increases. Thus, as λ_c decreases, V_{hn} keeps increasing and even when $\rho = 1$, it increases beyond O_h. Now, keeping ρ fixed, μ_{hn} is lower for some positive λ_c compared to the environment without CDS because high types end up selling CDS instead of buying bonds. When it was an interior solution, there was always enough entry so that the entry effect more than offset this congestion channel. However, as λ_c decreases further, the value of providing liquidity in the CDS market increases (V_{hn} increases) but at the boundary $\rho = 1$ everyone who could have entered has entered. So if

λ_c is too small (CDS search frictions too high), then the partial equilibrium effect dominates. As a result, bond liquidity and bond price is lower with CDS. $\square$

Proof of Proposition 5. Consider what (A.26) limits to for an arbitrary λ_b as $\lambda_c \to \infty$:

$$\frac{(1+\rho)F_h}{\gamma_d} - \left(\frac{S\lambda_b\mu_{hn}}{\lambda_b\mu_{hn}+\gamma_d} + \frac{F_l}{\gamma_u} + \mu_{hn}\right) = 0 \tag{A.41}$$

The LHS of (A.41) is positive at $\mu_{hn} = 0$, decreasing in μ_{hn}, and is negative for large μ_{hn}. Thus, for any ρ, μ_{hn} is finite as $\lambda_c \to \infty$. As a result, μ_{aob}, q_{bs} and q_{bb} are finite. Since $\mu_{ln} + \mu_{aoc} \to 0$, q_{cb} is also finite. But $q_{cs} = \lambda_c\mu_{hn} \to \infty$ Thus, $\Delta_{hoc} \to 0$.

When the solution is interior,

$$V_{hn}^{cds} = V_{hn}^{nocds} = O_h \tag{A.42}$$

Then, using $\Delta_{hoc} \to 0$ and (A.30):

$$\frac{x_b q_{bs}^{cds}\phi_h}{k^{cds}(\gamma_d + r)} = \frac{x_b q_{bs}^{nocds}\phi_h}{k^{nocds}(\gamma_d + r)} \tag{A.43}$$

Since this expression is uniquely determined by μ_{hn}, it has to be that:

$$\mu_{hn}^{cds} = \mu_{hn}^{nocds} \tag{A.44}$$

Thus, $q_{bb} = \lambda_b\mu_{hn}$ is the same as without CDS. Consequently, from (A.35)-(A.36) and (A.42), V_{hob} and V_{aob} are the same with or without CDS. Thus, when $\lambda_c \to \infty$, the bond price is the same as in the benchmark environment without CDS. For (A.44) to hold, from (A.41), the entry rate (hence the measure of high types) increases enough to exactly offset the total measure of low types $\frac{F_l}{\gamma_u}$: $\frac{(\rho^{cds} - \rho^{nocds})F_h}{\gamma_d} = \frac{F_l}{\gamma_u}$.

If entry is exogenous, $\lim_{\lambda_c \to \infty} p_b(\lambda_c) < p_b^{\text{no cds}}$ because the measure of high types (hence the measure of bond buyers) decreases due the existence of low types. $\square$

Proof of Proposition 6. The population measures evolve according to:

$$\dot{\mu}_{hn}(t) = (1+\rho)F_h + \gamma_u\mu_{hoc}(t) - [\gamma_d\mu_{hn}(t) + (q_{bs}(t) + q_{cb}(t))\,\mu_{hn}(t)] \tag{A.45}$$
$$\dot{\mu}_{ln}(t) = F_l - [\gamma_u\mu_{ln}(t) + q_{cs}\mu_{ln}(t)] \tag{A.46}$$
$$\dot{\mu}_{hob}(t) = q_{bs}\mu_{hn}(t) - \gamma_d\mu_{hob}(t) \tag{A.47}$$
$$\dot{\mu}_{aob}(t) = \gamma_d\mu_{hob}(t) - q_{bb}\mu_{aob}(t) \tag{A.48}$$
$$\dot{\mu}_{hoc}(t) = q_{cb}\mu_{hn}(t) - [\gamma_d\mu_{hoc}(t) + \gamma_u\mu_{hoc}(t)] \tag{A.49}$$
$$\dot{\mu}_{aoc}(t) = \gamma_d\mu_{hoc}(t) - [\gamma_u\mu_{aoc}(t) + q_{cs}\mu_{aoc}(t)] \tag{A.50}$$
$$\dot{\mu}_{lsc}(t) = q_{cs}\mu_{ln}(t) - \gamma_u\mu_{lsc}(t) \tag{A.51}$$

Value functions evolve according to:

$$\dot{V}_{hn}(t) = rV_{hn}(t) - [\gamma_d(0 - V_{hn}(t)) + q_{bs}(t)\phi\omega_b(t) + q_{cb}(t)(V_{hoc}(t) - V_{hn}(t))] \tag{A.52}$$

$$\dot{V}_{ln}(t) = rV_{ln}(t) - [\gamma_u(0 - V_{ln}(t)) + q_{cs}(t)(V_{lsc}(t) - V_{ln}(t))] \tag{A.53}$$

$$\dot{V}_{hob}(t) = rV_{hob}(t) - [\delta_b + x_b - y + \gamma_d(V_{aob}(t) - V_{hob}(t))] \tag{A.54}$$

$$\dot{V}_{aob}(t) = rV_{aob}(t) - [\delta_b - y + q_{bb}(t)\left(1 - \phi\right)\omega_b(t)] \tag{A.55}$$

$$\dot{V}_{hoc}(t) = rV_{hoc}(t) - [p_c(t) - (\delta_c - x_{cl}) - y + \gamma_d(V_{aoc}(t) - V_{hoc}(t)) + \gamma_u(V_{hn}(t) - V_{hoc}(t))] \tag{A.56}$$

$$\dot{V}_{aoc}(t) = rV_{aoc}(t) - [p_c(t) - \delta_c - y + q_{cs}(t)(0 - V_{aoc}(t)) + \gamma_u(0 - V_{aoc}(t))] \tag{A.57}$$

$$\dot{V}_{lsc}(t) = rV_{lsc}(t) - [-p_c(t) + (\delta_c + x_{ch}) - y + \gamma_u\left(0 - V_{lsc}(t)\right)] \tag{A.58}$$

Using the ODE for V_{hob} and V_{hn}:

$$\dot{\Delta}_{hob} = r\Delta_{hob} - [\delta_b + x_b - y - \left(\gamma_d + q_{bs}\phi\right)\omega_b - q_{cb}\phi\omega_c]$$

Together with the ODE for V_{aob}:

$$\dot{\omega}_b = -x_b + \left(r + \gamma_d + q_{bs}\phi + q_{bb}\left(1 - \phi\right)\right)\omega_b + q_{cb}\phi\omega_c \tag{A.59}$$

Analogously, we get the ODE for ω_c,

$$\dot{\omega}_c = -x_{cl} + q_{bs}\phi\omega_b + \left(r + \gamma_d + \gamma_u + q_{cb}\phi + q_{cs}(1 - \phi)\right)\omega_c \tag{A.60}$$

To solve for ω_b and ω_c, we write (A.59) and (A.60) in this form:

$$\begin{bmatrix} \dot{\omega}_b(t) \\ \dot{\omega}_c(t) \end{bmatrix} = -\begin{bmatrix} x_b \\ x_{cl} + x_{ch} - 2y \end{bmatrix} + A(t)\begin{bmatrix} \omega_b(t) \\ \omega_c(t) \end{bmatrix}$$

where

$$A(t) = \begin{bmatrix} r + \gamma_d + q_{bs}\phi + q_{bb}\left(1 - \phi\right) & q_{cb}\phi \\ q_{bs}\phi & r + \gamma_d + \gamma_u + q_{cb}\phi + q_{cs}(1 - \phi) \end{bmatrix}$$

Thus, the solution is:

$$\begin{bmatrix} \omega_b(t) \\ \omega_c(t) \end{bmatrix} = \int_t^\infty e^{-\int_t^s A(u)du}\begin{bmatrix} x_b \\ x_{cl} + x_{ch} - 2y \end{bmatrix} ds$$

From here, the solutions to the ODE for Δ_{hob} and V_{aob} are given by:

$$\Delta_{hob} = \frac{\delta_b + x_b - y}{r} - \int_t^\infty e^{-r(s-t)}\left(\left(\gamma_d + q_{bs}\phi\right)\omega_b + q_{cb}\phi\omega_c\right) ds$$

$$V_{aob} = \frac{\delta_b - y}{r} + \int_t^\infty e^{-r(s-t)} q_{bb}\left(1 - \phi\right)\omega_b ds$$

□

A.1 *A Simple Example of Hedging Benefits*

Let $\theta = 1$ denote a long position (exposed to risk) through the bond or CDS market, $\theta = 0$ no position, and $\theta = -1$ a short position (i.e. bought CDS). An agent with $\theta_b \in \{0, 1\}$ shares of the bond has a utility flow:[33]

$$\theta_b \left(\delta_b + x_t^b \right)$$

and an agent with CDS position $\theta_c \in \{-1, 0, 1\}$ has a utility flow:

$$-\theta_c \left(\delta_c + x_t^c \right) \tag{A.61}$$

where $x_t^b \in \{-x_b, 0, x_b\}$ and $x_t^c \in \{-x_{ch}, 0, x_{cl}\}$ are stochastic processes. I define an agent with $\{x_t^b = x_b, x_t^c = -x_{ch}\}$ as a high type, with $\{x_t^b = 0, x_t^c = 0\}$ as an average, and with $\{x_t^b = -x_b, x_t^c = x_{cl}\}$ as a low type.

The bond coupon flow, δ_b, can be interpreted as an expected coupon flow: with intensity η the bond defaults but otherwise pays \$1 of coupon. Hence, $\delta_b = (1 - \eta)\$1$. Similarly, δ_c can be interpreted as an expected insurance payment. A CDS contract pays out if there is default on the coupon payment: with intensity η CDS pays \$1 thus, $\delta_c = \eta\$1$. According to (A.61), a high type values this as $\delta_c - x_{ch}$ while a low type values this as $\delta_c + x_{ch}$. Thus, as a CDS seller ($\theta_c = 1$), a low type experiences a greater disutility *paying out* the insurance payment $-(\delta_c + x_{cl})$ than a high type $-(\delta_c - x_{ch})$. Conversely, as a CDS buyer ($\theta_c = -1$), a low type benefits more *receiving* the insurance payment $(\delta_c + x_{cl})$ than a high type $(\delta_c - x_{ch})$. Tables below show a simple example of how x_b, x_{ch}, and x_{cl} can depend on cash flow of the bond and CDS, and the default intensity of the bond. A more formal derivation in Section A.2 shows how, in an environment with risk averse agents, just two types of agents, and just the bond market, the hedging benefits are a function of the risk aversion parameter, the correlation between agents' idiosyncratic endowment and the bond, and riskiness of the bond.

Table 4: The Expected Valuation of the Bond Payoff

Consider an example where with intensity, η, the bond defaults and pays no coupon, otherwise pays \$1 of coupon. Hence, the expected coupon is $\delta_b = (1 - \eta)\$1$. The table shows valuations of the bond cash flow by high, average, and low types.

		Utility Valuation		
	Bond Payoff	High	Ave	Low
$1 - \eta$	\$1	1	1	1
η	\$0	ϵ_h	0	$-\epsilon_l$
Expected Valuation:		$\underbrace{(1-\eta)\,1}_{\delta_b} + \overbrace{\eta\epsilon_h}^{x_{bh}}$	$\underbrace{(1-\eta)}_{\delta_b}$	$\underbrace{(1-\eta)}_{\delta_b} - \overbrace{\eta\epsilon_l}^{x_{bl}}$

[33] For an expositional purpose, let us ignore y that is in section 2.

Table 5: The Expected Valuation of the CDS Payoff as a CDS Buyer

With intensity η the bond defaults and CDS pays \$1 and zero otherwise. Hence, the expected insurance payment is $\delta_c = \eta\$1$. The table shows a simple example of utility valuations of the cash flow as a CDS buyer by different types. In the default state, low types get an extra utility for extra \$1 than high types. Thus, in expectation, as a CDS buyer a low type benefits more *receiving* the insurance payment $(\delta_c + x_{cl})$ than a high type $(\delta_c - x_{ch})$.

	Cash Flow of CDS Buyer	Utility Valuations		
		High	Ave	Low
$1-\eta$	\$0	0	0	0
η	\$1	$1-\epsilon_h$	1	$1+\epsilon_l$
Expected Valuation:		$\overbrace{\eta}^{\delta_c} - \overbrace{\eta\epsilon_h}^{x_{ch}}$	$\overbrace{\eta}^{\delta_c}$	$\overbrace{\eta}^{\delta_c} + \overbrace{\eta\epsilon_l}^{x_{cl}}$

Table 6: The Expected Valuation of the CDS Payoff as a CDS Seller

With intensity η the bond defaults and CDS seller has to pay \$1. The table shows a simple example of utility valuations of the cash flow as a CDS seller by different types. In the default state, low types get an extra disutility for paying out the insurance than high types. Thus, in expectation, as a CDS seller a low type experiences a greater disutility *paying out* the insurance payment $-(\delta_c + x_{cl})$ than a high type $-(\delta_c - x_{ch})$.

	Cash Flow of CDS Seller	Utility Valuations		
		High	Ave	Low
$1-\eta$	\$0	0	0	0
η	$-$\$1	$-(1-\epsilon_h)$	-1	$-(1+\epsilon_l)$
Expected Valuation		$-(\overbrace{\eta}^{\delta_c} - \overbrace{\eta\epsilon_h}^{x_{ch}})$	$-\overbrace{\eta}^{\delta_c}$	$-(\overbrace{\eta}^{\delta_c} + \overbrace{\eta\epsilon_l}^{x_{cl}})$

A.2 *A Formal Derivation of Hedging Benefits*

In this section, I illustrate in a simpler environment a micro foundation for the liquidity shock x_b when the asset is risky and agents are risk averse. This derivation follows Vayanos and Weill (2008) and Duffie, Gârleanu, and Pedersen (2007). I simplify the baseline model in the paper by considering just two types (high and low) instead of three types (high, average, low), and no CDS markets. I simplify the notation by denoting continuous time dependence $y(t)$ as y_t.

Agents have CARA utility preferences with risk aversion parameter α and time preference rate of β. The risky asset has cumulative dividend process, D_t, of:

$$dD_t = \delta dt + \sigma_D dB_t \tag{A.62}$$

where B_t is a standard Brownian motion. Agents also have an idiosyncratic cumulative endowment process:[34]

$$de_t = \sigma_e \left[\rho_t dB_t + \sqrt{1-\rho_t^2} dZ_t\right]$$

where Z_t is another standard Brownian motion independent of B_t. The high and low types come in with the variable $\rho_t \in \{\rho_l, \rho_h\}$ that is a two-state Markov chain with $\rho_l > \rho_h$. If $\rho_t = \rho_l$, the agent is currently a low type agent which means her endowment process is highly correlated with the asset's dividend process, D_t, but if her type switches to a high type, $\rho_{t+\Delta t} = \rho_h$,

[34] The endowment process can have a trend component: $de_t = \mu_e dt + \sigma_e \left[\rho_t dB_t + \sqrt{1-\rho_t^2} dZ_t\right]$

her endowment process will be less correlated with D_t.[35] Analogous to the baseline model, a low type agent switches to high type with intensity γ_u, and high type to low type with intensity γ_u. We restrict the agent's asset position to $\theta_t \in \{\theta_n, \theta_o\}$ where $0 < \theta_n < \theta_o$. As there are two correlation types and two possible asset positions, there are total of four agent types: $\mathcal{T} = \{hn, ln, hob, lob\}$. hn and ln are high and low types, respectively, who both hold θ_n shares of the asset, while hob and lob are high and low types, respectively, who both hold θ_o shares of the asset. Table 7 illustrates the switching probabilities from $\tau_t \in \{hn, ln, hob, lob\}$ to $\tau_{t+\Delta t} \in \{hn, ln, hob, lob\}$.

		$\tau_{t+\Delta t}$			
		hn	ln	hob	lob
τ_t	hn	$(1 - \gamma_d dt - q_{bs} dt)$	$\gamma_d dt$	$q_{bs} dt$	$0dt$
	ln	$\gamma_u dt$	$(1 - \gamma_u dt)$	0	0
	hob	0	0	$(1 - \gamma_d dt)$	$\gamma_d dt$
	lob	0	$q_{bb} dt$	$\gamma_u dt$	$(1 - \gamma_u dt - q_{bb} dt)$

Table 7: Switching probabilities from τ_t to $\tau_{t+\Delta t} \in \{hn, ln, hob, lob\}$

An agent's optimization problem is:

$$J(W_0, \tau_0) = \max_{\{c_t\}} \mathbb{E} \int_0^\infty e^{-\beta t} u(c_t) dt \tag{A.63}$$

subject to:[36]

$$dW_t = (rW_t - c_t + \delta\theta_t)\, dt + (\sigma_D \theta_t + \rho_t \sigma_e)\, dB_t + \sigma_\eta \sqrt{1 - \rho_t^2} dZ_t - p_b d\theta_t \tag{A.64}$$

where W_t is the agent's wealth process, W_0 is given, p_b is the asset price. $J(W_0, \tau_0)$ is the maximized value of the objective function as a function of two state variables, the wealth process and the agent type $\tau \in \mathcal{T}$.

Equation (A.63) can be written recursively as:[37]

$$J(W_t, \tau_t) = \max_{c_0} u(c_t)\Delta t + (1 - \beta \Delta t)\, \mathbb{E} J(W_{t+\Delta t}, \tau_{t+\Delta t}) \tag{A.65}$$

[35] With three types, we could have a three-state Markov chain with, for example, $\rho_t \in \{-\rho, 0, \rho\}$ for some $\rho > 0$ where if $\rho_t = \rho$, an agent is low type, if $\rho_t = 0$, an agent's endowment has no correlation, and if $\rho_t = -\rho$ an agent is high type as her endowment is negatively correlated with the risky asset (and she would be willing to be exposed to the risky asset *relative* to the low type agent).

[36] (A.64) comes from $dW_t = (rW_t - c_t)\, dt + dD_t \theta_t + de_t - p_b d\theta_t$.

[37] This comes from observing that over a small time interval $[0, \Delta t]$, (A.63) can be written as:

$$J(W_0, \tau_0) = \mathbb{E} \textstyle\int_0^\infty e^{-\beta t} u(c_t^*) dt = u(c_0^*)\Delta t + e^{-\beta \Delta t} \mathbb{E} \left[\int_{\Delta t}^\infty e^{-\beta(t - \Delta t)} u(c_t^*) dt \right]$$

where $\{c_t^*\}$ is the optimal consumption path. The term inside the expectations operation is $J(W_{\Delta t}, \tau_{\Delta t})$, thus $J(W_0, \tau_0) = \max_{c_0} u(c_0)\Delta t + e^{-\beta \Delta t} \mathbb{E} J\left(W_{\Delta t}, \tau_{\Delta t}\right)$. Similarly if we start at $\{W_t, \tau_t\}$ and approximate $e^{-\beta \Delta t} \approx 1 - \beta \Delta t$, we get (A.65).

Deriving the Hamilton-Jacobi-Bellman (HJB) Equation

Next, we derive the Hamilton-Jacobi-Bellman (HJB) equation from (A.65). Subtract $(1-\beta\Delta t)\,J(W_t,\tau_t)$ from both sides and divide by Δt:

$$\beta J(W_t,\tau_t) = \max_{c_t} u(c_t) + (1-\beta\Delta t)\,\mathbb{E}\left[\frac{J(W_{t+\Delta t},\tau_{t+\Delta t}) - J(W_t,\tau_t)}{\Delta t}\right]$$

In the limit as $\Delta t \to 0$

$$\beta J(W_t,\tau_t) = \max_{c_t} u(c_t) + \mathbb{E}\left[\frac{dJ(W_t,\tau_t)}{dt}\right]$$

The next step is deriving the expectation of the total differential of $J(W_t,\tau_t)$. We approximate the total differential $dJ(W_t,\tau_t)$ by a Taylor expansion:

$$dJ(W_t,\tau_t) = J_W(W_t,\tau_t)dW_t + \frac{1}{2}J_{WW}(W_t,\tau_t)dW_t^2 + J_\tau(W_t,\tau_t)d\tau_t + \frac{1}{2}J_{\tau\tau}(W_t,\tau_t)d\tau_t^2$$

where dW_t is given by A.64. Thus,

$$\mathbb{E}dW_t = (rW_t - c_t + \mu_D\theta_t + \mu_\eta)\,dt - Pd\theta_t$$

$$\mathbb{E}dW_t^2 = (\sigma_D\theta_t + \sigma_\eta\rho_t)^2\,dt + \sigma_\eta^2\left(1-\rho_t^2\right)dt = \left((\sigma_D\theta_t)^2 + 2\sigma_D\theta_t\sigma_\eta\rho_t + \sigma_\eta^2\right)dt$$

$$\mathbb{E}J_\tau(W_t,\tau_t)d\tau_t = \begin{cases} \gamma_d dt\,(J(W_t,ln) - J(W_t,hn)) + q_{bs}dt\,(J(W_t - P\,(\theta_o-\theta_n)\,,hob) - J(W_t,hn)) & \text{if } \tau_t = hn \\ \gamma_u dt\,(J(W_t,hn) - J(W_t,ln)) & \text{if } \tau_t = ln \\ \gamma_d dt\,(J(W_t,lob) - J(W_t,hob)) & \text{if } \tau_t = hob \\ \gamma_u dt\,(J(W_t,hob) - J(W_t,lob)) + q_{bb}dt\,(J(W_t + P\,(\theta_o-\theta_n)\,,ln) - J(W_t,lob)) & \text{if } \tau_t = lob \end{cases}$$

As all the $d\tau_t$ terms involve dt term, $\mathbb{E}\frac{1}{2}J_{\tau\tau}(W_t,\tau_t)d\tau_t^2 = 0$.

When $\tau_t = lob$,

$$\begin{aligned} \mathbb{E}dJ(W_t,lob) &= J_W(W_t,lob)\,(rW_t - c_t + \delta\theta_o)\,dt + \frac{1}{2}J_{WW}(W_t,lob)\left((\sigma_D\theta_o)^2 + 2\sigma_D\theta_o\sigma_e\rho_t + \sigma_e^2\right)dt \\ &+ \gamma_u dt\,(J(W_t,hob) - J(W_t,lob)) + q_{bb}dt\,(J(W_t + P\,(\theta_o-\theta_n)\,,ln) - J(W_t,lob)) \end{aligned}$$

Thus, the Hamilton-Jacobi-Bellman (HJB) equation when $\tau_t = lob$:

$$\begin{aligned} \beta J(W_t,lob) = \max_{c_t} \quad & u(c_t) + J_W(W_t,lob)\,(rW_t - c_t + \delta\theta_o) && \text{(A.66)} \\ & + \frac{1}{2}J_{WW}(W_t,lob)\left((\sigma_D\theta_o)^2 + 2\sigma_D\theta_o\sigma_e\rho_t + \sigma_e^2\right) \\ & + \gamma_u\,(J(W_t,hob) - J(W_t,lob)) + q_{bb}\,(J(W_t + P\,(\theta_o-\theta_n)\,,ln) - J(W_t,lob)) && \text{(A.67)} \end{aligned}$$

The HJB equations for the other types are derived analogously.

Proposition 7. *Solutions for $J(W_t,\tau_t)$ are of the form:*

$$J(W_t,\tau_t) = -e^{-r\alpha(W_t + V_\tau + a)}$$

where V_τ $\tau \in \mathcal{T} = \{hn, ln, hob, lob\}$ are given by:

$$\begin{aligned}
rV_{lob} &= (k(\theta_0) - \theta_0 x_b) + \gamma_u \frac{1 - e^{-r\alpha(V_{hob} - V_{lob})}}{r\alpha} + q_{bb} \frac{1 - e^{-r\alpha(P(\theta_o - \theta_n) + V_{ln} - V_{lob})}}{r\alpha} \\
rV_{ln} &= (k(\theta_n) - \theta_n x_b) + \gamma_u \frac{1 - e^{-r\alpha(V_{hn} - V_{ln})}}{r\alpha} \\
rV_{hob} &= k(\theta_0) + \gamma_d \frac{1 - e^{-r\alpha(V_{lob} - V_{hob})}}{r\alpha} \\
rV_{hn} &= k(\theta_n) + \gamma_d \frac{1 - e^{-r\alpha(V_{ln} - V_{hn})}}{r\alpha} + q_{bs} \frac{1 - e^{-r\alpha(-P(\theta_o - \theta_n) + V_{ho} - V_{hn})}}{r\alpha}
\end{aligned}$$

and $k(\theta) = \delta\theta - \frac{1}{2} r\alpha \left(\sigma_D^2 \theta^2 + 2\sigma_D \theta \sigma_e \rho_h\right)$, $x_b = r\alpha\left(\rho_l - \rho_h\right)\sigma_D \sigma_e$ *and* $\bar{a} = \frac{1}{r}\left(\frac{log(r)}{\alpha} - \frac{r-\beta}{r\alpha} - \frac{1}{2} r\alpha\sigma_e^2\right)$.

Proof. Using the guessed functional form, $J(W_t, \tau_t) = -e^{-r\alpha(W_t + V_\tau + a)}$, and the first order condition of (A.66), we can solve for the optimal consumption rate for agent τ:[38]

$$c_\tau = -\frac{log\,(r)}{\alpha} + r\left(W + V_\tau + a\right)$$

Inserting the optimal consumption back into the HJB equation A.66 and using $J_W = r\alpha e^{-r\alpha(W + V_\tau + a)}$ and $J_{WW} = -r^2\alpha^2 e^{-r\alpha(W + V_\tau + a)}$:

$$\begin{aligned}
-\beta e^{-r\alpha(W + V_{lob} + a)} &= -e^{log(r) - r\alpha(W + V_{lob} + a)} + r\alpha e^{-r\alpha(W + V_{lob} + a)} \left(\frac{log\,(r)}{\alpha} - r\left(V_{lob} + a\right) + \delta\theta_o\right) \\
&- \frac{1}{2} r^2\alpha^2 e^{-r\alpha(W + V_{lob} + a)} \left(\left(\sigma_D \theta_o\right)^2 + 2\sigma_D \theta_o \sigma_e \rho_t + \sigma_e^2\right) + \gamma_u \left(-e^{-r\alpha(W + V_{hob} + a)} + e^{-r\alpha(W + V_{lob} + a)}\right) \\
&+ q_{bb} \left(-e^{-r\alpha(W + P(\theta_o - \theta_n) + V_{ln} + a)} + e^{-r\alpha(W + V_{lob} + a)}\right)
\end{aligned}$$

Cancel $e^{-ra(W+a)}$ and divide everything by $e^{-r\alpha V_{lob}}$:

$$\begin{aligned}
-\beta &= -r + r\alpha \left(\frac{log\,(r)}{\alpha} - r\left(V_{lob} + a\right) + \delta\theta_o\right) - \frac{1}{2} r^2\alpha^2 \left(\left(\sigma_D \theta_o\right)^2 + 2\sigma_D \theta_o \sigma_e \rho_t + \sigma_e^2\right) \\
&+ \gamma_u \left(-e^{-r\alpha(V_{hob} - V_{lob})} + 1\right) + q_{bb} \left(-e^{-r\alpha(P(\theta_o - \theta_n) + (V_{ln} - V_{lob}))} + 1\right)
\end{aligned}$$

Divide by $-r\alpha$:

$$\begin{aligned}
\beta \ \frac{1}{r\alpha} = &\ rV_{lob} + \gamma_u \frac{e^{-r\alpha(V_{hob} - V_{lob})} - 1}{r\alpha} + q_{bb} \frac{e^{-r\alpha(P(\theta_o - \theta_n) + (V_{ln} - V_{lob}))} - 1}{r\alpha} \\
&- \frac{log\,(r)}{\alpha} + ra + \frac{1}{\alpha} - \left(\delta\theta_o - \frac{1}{2} r\alpha \left(\sigma_D^2 \theta_o^2 + 2\sigma_D \theta_o \sigma_e \rho_t + \sigma_e^2\right)\right)
\end{aligned}$$

Rearranging, we find the expression for $\bar{a}$:

$$\begin{aligned}
0 = &\ rV_{lob} + \gamma_u \frac{e^{-r\alpha(V_{hob} - V_{lob})} - 1}{r\alpha} + q_{bb} \frac{e^{-r\alpha(P(\theta_o - \theta_n) + (V_{ln} - V_{lob}))} - 1}{r\alpha} \\
&+ r\bar{a} - r\frac{1}{r}\left(\frac{log\,(r)}{\alpha} - \frac{r - \beta}{r\alpha} - \frac{1}{2} r\alpha\sigma_e^2\right) - \left(\delta\theta_o - \frac{1}{2} r\alpha \left(\sigma_D^2 \theta_o^2 + 2\sigma_D \theta_o \sigma_e \rho_t\right)\right)
\end{aligned}$$

Thus, defining $\bar{a} \equiv \frac{1}{r}\left(\frac{log(r)}{\alpha} - \frac{r-\beta}{r\alpha} - \frac{1}{2} r\alpha\sigma_e^2\right)$, we get:

$$0 = rV_{lob} + \gamma_u \frac{e^{-r\alpha(V_{hob} - V_{lob})} - 1}{r\alpha} + q_{bb} \frac{e^{-r\alpha(P(\theta_o - \theta_n) + V_{ln} - V_{lob})} - 1}{r\alpha} - \left(\delta\theta_o - \frac{1}{2} r\alpha \left(\sigma_D^2 \theta_o^2 + 2\sigma_D \theta_o \sigma_e \rho_t\right)\right)$$

[38] FOC with respect to c_t is: $0 = \alpha e^{-\alpha c} - J_W(W_t, \tau_t)$. Using $J_W = r\alpha e^{-r\alpha(W + V_\tau + a)}$, $re^{-r\alpha(W + V_\tau + a)} = e^{-\alpha c}$. Rewrite it as: $e^{log(r)} e^{-r\alpha(W + V_\tau + a)} = e^{-\alpha c}$

Add and subtract $\frac{1}{2}r\alpha 2\sigma_D\theta_o\sigma_e\rho_h$:

$$\begin{aligned}0 = \quad & rV_{lob} + \gamma_u \frac{e^{-r\alpha(V_{hob}-V_{lob})}-1}{r\alpha} + q_{bb}\frac{e^{-r\alpha(P(\theta_o-\theta_n)+V_{ln}-V_{lob})}-1}{r\alpha} \\ & - \left(\delta\theta_o - \frac{1}{2}r\alpha\left(\sigma_D^2\theta_o^2 + 2\sigma_D\theta_o\sigma_e\rho_h\right)\right) + \theta_0 r\alpha\left(\rho_l-\rho_h\right)\sigma_D\sigma_e\end{aligned}$$

Define: $k(\theta) \equiv \delta\theta - \frac{1}{2}r\alpha\left(\sigma_D^2\theta^2 + 2\sigma_D\theta\sigma_e\rho_h\right)$ and $x_b \equiv r\alpha\left(\rho_l-\rho_h\right)\sigma_D\sigma_e$

$$0 = rV_{lob} + \gamma_u \frac{e^{-r\alpha(V_{hob}-V_{lob})}-1}{r\alpha} + q_{bb}\frac{e^{-r\alpha(P(\theta_o-\theta_n)+V_{ln}-V_{lob})}-1}{r\alpha} - \left(k(\theta_0)-\theta_0 x_b\right)$$

Rearranging:

$$rV_{lob} = \left(k(\theta_0)-\theta_0 x_b\right) + \gamma_u \frac{1-e^{-r\alpha(V_{hob}-V_{lob})}}{r\alpha} + q_{bb}\frac{1-e^{-r\alpha(P(\theta_o-\theta_n)+V_{ln}-V_{lob})}}{r\alpha} \tag{A.68}$$

It's similar to the other agent types. □

Comparison to the Baseline Model

In the limit as $\alpha \to 0$, the general value function (A.68) satisfies the value functions with risk-neutral agents of the baseline model of in the text of the paper. To see this, linearizing (A.68) (using $e^z - 1 \approx z$) for small α, we get:

$$rV_{lob} = \left(k(\theta_0)-\theta_0 x_b\right) + \gamma_u\left(V_{hob}-V_{lob}\right) + q_{bb}\left(V_{ln}-V_{lob}+p_b\left(\theta_o-\theta_n\right)\right) \tag{A.69}$$

(A.69) is analogous to the value functions of the baseline model with risk-neutral agents. Thus, the baseline model is a reduced form approximation of the more general specification with risk averse agents and risky assets.

The illiquidity shock or cost, $x_b = r\alpha\left(\rho_l-\rho_h\right)\sigma_D\sigma_e$, captures the risk aversion of agents (α), the riskiness of the asset (σ_D) and the endowment (σ_e), and the difference in the correlation of low and high types ($\rho_l - \rho_h$). The larger is any of these parameters, the larger is the illiquidity cost.

Figure 2: The Bond Illiquidity Discount

This figure illustrates the main result of the paper. It shows the difference in the bond illiquidity discount with and without CDS as a function of the CDS market efficiency (λ_c). With the existence of naked CDS buyers, the bond illiquidity discount (the solid blue line) is lower than the benchmark without CDS (dashed red line). If the CDS market is frictionless ($\lambda_c \to \infty$), the CDS market is redundant and does not affect bond market liquidity.

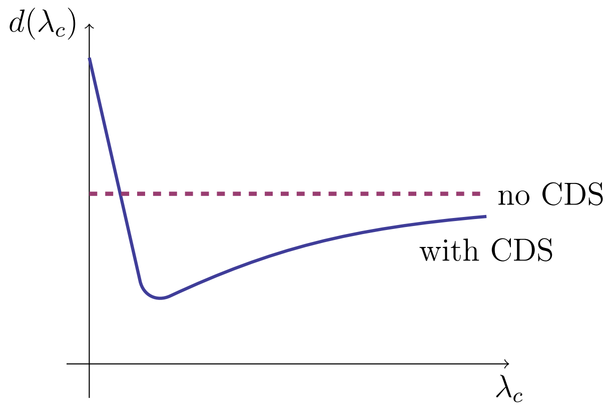


Figure 3: The Value of Trading as a High Type

The figure plots the value of trading as a long trader, V_{hn}, as a function of the entry rate (ρ). The value increases with the introduction of the CDS market (the curve shifts up) and is higher with search frictions present in the CDS market ($\lambda_c < \infty$) than without search frictions in the CDS market ($\lambda_c = \infty$). The equilibrium entry rate is determined by the intersection of V_{hn} and their outside option (the horizontal line at O_h).

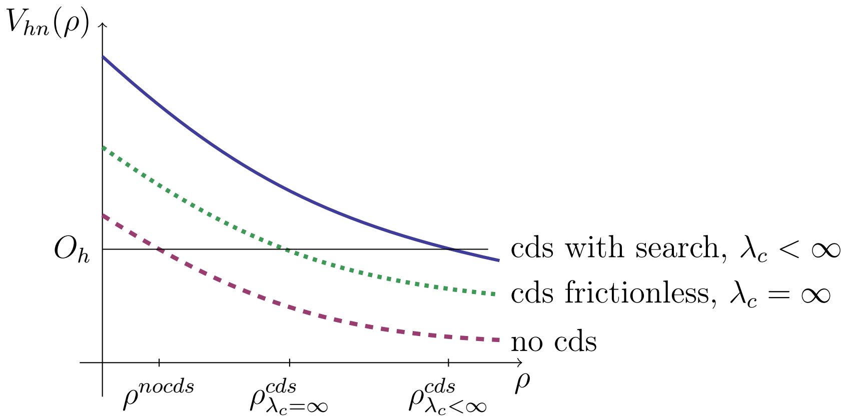


Figure 4: The Rate of Entry

The diagram illustrates how the introduction of the CDS market affects the entry rate, ρ, of long traders. By how much the entry rate increases depends on the total potential demand for CDS (i.e. the steady state measure of low types, $\frac{F_l}{\gamma_u}$, who in equilibrium want to short credit risk) and the CDS market matching efficiency, λ_c. The dashed line is the additional number of long traders in the economy due to the existence of naked CDS buyers. As the CDS market is frictionless, $\lambda_c \to \infty$, the increase in the measure of long traders exactly equals the demand for CDS (the horizontal line).

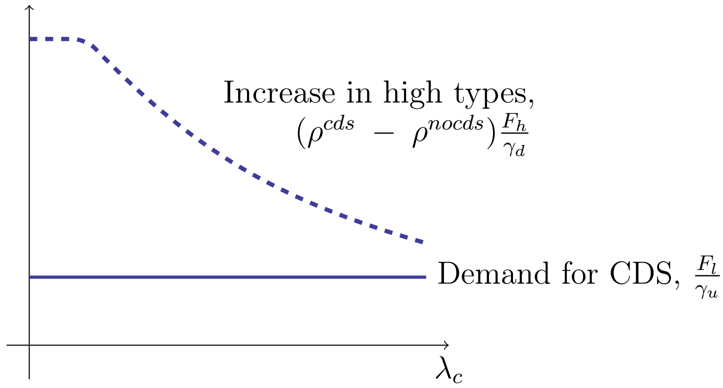
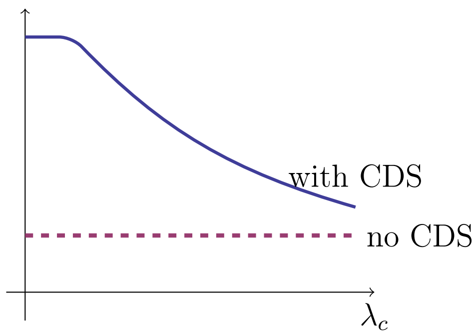


Figure 5: The Effect of CDS on Bond Market Composition

The figure compares the relative composition of buyers and sellers in the bond market with (solid line) and without CDS (dashed line) as a function of the CDS market efficiency (λ_c). The introduction of the CDS market increases the number of bond buyers (left panel) and decreases the number of bond sellers (right panel). If the CDS market is frictionless ($\lambda_c \to \infty$), the CDS market is redundant and does not affect the bond market composition.

No. of Bond Buyers, μ_{hn}



No. of Bond Sellers, μ_{aob}

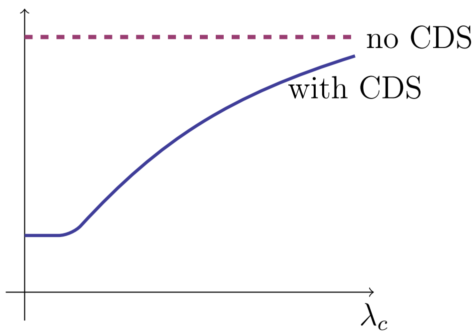


Figure 6: The Effect of CDS on Bond Volume

The figure compares the volume of trade in the bond market with (solid line) and without CDS (dashed line) as a function of the CDS market efficiency (λ_c). The introduction of the CDS market increases bond market volume. If the CDS market is frictionless ($\lambda_c \to \infty$), the CDS market is redundant and does not affect bond market volume.

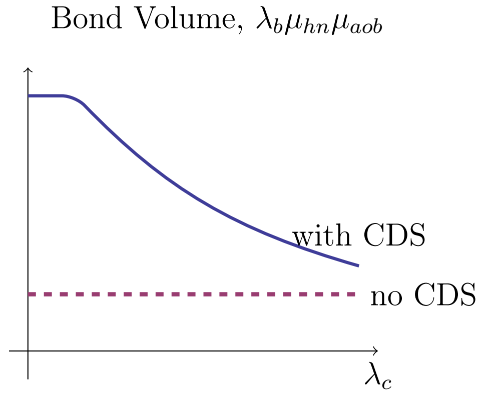


Figure 7: The Transition Dynamics of Types' Measures After a Temporary CDS Ban

A temporary naked CDS ban is modeled as a shock to the steady at time $t = 0$ that sets the number of naked CDS buyers to zero (as can be seen in the left panel). The figure plots the time varying equilibrium path back to the steady state number of CDS buyers (the left panel), bond buyers (the middle panel), and bond sellers (the right panel).

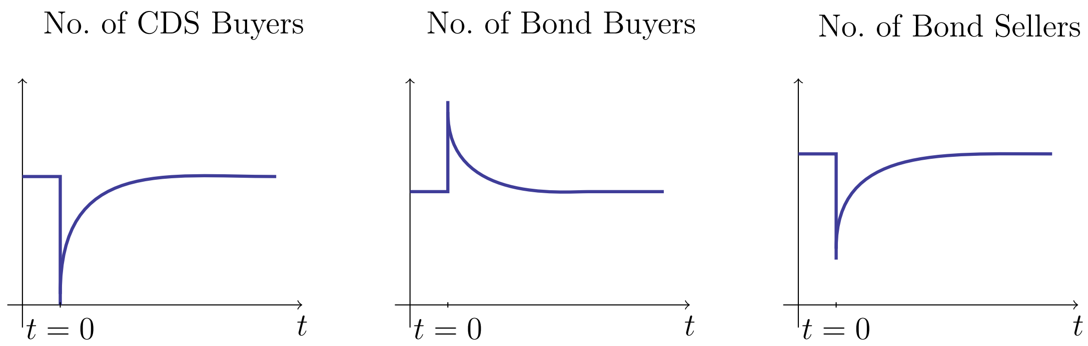


Figure 8: The Transition Dynamics of Bond Illiquidity

A temporary naked CDS ban is modeled as a shock to the steady at time $t = 0$ that sets the number of naked CDS buyers to zero. The figure plots the short run dynamics of the bond illiquidity discount. With a temporary naked CDS ban, the illiquidity discount temporarily decreases (i.e. liquidity increases).

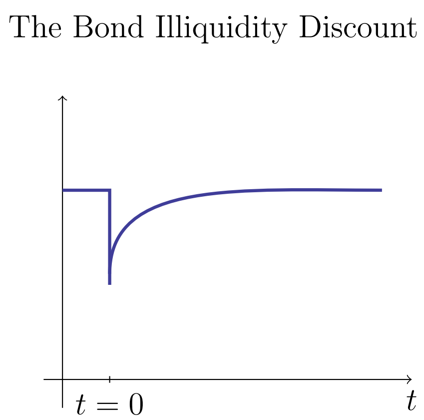


Figure 9: Cost of Entry

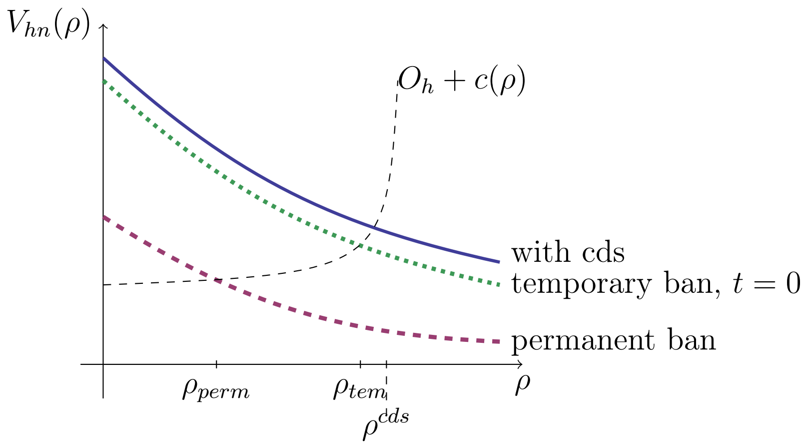
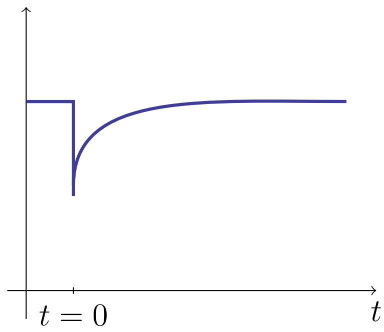


Figure 10: The Implicit Short-run Dynamics of the Cost of Entry $c(\rho(t))$



C APPENDIX: DATA TABLES

Table 8: Descriptive Statistics: Prices and Bid-Ask Spreads in Bond and CDS Markets

This table shows the descriptive statistics for the bond and CDS data for the period 2004Q1-2012Q1 across 65 sovereigns. Bond prices are quoted as a percent of the par (or face) value of the bond; for example, if the bond price is 95, the bond is trading at 95 cents on the dollar. "Bond Mid Price" is the average of the bid and the ask prices, and "Bond Price Bid-Ask (% of Par)" is the absolute bid-ask spread (the ask price minus the bid price). For example, if the ask and bid prices are 100.92 (% of par) and 100.00 (% of par), respectively, then "Bond Price Bid-Ask (% of Par)" would be 0.92 (% of par). "Bond Price Bid-Ask (% of Mid)" is the bid-ask spread as a percent of the mid price. Bond prices were also converted to yield-to-maturity. "Bond Mid Yield (%)" is the average of the bid and ask yields. Prices of CDS contracts are quoted in annualized percentages of the contract notional. Following market standards, they are reported in basis points. "CDS Mid (b.p.)" is the average of the bid and ask CDS prices (in basis points), "CDS Bid-Ask (b.p.)" is the absolute bid-ask spread in basis points, while "CDS Bid-Ask (% of Mid)" is the bid-ask spread as a percent of the mid price.

	Mean	St. Dev.	Min	Max	No obs.
(1) Bond Mid Price (% of Par)	106.56	13.75	27.94	166.67	1478
(2) Bond Price Bid-Ask (% of Par)	0.92	1.01	0.02	17.08	1478
(3) Bond Price Bid-Ask (% of Mid)	0.95	1.46	0.02	33.02	1478
(4) Bond Mid Yield (%)	5.37	2.93	-7.14	33.12	1478
(5) CDS Mid (b.p.)	205.20	438.87	1.73	10433.54	1478
(6) CDS Bid-Ask (b.p.)	15.11	43.73	1.00	1158.71	1478
(7) CDS Bid-Ask (% of Mid)	13.67	16.41	0.89	102.68	1478

Table 9: Descriptive Statistics: DTCC CDS Transactions Data

This table shows the descriptive statistics for the volume of trade in the CDS market (for an average day per quarter over 2009-2012) and the outstanding amount of CDS contracts and their total notional (gross and net) over 2008-2012. "Average Daily Number of Trades" is the daily total number of CDS trades; "Average Daily Notional" is the total notional of all trades per day in million $, this is effectively the daily volume of trade in the CDS market; "Daily Not'l (annual'd), % of Gross" is the daily CDS notional (annualized: daily CDS notional times 250 trading days) as percent of the outstanding gross notional, this is effectively CDS turnover. "Gross Government Debt" is the general government gross debt outstanding in million USD from World Bank Quarterly External Debt Statistics, and "Gross Not'l as % of Gross Debt" is the outstanding gross notional as percent of the outstanding gross government debt. Other variables are self explanatory. All CDS related data in this table comes from DTCC.

	Mean	St. Dev.	Min	Max	No obs.
(1) Average Daily Number of Trades	12.37	16.01	0.00	116.00	418
(2) Average Daily Notional (mln $)	162.66	240.02	2.50	1600.00	421
(3) Outstanding Gross Notional (mln $)	40007.57	47584.39	1659.32	310852.44	726
(4) Outstanding Net Notional (mln $)	3889.49	4789.88	251.42	27828.78	726
(5) Daily Not'l (annual'd), % of Gross	77.16	48.58	8.53	358.12	421
(6) Outstanding Number of Contracts	2829.47	2751.65	92.44	13324.67	726
(7) Gross Government Debt (bln $)	859.58	2511.51	5.75	16777.28	726
(8) Gross Not'l as % of Gross Debt	27.93	36.77	0.04	254.07	726

Table 10: Descriptive Statistics at the Country Level: Bond and CDS Price Data

This table shows for each country the average of the variables in Table 8. See Table 8 for the description and units. The column numbers correspond to the row numbers of Table 8.

	Bond Price			Bond Yield	CDS Price		
	(1) mid	(2) ba	(3) ba/mid	(4) mid	(5) mid	(6) ba	(7) ba/mid
Argentina	72.43	1.62	2.72	13.72	1239.13	38.14	2.42
Australia	104.05	0.31	0.28	4.93	60.69	6.07	10.87
Austria	104.74	0.42	0.39	3.83	46.33	3.55	41.33
Bahrain	98.54	0.75	0.76	5.73	249.16	19.50	8.25
Belgium	109.22	0.66	0.62	3.89	59.83	4.11	38.01
Brazil	119.65	0.92	0.77	6.92	213.88	6.88	2.88
Bulgaria	114.46	0.64	0.58	4.80	167.74	12.05	11.20
Chile	103.42	0.66	0.63	4.66	65.00	9.40	22.62
China	101.98	0.67	0.65	4.22	62.00	4.88	10.95
Colombia	115.95	1.59	1.39	7.10	211.30	9.34	4.33
Costa Rica	119.45	4.12	3.47	3.89	197.59	30.20	15.57
Croatia	101.19	0.59	0.59	5.16	166.80	13.85	13.42
Czech Republic	102.34	0.61	0.60	3.94	58.23	6.54	28.49
Denmark	108.94	0.39	0.35	2.89	57.49	5.49	11.66
Dominican Republic	110.47	2.70	2.48	6.76	300.21	69.52	23.75
Egypt	102.97	1.54	1.60	4.77	314.71	18.94	6.59
El Salvador	105.87	2.06	1.98	7.18	274.42	39.01	14.79
Finland	106.38	0.43	0.40	2.91	37.49	4.54	14.10
France	110.24	0.18	0.16	3.44	45.51	2.93	32.14
Germany	109.84	0.13	0.11	3.22	27.43	2.56	35.64
Greece	104.98	2.40	3.51	6.39	818.63	42.67	11.78
Guatemala	118.69	2.01	1.66	5.10	177.20	39.00	23.87
Hong Kong	103.67	0.57	0.55	3.55	41.00	6.39	25.31
Hungary	96.63	0.74	0.79	5.88	174.75	6.48	10.14
Iceland	93.33	1.23	1.39	7.96	217.21	26.14	29.66
Indonesia	108.68	0.86	0.82	7.61	233.24	14.31	5.46
Iraq	85.98	1.27	1.50	7.28	394.34	57.35	14.42
Ireland	95.69	0.97	1.06	5.81	357.07	14.03	5.29
Israel	105.57	0.94	0.88	4.09	87.65	10.58	16.24
Italy	109.38	0.50	0.46	4.08	93.76	3.82	12.89
Japan	104.00	0.17	0.17	1.11	73.68	4.70	8.44
Korea	107.47	0.19	0.20	1.82	91.72	5.15	7.86
Latvia	93.66	1.65	1.87	6.16	429.42	30.63	6.34
Lebanon	106.20	1.68	1.63	6.23	382.96	33.16	8.26
Lithuania	98.00	1.26	1.43	6.00	316.98	24.72	7.15
Malaysia	104.82	0.30	0.29	3.63	99.32	4.90	4.99
Mexico	114.38	0.82	0.71	5.63	121.50	4.63	4.50
Morocco	97.19	1.77	1.87	5.55	185.87	29.48	14.71
Netherlands	107.87	0.23	0.21	3.02	53.04	5.06	11.70
New Zealand	110.79	0.39	0.32	4.40	76.08	7.42	9.68
Norway	107.97	0.39	0.36	2.95	25.49	3.78	15.79
Pakistan	82.80	1.91	2.64	10.90	799.65	143.97	15.33
Panama	117.79	1.83	1.56	6.38	168.40	11.67	6.91
Peru	113.10	1.12	1.03	6.30	180.66	11.20	5.86
Philippines	113.87	0.85	0.75	7.15	252.18	10.60	3.81
Poland	100.51	0.70	0.75	4.68	89.43	5.89	15.13
Portugal	98.19	0.98	1.25	5.50	236.77	9.40	13.58
Qatar	134.87	1.11	0.83	5.49	74.20	10.98	22.23
Romania	96.85	1.18	1.30	6.66	333.36	18.21	5.00
Russia	141.36	1.08	0.89	6.31	176.96	5.48	3.39
Slovak Republic	102.15	0.83	0.81	4.05	61.13	7.68	27.72
Slovenia	101.36	0.64	0.65	4.17	125.56	11.55	9.90
South Africa	105.26	0.75	0.73	5.86	129.57	7.86	7.68
Spain	101.41	0.44	0.44	4.01	178.11	5.69	4.45
Sweden	112.51	0.51	0.43	3.29	48.22	4.96	12.27
Switzerland	115.25	0.88	0.75	1.37	46.37	8.18	17.34
Thailand	99.14	0.17	0.17	3.27	125.07	6.25	5.00
Tunisia	112.98	0.33	0.29	6.08	164.16	17.77	11.10
Turkey	109.08	0.83	0.80	6.47	245.00	6.95	2.49
Ukraine	93.94	1.09	1.46	8.38	623.08	40.56	5.71
United Kingdom	110.11	0.11	0.11	3.55	69.20	4.46	7.59
United States	112.41	0.04	0.04	1.12	44.22	5.53	12.95
Uruguay	114.84	1.55	1.41	4.53	165.57	58.68	36.77
Venezuela	94.68	1.52	1.82	10.20	762.27	25.27	3.53
Vietnam	102.48	0.95	0.94	6.46	240.06	16.83	7.91
Total	106.56	0.92	0.95	5.37	205.20	15.11	13.67

Table 11: Descriptive Statistics at the Country Level: DTCC CDS Transactions Data

This table shows for each country the average of the variables in Table 9. See Table 9 for the description and units. The column numbers correspond to the row numbers of Table 9.

	(1) Trades	(2) D Notl	(3) Gross Notl	(4) Net Notl	(5) Vol/Notl	(6) Contracts	(7) Debt	(8) Notl/Debt
Argentina	14.12	127.04	51128.71	2005.88	58.24	5402.24	188.84	27.08
Australia	10.25	124.96	12737.10	2340.57	175.90	1223.64	281.64	3.96
Austria	9.50	178.57	41462.82	6926.69	88.43	1788.86	280.22	14.66
Belgium	17.25	227.83	34689.50	5587.55	123.53	1692.91	468.85	7.30
Brazil	38.75	527.42	149945.74	13773.91	78.98	10976.56	1357.57	11.11
Bulgaria	4.25	34.95	17604.31	1174.65	46.10	1792.73	7.79	226.22
Chile	1.00	9.28	4103.50	542.42	48.62	433.87	19.01	24.20
China	22.14	207.20	36183.67	4467.73	107.63	3738.12	1544.44	2.43
Colombia	4.75	59.82	29955.13	2067.81	47.40	3100.72	99.13	30.58
Croatia	2.25	20.31	6957.01	645.65	63.36	940.07	25.55	27.00
Czech Republic	1.62	17.69	9335.36	997.29	40.09	780.84	75.92	12.18
Denmark	6.12	69.65	11402.03	2294.45	110.66	753.61	136.85	8.24
Egypt	4.62	22.66	3301.37	703.38	161.85	739.48	173.73	1.87
Finland	2.00	49.24	11795.68	2080.52	80.33	466.75	115.71	9.88
France	48.00	751.06	70524.73	13530.02	183.61	3107.82	2185.89	3.14
Germany	20.75	442.33	74806.12	14204.94	114.96	2273.48	2689.71	2.74
Greece	20.50	207.10	66021.98	6544.82	66.61	3331.57	443.55	14.76
Hong Kong	.	.	1705.10	586.40	.	125.00	77.59	2.20
Hungary	18.00	171.53	55362.56	3580.07	67.98	4734.15	105.87	52.29
Iceland	1.12	6.85	8041.10	899.64	22.67	1112.89	12.11	67.58
Indonesia	12.88	103.98	34771.02	2347.58	69.79	4404.34	185.00	18.89
Ireland	15.88	183.73	34660.07	4495.52	109.56	1844.76	189.18	18.09
Israel	.	.	7972.05	867.66	.	883.76	167.06	4.71
Italy	54.12	905.55	239173.95	22927.95	84.49	6439.61	2508.14	9.50
Japan	23.62	235.73	32841.52	5118.98	123.33	3094.66	11924.96	0.26
Korea	23.12	191.78	58609.57	4194.89	78.47	6289.65	333.95	17.75
Latvia	1.38	11.14	8331.06	714.38	30.89	1029.60	9.35	89.78
Lebanon	0.88	4.71	2005.05	455.91	56.65	320.46	53.27	3.76
Lithuania	1.12	8.31	5260.09	701.13	35.15	614.79	13.48	39.43
Malaysia	4.62	44.52	18993.22	1173.03	58.22	2380.99	135.32	14.29
Mexico	22.62	279.96	105715.77	6966.41	59.23	8762.12	453.09	23.27
Netherlands	4.38	72.67	16643.36	2796.76	84.04	790.74	509.03	3.24
New Zealand	0.62	5.51	2717.42	523.38	48.08	295.47	51.96	5.24
Norway	1.12	24.57	6416.15	979.87	79.21	285.46	218.75	2.87
Panama	1.38	10.58	6971.72	697.17	35.68	972.53	10.91	63.88
Peru	7.50	72.46	22014.63	1820.25	71.59	2264.87	36.77	59.83
Philippines	11.00	109.23	62354.69	2678.16	46.00	7256.49	85.65	74.09
Poland	9.88	109.15	29995.43	2105.51	76.54	2724.02	255.38	11.63
Portugal	22.12	281.60	56096.89	6853.02	104.09	2621.39	220.90	25.05
Qatar	3.00	27.77	6131.54	531.59	93.73	784.09	44.61	15.15
Romania	3.75	34.62	15876.14	1235.60	49.70	1662.19	49.92	32.64
Russia	24.38	250.19	104350.07	4944.82	59.21	7566.12	177.79	60.91
Slovak Republic	1.00	10.21	8968.89	926.96	25.48	699.42	35.92	24.92
Slovenia	0.75	7.79	4180.75	797.17	40.03	352.02	20.06	20.62
South Africa	10.25	103.94	38728.05	2199.24	62.20	4264.44	124.45	32.33
Spain	62.12	909.26	116133.54	14820.21	156.35	4801.36	893.63	12.71
Sweden	3.75	64.69	15142.33	2889.23	86.27	819.05	187.70	7.99
Thailand	4.75	41.41	18196.95	1146.42	57.15	2513.53	138.68	13.24
Tunisia	0.25	3.12	1998.25	285.63	36.28	306.24	19.27	10.36
Turkey	25.12	311.98	152490.26	5697.12	53.85	10097.13	298.35	51.23
Ukraine	7.50	78.70	45877.98	1636.15	44.92	3781.47	51.81	92.49
United Kingdom	18.00	248.77	43986.36	8077.81	103.85	2830.08	1733.67	2.43
United States	4.50	101.24	16403.97	3143.99	121.84	663.41	14319.44	0.11
Venezuela	13.62	135.78	50593.94	2003.41	60.89	5017.19	126.20	40.71
Vietnam	3.25	29.03	7980.77	613.96	79.06	1166.30	55.00	14.46
Total	12.37	162.66	40007.57	3889.49	77.16	2829.47	859.58	27.93

Table 12: Descriptive Statistics: the EU

The table shows the descriptive statistics for the weekly observations of the bond bid-ask spread (% of the mid price), CDS price (% of notional), CDS net notional (in billion USD) over the period October 2008 - March 2012 for the sample of European Union countries.

	Mean	St. Dev.	Min	Max	No obs.
Bond Bid-Ask (% of mid)	1.16	2.22	0.04	35.64	3913
CDS Price (% of Notl)	2.45	6.00	0.17	208.58	3932
CDS Net Notional (bln $)	5.40	5.98	0.45	29.46	3932

Table 13: The Bond Bid-Ask Spread and CDS Net Notional: the EU

The table reports the coefficient estimates from generalized least squares regressions with both country and time fixed effects for the sample of EU countries. The dependent variable is the bond market bid-ask price spread (% of the mid price). The main variable of interest is CDS net notional outstanding (*CDS Notional*) in billions of USD. Control variables are CDS price (% of notional) as a measure of credit risk*, CDS Price,* and gross government debt outstanding in billion USD, *Gross Debt*. Column (3) shows an alternative specification of CDS net notional (*CDS Notional/Debt*) as the log of the ratio of CDS net notional to gross debt outstanding. Standard errors are given in parentheses and allow disturbances to have heteroskedasticity, contemporaneous correlation across countries, and AR(1) serial correlation within countries.

	Dependent Variable: Bond Bid-Ask (1)	(2)	(3)
CDS price	0.106***	0.105***	0.106***
	(0.0129)	(0.0127)	(0.0130)
CDS Notional	-0.111***	-0.261***	
	(0.0174)	(0.0396)	
Gross Debt		0.00296***	
		(0.000461)	
CDS Notional/Debt			-0.357***
			(0.0492)
Week FE	Yes	Yes	Yes
Country FE	Yes	Yes	Yes
No. obs	3913	3913	3913
No. countries	24	24	24
R2	0.59	0.60	0.58

Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 14: Granger Causality: CDS Net Notional and Bond Bid-Ask Spreads

The table reports Granger causality test results from the underlying vector autoregressive regressions:

$$\Delta y_t = \beta_0 + \sum_{j=1}^{p-1} \Delta y_{t-j} + \Delta \epsilon_t$$

where y_t is a vector of three variables: 1) CDS price, 2) the absolute CDS bid-ask spread, and 3) the bond bid-ask (% of mid). The reported results are for the set of countries for which Johansen trace statistics cannot reject the null that there is no cointegration among the variables. The lag order, p, is selected to optimize Akaike information criterion for each country. The table reports p-values of the Wald test statistics of the null hypotheses that, in column 1, H0: the bond market bid-ask spread does not Granger-cause the CDS net notional, and in column 2, H0: CDS net notional does not Granger-cause the bond bid-ask spread. We see that for 5 out of 14, CDS net notional Granger-causes bond bid-ask spreads, while for only 2 out of 14, bond liquidity Granger-causes CDS net notional.

	Bond Causes CDS	CDS Causes Bond
Austria	0.04	0.20
Belgium	0.33	0.07
Croatia	0.19	0.03
Finland	0.61	0.07
Greece	0.95	0.90
Hungary	0.91	0.04
Ireland	0.67	0.19
Italy	0.54	0.04
Latvia	0.97	0.17
Lithuania	0.00	0.19
Poland	0.44	0.30
Slovak_Republic	0.65	0.84
Slovenia	0.42	0.66
Spain	0.74	0.46

Table 15: VECM: CDS Net Notional and Bond Bid-Ask Spreads

The table reports adjustment coefficients (λ_{CDS} and λ_{bond}) for the underlying VECM specification:

$$\Delta x_t = \lambda_x \left(x_{t-1} - \alpha_0 - \alpha_1 \mu_{t-1} - \alpha_2 d_{t-1}\right) + \sum_{j=1}^{p-1} \beta_{1j} \Delta x_{t-j} + \sum_{j=1}^{p-1} \delta_{1j} \Delta \mu_{t-j} + \sum_{j=1}^{p-1} \gamma_{1j} \Delta d_{t-j}$$

$$\Delta \mu_t = \lambda_{CDS} \left(x_{t-1} - \alpha_0 - \alpha_1 \mu_{t-1} - \alpha_2 d_{t-1}\right) + \sum_{j=1}^{p-1} \beta_{2j} \Delta x_{t-j} + \sum_{j=1}^{p-1} \delta_{2j} \Delta \mu_{t-j} + \sum_{j=1}^{p-1} \gamma_{2j} \Delta d_{t-j}$$

$$\Delta d_t = \lambda_{bond} \left(x_{t-1} - \alpha_0 - \alpha_1 \mu_{t-1} - \alpha_2 d_{t-1}\right) + \sum_{j=1}^{p-1} \beta_{3j} \Delta x_{t-j} + \sum_{j=1}^{p-1} \delta_{3j} \Delta \mu_{t-j} + \sum_{j=1}^{p-1} \gamma_{3j} \Delta d_{t-j}$$

where x is credit risk, μ is CDS net notional, and d is the bond bid-ask spread. The reported results are for the set of countries for which the the Johansen trace test statistics rejects the null hypothesis that the cointegration rank is at most zero and cannot reject that the null hypothesis that cointegration rank is at most 1. The lag order, p, is selected to optimize Akaike information criterion for each country.

	λ_{bond}	t-Stat	λ_{CDS}	t-Stat
Bulgaria	-0.18	-4.40	0.01	0.46
Czech Republic	-0.16	-6.17	0.02	2.15
Denmark	0.00	0.53	0.00	0.40
France	-0.09	-2.16	0.95	1.77
Germany	-0.59	-5.93	-0.65	-0.80
Netherlands	-0.45	-6.53	-0.22	-1.30
Portugal	-0.03	-1.10	-0.03	-2.47
Romania	0.01	0.61	0.00	3.73
United Kingdom	-0.31	-3.88	-1.55	-2.62

Table 16: Granger Causality: CDS Bid-Ask Spreads and Bond Bid-Ask Spreads

The table reports Granger causality test results from the underlying vector autoregressive regressions:

$$y_t = \beta_0 + \sum_{j=1}^{p} y_{t-j} + \epsilon_t$$

where y_t is a vector of three variables: 1) the first difference in CDS price, 2) the absolute CDS bid-ask spread, and 3) the bond bid-ask (% of mid). The lag order, p, is selected to optimize SBIC for each country. The table reports p-values of the Wald test statistics of the null hypotheses that, in column 1, H0: the bond market bid-ask spread does not Granger-cause the CDS bid-ask spread, and in column 2, H0: CDS bid-ask spread does not cause the bond bid-ask spread.

	Bond Causes CDS	CDS Causes Bond
Austria	0.08	0.01
Belgium	0.15	0.14
Bulgaria	0.00	0.00
Croatia	0.42	0.00
Czech_Republic	0.00	0.00
Denmark	0.22	0.15
Finland	0.53	0.26
France	0.05	0.00
Germany	0.00	0.00
Greece	0.00	0.00
Hungary	0.27	0.00
Ireland	0.10	0.20
Italy	0.01	0.01
Latvia	0.22	0.00
Lithuania	0.11	0.00
Netherlands	0.58	0.00
Poland	0.33	0.00
Portugal	0.00	0.00
Romania	0.00	0.00
Slovak_Republic	0.45	0.26
Slovenia	0.51	0.94
Spain	0.72	0.03
Sweden	0.18	0.09
United_Kingdom	0.67	0.01

Table 17: The Effect of the Permanent EU Ban on Contemporaneous Bond Illiquidity

This set of regressions explores the effect of the permanent EU-wide ban on naked CDS trading on bond market liquidity. The table shows coefficient estimates from generalized least squares regressions with both country and time fixed effects. The dependent variable is the bond market bid-ask spread (% of the mid price). The main variable of interest is *EU CDS Ban* dummy variable that equals one for country-date observations for which the CDS ban was in place. Control variables are CDS price as % of notional as a measure of credit risk*, CDS Price,* and gross debt outstanding in trillion USD, *Gross Debt*. Columns (1) and (2) compare whether including CDS price makes a difference. Columns (3)-(6) have country specific trends, while Column (7) allows for a group specific trend instead. Column (4) allows for a "treatment" intensity by incorporating an interaction between the ban dummy and the decrease in net notional between the ban period and before the ban period, $\Delta Notl$ (in billion USD). Column (5) excludes Greece as a potential outlier. Column (6) restricts the sample to OECD countries only. Standard errors are given in parentheses and allow disturbances to have heteroskedasticity, contemporaneous correlation across countries, and AR(1) serial correlation within countries.

	Dependent Variable: Bond Bid-Ask						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Gross Debt	-0.288***	-0.418***	-0.505***	-0.466**	-0.113***	-0.726***	-0.654***
	(0.104)	(0.0798)	(0.0818)	(0.232)	(0.0378)	(0.232)	(0.0860)
EU CDS Ban	0.271***	0.786***	0.652***	0.419*	0.314***	0.780**	0.959**
	(0.105)	(0.241)	(0.196)	(0.218)	(0.0482)	(0.335)	(0.442)
CDS Price		0.123***	0.0391	0.0401	0.328***	0.0384	0.122***
		(0.00423)	(0.0447)	(0.0360)	(0.0690)	(0.0359)	(0.00502)
EU CDS Ban$*\Delta$Notl				0.647**			
				(0.316)			
Week FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Country FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Country Trends	No	No	Yes	Yes	Yes	Yes	No
Group Trends	No	No	No	No	No	No	Yes
No. obs	2457	1802	1802	1560	1772	900	1802
No. countries	63	62	62	52	61	30	62
R2	0.57	0.87	0.90	0.90	0.89	0.90	0.88

Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 18: The Effect of the Permanent EU Ban on Past Bond Illiquidity

This set of regressions explores the effect of the permanent EU-wide ban on naked CDS trading on bond market liquidity *before* the ban. The table shows coefficient estimates from generalized least squares regressions with both country and time fixed effects. The dependent variable is lagged bond market bid-ask spread (% of the mid price). The main variable of interest is *EU CDS Ban*$*\Delta$*Notl* that is an interaction of the decrease in net notional and a dummy variable that equals one for country-date observations for which the CDS ban was in place. Control variables are lagged CDS price as % of notional as a measure of credit risk, *CDS Price,* and lagged gross debt outstanding in trillion USD, *Gross Debt*. Column (3) excludes Greece as an outlier. Standard errors are given in parentheses and allow disturbances to have heteroskedasticity, contemporaneous correlation across countries.

	Dependent Variable: Lagged Bond Bid-Ask		
	(1)	(2)	(3)
L4.CDS Price	0.203***	0.204***	0.633***
	(0.0516)	(0.0511)	(0.0267)
L4.Gross Debt	6.055**	5.982**	-11.54***
	(2.932)	(2.909)	(1.449)
ΔNotl	0.0166	0.0566	0.0144
	(0.0858)	(0.0635)	(0.0664)
EU CDS Ban$*\Delta$Notl		-0.334***	-0.279**
		(0.0613)	(0.115)
Week FE	Yes	Yes	Yes
Country FE	Yes	Yes	Yes
No. obs	156	156	153
No. countries	52	52	51
R2	0.99	0.99	0.98

Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 19: The Permanent EU Ban and Bond Illiquidity: Expanding the Data Sample Period

This set of regressions explores the effect of the permanent EU wide ban of naked CDS trading on bond market liquidity. The main specification in the paper used 4-months of data pre and post October 18, 2011 (the CDS ban legislation vote date). This table shows the effect of increasing the sample size from 4-months to 6, 9, and 12 months in columns (1)-(3), respectively. The table shows coefficient estimates from generalized least squares regressions with both country and time fixed effects. The dependent variable is the bond market bid-ask spread (% of the mid price). The main variable of interest is *EU CDS Ban* dummy variable that equals one for country date observations for which CDS ban was in place. Control variables are CDS price as % of notional as a measure of credit risk*, CDS Price,* and gross debt outstanding in trillion USD, *Gross Debt.* Greece is excluded as a potential outlier. Standard errors are given in parentheses and allow disturbances to have heteroskedasticity, contemporaneous correlation across countries, and AR(1) serial correlation within countries.

	Dependent Variable: Bond Bid-Ask		
	(1)	(2)	(3)
CDS Price	0.343***	0.343***	0.288***
	(0.0301)	(0.0299)	(0.0217)
Gross Debt	-0.0823***	-0.0307	-0.0372
	(0.0181)	(0.0401)	(0.0315)
banEU	0.320***	0.140*	0.0996**
	(0.0698)	(0.0718)	(0.0448)
Week FE	Yes	Yes	Yes
Country FE	Yes	Yes	Yes
Country Trends	Yes	Yes	Yes
No. obs	2313	3045	3781
No. countries	61	61	62
R2	0.87	0.86	0.83

Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 20: The Permanent EU Ban and Bond Illiquidity: Excluding Greece

This set of regressions repeats the main exercise in the paper by restricting the sample to OECD countries. It explores the effect of the permanent EU wide ban of naked CDS trading on bond market liquidity. The table shows coefficient estimates from generalized least squares regressions with both country and time fixed effects. The dependent variable is the bond market bid-ask spread (% of the mid price). The main variable of interest is *EU CDS Ban* dummy variable that equals one for country date observations for which CDS ban was in place. Control variables are CDS price as % of notional as a measure of credit risk, *CDS Price,* and gross debt outstanding in trillion USD, *Gross Debt.* Columns (1) and (2) compare whether including CDS price makes a difference. Columns (1)-(3) have country specific trends, while Column (4) has group specific trend. Columns (3) allows for "treatment" intensity by incorporating interaction between the ban dummy and the decrease in net notional between the ban period and before the ban period, $\Delta Notl$ (in billion USD). Standard errors are given in parentheses and allow disturbances to have heteroskedasticity, contemporaneous correlation across countries, and AR(1) serial correlation within countries.

	Dependent Variable: Bond Bid-Ask			
	(1)	(2)	(3)	(4)
Gross Debt	-0.0908	-0.113***	-0.109*	-0.103***
	(0.0608)	(0.0378)	(0.0630)	(0.0239)
EU CDS Ban	0.190**	0.314***	0.253**	0.436***
	(0.0953)	(0.0482)	(0.124)	(0.0853)
CDS Price		0.328***	0.434***	0.408***
		(0.0690)	(0.0761)	(0.0361)
EU CDS Ban$*\Delta$Notl			0.239***	
			(0.0803)	
Week FE	Yes	Yes	Yes	Yes
Country FE	Yes	Yes	Yes	Yes
Country Trends	Yes	Yes	Yes	No
Group Trends	No	No	No	Yes
No. obs	2418	1772	1530	1772
No. countries	62	61	51	61
R2	0.84	0.89	0.89	0.82

Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 21: The Permanent EU Ban and Bond Illiquidity: OECD Sample

This set of regressions repeats the main exercise in the paper by excluding Greece as a potential outlier. It explores the effect of the permanent EU wide ban of naked CDS trading on bond market liquidity. The table shows coefficient estimates from generalized least squares regressions with both country and time fixed effects. The dependent variable is the bond market bid-ask spread (% of the mid price). The main variable of interest is *EU CDS Ban* dummy variable that equals one for country date observations for which CDS ban was in place. Control variables are CDS price as % of notional as a measure of credit risk, *CDS Price,* and gross debt outstanding in trillion USD, *Gross Debt.* Columns (1)-(2) have country specific trends, while Column (3) has a group specific trend. Column (2) allows for "treatment" intensity by incorporating the interaction between the ban dummy and the decrease in net notional between the ban period and before the ban period, $\Delta Notl$ (in billion USD). Standard errors are given in parentheses and allow disturbances to have heteroskedasticity, contemporaneous correlation across countries, and AR(1) serial correlation within countries.

	Dependent Variable: Bond Bid-Ask		
	(1)	(2)	(3)
CDS Price	0.0384	0.0387	0.121***
	(0.0359)	(0.0358)	(0.0339)
Gross Debt	-0.726***	-0.759***	-0.754***
	(0.232)	(0.240)	(0.188)
EU CDS Ban	0.780**	0.606**	0.966**
	(0.335)	(0.247)	(0.393)
EU CDS Ban∗ΔNotl		0.582*	
		(0.300)	
Week FE	Yes	Yes	Yes
Country FE	Yes	Yes	Yes
Country Trends	Yes	Yes	No
Group Trends	No	No	Yes
No. obs	900	870	900
No. countries	30	29	30
R2	0.90	0.91	0.89

Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 22: Correlation between the change in notional during the ban and notional before the ban

This table checks the cross-country correlation between the decrease in CDS net notional during the ban and the level of net notional before the ban, *Notl(t-1)*. The change in net notional is constructed as the level of net notional averaged over the ban period minus the level of net notional averaged over the pre-ban period. The correlation controls for the pre-ban levels of debt outstanding, *Gross Debt(t-1)*, and credit risk measured by CDS price, *CDS Price(t-1)*. *EU* is a dummy variable that equals 1 for the EU countries. For the EU countries, the correlation is significant while for the non-EU countries there is no significant correlation.

	Dependent Variable: Change in Net Notional (1)
Notl(t-1)	-0.0312 (0.0361)
EU	-0.144 (0.238)
EU∗ Notl(t-1)	0.0838** (0.0394)
Gross Debt(t-1)	-0.0142 (0.0356)
CDS Price(t-1)	0.0346* (0.0201)
Constant	0.0673 (0.178)
No. obs	54
R2	0.24

Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 23: Collapsing the time series into pre and post ban.

This set of regressions collapses the time series data into an average pre and post ban for each country. The table shows coefficient estimates from OLS regressions with country fixed effects. The dependent variable is the bond market bid-ask spread (% of the mid price). The main variable of interest is *EU* dummy variable that equals one for countries subject to the CDS ban. Control variables are CDS price as % of notional as a measure of credit risk*, CDS Price,* and gross debt outstanding in trillion USD, *Gross Debt.* Standard errors are given in parentheses. They allow disturbances to have heteroskedasticity and are clustered at the EU level. Column (1) uses the whole sample of countries, and column (2) restricts to OECD countries only.

	Dependent Variable: Bond Bid-Ask	
	(1)	(2)
EU	0.0810	0.139***
	(0.211)	(0.004)
Gross Debt	0.147	-0.0459*
	(0.135)	(0.054)
CDS Price	0.276**	0.269***
	(0.028)	(0.001)
No. obs	60	30
R2	0.75	0.97

p-values in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 24: The Effect of the Temporary German Ban on Bond Illiquidity

This set of regressions explores the effect of May 2010 German ban on naked CDS trading on bond market liquidity. The table shows coefficient estimates from generalized least squares regressions with both country and time fixed effects. The dependent variable is the bond market bid-ask spread (% of the mid price). The main variable of interest is *CDS Ban* dummy variable that equals one for country-date observations for which the CDS ban was in place. Control variables are CDS price as % of notional as a measure of credit risk, *CDS Price,* and gross debt outstanding in trillion USD, *Gross Debt.* $\Delta Notl$ is the decrease in net notional between the ban period and before the ban period and captures "treatment" intensity. Columns (1) and (2) compare whether including CDS price makes a difference. Columns (3)-(5) have country specific trends, while Column (6) allows for a group specific trend instead. Column (4) allows for a "treatment" intensity by incorporating an interaction between the ban dummy and the decrease in net notional between the ban period and before the ban period, $\Delta Notl$ (in billion USD). Column (5) excludes Greece as a potential outlier. Standard errors are given in parentheses and allow disturbances to have heteroskedasticity, contemporaneous correlation across countries, and AR(1) serial correlation within countries.

	Dependent Variable: Bond Bid-Ask					
	(1)	(2)	(3)	(4)	(5)	(6)
Gross Debt	2.033***	1.453***	0.425*	0.570	0.473	-0.0634
	(0.120)	(0.309)	(0.237)	(0.562)	(0.310)	(0.0488)
CDS Ban	-0.216***	-0.217***	-0.233***	-0.143***	-0.0757**	-0.219***
	(0.0572)	(0.0589)	(0.0584)	(0.0482)	(0.0382)	(0.0575)
CDS Price		0.235*	0.110	0.114	0.151**	0.229*
		(0.124)	(0.148)	(0.139)	(0.0666)	(0.125)
CDS Ban$*\Delta$Notl				-0.304**		
				(0.119)		
Week FE	Yes	Yes	Yes	Yes	Yes	Yes
Country FE	Yes	Yes	Yes	Yes	Yes	Yes
Country Trends	No	No	Yes	Yes	Yes	No
Group Trends	No	No	No	No	No	Yes
No. obs	816	740	740	740	706	740
No. countries	24	24	24	24	23	24
R2	0.74	0.81	0.85	0.85	0.93	0.81

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Figure 11: The Permanent EU Naked CDS Ban and the Amount of CDS Purchased, 2011.01 - 2012.08

The solid line plots the total CDS purchased (CDS net notional, $bln) across countries that were subject to the EU ban. The dashed line plots the total for countries that were not affected by the ban and CDS could still be purchased. The vertical line is drawn at October 18, 2011 and shows when the EU passed the naked CDS ban legislation.

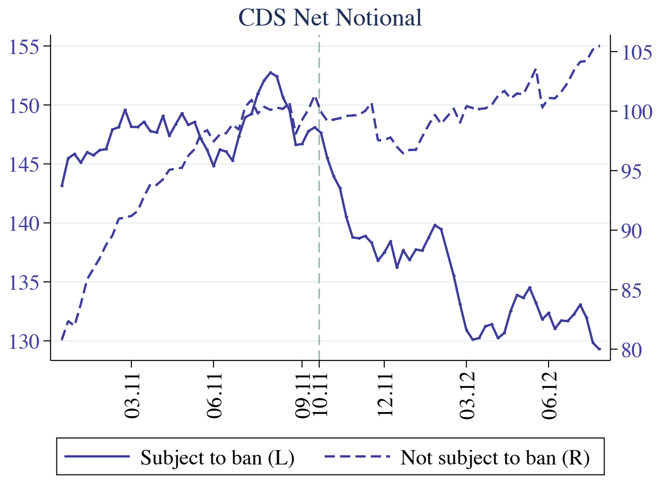


Figure 12: The Permanent EU Naked CDS Ban and Bond Illiquidity, 2011.01 - 2012.08

The vertical line drawn at October 18, 2011 shows when the EU passed the naked CDS ban legislation. The solid line plots the cross-country average bond bid-ask spread (% of the mid price) for the countries subject to the ban (the EU countries). The dashed line plots the average bond bid-ask spread for countries that were not affected by the ban (outside the EU). We see that the countries affected by the ban experienced an increase in their bond bid-ask spread.

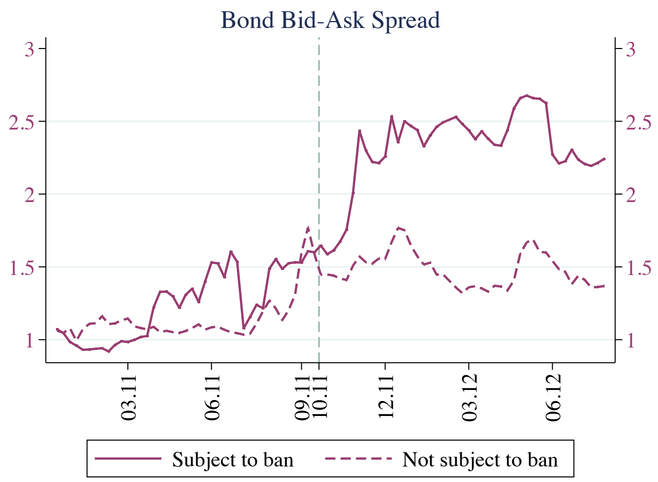


Figure 13: The Temporary CDS Ban and Bond Illiquidity, Mar 2010 - Aug 2010

The solid line plots the cross-country average bond bid-ask spread (% of the mid price) for the EU countries that were subject to the ban (i.e. Eurozone countries). The dashed line shows the average for the EU countries not affected by the ban (i.e. naked CDS referencing these countries could still be purchased). The vertical lines are drawn at the week before and after the German ban is instituted. We see that the countries affected by the ban experienced an immediate decrease in their bond bid-ask spread.

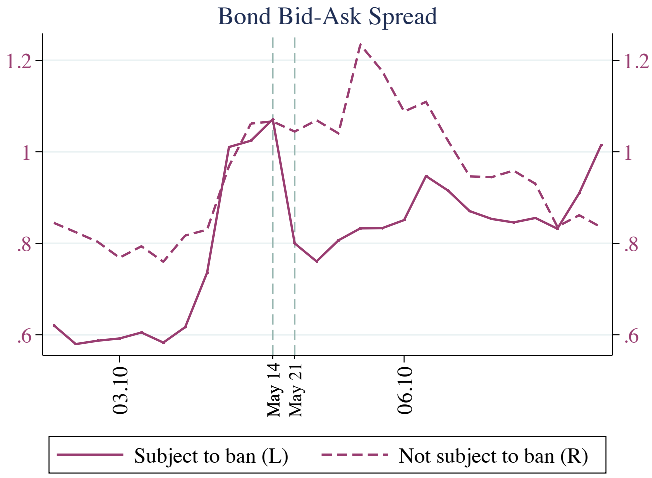


Figure 14: The Temporary German CDS Ban and the Amount of CDS Purchased, Mar 2010 - Aug 2010

The solid line plots the time series of the total CDS net notional ($billion) across EU countries that were subject to the ban (i.e. Eurozone countries). The dashed line shows the total for EU countries that were not affected by the ban (i.e. naked CDS referencing these countries could still be purchased). The vertical lines are drawn at the week before and after the German ban is instituted.

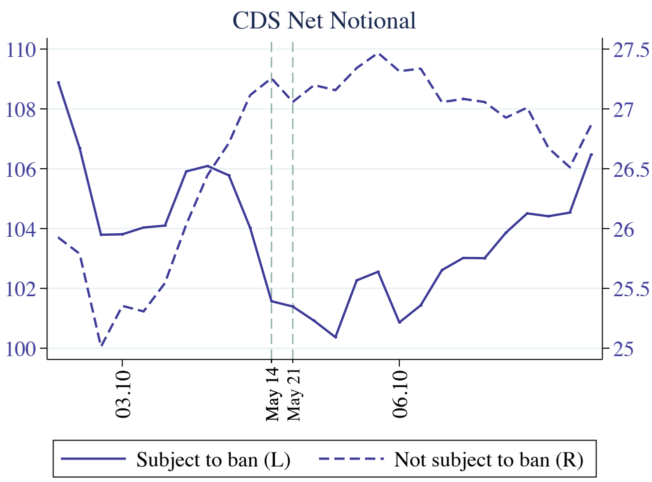


Figure 15: Bond (il)liquidity and CDS Net Notional, Italy (Dec 2008 - Aug 2012)

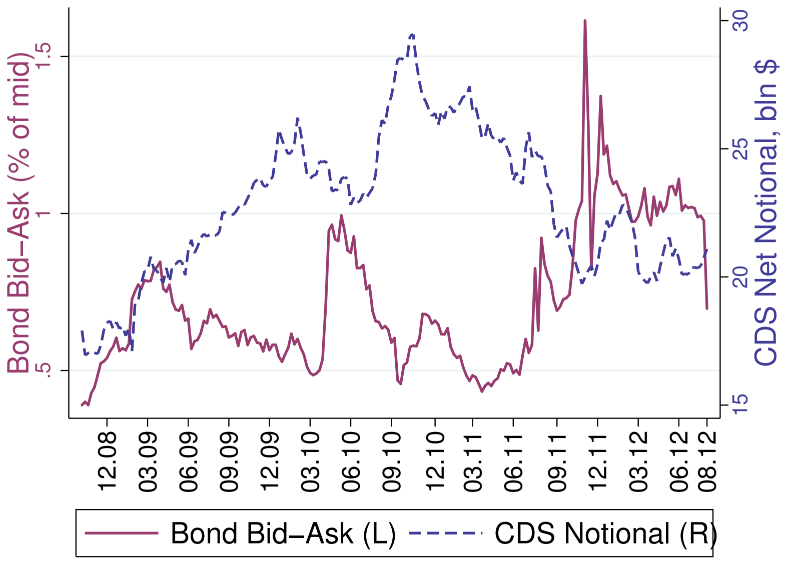


Table 25: Anecdotal Evidence of How the EU Ban Affected the Bond Market

In February 2013, the European Securities and Market Authority (ESMA) surveyed market participants on the effects of the EU naked CDS ban. Below are some responses to Question 15 in the survey that asked *Have you noticed any effect of the prohibition on entering into an uncovered sovereign CDS transaction on the price and on the volatility of the sovereign debt instruments*? For more information on the survey and the responses received from private institutions and industry associations see: http://www.esma.europa.eu/consultation/Call-evidence-evaluation-Regulation-short-selling-and-certain-aspects-credit-default-sw#responses and ESMA (2013).

The German Banking Industry Committee:

> "The market has become less liquid; the bid-offer spread has widened. Volatility is unchanged, but has tended to shift to the spot/cash markets."

The Association for Financial Markets in Europe (AFME) and the International Swaps and Derivatives Association (ISDA):

> "Market participants have already observed that seemingly due to the SSR Regulation (restrictions it imposed on sovereign debt and sovereign CDS markets), Asian participation in the European bond market fell by around 50% immediately after the introduction of the SSR, thus demonstrating neatly one adverse impact of the SSR in general in driving investors away."

> "Some buy side market participants have already remarked that even though there is still liquidity in sovereign debt, it is more difficult to source this liquidity."

Alternative Investment Management Association (AIMA) and Managed Funds Association (MFA):

> "Some of our members have reported that they have stopped trading European sovereign CDS and bonds, given the regulatory and reputational risks."

> "Restrictions on CDS positions over the medium term will generally make it more difficult for sovereign issuers to borrow through long-dated securities, leading to a shortening of the average maturity profile of sovereign issuance as investors seek to limit their risk exposure, thereby increasing the vulnerability of sovereigns to short term liquidity and funding crises. This sentiment is reflected in the responses to AIMA and MFA's poll of their members."

> "At worst, the ban could ultimately undermine liquidity in the underlying sovereign debt markets, undermining the ability of sovereigns to raise finance through debt issuance."

Deutsche Bank

> "We observed anecdotally that as investors began to understand the details of the regulation, cash volumes reduced with a resultant increase in volatility, although this was not significant."

ESSAY 2: CDS AS SOVEREIGN DEBT COLLATERAL

Abstract

A defining friction of sovereign debt is the lack of collateral that can back sovereign borrowing. This paper shows that credit default swaps (CDS) can serve as collateral and thereby support more sovereign borrowing. By giving more bargaining power to lenders in ex-post debt renegotiations, CDS becomes a commitment device for lenders to extract more repayment from the debtor country. This ex-post disciplining effect during debt renegotiations better aligns the sovereign's ex-ante incentives with that of the lender. CDS alleviates agency frictions that are present in any lending contracts but are particularly difficult to mitigate in sovereign debt context.

1 INTRODUCTION

With the recent sovereign debt crisis in Europe, sovereign credit default swaps (CDSs) have been blamed by politicians and regulators for increasing borrowing costs and exacerbating the debt crisis.[39] In May 2010, regulators in Germany temporarily banned the purchase of CDS on Euro zone government bonds by those who do not own the underlying bond, and in October 2011 the ban was made permanent and applicable across the European Union. An implicit argument in these criticisms and policy actions is that CDSs can somehow affect the underlying borrower. Most of the existing literature on credit derivatives, however, focuses on the incentive problems between the lender and the insurer while abstracting from the effect on the borrower. But why might CDSs matter for the borrower? How might the lender's insurance activity affect the incentives and the welfare of the sovereign borrower?

This paper's answer is straightforward. The existence of CDS can give lenders more leverage in ex-post renegotiations. This alleviates ex-ante borrowing constraints, provides more external capital to the debtor coun-

[39] Credit default swaps are over-the-counter derivative contracts where the seller of the contract pays the buyer of the contract a pre-specified amount (called notional) when a credit event occurs (such as default by a firm or a government). In return, the buyer of the contract pays a periodic fee until either the contract matures or a credit event occurs. The contract specifies, among other things, the reference entity, the contract maturity date, the notional amount, and the events that constitute as a credit event.

try, increases investment and, therefore, welfare. CDS can be beneficial for the borrower. The effect of CDS on ex-post renegotiations is especially crucial in sovereign debt context because it is difficult to collateralize sovereign debt.

We derive this result using a framework with an agency problem between the lender and the sovereign borrower. The model features two frictions characteristic to sovereign debt. First, due to the lack of international law for sovereign bankruptcy, sovereign governments often get away without fully repaying their debt and this debt reduction is achieved through renegotiations with lenders.[40] Second, due to asymmetric information about the sovereign's actions - such as investment - which in turn affect the repayment ability of the sovereign, the incentives of the borrower and the lender are misaligned. The asymmetric information, together with the inability of the lender to credibly deny any debt reduction, gives rise to a moral hazard problem on the part of the borrower: knowing that in a low-output state a debt reduction will be reached, the borrower does not invest enough to avoid a low future output.

In this setting, we find that insurance serves as a commitment device for the lender. By giving credibility to lender's threat to walk away from debt renegotiations and let the sovereign default, CDS improves the lender's bargaining power during debt renegotiations. This increased leverage of the lender enables him to extract more repayment even in bad states of the world, which in turn allows the lender to ex-ante offer better loan contracts (e.g. with lower borrowing cost) compared to an uninsured lender. As bad output states are even less attractive to the borrower, the incentives of the borrower and the lender are better aligned and the borrower invests more efficiently. Thus, insurance has a disciplining effect on the borrower: it increases investment and lowers the probability of default and cost of borrowing. As a result, CDS alleviates the moral hazard problem and improves welfare.

These results are based on the assumption that lenders are the only agents purchasing insurance. In reality, there are investors, so called 'naked buyers', who purchase insurance but do not own the underlying bond. How robust are our findings if we allow for investors that purchase insurance but do not lend to the sovereign? We find that the existence of naked buyers impacts the debtor country only if the CDS market is concentrated (e.g. a monopoly) but not if it is perfectly competitive as we had assumed up to this point. If the insurer is a monopolist, he can indirectly affect the borrower's investment through the insurance contract offered to the lender. The insurance seller not only earns a profit from insuring the lender but also from insuring the naked buyer where, by

[40] Benjamin and Wright (2009) constructed a database covering 90 defaults and renegotiations by 73 countries that occurred over the period of 1989-2006. Argentina, for example, defaulted in 2001 on its $94 billion international bonds, which, according to Benjamin and Wright (2009) estimates, led to creditor losses of 63% of the value of the debt.

assumption, the latter is affected by the borrower's investment.[41] The insurance seller designs the insurance contract so as to induce the level of investment that maximizes his total profit. Thus, the existence of naked buyers can result in over-investment. Importantly, our analysis points to the importance of the CDS market structure for whether the existence of naked buyers affects the borrower.

Related Literature

This paper is related to sovereign debt models with agency frictions. We model the asymmetric information friction similar to Atkeson (1991) and Gertler and Rogoff (1990), while the debt renegotiation framework is similar to Yue (2010). This paper shows how derivative financial contracts that exist today can help mitigate agency frictions that sovereign debt contracts are mired with.

The framework of our model is similar to the standard moral hazard model in corporate finance with a lender and an entrepreneur who needs financing for a project (see, for example, Repullo and Suarez (1998), Tirole (2006)).[42] If the entrepreneur exerts effort, the project is more likely to succeed, but the entrepreneur's effort is noncontractible. There is an intermediate date at which the lender can choose to either liquidate the project, if he suspects that the entrepreneur shirked, or continue the project. If the lender can credibly commit to liquidate the project, the entrepreneur will exert more effort. But the lender's threat to liquidate is not credible ex-post, thus creating a moral hazard. Various mechanisms have been suggested that can make the lender's threat credible and discipline the borrower. Hart and Moore (1995), for example, suggest making the original lenders senior to new lenders who might come in and provide funding to continue the project. Dewatripont and Tirole (1994) argue for a diversity of tough and soft claim-holders and Berglof and von Thadden (1994) show that short term lending can have a disciplining effect. Our paper suggests that insurance against default is another form of a such disciplining mechanism.

There is a growing literature on CDS. Most of the corporate CDS literature addresses asymmetric information between lenders and insurers about the loans on which the lenders purchase insurance.[43] However,

[41] The naked buyer, we assume, has cash flows that are positively correlated to that of the sovereign's: his cash flow is high in the state where the sovereign's output is high and low in the sovereign's low output state. Then, by assumption, the probability that the naked buyer has a high cash flow increases with the borrower's investment; in other words, the borrower's investment affects the utility of the naked buyer. Thus, the naked buyer's utility (and hence the profit extracted by the insurer) depends not only on the naked buyer's own insurance but also on the insurance purchased by the lender since the latter affects the borrower's investment.

[42] See the discussion in Tirole (2006), section 5.5.

[43] See, for example, Duffee and Zhou (2001), Morrison (2005), Thompson (2007), Parlour and Winton (2009). See Acharya and Johnson (2007) for empirical support for asymmetric information problems between lenders and insurers. For an overview of the CDS market, see Stulz (2010) and Weistroffer, Speyer, Kaiser, and Mayer (2009);

sovereign debt is less prone to this type of asymmetric information since if information about a country is available to international lenders, then it is likely to be available to insurers. In contrast, our paper focuses on the effect of CDSs on the borrower-lender relationship. As our paper sheds light on CDS's effect on the probability of default, it is also related to papers that study its effect on financial stability. Instefjord (2005) finds that CDSs can lead to banks taking on more risk. Allen and Carletti (2006) show that if banks face the same liquidity demand, credit risk transfer is beneficial, but if banks face an idiosyncratic risk, then credit risk transfer can increase the risk of financial crises.

This paper, however, is most closely related to Arping (2013) and Bolton and Oehmke (2011) who both show the disciplining effect of CDSs in a framework with firm agency problems. The agency problems in these models are moral hazard (noncontractible effort) in Arping (2013) and strategic default (nonverifiable cash flow) in Bolton and Oehmke (2011). In both, the threat of liquidation motivates the borrower to repay, however the possibility of renegotiating before the lender's liquidation decision creates a credibility issue on the part of the lender to actually carry out the liquidation ex-post. Thus, similar to our paper, CDS plays as a commitment device to carry out the threat if necessary.

The novelty of this paper lies in the application to sovereign debt. These models are specific to corporate debt and bankruptcy and, hence, do not capture the salient features of sovereign debt that distinguish it from corporate debt. First, although the threat to liquidate is a standard modeling feature in corporate finance literature, in sovereign debt, there is no such concept as going to a court to liquidate or take control over a country or a government. Even if that were possible, in a corporate setting all the assets of a firm can serve as collateral, whereas in a sovereign context the amount of seizable assets is negligible. Instead, what impels sovereigns to repay is reputational costs of default such as the potential increase in borrowing costs, loss of trade relations, and domestic banking crises.

Second, Bolton and Oehmke (2011)'s strategic default setting results in the borrower trying to default and renegotiate when the borrower's cash flow is high. In sovereign debt context, sovereigns instead default and renegotiate during recessions when they are actually facing repayment difficulties than in high revenue states. Sovereign debt agency frictions are better captured with a moral hazard problem where the sovereign does not take sufficient measures (e.g. enough investment or policy effort to improve macroeconomic performance) to avoid repayment difficulties in the future.

Third, to create an interim renegotiation stage, Arping (2013) makes effort observable (although still not contractible) which is the key assumption of his model that drives his results. Since in our model the borrower's investment is neither detectable nor enforceable, it allows us to

and for an overview more specific to sovereign CDS, see Ranciere (2001), Packer and Suthiphongchai (2003) and Verdier (2004).

analyze the impact of CDS in a standard moral hazard agency framework. Furthermore, debt renegotiation in Arping (2013) occurs before output is realized but sovereign debt renegotiations occur after the sovereign finds out that it will be facing repayment difficulties (i.e. after the output realizes). Thus, our model better captures these crucial differences between sovereign and corporate debt. Nevertheless, our paper compliments the existing literature and is able to show that the disciplining effect of CDS carries over to an environment that is specific to sovereign debt.

The next section lays out, first, the frictionless benchmark economy, followed by private information environment where we characterize the moral hazard problem. Then we introduce the insurance market and give our main result that demonstrates CDS's disciplining role. In appendix B.1, we show that insurance is inconsequential unless both private information and bargaining are present. Section 3 relaxes the perfectly competitive environment by considering a monopolistic insurer and looks at how the existence of naked buyers affects the debtor country. The last section concludes while the proofs of the results are relegated to the appendix.

2 MODEL

Consider a small open economy representing the debtor country. There are two dates: t={0,1}. The borrower is risk neutral and does not discount, $U(c_0, c_1) = c_0 + E_0 c_1$, and can invest I at date 0 to earn a random output at date 1. The distribution of the output depends on the amount invested at date 0: output is high, $y_1 = y_h$, with probability $\pi(I)$ and low, $y_1 = y_l$, with probability $(1 - \pi(I))$ where $\pi'(I) > 0$, $\pi''(I) < 0$, $\pi(0) = 0$, and $\pi'(0)(y_h - y_l) > 1$.[44] Thus, the probability of the high state is increasing in investment. The borrower has zero endowment at date 0, but can borrow by trading one-period zero-coupon bonds with risk neutral competitive foreign lenders. We denote the face value and the price of the bond as B and q respectively. $B > 0$ means the sovereign country is a net borrower: he receives $qB \geq 0$ consumption goods at date 0 and has to repay B at date 1 regardless of the state realized. The raised funds, qB, can be used for either consumption or investment, thus his date 0 consumption is: $c_0 = qB - I$.

The borrower has an incentive to repay at date 1 because if he defaults in full, he loses a fraction of his output, py_l. This cost of default can be interpreted as a reduced form for costs associated with temporary exclusion from credit markets, trade disruptions, or foreign investors' lack of confidence. If the borrower repays the loan, his consumption is:

$$c_1^{nd} = y_1 - B$$

[44] The last condition implies that positive investment is efficient.

While if he defaults in full, his consumption is:

$$c_1^d = y_1(1 - p)$$

We assume that the cost of default in the good output state is high enough to deter any default, while too low in the bad state. Thus a default can occur only in the bad state, which is an assumption consistent with the sovereign debt stylized fact that debtor countries typically default when they are in a recession.[45] In particular, the borrower defaults in the bad state by negotiating a debt reduction with the lender. If such a debt reduction agreement is reached, the borrower is able to avert the default cost. An interpretation of this assumption is that the ability of the sovereign to reach an agreement with the lenders is perceived positively by the market and prevents further loss of confidence by investors or trade partners.[46] From here on, we refer to defaults to mean partial defaults that occur through debt renegotiation when y_l is realized and the ex-ante probability of default is $1 - \pi(I)$. If an agreement is reached, $1 - \alpha$ and α are the shares of y_l going to the borrower and the lender respectively.[47]

Figure 16 shows the timeline of the model as well as the subgame that gets played out at date 1 once the borrower has borrowed and invested at date 0. When y_l is realized, a debt reduction agreement will be reached (as pointed out by the arrow) since the lender will prefer getting αy_l over nothing and for the borrower the bargaining outcome will be at least better than suffering the output loss py_l. While, if y_h is realized, the borrower will repay in full as indicated by the arrow.

The loan contract signed by the borrower and the lender at date 0 will take into account the bargaining outcome of the subgame when y_l is realized at date 1. The borrower's expected utility is:

$$c_0 + \beta E_0 c_1 = (qB - I) + \pi(I)(y_h - B) + (1 - \pi(I))y_l(1 - \alpha)$$

And the lender's zero-profit condition is:

$$\pi(I)B + (1 - \pi(I))\alpha y_l = qB$$

Next, we proceed by characterizing the outcome of the ex-post bargaining problem which will then be used to solve for the equilibrium loan contract and investment.

[45] This fact can be reproduced in a dynamic model with a risk averse borrower. For a risk averse borrower, when output is already low, it hurts more to further lower consumption by paying off his debt. As our model is a two-period model with a risk neutral borrower, we resort to assuming this result. Nevertheless, we show in appendix B.2 that the results of this section hold in a more general setting where the borrower defaults and bargains both in the high and low states.

[46] A more general approach would be to have two types of default costs: one for partial default and a more costly one for full default, but for our purposes only the relative difference in default costs matters. Thus, we can assume the cost of partial default (i.e. for reaching a renegotiation agreement) is zero.

[47] This assumption is made for simplicity since usually debt renegotiations are over reductions of the actual debt, i.e. the borrower pays back α share of the original debt, B. Thus, more general approach would be for them to bargain over $min(B, y_l)$.

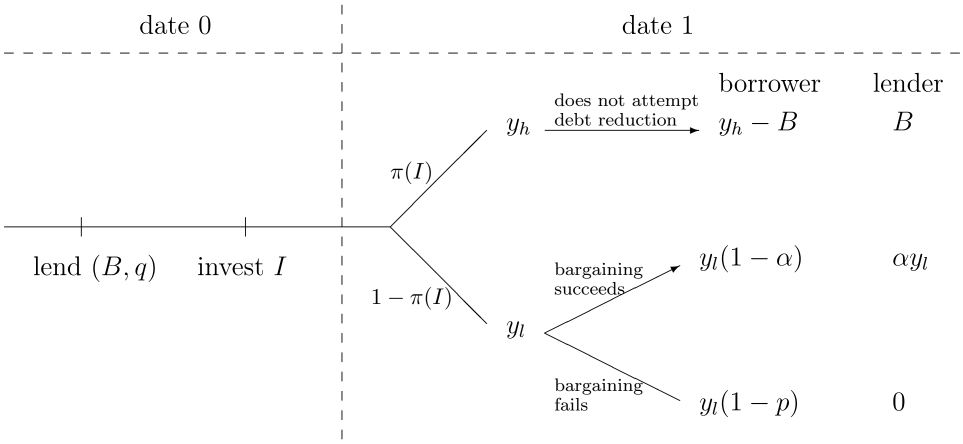


Figure 16: Model timeline

2.0.1 *The Nash Bargaining Problem*

Following the general approach of Yue (2010), we model the partial default (i.e. debt renegotiation or restructuring) that occurs when y_l is realized as an outcome from a Nash bargaining problem. We assume that the lender and the borrower have an equal bargaining power. If they fail to reach an agreement, the threat point for the borrower is being penalized by py_l and for the lender it is getting nothing. Thus, their respective surpluses from bargaining are:

$$\Delta_B(a) = (1-a)y_l - (1-p)y_l$$
$$\Delta_L(a) = ay_l$$

The Nash bargaining solution is given by:

$$\begin{aligned} \alpha = &\arg\max_{a\in[0,1]} \Delta_B(a)\Delta_L(a) \\ \text{s.t.} \quad &\Delta_B(a) \geq 0 \quad \text{i.e.} \quad ay_l \leq py_l \\ &\Delta_L(a) \geq 0 \quad \text{i.e.} \quad ay_l \geq 0 \end{aligned}$$

Solving the above problem we get:

$$\alpha = \frac{1}{2}p$$

To interpret this result, since the lender's threat to the borrower is that the borrower's endowment will decrease by py_l, they are really bargaining over how to split py_l. Hence the outcome from bargaining is to split py_l in half because we have assumed that they have an equal bargaining power.

Now let us characterize the equilibrium debt level, bond price, and investment under the full information setting which we call the first best allocation. Because the lenders are competitive they compete to maximize the borrower's utility.

Definition 3. *The first best contract* (q^{FB}, B^{FB}, I^{FB}) is the optimal contract achieved under full information, bargaining setting. It maximizes the borrower's payoff subject to the lender's zero-profit condition:

$$\max_{I,q,B} \quad (qB - I) + \pi(I)(y_h - B) + (1 - \pi(I))y_l(1 - \alpha) \qquad (\mathcal{P}^{\mathcal{FB}})$$
$$\text{s.t.} \quad \pi(I)(B) + (1 - \pi(I))\alpha y_l = qB \qquad (2.1)$$
$$qB - I \geq 0$$

where $\alpha = \frac{1}{2}p$.

Solving this, I^{FB} and B^{FB} are given by:

$$\pi'(I^{FB})(y_h - y_l) = 1 \qquad (2.2)$$

$$B^{FB} = \frac{I^{FB} - (1 - \pi(I^{FB}))\alpha y_l}{\pi(I^{FB})}$$
$$c_0 = q^{FB}B^{FB} - I^{FB} = 0$$

Equation (2.2) says that the borrower invests such that the marginal benefit of increasing investment (an increase in date 1 consumption) equals the marginal cost (a decrease in date 0 consumption). If I was lower than I^{FB}, an additional investment would yield extra date 1 consumption that more than compensates for the decrease in date 0 consumption.[48][49] Next let us introduce private information and characterize the moral hazard problem it creates.

2.1 *Private Information*

Under private information, the borrower's output is observable, but his investment and consumption decision is not observable, and hence cannot be contracted upon. In this case, if the borrower is offered the first best loan contract (B^{FB}, q^{FB}), the borrower will not invest I^{FB} since his optimization problem is:

$$\max_{I} \quad (q^{FB}B^{FB} - I) + \pi(I)(y_h - B^{FB}) + (1 - \pi(I))y_l(1 - \alpha)$$

[48] In this setup, there is no point in borrowing more than I^{FB}, investing I^{FB}, and consuming the rest, $qB - I^{FB}$, since the borrower will have to repay this amount back in full in the high state. Thus, we can safely restrict our attention to the case where he will borrow $qB = I^{FB}$ and invest all of it without consuming any.

[49] Note that the lender is ex-ante competitive but ex-post non-competitive and hence bargain with the borrower. This is a feature common in sovereign debt bargaining models. A motivation for this is that potential new lenders, afraid that a troubled borrower is going to default on somebody (including themselves), do not lend until they see the borrower repay the incumbent lender. Thus, until some repayment is made to the incumbent lender, the borrower cannot access the competitive loan market and the incumbent lender has some market power over the borrower. Kovrijnykh and Szentes (2007) show formally how this lender's switch from competitive in the pre-default stage to noncompetitive ex-post can arise endogenously when old debt is senior to new debt. Nevertheless, we have checked that the main intuition of our model that CDS can alleviate moral hazard would still hold if the lender have as much bargaining power ex-ante as ex-post.

where FOC with respect to I gives:

$$\pi'(I)(y_h - y_l - (B^{FB} - \alpha y_l)) = 1 \tag{2.3}$$

Comparing (2.3) with (2.2), we see that as long as $B^{FB} \geq \alpha y_l$, due to the concavity of $\pi(I)$, the borrower will invest less than I^{FB} and consume the rest.

To see the intuition, with the loan contract (q^{FB}, B^{FB}) the borrower has secured himself the consumption profile of $y_h - B^{FB}$ in the good state and $y_l(1-\alpha)$ in the bad state regardless of his investment. Thus, he can cheat and consume, unobserved by the lender, some of the $q^{FB}B^{FB}$ that he was supposed to invest. This is the moral hazard problem.

Before characterizing the second best contract, the functional form of $\pi(I)$ is assumed to be: $\pi(I) = \sqrt{I}$ where I is investment as a fraction of the steady state output.[50][51] Given this functional form, the parameter condition under which the borrower has an incentive to invest less than the first best, $B^{FB} \geq \alpha y_l$, is:[52]

$$(y_h - y_l)^2 \geq 2py_l \tag{2.4}$$

We assume that condition (2.4) holds, hence, the borrower has an incentive to cheat and there is moral hazard problem. Since the lender will not break even with the first best contract, the lender has to offer a contract that accounts for a such behavior of the borrower (i.e. it has to be incentive compatible) while still maximizing the borrower's utility and the lender himself breaks even.

Definition 4. *The second best (SB)* loan contract (q^{SB}, B^{SB}) and investment, I^{SB}, under private information and bargaining is an incentive compatible contract given by the solution to Program $\mathcal{P}^{SB}$:[53]

$$\max_{q,B,I} \quad qB - I + \pi(I)(y_h - B) + (1-\pi(I))(y_l - \alpha y_l) \tag{$\mathcal{P}^{SB}$}$$

$$\text{s.t.} \quad \pi'(I)(y_h - B - (1-\alpha)y_l) = 1 \tag{IC_B}$$

$$\pi(I)B + (1-\pi(I))\alpha y_l = qB \tag{IR_L}$$

$$c_0 = qB - I \geq 0$$

where, as before, $\alpha = \frac{1}{2}p$. (IR_L) is the lender's zero profit condition and (IC_B) is the incentive compatibility constraint.

[50] All the other variables are in steady state output units as well.

[51] For a general specification, $\pi(I) = I^\gamma$ where $\gamma < 1$, there is no analytical solution, but we have computationally checked that the main results of our paper still hold. For example, Figure 19 looks qualitatively the same for $\gamma < 1$.

[52]

$$B^{FB} \geq \alpha y_l \quad \Leftrightarrow \quad \frac{I^{FB} - (1-\pi(I^{FB})\alpha y_l}{\pi(I^{FB})} \geq \alpha y_l \quad \Leftrightarrow \quad I^{FB} - \alpha y_l + \pi(I^{FB})\alpha y_l \geq \pi(I^{FB})\alpha y_l$$

$$\Leftrightarrow \quad I^{FB} \geq \alpha y_l. \quad \text{For } \pi(I) = I^\gamma \text{ and } \gamma = 0.5 \quad \sqrt{I^{FB}} = \frac{1}{2}(y_h - y_l) \Rightarrow \quad (y_h - y_l)^2 \geq 2py_l$$

[53] The appendix shows that the constraint $c_0 \geq 0$ will hold with equality.

Comparing (2.2) with (IC_B), we see that since $\pi(I)$ is concave, the second best investment will be less than the first best as long as: $B^{SB} \geq \frac{1}{2}py_l$. Thus, the moral hazard problem constrains borrowing and results in an investment less than the first best. The utility achieved under private information will always be less than under full information because of the extra (IC) constraint.

2.2 *Credit insurance*

Up to now, the results on moral hazard are standard.[54] Now let us introduce an insurance market where the lender can buy a protection against default from risk neutral competitive CDS insurers. An insurance contract with notional, i, insures the lender up to the amount i in case of a credit event. A credit event here is defined as a full default by the borrower, in other words the lender has not agreed to any debt reduction. We assume the lender's insurance activity is observable by the borrower.[55]

When negotiations fail, the lender now receives i instead of getting nothing; thus, an insurance improves his outside option and the lender's surplus from bargaining is now:

$$\Delta_L(a) = ay_l - i$$

Figure **??** reflects this change.

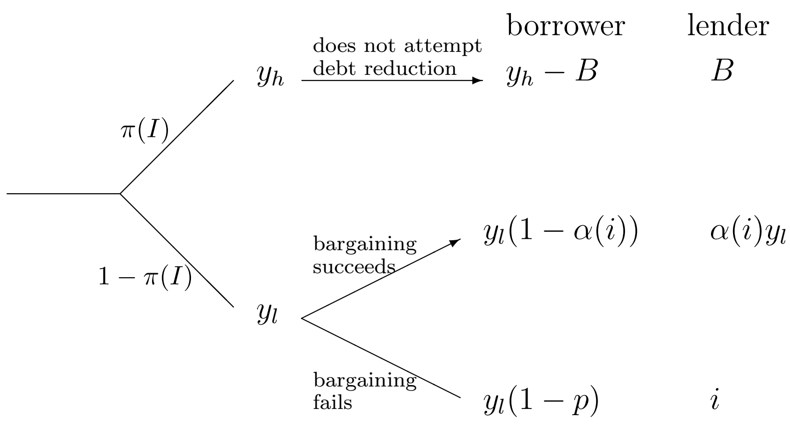


Figure 17: Date 1 subgame with insurance

[54] See, for instance, Atkeson (1991) and Gertler and Rogoff (1990).

[55] We implicitly rule out the lender and the borrower together falsifying the credit event so that the lender still gets paid by the borrower some amount while lying to the insurer about the credit event and getting an insurance payment. Thus, we assume a credit event is contractible. A similar situation that we rule out is the lender and the insurer negotiating ex-post so that the insurer pays less than what was contracted; in return, the lender does not reject debt restructuring and gets paid by the borrower also.

Now when the low output y_l is realized, the bargaining outcome depends on the amount of insurance purchased, i. Solving the bargaining problem as before, the share the lender extracts from the borrower is:

$$\alpha(i) = \frac{1}{2}p + \frac{1}{2y_l}i \quad \text{if } i \leq py_l \tag{2.5}$$

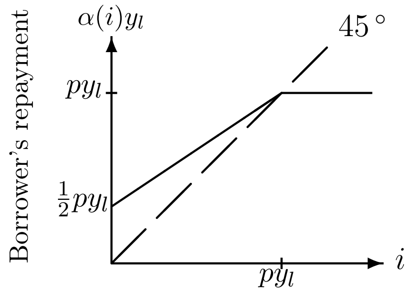


Figure 18: Nash Bargaining Solution

Figure 18 shows the solution graphically. If $i = 0$, we are back to the no insurance case. On the interval $i \in [0, py_l)$, as the amount of insurance purchased increases, the lender's share increases and will be strictly larger than the insurance itself i (i.e. the 45° line). The lender will strictly prefer restructuring over insurance. When $i = py_l$, $\alpha(i)y_l$ crosses the 45° line and the payment from the borrower (in case of restructuring) will exactly equal the insurance amount (in case of default). Thus, the lender will be indifferent between receiving the py_l from the borrower (by accepting restructuring) or from the insurer (by rejecting restructuring) and we model the lender's decision as choosing the best mixed strategy. Whether the lender accepts or rejects, the borrower's endowment decreases by the same amount (because the borrower either has repaid py_l or defaulted and was penalized by py_l). If $i > py_l$, the most the lender can get from the borrower is py_l, thus the lender will reject any restructuring and trigger insurance, and the borrower will suffer an output loss of py_l.

To characterize the zero-profit conditions of the lender and the insurer, we first subdivide i into three intervals based on what the lender will do on each interval as discussed above: 1) $i \leq py_l$, the lender always accepts restructuring, 2) $i = py_l$, the lender is indifferent between accepting or rejecting and getting insurance, and 3) $i > py_l$, the lender claims insurance. The lender's zero-profit condition when he purchases insurance is:

$$\pi(I)B + (1 - \pi(I))c^L = qB + m \tag{2.6}$$

where c^L is the lender's consumption in the low state. On the first interval, since the lender restructures and does not claim insurance, the payment from the insurer is zero. Thus, $m = (1 - \pi(I))0 = 0$, $c^L = \alpha(i)y_l$, and $c^B = (1 - \alpha(i))y_l$ where c^B is the borrower's consumption in the low output state.[56] When indifferent, the lender uses a mixed strategy (ω, $(1-\omega)$) where ω is the probability he will accept restructuring and $(1-\omega)$ is the probability he will claim insurance. Insurance premium then is $m = (1-\pi(I))(1-\omega)i$ while $c^L = \omega\alpha(i)y_l + (1-\omega)i$ and $c^B = \big(\omega(1-\alpha(i)) + (1-\omega)(1-p)\big)y_l = (1-p)y_l$. On the third interval, since the borrower defaults, the insurer pays the lender the full i ($c^L = i$) and $m = (1-\pi(I))i$ while $c^B = (1-p)y_l$.

We now characterize the equilibrium when an insurance market exits.

Definition 5. The optimal loan contract (q, B), I, and i when there exists an insurance market is such that it is incentive compatible for the borrower and maximizes the borrower's utility subject to the zero profit conditions of the lender and the insurer.

$$\begin{aligned}
\max_{q,B,I,i,\omega} \quad & (qB - I) + \pi(I)(y_h - B) + (1-\pi(I))c^B \qquad (\mathcal{P}^{\mathcal{SB},\text{ins}}) \\
\text{s.t.} \quad & \pi'(I)(y_h - B - c^B) = 1 \\
& \pi(I)B + (1-\pi(I))c^L = qB + m \\
& c_0 = qB - I \geq 0
\end{aligned}$$

where m, c^B, and c^L in each of the three intervals of i are:

$$m = \begin{cases} (1-\pi(I))0 = 0 \\ (1-\pi(I))(1-\omega)i \\ (1-\pi(I))i \end{cases} \qquad c^B = \begin{cases} (1-\alpha(i))y_l & \text{if } i < py_l \\ (1-p)y_l & \text{if } i = py_l \\ (1-p)y_l & \text{if } i > py_l \end{cases}$$

$$c^L = \begin{cases} \alpha(i)y_l & \text{if } i < py_l \\ \omega\alpha(i)y_l + (1-\omega)i & \text{if } i = py_l \\ i & \text{if } i > py_l \end{cases}$$

and $\alpha(i)$ is given by (2.5).

From solving the above problem, we arrive at the main results of our paper that compares the effects of insurance to the second best without insurance:

Proposition 8. *The optimal insurance is:* $i^* = min\{\frac{1}{2}(y_h - y_l)^2 - py_l, py_l\}$. *Specifically:*

$$\text{if } (y_h - y_l)^2 < 4py_l, \text{ then it's an 'interior' solution: } i^* = \frac{1}{2}(y_h - y_l)^2 - py_l \tag{2.7}$$

$$\text{if } (y_h - y_l)^2 \geq 4py_l, \text{ then it's a 'corner' solution: } i^* = py_l \tag{2.8}$$

Proof. See appendix □

Figure 19 demonstrates the result by showing the borrower's utility as a function of i. In Figure 19(a), condition (2.7) holds, in which case i^* is given by an interior solution on the interval $[0, py_l]$. In this case, the

[56] See the end of section 2.1 for more discussion about insurance premium equalling zero.

constraint $i_l \leq p_l y_l$ does not bind, allowing i_l to be as high as it needs to be, and there is a complete alleviation of moral hazard: $I^{ins} = I^{FB}$. Whereas in Figure 19(b), condition (2.8) holds and the borrower's utility is strictly increasing in i on the interval $[0, py_l]$. Thus, the optimal insurance is given by the corner solution $i^* = py_l$, in which case moral hazard is only partially alleviated. When $i = py_l$, the lender plays a mixed strategy, but the borrower's utility is increasing in ω, hence the best mixed strategy is the degenerate one: $\omega = 1$. When $i > py_l$, the borrower would be worse off than the second best. This is because py_l - the amount the borrower gets penalized by - is a deadweight cost that no one benefits from; it is better if it instead gets used to repay the loan.

Proposition 9. *Comparing Program $\mathcal{P}^{\mathcal{SB},ins}$ with the benchmark without CDS:*

(i) The borrower is better off: $U^{ins} \geq U^{SB}$.

(ii) Investment increases: $I^{ins} \geq I^{SB}$, and the probability of default $(1 - \pi(I))$ decreases.

(iii) The borrower is more indebted: $B^{ins} \geq B^{SB}$.

(iv) The bond price increases: $q^{ins} \geq q^{SB}$, or equivalently, the borrowing cost decreases.

Proof. See appendix □

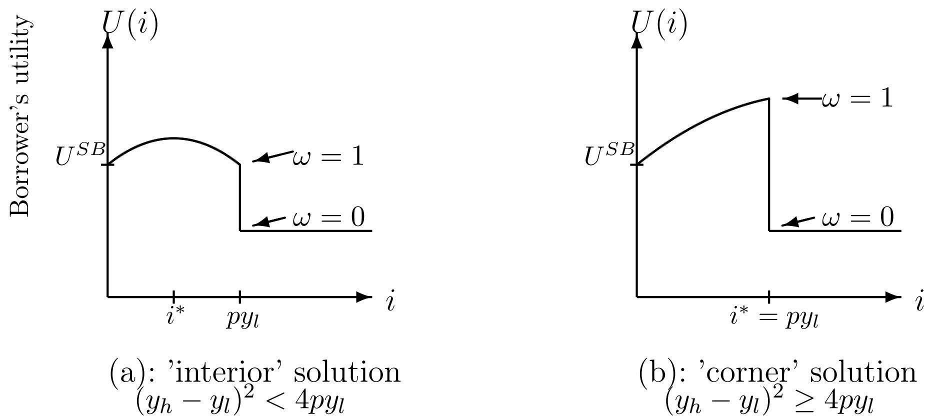


Figure 19: Optimal Insurance

The intuition for Proposition 9(i) is as follows. As long as there is the moral hazard problem given by condition (2.4), the existence of insurance improves the borrower's welfare. This is because it turns out that (2.4) is also the condition for $U'(i = 0) > 0$, and hence a positive level of insurance is pareto optimal. Condition (2.4) is satisfied if, for instance, y_l is small compared to y_h or the volatility of output is high.

Proposition 9(ii) shows that the lender's insurance activity disciplines the borrower. In the bad state, if the borrower's offer is not high enough compared to lender's insurance, debt reduction negotiation will fail and the borrower will be penalized. To avoid being penalized, the borrower will have to pay more than when insurance did not exist. Consequently, the low output state looks less attractive to the borrower, thus he will invest more (i.e. closer to the first best amount) to avoid it.

Since the borrower finds it optimal to invest more to avoid the bad outcome, Proposition 9(iii) illustrates that it requires more borrowing (an increase in the face value of the bond). Thus, as CDS alleviates agency frictions, the sovereign is able to raise more capital.

As CDS's disciplining effect induces the borrower to invest more, Proposition 9(iv) shows that the probability of the high state (hence full repayment) increases. Moreover, the borrower pays more in the low state. As a result, the bond price is higher and the sovereign's borrowing cost is lower.

Discussion of the corner solution. When $i^* = py_l$ (Figure 19(b)), the lender buys just enough insurance to make him indifferent between accepting the debt restructuring $\alpha(i^*) = py_l$ and rejecting it and triggering an insurance payment. This amount of insurance lets him extract the maximum possible amount from the borrower. Although ex-post he will be indifferent between accepting or rejecting the debt restructuring, ex-ante it is optimal to always get repaid by the borrower and not file a claim with the insurer (i.e. play the degenerate mixed strategy: $\omega = 1$). That way the insurance premium is the cheapest possible (zero, to be specific), hence the borrowing cost is the lowest possible. In Figure 4(b), we can see that any $\omega < 1$ would not be an equilibrium since for such ω, there is an ϵ such that $U(i^* - \epsilon) > U(\omega | i = py_l)$. Thus, we implicitly assume that the lender can credibly commit to, ex-post, always accept the payment from the borrower and not the insurer although he is indifferent.[57] In the end, the reason an insurance makes a difference is that, before with no insurance as an outside option, the lender could not credibly reject a partial repayment and punish the borrower. But now he can credibly reject any restructuring offer less than the insurance purchased.

Remark on insurance premium. Insurance in this context becomes a costless mechanism to extract the maximum repayment possible from the borrower. The reason for the zero price for an insurance contract is due to the assumptions that the only credit event is a full default and that there are only two output realizations where in the low output state the borrower ends up, in equilibrium, paying partially. Thus, in equilibrium, there is never a full default and the insurer never has to make any payment. But the zero cost of insurance does not have to be taken literally. As shown in Figure 20, if the support of the output has a state where the output realized is zero and the sovereign does not have any means to pay, then there is an automatic default in that state. Thus, there is always a state in which insurance will be paid out to the lender so that the insurance premium will be positive. Although this kind of

[57] If the lender's credibility is an issue, a policy implication could be to limit $i \leq py_l - \epsilon$ where ϵ is a small number. Then, the insurance purchased would be $i = py_l - \epsilon$, which would be slightly less than the share achieved in negotiations with that amount of insurance as the outside option: $\alpha(i) = py_l - \frac{\epsilon}{2}$. The outcome in this case will still be better than the second best.

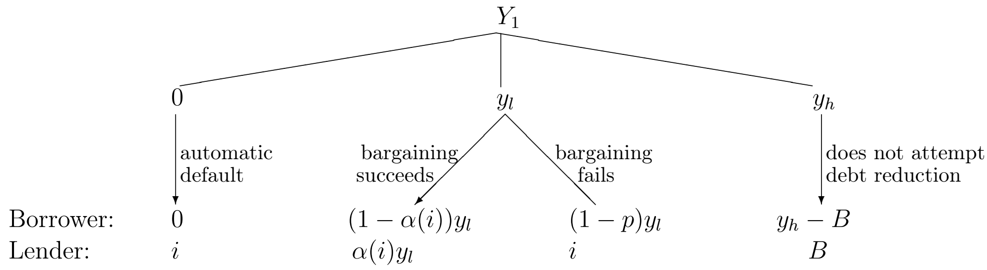


Figure 20: Date 1 subgame with insurance with three states

general setting may be desirable, the main results are likely to be the same.

2.3 *Restructuring*

So far we have assumed that only a full default is a credit event. In this section, we relax this assumption and allow debt restructuring to be a credit event also.[58]

If there is a debt restructuring and the lender files a claim with the insurer, the payment made from the insurer is the incured loss up to i which is the difference between i and the recovery value. In particular, if αy_l is the recovery value (what the lender receives from the borrower through debt restructuring), then the insurer pays the remaining $i - \alpha y_l$.[59] Thus, when a debt restructuring is a credit event, the lender is always indifferent between accepting the partial repayment αy_l and getting the remaining $i - \alpha y_l$ from the insurer versus completely rejecting a debt

[58] Market participants follow credit event definitions developed by the International Swaps and Derivative Association (ISDA) as a legal framework. In ISDA definitions, restructuring is included as a credit event in sovereign CDSs as long as it was due to a deterioration in the creditworthiness of the sovereign. But a voluntary restructuring is not included and there is an ambiguity in terms of what constitutes as a voluntary. It is common now for sovereigns to offer voluntary exchange offers as a way to reduce debt payments. Even though the lender often has no other option but to accept the offer and the exchange entails a deterioration in the creditworthiness of the sovereign (e.g. an extension in the maturity or a reduction in the interest payments), the creditor could be considered to have agreed to the restructuring. There have not been many cases of restructuring and subsequent CDS payment triggers to help us draw conclusions. One exception is the New York court case, Eternity Global Master Fund vs. Morgan Guaranty, regarding Argentina's debt restructuring in November of 2001 before its actual default in December of 2001. The november debt exchange entailed both lower yields and longer maturity. Following the debt exchange, Eternity demanded payments from Morgan Guaranty, the protection seller, arguing that there was a debt restructuring but Morgan Guaranty argued that it was not a credit event because it was voluntary. Eventually, the court ruled on Morgan's side. See Verdier (2004) for more discussion.

[59] This is analogous to cash settlement.

reduction (and having the borrower default in full) and getting the insured amount, i, in full from the insurer:

$$c^L = \begin{cases} \alpha y_l + (i - \alpha y_l) = i & \text{if accepts restructuring} \\ i & \text{if rejects restructuring} \end{cases}$$

Thus, α cannot be uniquely determined from the bargaining problem because the lender's surplus from bargaining is always zero.

However, the borrower's optimal repayment in the low state can be uniquely determined from the perspective of date 0. Let us denote the repayment in the bad state as $\tilde{\alpha}$, where the tilde is to notationally distinguish it from α, which was determined from the bargaining problem. Since the lender is always indifferent, suppose he accepts restructuring with a probability ω and rejects it with a probability $1-\omega$. Then we can solve solve $\tilde{\alpha}$ along with the optimal loan contract, investment, ω, and insurance that maximizes the borrower's utility subject to the incentive compatibility constraint and the zero-profit conditions of the lender and the insurer:

$$\begin{aligned} \max_{q,B,I,\tilde{\alpha},\omega,i} \; & (qB - I) + \pi(I)(y_h - B) + (1-\pi(I))\{\omega(1-\tilde{\alpha})y_l + (1-\omega)(1-p)y_l\} \\ \text{s.t.} \quad & \pi'(I)\big(y_h - B - \{\omega(1-\tilde{\alpha})y_l + (1-\omega)(1-p)y_l\}\big) = 1 \\ & \pi(I)B + (1-\pi(I))\big(\omega(\alpha y_l + (i - \tilde{\alpha} y_l)) + (1-\omega)i\big) = qB + m \\ & m = (1-\pi(I))\big(\omega(i - \tilde{\alpha} y_l) + (1-\omega)i\big) \\ & c_0 = qB - I \geq 0 \\ & i \geq \tilde{\alpha} y_l \end{aligned}$$

Solving this problem, the optimal $\tilde{\alpha}$ and ω are:

$$\omega^* = 1 \quad \text{and} \quad \tilde{\alpha}^* = min\left(\frac{(y_h - y_l)^2}{4y_l}, p\right)$$

Remember that when restructuring was not a credit event, the optimal insurance was $i^* = min\{\frac{1}{2}(y_h - y_l)^2 - py_l, py_l\}$, so that $\alpha(i^*) = \frac{1}{2}p + \frac{i^*}{2y} = min\big(\frac{(y_h-y_l)^2}{4y_l}, p\big)$. Thus, the optimal repayment when y_l is realized is exactly the same as when restructuring was not a credit event and there is as much alleviation of moral hazard as before. Thus, the optimal investment and utility achieved does not depend on whether restructuring is a credit event or not.

Although ex-post, at date 1, the lender would be indifferent between accepting any restructuring $\tilde{\alpha} \leq \tilde{\alpha}^*$ (since he will be compensated by the insurer up to i on the remaining $(i - \tilde{\alpha} y_l)$), ex-ante it is optimal to not accept any $\tilde{\alpha} < \tilde{\alpha}^*$ so that there is as much alleviation of moral hazard as possible. This outcome hinges upon the lender's credibility to ex-post do what was ex-ante optimal. Nevertheless, how does insurance make a difference in this case? Before, a lender's threat that he will reject any $\tilde{\alpha} < \tilde{\alpha}^*$ was not credible because without insurance he strictly preferred accepting restructuring over rejecting it. But now, insurance makes the lender's threat credible precisely because the lender is now indifferent.

The only difference in results from allowing restructuring to be a credit event is that there is no unique equilibrium insurance level as long as $i \geq \tilde{\alpha}^* y$. Suppose $i > \tilde{\alpha}^* y$ and consider the zero-profit conditions of the lender and the insurer after substituting in $\omega^* = 1$:

$$\pi(I)B + (1 - \pi(I))(\tilde{\alpha}^* y_l + (i - \tilde{\alpha}^* y_l)) = qB + m \tag{2.9}$$

$$m = (1 - \pi(I))(i - \tilde{\alpha}^* y_l) \tag{2.10}$$

From (2.10), the higher the payment from the insurer, $i - \tilde{\alpha}^* y$, the higher the insurance price, m. But, from (2.9), the increase in the insurance price gets exactly offset by the increase in the lender's consumption when y_l is realized. So there would be no point in purchasing $i > \tilde{\alpha}^* y$.

3 NAKED CDS BUYERS

We have assumed so far that the lenders are the only ones that purchase insurance when in reality there are 'naked buyers' who purchase insurance but do not own the underlying bond. In this section we consider how the results of the previous section change with the existence of naked buyers.

We assume that the naked buyer has cash flows correlated with the output of the sovereign: his endowment is high in state H and low in state L. He is risk averse and buys sovereign CDS to insure against the low output state. These assumptions are motivated by the following example: suppose that an investor has an investment in a Greek firm but CDSs on the firm itself do not exist or are relatively illiquid. He could instead purchase CDS on a Greek government bond because during the states of the world where the government is struggling, the private sector is likely to be struggling as well.[60]

If the CDS market is perfectly competitive, as in the previous section, then the existence of naked buyers does not affect the lender-borrower contract. Even if the insurer is a monopolist but the naked buyer is risk neutral, then again the existence of naked buyers does not make any difference. Thus, we assume that the CDS market is imperfectly competitive (the CDS seller is a monopolist, to be specific) and that the

[60] There could be other reasons for naked buying. One reason could be due to heterogenous beliefs about the likelihood of default: an investor might think that a particular government or company is more or less likely to default than is suggested by CDS prices. Another reason could be due to the fact that CDS trading is often done through dealers who buy and sell CDS without holding the underlying security. For example, suppose bank X with Greek government bonds wants to decrease its exposure to default risk and purchases CDS from bank A. Bank A wants to hedge its increased exposure so it purchases CDS on Greek government bond from another party, Bank B. Bank B does the same as Bank A and purchases CDS from Bank Y. Bank Y, on the other hand, is willing to bear the risk of Greek default and hence does not purchase CDS. In this example, banks X and Y were the end users of CDS while banks A and B acted as the dealers and would be considered 'naked buyers' as they purchased CDS without actually holding the underlying security. Dealers contribute to the liquidity of CDS market as they eliminate the need for Bank X, in our example, to directly find the end user, Bank Y, who is willing to take the opposite position of Bank X.

naked buyer is risk averse. We first show how the problem changes when the insurer is a monopolist instead of perfectly competitive and this will be our new "benchmark." Then we introduce naked buyers and compare the result with that of the benchmark scenario without the naked buyers.

3.1 *Benchmark: A Monopolist Insurer*

The lender's problem is the same as before: he is choosing the loan contract that maximizes the borrower's utility and is incentive compatible for the borrower. But now the monopolist insurer makes the lender take-it-or-leave-it offer for insurance contract so that the lender takes the price and the quantity of insurance as given. Let (i_l, m_l) denote the insurance contract bought by the lender. Then, the lender's problem is:

$$
\begin{aligned}
U(i_l, m_l) \equiv \max_{q,B,I} \quad & (qB - I) + \pi(I)(y_h - B) + (1 - \pi(I))(1 - \alpha(i_l))y_l \\
\text{s.t.} \quad & \pi'(I)(y_h - B - (1 - \alpha(i_l))y_l) = 1 \\
& \pi(I)B + (1 - \pi(I))\alpha(i_l)y_l = qB + m_l \\
& c_0 = qB - I \geq 0 \\
& i_l \leq py_l
\end{aligned}
$$

Note that the resulting investment, debt level, and bond price will be functions of insurance level i_l and price m_l: $I(i_l, m_l)$, $B(i_l, m_l)$, $q(i_l, m_l)$.

The equilibrium insurance level and insurance price will be determined by the monopolist's profit maximization problem. The insurer maximizes his profit subject to the lender's individual rationality constraint: if the lender purchases insurance, it has to make him (and hence the borrower) at least better off than without the insurance. The insurer's profit is just the price charged for the insurance as we have previously explained at the end of section 2.2.

Definition 6. The equilibrium insurance bought, i_l^{mon}, and premium charged, m_l^{mon}, when the insurer is a monopolist will be the solution to Program $\mathcal{P}^{Benchmark}$:

$$
\begin{aligned}
\{i_l^{mon}, m_l^{mon}\} \equiv \arg\max_{i_l, m_l} \; & m_l \\
\text{s.t.} \quad & U(i_l, m_l) \geq U^{SB}
\end{aligned}
\tag{$\mathcal{P}^{Benchmark}$}
$$

where U^{SB} is the borrower's utility achieved when the lender does not purchase any insurance, $U^{SB} = U(i_l = 0, m_l = 0)$, and is the same as the second best of the previous section.

From the equilibrium insurance level and insurance price we can derive the equilibrium investment level $I^{mon} \equiv I(i_l^{mon}, m_l^{mon})$. Proposition 10 compares the equilibrium investment level when the insurer is a monopolist, I^{mon}, with the equilibrium investment level when the insurer is perfectly competitive, I^{ins}:

Proposition 10.

$$\begin{array}{lll} I^{SB} \leq I^{mon} = I^{ins} = I^{FB} & \textit{if} & 2p_l y_l \leq (y_h - y_l)^2 \leq 4p_l y_l \\ I^{SB} \leq I^{mon} = I^{ins} < I^{FB} & \textit{if} & (y_h - y_l)^2 > 4p_l y_l \end{array}$$

Proof. See Appendix □

Thus the equilibrium investment level is exactly the same as when the insurer was competitive ($I^{mon} = I^{ins}$). Consequently, $I^{mon} \geq I^{SB}$ which means insurance still alleviates the moral hazard problem and increases the social welfare. The increase in social welfare is exactly the same as before but, in contrast to the previous section, the increased welfare only goes to the insurer and none to the borrower. The insurer achieves this through a higher insurance price while keeping the borrower's payoff the same as the second best. Because the lender is now paying a higher price for the insurance, the lender accounts for this in the bond price he charges the borrower; thus compared to the last section borrowing cost is higher when the insurer is a monopolist.

3.2 *Naked CDS Buyers*

We now introduce naked buyers who are risk averse and have cash flows that are correlated with that of the sovereign's: its cash flow is likely to be high when the sovereign has a high output state realization and vice versa. The naked buyer's cash flow is c_h with probability $\pi(I)$ and c_l with probability $1 - \pi(I)$. Thus, implicit in this assumption is that the sovereign's action (i.e. investment) affects the naked buyer's cash flow: the probability of the high cash flow state is increasing in the sovereign's investment. We assume the naked buyer buys a type of CDS where a restructuring is considered a credit event. If the naked buyer buys a CDS where only full default is a credit event, then it will not provide him with any insurance since a full default never occurs in equilibrium. Let (i_n, m_n) denote the insurance contract sold to the naked buyer. Thus, when y_l is realized and there is a debt restructuring, the insurance pays i_n minus the recovery value $\alpha(i_l)y_l$ to the naked buyer. When y_h is realized there is no credit event and hence no payments from the insurer. The insurance premium, m_n, is paid in both states. The naked buyer's consumption is:

$$c = \begin{cases} c_h - m_n & \text{if } y_h \\ c_l + i_n - \alpha(i_l)y_l - m_n & \text{if } y_l \end{cases}$$

Therefore, the naked buyer has the following expected utility:

$$U^N(i_n, m_n) = \pi(I)u(c_h - m_n) + (1 - \pi(I))u(c_l + i_n - \alpha(i_l)y_l - m_n)$$

The insurer now maximizes his profit over two sets of insurance contracts: one for the lender (i_l, m_l) and one for the naked buyer (i_n, m_n). Each insurance contract has to be individually rational: the lender is at least better off with (i_l, m_l) than without it, and the naked buyer is also at least better off with insurance (i_n, m_n) than without it. We assume that the insurer can tell apart between the lender and the naked buyer such that incentive compatibility constraints (that the naked buyer will prefer the contract designed for him rather than the one for the lender and vice versa) are not imposed.[61]

Definition 7. The equilibrium insurance contracts (i_l^*, m_l^*) and (i_n^*, m_n^*) and hence the equilibrium investment level when there is a naked buyer who buys insurance are given by the solution to Program $\mathcal{P}^{spec}$:

$$\max_{i_l, m_l, i_n, m_n} \; m_l + m_n - (1 - \pi(I))(i_n - \alpha(i_l) y_l) \tag{$\mathcal{P}^{spec}$}$$

$$\text{s.t.} \quad U(i_l, m_l) \geq U^{SB} \tag{IR^L}$$

$$U^N(i_n, m_n) \geq U^N(0, 0) \tag{IR^N}$$

The main result of this section is given next.

Proposition 11. *If the parameter condition is such that* (A.9) *holds, then in equilibrium there is an over-investment ($I^* \geq I^{mon}$) relative to the benchmark case ($\mathcal{P}^{Benchmark}$) without the naked buyers and the cost of borrowing is lower ($q^* \geq q^{mon}$).*

Proof. See appendix. □

To see the intuition, let us first simplify Program $\mathcal{P}^{spec}$. The appendix shows that both of the individual rationality constraints of Program $\mathcal{P}^{spec}$ bind. Moreover, $i_n - \alpha(i_l) y_l = c_h - c_l$. Denote $\sigma_c \equiv c_h - c_l$. Then after some algebra, Program $\mathcal{P}^{spec}$ becomes:

$$\max_{i_l, m_n} \quad m_l(i_l) + m_n - (1 - \pi(I(i_l)))\sigma_c$$

$$\text{s.t.} \quad u(c_h - m_n) = \pi(I(i_l)) u(c_h) + (1 - \pi(I(i_l))) u(c_l) \tag{3.1}$$

We can further simplify by solving for m_n from eq. (3.1) and substituting it into the objective function. Program $\mathcal{P}_{spec}$ boils down to maximizing over only one variable which is the lender's insurance level:

$$\max_{i_l} \quad m_l(i_l) + m_n(i_l) - (1 - \pi(I(i_l)))\sigma_c$$

[61] This is because the problem becomes analytically intractable with the lender's incentive compatibility constraint. By design the naked buyerÕs insurance contract is incentive compatible: the naked buyer will prefer the contract designed for him rather than the one for the lender. In equilibrium there is only a debt restructuring and never a full default and the naked buyerÕs insurance contract includes restructuring as a credit event while the lenderÕs does not. We have checked numerically that imposing lender's incentive compatibility constraints do not qualitatively change our results.

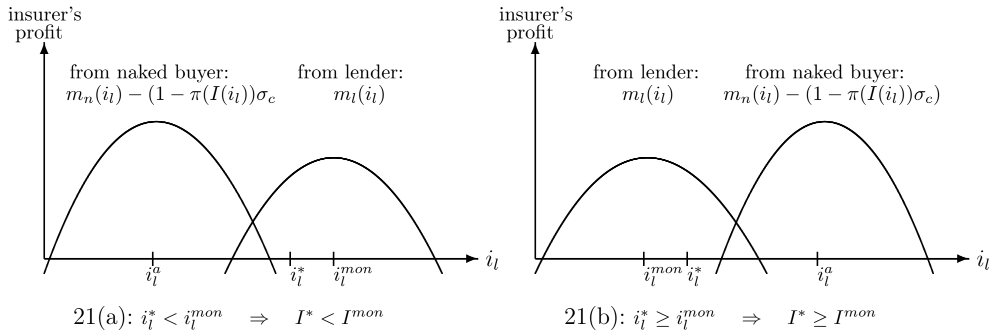


Figure 21: Naked Buyers

The objective function of the insurer (i.e. his total profit) is comprised of profits from insuring both the lender, $m_l(i_l)$, and the naked buyer, $m_n(i_l) - (1 - \pi(I(i_l)))\sigma_c$. Figure 21 shows a stylized representation of the insurer's problem: it plots the insurer's profit as a function of the insurance sold to the lender (i_l). In panel (a), the parameter conditions are such that the profit function from insuring the naked buyer is to the left of the lender's meaning that the maximum profit received from the naked buyer occurs at a lower i_l than the profit from the lender. To determine the optimal insurance level for the lender, i_l^*, we know from looking at the graph that i_l^* will not be less than the level that maximizes the profit from insuring the naked buyer (denoted by i^a) because if it is, then by increasing i_l the insurer can increase the profit he receives both from the naked buyer and the lender.

Similarly, i_l^* cannot be greater than i_l^{mon}, the level that maximizes the profit from the lender. Thus, i_l^* will be between i^a and i_l^{mon}. In fact, i_l^* will be where the marginal benefit of an extra i_l (increase in profit received from the lender) equals the marginal cost (decrease in profit from the naked buyer). More importantly, i_l^* will be less than i_l^{mon} which was the equilibrium insurance in the no-naked-buyers case: $m_l'(i_l^{mon}) = 0$. Since investment is an increasing function of i_l, the equilibrium investment is lower than without the naked buyer: $I^* < I^{mon}$. The equilibrium investment of the no-naked-buyers case resulted in as much alleviation of moral hazard as possible (e.g. $I^{mon} = I^{FB}$ if corner solution); thus when $I^* < I^{mon}$ the existence of a naked buyer impedes the alleviation of moral hazard. In panel (b), the parameter conditions are the opposite of panel (a). By the same arguments, we have that $i_l^* \geq i_l^{mon}$ and hence an over-investment.

What is going on is that due to insuring the naked buyer, the insurer profits from insuring not only the lender but also the naked buyer where the profit from the latter is affected by the borrower's investment. Since the insurer can indirectly control the borrower's investment through the insurance contract offered to the lender, (i_l, m_l), the insurer chooses (i_l, m_l) and hence the borrower's investment to maximize the sum of the two profits. Depending on the naked buyer's preferences and cash flows c_h and c_l, it can be more profitable to induce a higher borrower investment as in panel (b) or a lower borrower investment as in panel (a). For example, when the cash flow of the naked buyer in the low state (c_l) is very low, the insurer will have to pay out a larger claim, σ_c, to the naked buyer in the low state since the optimal insurance for the naked buyer completely smoothes his consumption across states. In this case, it is more profitable to the insurer to induce a higher borrower investment so that the state in which he has to make a large net transfer occurs with a lower probability.

4 CONCLUSION

Motivated by the concerns raised over the use of credit default swaps during the recent European sovereign debt crises, we ask: could credit default swaps be beneficial for the debtor country, and if so, why? We find that CDSs can be beneficial for the borrower because they can serve as a disciplining mechanism in an environment with debtor moral hazard and debt renegotiation. Specifically, the moral hazard problem arises from the assumption of private information about the borrower's investment, and, as a consequence, results in credit rationing and suboptimal investment level. In this framework, we find that insurance serves as a commitment device for a lender to credibly reject low levels of repayment from the borrower, thereby, increasing the lender's bargaining power in ex-post debt renegotiations. The increased bargaining power of the lender, in turn, alleviates ex-ante borrowing constraints, provides more external capital to the debtor country, and increases investment and, hence, welfare. Thus, the existence of an insurance market alleviates the moral hazard problem by better aligning the lender and borrower's incentives.

Using this framework, we also analyze the effect of naked buyers who do not lend directly to the sovereign. If there are naked buyers who purchase insurance, our analysis shows that the market structure of the insurer's market could be important. If the insurer's market is relatively competitive, our model suggests that the existence of naked buyers should have no impact on the debtor country. While if the insurer's market is concentrated, the existence of naked buyers could either lead to an over-investment or impede the alleviation of moral hazard. Nevertheless, this paper raises some issues in favor of CDS, thus putting a larger onus on those who call for regulation to come up with serious analyses of the detrimental aspects of CDSs.

A APPENDIX: PROOFS

Claim 1. $c_0 = 0$ in Program $\mathcal{P}^{SB}$ of section 2.1.[62]

Proof. The Lagrangian is given by:

$$\begin{aligned}\mathcal{L} = qB - I + \pi(I)(y_h - B) + (1-\pi(I))(y_l - \alpha y_l)\\ + \mu[\pi'(I)(y_h - B - (1-\alpha)y_l) - 1]\\ + \psi[\pi(I)B + (1-\pi(I))\alpha y_l - qB]\\ + \lambda[qB - I]\end{aligned}$$

The first order conditions with respect to I, B, and q are:

$$-1+\pi'(I)(y_h-B-(1-\alpha)y_l)+\mu\pi''(I)(y_h-B-(1-\alpha)y_l)+\psi\pi'(I)(B-\alpha y_l)-\lambda=0 \quad (FOC_I)$$

$$q - \pi(I) - \mu\pi'(I) + \psi[\pi(I) - q] + \lambda q = 0 \qquad (FOC_B)$$

$$B - \psi B + \lambda B = 0 \qquad (FOC_q)$$

$$\lambda(qB - I) = 0$$

Suppose $\lambda = 0$, then from the FOC with respect to q,

$$\psi = 1$$

From the FOC with respect to I, $\mu\pi''(I)/\pi'(I) = -\psi\pi'(I)(B - \alpha y_l) < 0$ which implies $\mu > 0$ since $\pi''(I) < 0$. However, from the FOC with respect to B, $\mu\pi'(I) = 0$ but this is a contradiction since $\pi'(I) > 0$ and $\mu > 0$.

We first solve for the optimal investment and the utility achieved without insurance (i.e. the second best) which will be used to compare to the case with insurance. Then Proposition 8 will follow from Lemmas 3 and 4 shown below. First, solving the second best contract, we get:

$$\begin{cases}\pi'(I)(y_h - B - (1-\alpha^{SB})y_l) = 1\\ \pi(I)B + (1-\pi(I))\alpha^{SB}y_l = I\end{cases}$$

Rearranging it we get,

$$\begin{cases}B = -\frac{1}{\pi'(I)} + y_h - (1-\alpha^{SB})y_l\\ B = \frac{I-(1-\pi(I))\alpha^{SB}y_l}{\pi(I)}\end{cases}$$

Together they imply,

$$\sqrt{I^{SB}} = \frac{y_h - y_l + \sqrt{(y_h-y_l)^2 + 12\alpha^{SB}y_l}}{6}$$

and consequently the borrower's utility is:

$$U^{SB} = \frac{(y_h-y_l)^2}{9} + \frac{(y_h-y_l)\sqrt{(y_h-y_l)^2+12\alpha^{SB}y_l}}{9} - \frac{\alpha^{SB}y_l}{3} + y_l$$

□

Lemma 3. *Let i_1 be the optimal insurance on the interval $i \le py_l$. If $(y_h - y_l)^2 \ge 4py_l$, then $\alpha = p$ and $i_1 = py_l$. If $(y_h-y_l)^2 < 4py_l$, then $\alpha = \frac{(y_h-y_l)^2}{4y_l}$, and $i_1 = \frac{1}{2}(y_h-y_l)^2 - py_l$*

Proof. We first find the optimal investment, the loan, and the utility level for a fixed insurance i on the interval $[0, py_l]$ and denote them as $I_1(i)$, $B_1(i)$, and $U_1(i)$, respectively. Then we find i that maximizes $U_1(i)$ on the interval $[0, py_l]$, i.e. $i_1 \equiv \arg\max U_1(i)$.

[62] The same argument holds for Program $\mathcal{P}^{SB,ins}$.

$I_1(i)$, $B_1(i)$ for a fixed i on $[0, py_l]$ are given by the solution to:

$$\begin{cases} \pi'(I)(y_h - B - (1-\alpha(i))y_l) = 1 \\ \pi(I)B + (1-\pi(I))\alpha(i)y_l = I \end{cases}$$

Rearranging it,

$$\begin{cases} B = -\frac{1}{\pi'(I)} + y_h - (1-\alpha(i))y_l \\ B = \frac{I-(1-\pi(I))\alpha(i)y_l}{\pi(I)} \end{cases}$$

Together they imply:

$$\sqrt{I_1(i)} = \frac{1}{6}(y_h - y_l + \sqrt{(y_h-y_l)^2 + 12\alpha(i)y_l}) \tag{A.1}$$

The utility achieved for a given i is then:

$$\begin{aligned} U_1(i) =& \pi(I)y_h - \pi(I)B + (1-\pi(I))(y_l - \alpha(i)y_l) \\ =& \frac{1}{9}(y_h-y_l)^2 + \frac{1}{9}(y_h-y_l)\sqrt{(y_h-y_l)^2 + 12\alpha(i)y_l} - \frac{1}{3}\alpha(i)y_l + y_l \end{aligned}$$

We now solve for i such that $\frac{\partial U_1}{\partial i} = \frac{\partial U_1}{\partial \alpha}\frac{\partial \alpha}{\partial i} = 0$. Since $\alpha(i) = \frac{1}{2}p + \frac{1}{2y_l}i$, $\frac{\partial \alpha}{\partial i} > 0$. Then α such that $\frac{\partial U_1}{\partial \alpha} = 0$ is given by:

$$\alpha = \frac{(y_h-y_l)^2}{4y_l}$$

This implies:

$$\alpha = \begin{cases} \frac{(y_h-y_l)^2}{4y_l} & \text{if} \quad (y_h-y_l)^2 < 4py_l \\ p & \text{if} \quad (y_h-y_l)^2 \geq 4py_l \end{cases}$$

and consequently,

$$i_1 = \begin{cases} \frac{1}{2}(y_h-y_l)^2 - py_l & \text{if} \quad (y_h-y_l)^2 < 4py_l \\ py_l & \text{if} \quad (y_h-y_l)^2 \geq 4py_l \end{cases}$$

□

Lemma 4. *Let $\bar{U_1}$ and $\bar{U_2}$ be the utilities achieved with the optimal insurance on the intervals $i \leq py_l$ and $i \geq py_l$, respectively. Then $\bar{U_1} \geq \bar{U_2}$.*

Proof. Let $\bar{I_1}$, $\bar{B_1}$, and $\bar{U_1}$ be the values achieved with the optimal insurance i_1, i.e. $\bar{I_1} \equiv I_1(i_1)$, $\bar{B_1} \equiv B_1(i_1)$, and $\bar{U_1} \equiv U_1(i_1)$. Likewise, let $\bar{I_2}$, $\bar{B_2}$, and $\bar{U_2}$ be the optimal values on $i \geq py_l$. We show the proof by the following four steps: (i) solve for $\bar{I_2}$, (ii) show $\bar{B_1} \leq \bar{B_2}$, (iii) show $\bar{I_1} \geq \bar{I_2}$, and then as a consequence: (iv) $\bar{U_1} \geq \bar{U_2}$.

Step (i). Solve for $\bar{I_2}$

We do the same as in Lemma 3 but constraining insurance to be at least greater than py_l. We first solve for the optimal investment $I_2(i)$ for a fixed insurance i for $i \geq py_l$:

$$\begin{cases} \pi'(I)(y_h - B - (1-p)y_l) = 1 \\ \pi(I)B = I \end{cases}$$

Rearranging it we get,

$$\begin{cases} B = -\frac{1}{\pi'(I)} + y_h - (1-p)y_l \\ B = \frac{I}{\pi(I)} \end{cases}$$

Together they imply,

$$\sqrt{I_2(i)} = \frac{y_h - y_l + py_l}{3} = \frac{y_h - y_l}{3} + \frac{py_l}{3} \tag{A.2}$$

Note that investment in this case does not depend on i. Thus, $\bar{I}_2$ is given by (A.2).

Step (ii). Proof of $\bar{B}_1 \leq \bar{B}_2$

$$\bar{B}_2 = \frac{\pi(I)}{I} = \frac{y_h - y_l + py_l}{3}$$

For the interior case, where $\frac{1}{4}(y_h - y_l)^2 \leq py_l$*:*

$$\bar{B}_1 = -2\sqrt{I_1} + y_h - (y_l - \alpha(i_1)y_l) = \alpha y_l = \frac{1}{4}(y_h - y_l)^2 \leq py_l$$

$$\bar{B}_2 \geq \frac{(y_h - y_l)^2 + py_l}{3} \geq \frac{3py_l}{3} = py_l$$

where the last inequality is due to the moral hazard condition. Thus, $\bar{B}_1 \leq \bar{B}_2$.
For the corner case:

$$\bar{B}_1 = -\frac{(y_h - y_l) + \sqrt{(y_h - y_l)^2 + 12py_l}}{3} + y_h - y_l + py_l$$

$$\Rightarrow \quad \bar{B}_1 - \bar{B}_2 = \frac{1}{3}(y_h - y_l) - \frac{\sqrt{(y_h - y_l)^2 + 12py_l}}{3} + \frac{2}{3}py_l$$

Suppose $\bar{B}_1 - \bar{B}_2 > 0$, then:

$$(y_h - y_l) + 2py_l > \sqrt{(y_h - y_l)^2 + 12py_l}$$

Rearranging it,

$$py_l(y_h - y_l) + (py_l)^2 > 3py_l \tag{A.3}$$

But the LHS of (A.3) is:

$$py_l(y_h - y_l) + (py_l)^2 \leq py_l(y_h - y_l) + py_l = py_l(y_h - y_l + 1) \leq 2py_l$$

which is a contradiction, thus, $\bar{B}_1 \leq \bar{B}_2$.

Step (iii). Proof of $\bar{I}_1 \geq \bar{I}_2$
For the interior solution case:

$$\alpha(i_1)y_l = \frac{1}{4}(y_h - y_l)^2$$

$$\sqrt{\bar{I}_1} = \sqrt{I_1(i_1)} = \frac{1}{6}(y_h - y_l + \sqrt{4(y_h - y_l)^2}) = \frac{y_h - y_l}{2} = \frac{y_h - y_l}{3} + \frac{y_h - y_l}{6}$$

From the moral hazard condition $(y_h - y_l)^2 \geq 2py_l$ and using the fact that $y_h - y_l \leq 1$:

$$\frac{y_h - y_l}{6} \geq \frac{py_l}{3}$$

For the corner solution case where $(y_h - y_l)^2 \geq 4py_l$*:*

$$\alpha(i_1)y_l = py_l$$

$$\pi'(\bar{I}_1)(y_h - \bar{B}_1 - (1-p)y_l) = 1$$

$$\pi'(\bar{I}_2)(y_h - \bar{B}_2 - (1-p)y_l) = 1$$

Thus, due to the concavity of $\pi(I)$, when $\bar{B}_1 \leq \bar{B}_2$, we have $\bar{I}_1 \geq \bar{I}_2$.

Step (iv). Proof of $\bar{U}_1 \geq \bar{U}_2$

Using (i), (ii), and (iii) it is straightforward to see the result from:

$$\bar{U_1} = \pi(\bar{I_1})(y_h - \bar{B_1}) + (1 - \pi(\bar{I_1}))(1-p)y_l = \pi(\bar{I_1})(y_h - \bar{B_1} - (1-p)y_l) + (1-p)y_l$$
$$\bar{U_2} = \pi(\bar{I_2})(y_h - \bar{B_2}) + (1 - \pi(\bar{I_2}))(1-p)y_l = \pi(\bar{I_2})(y_h - \bar{B_2} - (1-p)y_l) + (1-p)y_l$$

□

Proof of Proposition 9(i) . U^{ins} is the maximum utility achieved when an insurance market exits. Thus, $U^{ins} = \bar{U_1}$ since we have shown that $\bar{U_1} \geq \bar{U_2}$. But using the solution of the second best contract we have: $U^{SB} = U_1(i)|_{i=0}$, but $U_1(i)|_{i=0} \leq \max_i U_1(i) \equiv \bar{U_1}$ and the result follows. □

Proof of Proposition 9(ii). From the solution to the second best contract: $I^{SB} = I_1(i = 0)$. But $I_1(i)$ is increasing in insurance on $[0, py_l]$ from (A.1) in Lemma 1.1. □

Proof of Proposition 9(iii).

$$\begin{aligned} B(i) &= -\frac{1}{\pi'(I(i))} + y_h - (1-\alpha(i))y_l = -2\sqrt{I(i)} + y_h - y_l + \frac{1}{2}py_l + \frac{1}{2}i \\ &= -\frac{1}{3}(y_h - y_l + \sqrt{(y_h-y_l)^2 + 6py_l + 6i}) + y_h - y_l + \frac{1}{2}py_l + \frac{1}{2}i \end{aligned}$$

We find that $B(i)$ is convex since

$$B''(i) = 3((y_h - y_l)^2 + 6py_l + 6i)^{-\frac{3}{2}} > 0$$

We will that show i^* is less than i^{min} where $B'(i^{min}) = 0$

$$B'(i) = -\frac{1}{\sqrt{(y_h-y_l)^2 + 6py_l + 6i}} + \frac{1}{2} = 0$$

This implies,

$$i^{min} = \frac{2}{3} - \frac{y_h - y_l}{6} - py_l$$

For the interior solution case where $i^* = \frac{y_h - y_l}{2} - py_l$, $(y_h - y_l)^2 \leq 1$. Hence,

$$\frac{y_h - y_l}{2} \leq \frac{2}{3} - \frac{y_h - y_l}{6}$$

For the corner solution case, since $i^* = py_l \leq \frac{y_h - y_l}{2} - py_l$, the same argument as for the interior solution holds. □

Proof of Proposition 9(iv). Since $q = \frac{I}{B}$, the proof is a corollary of Propositions 9(ii) and 9(iii). □

Proof of Proposition 10. First, we show that the lender's IR constraint binds in Program $\mathcal{P}^{Benchmark}$:

$$\begin{aligned} \max_{i_l, m_l} \quad & m_l \\ \text{s.t.} \quad & U(i_l, m_l) \geq U^{SB} \end{aligned}$$

where $U^{SB} = (y_h - y_l)^2/9 + (y_h - y_l)\sqrt{(y_h - y_l)^2 + 6p_l y_l}/9 - p_l y_l/6 + y_l$.

In the lender's problem, the constraint $qB - I \geq 0$ binds as before, so B and I can be solved as functions of i_l and m_l from:

$$\begin{cases} \pi'(I)(y_h - B - (1-\alpha(i_l))y_l) = 1 \\ \pi(I)B + (1-\pi(I))\alpha(i_l)y_l = I + m_l \end{cases}$$

To simplify the complexity of the notations, let us define:

$$\sigma_y \equiv y_h - y_l$$

$$u \equiv U^{SB} - y_l$$

$$x(i_l, m_l) \equiv \sqrt{I(i_l, m_l)}$$

I and U are given by:

$$\sqrt{I(i_l, m_l)} = \frac{\sigma_y + \sqrt{\sigma_y^2 + 12(\frac{1}{2}p_l y_l + \frac{1}{2}i_l - m_l)}}{6}$$

$$\begin{aligned} U(i_l, m_l) &= -m_l - I(i_l, m_l) + \sqrt{I(i_l, m_l)}\sigma_y + y_l \\ &= -m_l - x(i_l, m_l)^2 + x(i_l, m_l)\sigma_y + y_l \end{aligned}$$

Then Program $\mathcal{P}^{Benchmark}$ becomes:

$$\begin{aligned} \max_{i_l, m_l} \quad & m_l \\ \text{s.t.} \quad & -m_l - x(i_l, m_l)^2 + x(i_l, m_l)\sigma_y + y_l \geq U^{SB} \\ & i_l \leq p_l y_l \end{aligned}$$

The first order conditions are:

$$FOC_{m_l}: \quad 1 + \lambda_1(-1 - 2x x_m + x_m \sigma_y) = 0$$

$$FOC_{i_l}: \quad \lambda_1(-2x x_{i_l} + x_{i_l}\sigma_y) + \lambda_2 = 0$$

where λ_1 and λ_2 are the Lagrange multipliers on the first and second constraints respectively. We see from FOC_{m_l} that if $\lambda_1 = 0$, we get a contradiction. Thus, the constraint binds.

Next, we see from FOC_{i_l} that $x = \frac{\sigma_y}{2}$, that is $I = I^{FB}$. To solve for the optimal insurance, we first solve for m_l as a function of i_l. It can be shown that:

$$m_l(i_l) = -\frac{3}{2}\left(u + \frac{1}{6}(p_l y_l + i_l)\right) + \frac{\sigma_y}{2}\sqrt{2u + p_l y_l + i_l}$$

Then Program $\mathcal{P}^{Benchmark}$ boils down to:

$$\begin{aligned} \max_{i_l} \quad & m_l(i_l) \\ & i_l \leq p_l y_l \end{aligned}$$

Taking the first order condition with respect to i_l, optimal i_l^{mon} is given by:

$$1 = \sigma_y \frac{1}{\sqrt{p_l y_l + i_l^{mon} + 2u}}$$

From here,

$$i_l^{mon} = \min\{-2u + \sigma_y^2 - p_l y_l, p_l y_l\}$$

To check again that $\sqrt{I^{mon}} = \frac{1}{2}\sigma_y$ for the interior case:

$$m(i_l^{mon}) = -\frac{3}{2}\left(\frac{1}{6}\sigma_y^2 + \frac{2}{3}u\right) + \frac{1}{2}\sigma_y^2 = \frac{1}{4}\sigma_y^2 - u$$

$$\frac{1}{2}(p_l y_l + i_l^{mon}) - m_l(i_l^{mon}) = -u + \frac{1}{2}\sigma_y^2 - \frac{1}{4}\sigma_y^2 + u = \frac{1}{4}\sigma_y^2$$

Plug the above in the expression for I:

$$\sqrt{I(i_l^{mon}, m_l)} = \frac{\sigma_y\sqrt{\sigma_y^2 + 3\sigma_y^2}}{6} = \frac{1}{2}\sigma_y$$

The corner solution case.

$$U(i_l, m_l) \geq U^{SB}$$

$$-I^{mon} + (y_h - y_l)\sqrt{I^{mon}} - m_l - y_l \geq -I^{SB} + (y_h - y_l)\sqrt{I^{SB}} - y_l$$

$$-I^{mon} + (y_h - y_l)\sqrt{I^{mon}} \geq m_l - I^{SB} + (y_h - y_l)\sqrt{I^{SB}} \tag{A.4}$$

From (A.4) we see that the borrower over-invests,

$$I^{mon} \geq I^{SB}$$

□

Proof of Proposition 11. To simplify the notation, define: $\sigma_c \equiv c_h - c_l$, $x_m \equiv \frac{\partial x(i_l,m_l)}{\partial m_l}$, and $x_i \equiv \frac{\partial x(i_l,m_l)}{\partial i_l}$. The proof consists of four steps.

Step 1: We show that in Program $\mathcal{P}_{spec}$ the individual rationality constraints bind and that $i_n - \alpha(i_l)y_l = c_h - c_l$.

$$\begin{aligned}
\max_{i_l,m_l,i_n,m_n} \quad & m_l + m_n - (1 - x(i_l, m_l))(i_n - (p_l y_l + i_l)/2 \\
\text{s.t.} \quad & -m_l - x(i_l, m_l)^2 + x(i_l, m_l)\sigma_y + y_l \geq U^{SB} \\
& x(i_l, m_l)\sqrt{c_h - m_n} + (1 - x(i_l, m_l))\sqrt{c_l + i_n - (p_l y_l + i_l)/2 - m_n} \geq \\
& \qquad x(i_l, m_l)u(c_h) + (1 - x(i_l, m_l))u(c_l) \\
& i_l \leq p_l y_l
\end{aligned}$$

Let λ_1, λ_2, and λ_3 be the Lagrange multipliers of the above three constraints respectively. The first order conditions are:
FOC_{m_l} :

$$\begin{aligned}
1 + x_m(i_n - (p_l y_l + i_l)/2) + \lambda_1(-1 - 2xx_m + x_m\sigma_y) + \lambda_2 x_m\big(\sqrt{c_h - m_n} - \\
\sqrt{c_l + i_n - (p_l y_l + i_l)/2 - m_n} - (\sqrt{c_h} - \sqrt{c_l})\big) = 0
\end{aligned}$$

FOC_{i_l} :

$$\begin{aligned}
& x_i\big(i_n - (p_l y_l + i_l)/2\big) + (1 - x)/2 + \lambda_1(-2xx_i + x_i\sigma_y) + \lambda_2\Bigg(x_i\big(\sqrt{c_h - m_n} \\
& - \sqrt{c_l + i_n - (p_l y_l + i_l)/2 - m_n} - (\sqrt{c_h} - \sqrt{c_l})\big) - \frac{(1 - x)/2}{2\sqrt{c_l + i_n - (p_l y_l + i_l)/2 - m_n}}\Bigg) + \lambda_3 = 0
\end{aligned}$$

FOC_{m_n} :

$$1 - \frac{1}{2}\lambda_2\Big(\frac{x}{\sqrt{c_h - m_n}} + \frac{1 - x}{\sqrt{c_l + i_n - (p_l y_l + i_l)/2 - m_n}}\Big) = 0$$

FOC_{i_n} :
$$(1-x)=\lambda_2\frac{1-x}{2\sqrt{c_l+i_n-(p_ly_l+i_l)/2-m_n}}$$
Then from either FOC_{m_n} or FOC_{i_n}, $\lambda_2\neq 0$. And from both FOC_{m_n} and FOC_{i_n} we have that:
$$\sqrt{c_h-m_n}=\sqrt{c_l+i_n-(p_ly_l+i_l)/2-m_n}$$
This implies: $i_n-(p_ly_l+i_l)/2=c_h-c_l$ and $\lambda_2=2\sqrt{c_h-m_n}$. Suppose $\lambda_1=0$, then using $\lambda_2=2\sqrt{c_h-m_n}$, FOC_{m_l} becomes:
$$1+x_m\sigma_y-2x_m\sqrt{c_h-m_n}(\sqrt{c_h}-\sqrt{c_l})=0 \tag{A.5}$$
Rearranging,
$$\sqrt{c_h-m_n}=\frac{x_i\sigma_c-\frac{1}{2}}{2x_i(\sqrt{c_h}-\sqrt{c_l})} \tag{A.6}$$
And since $\lambda_2\neq 0$, $\sqrt{c_h-m_n}=x\sqrt{c_h}+(1-x)\sqrt{c_l}$. Plug this in to (A.6):
$$2(x(\sqrt{c_h}-\sqrt{c_l})^2+\sqrt{c_h}(\sqrt{c_h}-\sqrt{c_l}))=\sigma_c-\sqrt{\sigma_y^2+12(\frac{1}{2}p_ly_l+\frac{1}{2}i_l-m_l)} \tag{A.7}$$
Define $a\equiv\sqrt{\sigma_y^2+12(\frac{1}{2}p_ly_l+\frac{1}{2}i_l-m_l)}$ then from (A.7),
$$(\frac{\sigma_y}{3}+\frac{a}{3})(\sqrt{c_h}-\sqrt{c_l})^2+2\sqrt{c_h}(\sqrt{c_h}-\sqrt{c_l})=\sigma_c-a$$
Rearranging,
$$a=\frac{\sigma_c-2\sqrt{c_h}(\sqrt{c_h}-\sqrt{c_l})-\frac{\sigma_y}{3}(\sqrt{c_h}-\sqrt{c_l})^2}{\frac{1}{3}(\sqrt{c_h}-\sqrt{c_l})^2+1}=\frac{-(\sqrt{c_h}-\sqrt{c_l})^2-\frac{\sigma_y}{3}(\sqrt{c_h}-\sqrt{c_l})^2}{\frac{1}{3}(\sqrt{c_h}-\sqrt{c_l})^2+1}\leq 0$$
which is a contraction.

Step 2: After the simplifications from step 1, in this step we derive the optimal insurance contracts (i_l, m_l), (i_n, m_n) and λ_1.
They are given by the following five equations:
$$\begin{aligned}
FOC_{m_l}:&\quad 1+x_{m_l}\sigma_c+\lambda_1(-1-2xx_m+x_m\sigma_y)-2\sqrt{c_h-m_n}x_m(\sqrt{c_h}-\sqrt{c_l})=0\\
FOC_{i_l}:&\quad \sigma_c+\lambda_1(-2x+\sigma_y)-2\sqrt{c_h-m_n}(\sqrt{c_h}-\sqrt{c_l})=0\\
FOC_{i_n}\ \&\ FOC_{m_n}:&\quad i_n=\frac{1}{2}(py+i_l)+\sigma_c\\
&\quad -m_l-x^2+x\sigma_y-u=0\\
&\quad \sqrt{c_h-m_n}=x(\sqrt{c_h}-\sqrt{c_l})+\sqrt{c_l}
\end{aligned} \tag{A.8}$$
From FOC_{m_l} and FOC_{i_l}:
$$\begin{aligned}1+x_m\sigma_c+\frac{2\sqrt{c_h-m_n}(\sqrt{c_h}-\sqrt{c_l})-\sigma_c}{-2x+\sigma_y}(-1)+(2\sqrt{c_h-m}(\sqrt{c_h}-\sqrt{c_l})-\sigma_c)x_m\\
-2\sqrt{c_h-m_n}x_m(\sqrt{c_h}-\sqrt{c_l})=0\end{aligned}$$
Rearranging,
$$2\sqrt{c_h-m_n}(\sqrt{c_h}-\sqrt{c_l})-\sigma_c=-2x+\sigma_y$$
Then the above together with (A.8) implies
$$2x(\sqrt{c_h}-\sqrt{c_l})^2+2\sqrt{c_l}(\sqrt{c_h}-\sqrt{c_l})-\sigma_c=-2x+\sigma_y$$

which rearranged becomes,

$$x = \frac{\frac{1}{2}(\sigma_y\sigma_c) - \sqrt{c_l}(\sqrt{c_h} - \sqrt{c_l})}{1 + (\sqrt{c_h} - \sqrt{c_l})^2}$$

Define $g \equiv \frac{\frac{1}{2}(\sigma_y\sigma_c) - \sqrt{c_l}(\sqrt{c_h} - \sqrt{c_l})}{1 + (\sqrt{c_h} - \sqrt{c_l})^2}$, then

$$\begin{aligned} x(i_l) = x(i_l, m_l(i_l)) &= \left(\sigma_y \sqrt{\sigma_y^2 + 6(p_l y_l + i_l) - 12 m_l(i_l))}\right)/6 \\ &= \left(\sigma_y \sqrt{\sigma_y^2 + 9(p_l y_l + i_l) + 18u - 2\sigma_y \sqrt{9(p_l y_l + i_l) + 18u}}\right)/6 \end{aligned}$$

Setting $x(i_l)$ equal to g, i_l can be solved as:

$$i_l = 4g^2 - 2u - p_l y_l$$

Thus, the optimal insurance contract is given by:

$$\begin{aligned} i_l^* =& \min\{4g^2 - 2u - p y_l, p_l y_l\} \\ m_l^* =& \begin{cases} -u - g^2 + \sigma_y g & \text{if} \quad i_l \leq p_l y_l \\ -\frac{3}{2}\left(u + \frac{1}{3} p_l y_l\right) + \frac{\sigma_y}{2}\sqrt{2u + 2p_l y_l} & \text{if} \quad i_l = p_l y_l \end{cases} \\ i_n^* =& \frac{1}{2}(p_l y_l + i) + \sigma_c = \min\{2g^2 - u + \sigma_c, p_l y_l + \sigma_c\} \\ m_n^* =& c_h - (x(\sqrt{c_h} - \sqrt{c_l}) + \sqrt{c_l})^2 \end{aligned}$$

Step 3: Over and under-investment

Consider the following four cases:

$$\begin{array}{ll} \text{Case 1:} & \max\{\sigma_y^2 - 2u - p y_l, 4g^2 - 2u - p y_l\} \leq p y_l \\ \text{Case 2:} & 4g^2 - 2u - p y_l \leq p y_l \leq \sigma_y^2 - 2u - p y_l \\ \text{Case 3:} & \sigma_y^2 - 2u - p y_l \leq p y_l \leq 4g^2 - 2u - p y_l \\ \text{Case 4:} & p y_l \leq \min\{\sigma_y^2 - 2u - p y_l, 4g^2 - 2u - p y_l\} \end{array}$$

Case 1. In this case, both i^{mon} and i_l^* are given by the respective interior solutions. There is an over-investment if $I^{mon} = \frac{1}{2}\sigma_y \leq g = I^*$, an under-investment if otherwise. For example, if $c_h = y_h$ and $c_l = y_l$, the condition $g \geq \frac{1}{2}\sigma_y$ boils down to whether $1 \geq \sigma_y$.
Case 2. Here, $i^{mon} = p_l y_l$, i.e. it is a corner solution while i_l^* is an interior solution. Then:

$$I^{mon} = \frac{\sigma_y}{6} + \frac{1}{6}\sqrt{\sigma_y^2 + 18(p_l y_l + u) - 2\sigma_y\sqrt{18(p_l y_l + u)}} \qquad \text{and} \qquad I^* = g$$

Thus, there is an over-investment if

$$g \geq \frac{\sigma_y}{6} + \frac{1}{6}\sqrt{\sigma_y^2 + 18(p_l y_l + u) - 2\sigma_y\sqrt{18(p_l y_l + u)}} \tag{A.9}$$

and an under-investment if otherwise.
Case 3. In this case, $\sigma_y \leq 4g^2$, thus $i^{mon} \leq i^*$. Also $m_l(i_l)$ attains maximum at $i^{mon}(i_l)$ and is concave, thus $m_l(i^*) \leq m_l(i^{mon})$. Hence, $I^{mon} \leq I^*$
Case 4. $i^{mon} = i^* = p_l y_l$, thus again $m_l(i^{mon}) = m_l(i^*)$, hence $I^{mon} = I^*$.

Step 4: Cost of borrowing
Cases 1 and 3. Since $i_l^{mon} = \sigma_y^2 - 2u - p_l y_l$,

$$\pi'(I^{mon})(y_h - B^{mon} - (1 - \alpha(i^{mon}))y_l) = 1$$

As $\pi'(I^{mon})(y_h - y_l) = 1$, we get,

$$B^{mon} = \alpha(i^{mon})y_l$$

Using the solution to the bargaining outcome,

$$B^{mon} = \alpha(i^{mon})y_l = \frac{1}{2}py_l + \frac{1}{2}i^{mon}$$

Compare this to the bond face level without naked CDS buyers:

$$B^* = -2\sqrt{I^*} + y_h - y_l + \frac{1}{2}py_l + \frac{1}{2}i^*$$

The difference is given by:

$$B^{mon} - B^* = \frac{1}{2}(i^{mon} - i^*) + 2\sqrt{I} - (y_h - y_l)$$

Thus, $B^{mon} \geq B^*$ if an over-investment and since $q = \frac{I}{B}$, the cost of borrowing is lower.
Case 2.

$$B^{mon} = -2\sqrt{I^{mon}} + (y_h - y_l) + \frac{1}{2}p_l y_l + \frac{1}{2}i^{mon}$$

Again compare this to the bond face level without naked CDS buyers:

$$B^{mon} - B^* = -2(\sqrt{I^{mon}} - \sqrt{I^*}) + \frac{1}{2}(p_l y_l - i^*)$$

If an over-investment: $-(\sqrt{I^{mon}} - \sqrt{I^*}) \geq 0$ which implies $B^{mon} - B^* \geq 0$. Thus, again cost of borrowing is lower. If an under-investment: it is analytically intractable in this case to show the result about cost of borrowing, so we resort to checking this computationally.
Case 4:
$I^{mon} = I^*$, thus $q^{mon} = q^*$ and the cost of borrowing is the same. □

B APPENDIX: EXTENSIONS

B.1 *The need for both bargaining and private information*

Bargaining but no private information

The equilibrium under full information, bargaining setting without the insurance market is already given by Program $\mathcal{P}^{\mathcal{FB}}$. The equilibrium when there exists an insurance maximizes the borrower's utility subject to the zero-profit conditions of the lender and the insurer:

$$\begin{aligned} \max_{q,B,I,i,\omega} \quad & (qB - I) + \pi(I)(y_h - B) + (1 - \pi(I))c^B \\ \text{s.t.} \quad & \pi(I)B + (1 - \pi(I))c^L = qB + m \\ & c_0 = qB - I \geq 0 \end{aligned} \tag{$\mathcal{P}^{\mathcal{FB},\text{ins}}$}$$

where m, c^B, and c^L in each of the three intervals of i are:

$$m = \begin{cases} (1 - \pi(I))0 = 0 \\ (1 - \pi(I))(1 - \omega)i \\ (1 - \pi(I))i \end{cases} \quad c^B = \begin{cases} (1 - \alpha(i))y_l \\ (1 - p)y_l \\ (1 - p)y_l \end{cases} \quad c^L = \begin{cases} \alpha(i)y_l & \text{if } i < py_l \\ \omega\alpha(i)y_l + (1 - \omega)i & \text{if } i = py_l \\ i & \text{if } i > py_l \end{cases}$$

and $\alpha(i)$ is given by (2.5).

Solving this problem, the optimal investment is given by:

$$\pi'(I)(y_h - y_l) = 1 \tag{B.1}$$

which is exactly the same as when the insurance market did not exist. Moreover, the utility of the borrower, given by $U = -I + \pi(I)y_h + (1 - \pi(I))y_l$, does not depend on insurance and is also exactly as it was before without the insurance. Thus, the lender's credit insurance activity does not matter.

The intuition. First, the lender will not buy insurance that is more than py_l. Since the borrower will at most pay py_l, when $i > py_l$, the lender will prefer full default so that he can get $i > py_l$ from the insurer. However, this increases the cost of insurance and, due to the lender's break-even condition, the lender passes down to the borrower the insurance cost through higher borrowing cost.[63]

By the same argument, $\omega = 1$. Thus, we can narrow down i to $i \leq py_l$ in which case $m = 0$, $c^L = \alpha(i)y_l$, and $c^B = (1 - \alpha(i))y_l$. Substituting them in, Program $\mathcal{P}^{\mathcal{FB},\text{ins}}$ boils down to:

$$\begin{aligned}
\max_{q,B,I,i} \quad & (qB - I) + \pi(I)(y_h - B) + (1 - \pi(I))(1 - \alpha(i))y_l \\
\text{s.t.} \quad & \pi(I)B + (1 - \pi(I))\alpha(i)y_l = qB \\
& c_0 = qB - I \geq 0
\end{aligned}$$

From here, it is straight forward to see that investment is given by (B.1). However, B and q will be functions of i:[64]

$$B(i) = \frac{1}{\pi(I)}\big(I - (1 - \pi(I))\alpha(i)y_l\big) \qquad q = \frac{I}{B(i)}$$

$$q = \frac{I}{B(i)}$$

For $i = 0$, we are back to Program $\mathcal{P}^{\mathcal{FB}}$, while for $0 < i \leq py_l$ all insurance does is make the borrower pay more in the low-output state. Paying more in the low-output state lowers the borrowing cost, or, equivalently, increases the bond price. So without borrowing as much (i.e. B is lower) he is able to raise the same funds, qB, such that: $qB = I^{FB} = I^{FB^*}$. But the borrower's investment and utility does not change with i and, hence, there is no unique optimal insurance level as long as $i \leq py_l$. In the end, insurance does not matter because we still do not have a friction that constrains borrowing and results in a suboptimal investment.

Private information but no bargaining

We now shut off bargaining and show that the CDS market does not make a difference in a private information but full default setting. The optimal contract without the existence of an insurance market is given by the solution to:

$$\begin{aligned}
\max_{q,B,I} \quad & qB - I + \pi(I)(y_h - B) + (1 - \pi(I))y_l(1 - p) \\
\text{s.t.} \quad & \pi'(I)(y_h - B - y_l(1 - p)) = 1 \\
& \pi(I)B + (1 - \pi(I))0 = qB \\
& c_0 = qB - I \geq 0
\end{aligned} \tag{B.2}$$

[63] The py_l that the borrower gets penalized by is a deadweight cost that no one benefits from; it is better if it instead gets used to repay back the loan.

[64] We can again safely assume $qB - I = 0$ as before.

Now with an insurance market:

$$
\begin{aligned}
\max_{q,B,I,i} \quad & qB - I + \pi(I)(y_h - B) + (1-\pi(I))y_l(1-p) \\
\text{s.t.} \quad & \pi'(I)(y_h - B - y_l(1-p)) = 1 \\
& \pi(I)B + (1-\pi(I))i = qB + m && \text{(B.3)} \\
& m = (1-\pi(I))i && \text{(B.4)} \\
& c_0 = qB - I \geq 0
\end{aligned}
$$

Note that (B.3) and (B.4) together gives you exactly (B.2). Thus, the two problems are the same and the lender's insurance activity does not affect the borrower's behavior. This is because without bargaining, the lender's credibility to penalize is no longer in question: the borrower automatically gets penalized when y_l is realized.

B.2 *Default and Bargaining in Both States {H, L}*

In this section, we relax the assumption that the borrower and the lender do not bargain when y_h is realized. We first characterize the first and the second best environment without CDS and the next subsection analyzes how CDS affects the investment level. Figure **??** illustrates the change.

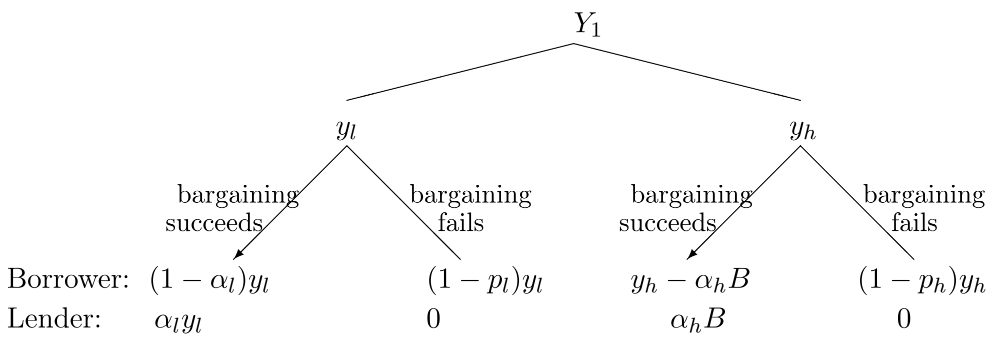


Figure 22: Bargaining in both states

The bargaining problem in the low state is the same as before. The bargaining problem in the high state is given by: $\max_{\alpha_h} \Delta_B \Delta_L$. Consider the product of the bargaining surpluses:

$$\Delta_B \Delta_L = \big(y_h - \alpha_h B - (1-p_h)y_h\big)\alpha_h B = (-\alpha_h B + p_h y_h)\alpha_h B = -B^2\alpha_h^2 + p_h y_h B \alpha_h$$

Maximizing this with respect to α_h, we get $\alpha_h = \frac{p_h y_h}{2B}$, which implies (B.5).

$$\alpha_h B = \frac{p_h y_h}{2} \tag{B.5}$$

The first best environment is characterized as:

$$
\begin{aligned}
\max_{q,B,I} \quad & qB - I + \pi(I)(y_h - \alpha_h B) + (1-\pi(I))(1-\alpha_l)y_l \\
s.t. \quad & \pi(I)\alpha_h B + (1-\pi(I))\alpha_l y_l = qB \\
& qB - I \geq 0
\end{aligned}
$$

Similarly, the second best is the following:

$$\begin{aligned}
\max_{q,B,I} \quad & qB - I + \pi(I)(y_h - \alpha_h B) + (1-\pi(I))(1-\alpha_l)y_l \\
s.t. \quad & \pi'(I)(y_h - \alpha_h B - (1-\alpha_l)y_l) = 1 \\
& \pi(I)\alpha_h B + (1-\pi(I))\alpha_l y_l = qB \\
& qB - I \geq 0
\end{aligned}$$

Substituting in the solutions to the bargaining problem in to the second best environment:

$$\begin{aligned}
\max_{q,B,I} \quad & qB - I + \pi(I)(y_h - \frac{1}{2}p_h y_h) + (1-\pi(I))(y_l - \frac{1}{2}p_l y_l) \\
s.t. \quad & \pi'(I)\big(y_h - \frac{1}{2}p_h y_h - (y_l - \frac{1}{2}p_l y_l)\big) = 1 \\
& \pi(I)\alpha_h B + (1-\pi(I))\alpha_l y_l = qB \\
& qB - I \geq 0
\end{aligned} \tag{B.6}$$

With CDS

We now allow the lender to buy CDS insurance. Figure **??** illustrates the change in the ex-post bargaining problem.

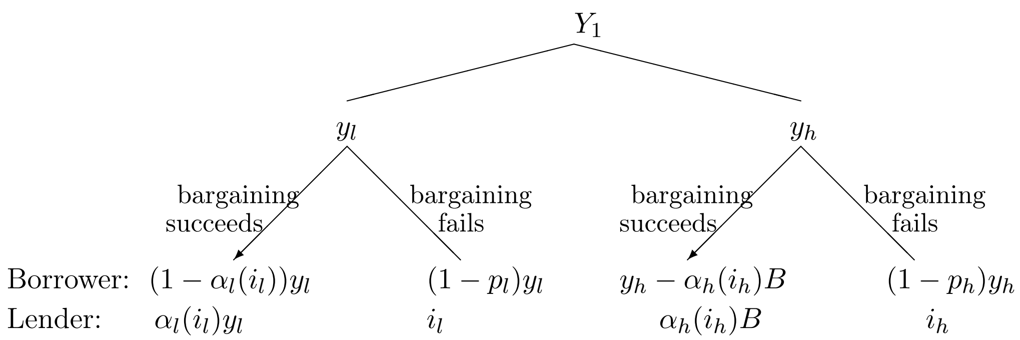


Figure 23: Bargaining in both states with insured lender

The product of the bargaining surpluses in the high state changes to:

$$\begin{aligned}
\Delta_B \Delta_L &= \big(y_h - \alpha_h B - (1-p_h)y_h\big)\big(\alpha_h B - i_h\big) \\
&= (-\alpha_h B + p_h y_h)(\alpha_h B - i_h) \\
&= -B^2\alpha_h^2 + (p_h y_h B + i_h B)\alpha_h - p_h y_h i
\end{aligned}$$

Maximizing this with respect to α_h, we get $\alpha_h(i_h) = \frac{p_h y_h + i_h}{2B}$ which implies (B.7).

$$\alpha_h(i_h)B = \frac{p_h y_h}{2} + \frac{i_h}{2} \tag{B.7}$$

The second best equilibrium with CDS is characterized by the following problem.

$$\begin{aligned}
\max_{q,B,I,i_h,i_l} \ & qB - I + \pi(I)(y_h - \frac{1}{2}(p_h y_h + i_h)) + (1-\pi(I))(y_l - \frac{1}{2}(p_l y_l + i_l)) \\
s.t. \quad & \pi'(I)(y_h - \alpha_h B - (1-\alpha_l)y_l) = 1 \\
& \pi(I)\alpha_h B + (1-\pi(I))\alpha_l y_l = qB \\
& qB - I \geq 0 \\
& i_l \leq p_l y_l \\
& i_h \leq p_h y_h
\end{aligned}$$

Substituting in the solutions to the bargaining problem in both states:

$$\begin{aligned}
\max_{q,B,I,i_h,i_l} \ & qB - I + \pi(I)(y_h - \frac{1}{2}(p_h y_h + i_h)) + (1-\pi(I))(y_l - \frac{1}{2}(p_l y_l + i_l)) \\
s.t. \quad & \pi'(I)\big(y_h - \frac{1}{2}(p_h y_h + i_h) - (y_l - \frac{1}{2}(p_l y_l + i_l)\big) = 1 && \text{(B.8)} \\
& \pi(I)\big(\frac{1}{2}p_h y_h + \frac{1}{2}i_h\big) + (1-\pi(I))\big(\frac{1}{2}p_l y_l + \frac{1}{2}i_l\big) = qB && \text{(B.9)} \\
& qB - I \geq 0 \\
& i_l \leq p_l y_l \\
& i_h \leq p_h y_h
\end{aligned}$$

Note that if $i_h = i_l$, comparing (B.8) with the second best equivalent without CDS (B.6), the borrower will choose the same investment level as in the second best; in other words, the lender's insurance activity will not matter. This is because the borrower's consumption in both states goes down by exactly the same amount ($\frac{1}{2}i_h$ or $\frac{1}{2}i_l$). Thus, the optimal i_l and i_h will have to be different to induce the borrower to invest an amount other than the second best.

We now solve for the optimal insurance level. Comparing (B.8) with the second best equivalent (B.6) rewritten here:

$$\begin{aligned}
&\pi'(I)\big(y_h - y_l - \frac{1}{2}(p_h y_h - p_l y_l) - \frac{1}{2}(i_h - i_l)\big) = 1 \\
&\pi'(I)\big(y_h - y_l - \frac{1}{2}(p_h y_h - p_l y_l)\big) = 1
\end{aligned}$$

we see that due to the concavity of $\pi(I)$, CDS increases investment and thereby alleviates moral hazard only if $-\frac{1}{2}(i_h - i_l) \geq 0$ or $i_l > i_h$. In fact the bigger the difference $i_l - i_h$ is, the bigger the investment. I is increasing in i_l and we can set $i_h = 0$.

Substituting (B.9) into the objective function and canceling terms, we get:

$$\begin{aligned}
\max_{I,i_l} \ & - I + \pi(I)y_h + (1-\pi(I))(1-y_l) && \text{(B.10)} \\
s.t. \quad & \pi'(I)\big(y_h - \frac{1}{2}p_h y_h - (y_l - \frac{1}{2}(p_l y_l + i_l)\big) = 1 && \text{(B.11)} \\
& i_l \leq p_l y_l
\end{aligned}$$

Since we have assumed that $\pi(I) = \sqrt{I}$, (B.11) implies:

$$\sqrt{I} = \frac{1}{2}\left(y_h - y_l - \frac{1}{2}(p_h y_h - p_l y_l) + \frac{1}{2}i_l\right)$$

Substituting the above equation into the objective function (B.10) and maximizing with respect to i_l we get:

$$i_l = min\{p_h y_h - p_l y_l, p_l y_l\}$$

Thus, the borrower's utility is increasing in i_l up until $i_l = p_h y_h - p_l y_l$. When $i_l = p_h y_h - p_l y_l$, the moral hazard is completely alleviated since $I(i_l) = I^{FB}$. However, we have the constraint $i_l \leq p_l y_l$ and if the parameters are such that $p_l y_l \leq p_h y_h - p_l y_l$, then the constraint will bind and the optimal i_l equals $p_l y_l$ and $U^{FB} \geq U^{ins} \geq U^{SB}$. Nevertheless, $U^{ins} \geq U^{SB}$ and the main result of the paper that the lender's insurance activity has a disciplining effect holds in this slightly more general setting. An issue here is the fact that q and B are not identified separately because B is not a control variable anymore. Because of bargaining in both states, how much the borrower ends up repaying is fixed: $\frac{1}{2}p_h y_h$ in the high state and $\frac{1}{2}p_l y_l$ in the low state regardless of the investment level or how much was borrowed initially qB.

B.3 *Uncompetitive Lender*

We have assumed in the benchmark model that lenders are competitive. In this section we relax this assumption and instead consider a lender who has a bargaining power and extracts a positive expected surplus from the loan contract. First, consider the ex-post renegotiation where the bargaining power of the lender and the borrower are β and $1-\beta$, respectively:

$$\max_{\alpha} \Big((1-\alpha)y_l - (1-p)y_l\Big)^{\beta} \Big(\alpha y\Big)^{1-\beta}$$

Solving for α,

$$\beta(-\alpha y + py)^{\beta-1}(\alpha y)^{\beta}(-y) + (1-\beta)(-\alpha y + py)^{\beta}(\alpha y)^{-\beta} y = 0$$

This boils down to

$$\alpha = (1-\beta)p$$

We now use this ex-post renegotiation outcome to solve for the first-best investment level. Thus, consider the first-best environment without private information:

$$\max_{q,B,I,i} \quad U^{\beta} L^{1-\beta} \tag{B.12}$$

$$\text{st:} \quad qB - I \geq 0 \tag{B.13}$$

$$L \geq 0 \tag{B.14}$$

$$U \geq 0 \tag{B.15}$$

where

$$U = qB - I + \pi(I)(y_h - B) + (1-\pi(I))(1-\alpha)y_l - y_l$$
$$L = \pi(I)B + (1-\pi(I))\alpha y_l - qB$$

Throughout I assume that the parameter conditions are such that $U \geq 0$ and $L \geq 0$ are satisfied. Also, let $qB = I$:

$$\max_{B,I} \quad \Big(\pi(I)(y_h - B) + (1-\pi(I))(1-\alpha(i))y_l - y_l\Big)^{\beta} \Big(\pi(I)B + (1-\pi(I))\alpha(i)y_l - I\Big)^{1-\beta} \tag{B.16}$$

Define $x \equiv \pi(I)$, and with the change of variables, we get:

$$\max_{B,x} \quad \Big(x(y_h - B) + (1-x)(1-\alpha)y_l - y_l\Big)^{\beta} \Big(xB + (1-x)\alpha y_l - x^2\Big)^{1-\beta} \tag{B.17}$$

The first order conditions with respect to B:

$$\beta U^{\beta-1} L^{1-\beta}(-x) + (1-\beta) U^{\beta} L^{-\beta} x = 0 \tag{B.18}$$

This implies: $\beta U^{-1} L = (1-\beta)$. The first order condition with respect x is:

$$FOC_x: \quad \beta U^{\beta-1} L^{1-\beta}(y_h - B - (1-\alpha)y_l) + (1-\beta) U^{\beta} L^{-\beta}(B - \alpha y_l - 2x) = 0 \tag{B.19}$$

These two FOCs combine to get: $x = \pi(I) = \frac{\sigma}{2}$

We now consider the environment with private insurance and show that the borrower's investment is lower than the first-best.

$$\max_{B,I} \quad \Big(\pi(I)(y_h - B) + (1-\pi(I))(1-\alpha)y_l - y_l\Big)^{\beta} \Big(\pi(I)B + (1-\pi(I))\alpha y_l - I\Big)^{1-\beta} \tag{B.20}$$

$$\text{st:} \quad \pi'(I)(y_H - B - (1-\alpha)y_L) = 1 \tag{B.21}$$

With the change of variables,

$$\max_{B,x} \quad \Big(x(y_h - B) + (1-x)(1-\alpha)y_l - y_l\Big)^{\beta} \Big(xB + (1-x)\alpha y_l - x^2\Big)^{1-\beta} \tag{B.22}$$

$$\text{st:} \quad B = -2x + \sigma + \alpha y_l \tag{B.23}$$

Substituting in the expression for B into the borrower utility,

$$U = x(2x + (1-\alpha)y_l) + (1-\alpha)y - x(1-\alpha)y_l - yl = 2x^2 - \alpha y_l$$

Using the constrain in the lender utility,

$$L = -2x^2 + \sigma x + \alpha y_l x + (1-x)\alpha y_l - x^2 = -3x^2 + \sigma x + \alpha y_l \tag{B.24}$$

$$= -3\left(x - \frac{\sigma + \sqrt{\sigma^2 + 12\alpha y_l}}{6}\right)\left(x - \frac{\sigma - \sqrt{\sigma^2 + 12\alpha y_l}}{6}\right) \tag{B.25}$$

We show by contradiction that the borrower's investment is lower than the first-best. Suppose $x = \pi(I) > \frac{1}{2}\sigma$. The moral hazard condition boils down to:

$$\sigma^2 \geq 4\alpha y_l \quad \Rightarrow \quad 12\alpha y_l \leq 3\sigma^2 \quad \Rightarrow \quad \frac{\sigma + \sqrt{\sigma^2 + 12\alpha y_l}}{6} \leq \frac{1}{2}\sigma$$

Then $L < 0$ but this is a contradiction. Thus, $I^{SB} \leq I^{FB}$.

Thus far we did not have CDS. Now let us allow lenders to buy insurance,

$$\max_{q,B,I,i} \quad U^{\beta} L^{1-\beta} \tag{B.26}$$

$$\pi'(I)(y_H - B - (1-\alpha(i))y_L) = 1 \tag{B.27}$$

$$qB - I \geq 0 \tag{B.28}$$

$$i \leq py \tag{B.29}$$

Again we assume $qB - I \geq 0$ binds,

$$\max_{B,I,i} \quad \Big(\pi(I)(y_h - B) + (1-\pi(I))(1-\alpha(i))y_l\Big)^{\beta}\Big(\pi(I)B + (1-\pi(I))\alpha(i)y_l - I\Big)^{1-\beta} \tag{B.30}$$

$$\pi'(I)(y_H - B - (1-\alpha(i))y_L) = 1 \tag{B.31}$$

$$i \leq py \tag{B.32}$$

With the change of variables,

$$\max_{B,x,i} \quad \Big(x(y_h - B) + (1-x)(1-\alpha(i))y_l\Big)^{\beta}\Big(xB + (1-x)\alpha(i)y_l - I\Big)^{1-\beta} \tag{B.33}$$

$$\text{st:} \quad B = -2x + \sigma + \alpha y_l \tag{B.34}$$

$$i \leq py \tag{B.35}$$

Substituting in the objective function the expression for B:

$$\max_{x,i} \quad \Big(2x^2 - \alpha(i)y_l\Big)^{\beta}\Big(-3x^2 + \sigma x + \alpha(i)y_l\Big)^{1-\beta} \tag{B.36}$$

$$\text{st:} \quad i \leq py \tag{B.37}$$

We can analogously derive the renegotiation outcome,

$$\alpha(i)y_l = \beta i + (1-\beta)py_l$$

Thus, the first order conditions are,

$$\begin{aligned} FOC_x: &\quad \beta U^{\beta-1}L^{1-\beta}4x + (1-\beta)U^{\beta}L^{-\beta}(-6x+\sigma) = 0 \\ FOC_i: &\quad \beta U^{\beta-1}L^{1-\beta}(-\beta) + (1-\beta)U^{\beta}L^{-\beta}(\beta) = 0 \end{aligned}$$

They imply,

$$\begin{aligned} \beta U^{-1}L4x &= (1-\beta)(6x-\sigma) \\ \beta U^{-1}L &= (1-\beta) \end{aligned}$$

Combining these two, we get:

$$x = \pi(I) = \frac{1}{2}\sigma$$

Thus, the borrower's investment level in the second best environment with CDS is the same as the first best environment. This shows that the main result of the paper holds even when the lender has a bargaining power.

ESSAY 3: CURRENCY RISK AND PRICING KERNEL VOLATILITY

Joint with Federico Gavazzoni and Chris Telmer

Abstract

A basic tenet of lognormal asset pricing models is that a risky currency is associated with *low* pricing kernel volatility. Empirical evidence indicates that a risky currency is associated with a relatively *high* interest rate. Taken together, these two statements associate high-interest-rate currencies with low pricing kernel volatility. We document evidence suggesting that the opposite is true, thus contradicting a fundamental empirical restriction of lognormal models. Our identification strategy revolves around using interest rate *volatility* differentials to make inferences about pricing kernel volatility differentials. In most lognormal models the two are monotonic functions of one another. A risky currency, therefore, is one with relatively low pricing kernel volatility *and* relatively low interest rate volatility. In the data, however, we see the opposite. High interest rates are associated with *high* interest rate volatility. This indicates that lognormal models of currency risk are inadequate and that future work should emphasize distributions in which higher moments play an important role. Our results apply to a fairly broad class of models, including Gaussian affine term structure models and many recent consumption-based models.

1 INTRODUCTION

Currency risk is tricky. Unlike many financial securities, volatility and risk can be very different things. Consider the following anecdote. Producers of apples and bananas face supply shocks. The supply shocks become manifest in the relative price of fruit, the cost of one banana in units of apples. The producers, therefore, face price risk ... a positive supply shock to apples decreases the relative price of apples. Now consider volatility. Suppose that the apple shocks are volatile relative to the banana shocks. Apple producers notice two things: (i) the relative price of *bananas* isn't doing them any favors; it goes down whenever they get a bad shock, (ii) the supply shocks that they face are *dominating* the variability in the relative price. These apple producers therefore view the price of bananas as being *risky*. If you offer them a derivative security that has its value tied to the price of bananas, they will demand a risk premium in order to buy it. Banana producers, of course, face a reciprocal sort of situation. They notice that the relative price of *bananas* goes in their favor when they get a bad shock. A banana derivative security represents a hedge for them and they will pay an insurance premium in order to buy it. However, because the supply shocks that they face have relatively small volatility, this insurance premium will be small relative to the risk premium demanded by the apple producers. The equilibrium price will *tend to* — remember, this is an anecdote — feature a positive risk premium for bananas. Herein lies the tricky business. The *high* volatility commodity, apples, enjoys relatively *low* relative price risk.

If you replace apples and bananas with the pricing kernel for dollars and pounds, and the relative price of bananas with the price of one pound in units of U.S. dollars, you now understand what currency risk is in *any* lognormal model. The *currency* with the relatively volatile pricing kernel will pay a relatively low risk premium (in fact, negative). Many recent statistical models of the term structure of interest rates fall into this category. If you assume that financial markets are complete you can go further and replace "currency" with "country." In *any* complete-markets, lognormal equilibrium model, the *country* with the relatively volatile nominal marginal rate of substitution will have a currency that pays a negative risk premium. Many recent consumption-based asset pricing models fall into this category. High volatility in marginal utility growth coincides with low currency risk.

Algebraically, currency risk in lognormal models takes the form

$$E_t\, s_{t+1} - f_t = \frac{1}{2}\left(Var_t \log m_{t+1} - Var_t \log m^*_{t+1}\right), \qquad (1.1)$$

where s_t and f_t are the log spot and forward exchange rates (price of pounds in units of dollars), and m_t and m^*_t are respectively, the dollar and pound pricing kernels (derivation provided below). The left-hand-side is the (continuously compounded) expected excess return on pounds. We call it the *currency risk premium on the pound.* The right-hand-side is the pricing kernel conditional volatility differential. Eq. (1.1) says that the pound will pay a positive risk premium if its pricing kernel has relatively low volatility. Empirically, the LHS seems to be increasing in the pound less dollar interest rate differential. This is the "carry trade evidence." The question we ask is whether or not lognormal models are consistent with the carry trade evidence. This amounts to evaluating the restriction that *high* interest rate currencies have *low* pricing kernel volatility.

The pricing kernel, of course, is not observable. So neither is the volatility difference on the RHS of Eq. (1.1). Many previous papers have attempted to relate the LHS to the volatility of the currency depreciation rate, $s_{t+1} - s_t$. As we'll see below, this amounts to relating the risk premium to the volatility of the difference,

$$Var_t\big(s_{t+1} - s_t\big) = Var_t\big(\log m_{t+1} - \log m^*_{t+1}\big),$$

not the difference in the volatility from Eq. (1.1). So, strictly speaking, it's the wrong measure of volatility. Other papers have have implicitly evaluated Eq. (1.1) by formulating a specific model of m_t and m_t^* and asking whether or not the model can account for the carry trade evidence. If it can then, if the model is in the lognormal class that we consider (many have been), Eq. (1.1) holds by construction.

Our approach is to try to say something that is not so strongly tied to one particular model. We do so by associating the RHS with the interest rate *volatility* differential in the following way. We outline conditions under which

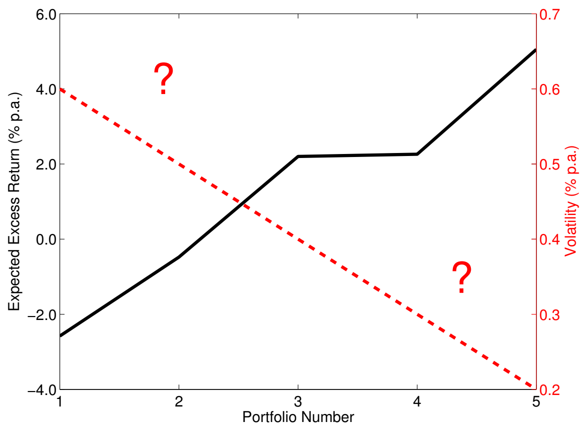
$$Var_t(m_{t+1}) > Var_t(m^*_{t+1}) \Leftrightarrow Var_t(i_{t+1}) > Var_t(i^*_{t+1}) \tag{1.2}$$

If this condition is true, then we can say that currency risk is associated with a *negative* interest-rate volatility differential. A risky currency (in a lognormal model) is one with relatively *low* interest rate volatility. This is the restriction that we actually evaluate.

Figure 24 illustrates our question graphically. It is an adaptation of Table 1 from Lustig, Roussanov, and Verdelhan (2011) (LRV). We construct five portfolios of currencies by constructing a monthly sort by the level of interest rates. Figure 24 reports the annualized average excess return, vis-a-vis the U.S. dollar, on each of the currency portfolios. We see what LRV, Burnside, Eichenbaum, Kleshchelski, and Rebelo (2011) and many others have documented; the 'carry-trade' of funding a long-position in a high-interest-rate currency by borrowing in a low-interest-rate currency seems to pay a positive excess expected return. Our question, then, is this. If Figure 1 also plots, for each portfolio, the average interest rate volatility of all the currencies in the portfolio, then, according to Eq. (1.2), the line should be decreasing. Is it?

Figure 24: Currency Risk and Return

The solid black line (left axis) reports the sample mean of the excess return on 5 interest-rate sorted currency portfolios, similar to that reported in Lustig, Roussanov, and Verdelhan (2011). The red dashed line (right axis) is, qualitatively, what lognormal pricing kernel models predict that pricing kernel and interest rates volatility should look like.



We find evidence suggesting that it is not. High expected return seems to be associated with high pricing kernel volatility. This implies one of two things. Either there is something wrong with method of inferring what the pricing kernel volatility differential in Eq. (1.1) looks like, or there's something wrong with conditional lognormality. We argue that the latter is most likely. We conclude that models of currency risk should incorporate departures from lognormality. Our results are supportive of recent work by Brunnermeier, Nagel, and Pedersen (2008) and others that emphasize conditional skewness and leptokurtosis.

It is important to understand that, while Eqns. (1.1) and (1.2) hold for any lognormal model, the particular pricing kernels for which they do must be carefully defined. Section 2 therefore begins with some background and a precise statement of which models our results restrict, and which they do not. Section 3 develops the overall affine Gaussian structure and articulates our main results using two theorems. Section 4 describes our data, Section 5 our results, and Section 6 concludes.

2 NOTATION AND BASIC APPROACH

We begin with a simple derivation of Eq. (1.1). The pricing kernel for claims denominated in U.S. dollars (USD) is m_{t+1}, so that $E_t\, m_{t+1} R_{t+1} = 1$ for all (gross) USD-denominated asset returns, R_{t+1}, realized between dates t and $t+1$. The analogous pricing kernel for claims denominated in foreign currency (say, British pounds, GBP) is m^*_{t+1} and GBP-denominated returns are R^*_{t+1}. Both m_t and m^*_t exist by virtue of no-arbitrage, but are only unique if markets are complete. For any of these pricing kernels, one period, continuously-compounded USD and GBP interest rates, i_t and i^*_t, satisfy

$$i_t = -\log E_t\, m_{t+1} \tag{2.1}$$

$$i^*_t = -\log E_t\, m^*_{t+1}. \tag{2.2}$$

The date-t nominal spot exchange rate, USD per GBP, is S_t. Since the USD pricing kernel must also price USD-denominated returns on GBP-denominated assets, no-arbitrage implies that, for any m_{t+1},

$$E_t\left(m_{t+1}\frac{S_{t+1}}{S_t}R^*_{t+1}\right) = 1. \tag{2.3}$$

Consider one particular USD pricing kernel, m_{t+1}, along with the observable process S_{t+1}/S_t. Define

$$m^*_{t+1} = m_{t+1}\,\frac{S_{t+1}}{S_t} \tag{2.4}$$

and note that, by construction, $E_t\, m^*_{t+1} R^*_{t+1} = 1$, so that this particular m^*_{t+1} is a legitimate pricing kernel GBP-denominated claims. If markets are incomplete, then there are other legitimate GBP pricing kernels. This is discussed in the next section. Express Eq. (2.4) in terms of natural logarithms:

$$s_{t+1} - s_t = \log m^*_{t+1} - \log m_{t+1}, \tag{2.5}$$

where $s_t \equiv \log S_t$, so that the LHS is the (continuously-compounded) depreciation rate of the USD. The one-period forward exchange rate and its logarithm are F_t and f_t. Subtract Eq. (2.2) from Eq. (2.1) and invoke covered interest parity, $f_t - s_t = i_t - i^*_t$:

$$f_t - s_t \;\; = \;\; -\log E_t\, m_{t+1} - (-\log E_t\, m^*_{t+1}). \tag{2.6}$$

Take the conditional mean of Eq. (2.5), and subtract from it Eq. (2.6):

$$\begin{aligned} E_t\, s_{t+1} - f_t &= \big(\log E_t\, m_{t+1} - E_t \log m_{t+1}\big) - \big(\log E_t\, m^*_{t+1} - E_t \log m^*_{t+1}\big) \\ &= \frac{1}{2}\big(\mathit{Var}_t \log m_{t+1} - \mathit{Var}_t \log m^*_{t+1}\big), \end{aligned} \tag{2.7}$$

where the last equation holds if we assume that m_t and m^*_t are jointly, conditionally lognormal.[65] It is the same as Eq. (1.1) from the introduction. The LHS is the *risk premium on GBP*: the (continuously-compounded) expected excess return on borrowing USD and investing the proceeds in GBP. The RHS says that, in this class of models, a necessary condition for GBP to be risky is that its pricing kernel exhibit relatively *low* volatility.

Note that a great deal of empirical work — from older papers such as Domowitz and Hakkio (1985) to newer papers such as Chernov, Graveline, and Zviadadze (2012) — has focused on the conditional variance of the exchange rate, $\mathit{Var}_t(s_{t+1} - s_t)$. Similarly, Brunnermeier, Nagel, and Pedersen (2008) focus on $\mathit{Skew}_t(s_{t+1} - s_t)$. While these moments are certainly interesting in-and-of-themselves, the combination of Eqns. (2.5) and (2.7) indicate that they emphasize moments of the difference, whereas currency risk is more directly about the difference of the moments. Sorting this out (in a new way) is the focal point of our paper.

2.1 *Incomplete Markets*

Eq. (2.7) holds by no-arbitrage for *given* processes m_t and S_{t+1}/S_t, and for the *particular* no-arbitrage pricing kernel, m^*_t, defined by Eq. (2.4). If markets are complete then these things are all unique and any two will tell you the third. If not, then there is a multiplicity that one must consider.

Our approach is to fix models for m_t and m^*_t and then use Eq. (2.4) to compute the depreciation rate, $S_{t+1}/S_t = m^*_{t+1}/m_{t+1}$. The multiplicity can be represented by a random variable η_{t+1} such that $\exp(\eta_{t+1})m^*_{t+1}/m_{t+1}$

[65] Backus, Foresi, and Telmer (2001), page 286, Eq. (12), shows the more general expression, involving higher-order moments. This will play a key role later, in Section #.

is also a legitimate depreciation rate. Complete markets imposes that $\eta_{t+1} = 0$. No arbitrage imposes that

$$E_t\big(m_{t+1}R_{t+1}\big) = E_t\big(m_{t+1}e^{-\eta_{t+1}}R_{t+1}\big) = 1$$
$$E_t\big(m^*_{t+1}R^*_{t+1}\big) = E_t\big(m^*_{t+1}e^{\eta_{t+1}}R^*_{t+1}\big) = 1$$

for all asset returns, R_{t+1} and R^*_{t+1}, denominated in USD or GBP, respectively. These conditions restrict the admissible η_t processes in important ways but still leave open a potentially large set of admissible exchange rate processes that are consistent with no-arbitrage and the fixed processes m_t and m^*_t. However, if we restrict attention to affine term structure models, then all elements of this set are observationally equivalent and we can set $\eta_t = 0$ without loss of generality (Backus, Foresi, and Telmer (2001), Proposition 1 and subsequent discussion on page 289). In this sense, our results apply to *any* Gaussian affine term structure model of currency risk (*e.g.*, Backus, Foresi, and Telmer (2001), Bakshi and Chen (1997), Bansal (1997), Brenna and Xia (2006), Frachot (1996), Lustig, Roussanov, and Verdelhan (2011), and Saá-Requejo (1994)).

What exactly does this mean? The presumption of a two-currency affine term structure model is that all predictable and unpredictable movements in log bond prices and the depreciation rate are spanned by the model's state variables and innovations, respectively. Hence, the residual η_t obeys the same affine structure as m_t and m^*_t. Hence, we can redefine m^*_t, say, to encapsulate the residual, use the result in combination with the original m_t and Eq. (2.4) to define S_{t+1}/S_t, and the mapping between identifiable parameters and moments of the data will be identical to the case of $\eta_t = 0$. Basically, if we restrict ourselves to Gaussian affine models in which parameter values are identified only via *moments* of bond prices and exchange rates, then, conditional on the specifications for m_t and m^*_t, the distinction between complete and incomplete markets is moot. From a macroeconomic perspective, this might seem like throwing out the baby with the bathwater.[66] From the perspective of the large literature on statistical term structure models, perhaps not.

2.2 *Changing Units Versus Changing Countries*

When can we associate Eq. (1.1) with *countries*, not just *currencies*? Only when markets are complete. Only then does Eq. (**??**) describe an equilibrium condition in which marginal rates of substitution of domestic and foreign representative agents are equated, pointwise, via the change of units process, S_{t+1}/S_t. This is, of course, a restrictive assumption. But it is employed by a large majority of recent consumption-based models that employ either the Campbell and Cochrane (1999) or the Bansal and Yaron (2004) machinery to a multi-country setting. Recent examples include Alvarez, Atkeson, and Kehoe (2009), Bansal and Shaliastovich

[66]The bathwater is the difficult job of identifying η_t using, say, consumption data. The baby is what we might learn by doing so.

(2013), Colacito and Croce (2011), Stathopoulos (2012) and Verdelhan (2010). These are models in which Eq. (2.4) describes a unique relationship between marginal rates of substitution and exchange rates, and in which the pricing kernels are jointly lognormal (thanks in some cases to linearizations such as that from Hansen, Heaton, and Li (2008)). In most cases these models *work* in the sense that calibrated versions of them are consistent with the standard set of carry-trade facts. This means that, necessarily, Eq. (1.1) applies and that high risk is associated with low pricing kernel volatility. We will show that, for this class of models, data on interest rate volatility differentials calls this relationship into question and thus poses a challenge to these models.

2.3 *A Very Rough Approximation*

Eq. (2.7) tells us that the currency risk premium is the difference between two unobservable variables. Many papers, of course, have written down an explicit (lognormal) model of m_t and m_t^* under which this difference is a function of observables. A successful model has been one with an interest rate differential, $i_t - i_t^*$, that is *negatively* correlated with the volatility difference, $Var_t \log m_{t+1} - Var_t \log m_{t+1}^*$. Our goal, instead, is to derive an empirical measure of this variance difference that applies more generally than to just one, particular (lognormal) model. We begin with a very informal derivation, intended to get at the main idea. In the next section we provide a tight, formalized analysis showing that what we do here applies much more generally.

With lognormality, Eq. (2.1) implies that the period-ahead domestic interest rate is

$$i_{t+1} \quad = \quad -E_{t+1} \log m_{t+2} - \frac{1}{2} Var_{t+1} \log m_{t+2}. \tag{2.8}$$

Suppose that variation in conditional mean is relatively small. This is not as unreasonable as it might sound, at least for our question. Backus, Foresi, and Telmer (2001) show that Fama (1984) necessary conditions for resolving the forward premium anomaly translate into conditions stating that 'variance in pricing kernel variance must be larger than variance in pricing kernel means.' Papers by and show that something similar is implied by the Campbell-Shiller, Fama-Bliss term structure regressions. Thus ignoring the first term in Eq. (2.8) we have

$$Var_t\, i_{t+1} \approx \frac{1}{4} Var_t\, Var_{t+1} \log m_{t+2}.$$

Suppose that the conditional variance of the conditional variance isn't too different from the the conditional variance. For some processes, this is sensible. For example, for the canonical square-root process, $x_{t+1} = \mu + \varphi x_t + \sigma x_t^{1/2} \varepsilon_{t+1}$, we have that $Var_t\, Var_{t+1} x_{t+2} = \sigma^2\, Var_t x_{t+1} = \sigma^4 x_t$. For other processes — *e.g.*, an autoregression with stochastic volatility, but where the volatility is homoskedastic — it makes no sense. Ignoring the latter,

$$Var_t\, i_{t+1} \approx \frac{1}{4}\, Var_t \log m_{t+1}. \tag{2.9}$$

This is the main idea. Perhaps we can learn something about pricing kernel volatility by simply estimating interest rate volatility?

Suppose that this is valid. Substitute it into Eq. (2.7):

$$E_t\, s_{t+1} - f_t \approx 2\big(Var_t\, i_{t+1} - Var_t\, i^*_{t+1}\big).$$

In most lognormal models the LHS is a linear function of the interest rate differential:

$$a + b\big(i_t - i^*_t\big) \approx 2\big(Var_t\, i_{t+1} - Var_t\, i^*_{t+1}\big). \tag{2.10}$$

There is overwhelming empirical evidence indicating that $b < 0$. This is the 'Uncovered Interest Rate Parity (UIP)' regression evidence first found by Bilson (1981), Fama (1984) and Tryon (1979). It is the basis for the foreign currency 'carry trade.' Assume that $a = 0$.[67] Eq. (2.10) implies that, according to lognormal models, (i) the interest rate differential, $i_t - i^*_t$ and the volatility differential, $Var_t\, i_{t+1} - Var_t\, i^*_{t+1}$, should have the opposite sign, and (ii) they should be negatively correlated.

Figure 26 shows results that are representative of our main findings. It shows that, for a classic 'carry trade' pair of currencies — USD and the Australian dollar (AUD) — restrictions (i) and (ii) seem to be strongly at odds with the data. Much more often than not, the LHS and RHS of Eq. (2.10) have the *same* sign. Moreover, they are positively correlated at 0.52.

Interest rate differentials and interest rate volatility differentials appear to be positively related. The high-interest rate currency — the carry-trade recipient currency that pays a positive risk premium — also appears to be the high interest-rate-volatility currency. This isn't the case for every currency pair and time period but, as we exhaustively demonstrate in Section 5, it is much more the rule than the exception. Our approximation, Eq. (2.9), suggests that we can restate this. The high-interest rate currency appears to be the high pricing-kernel-volatility currency. This is a stark contradiction of any lognormal model of currency risk.

We now tighten up the approximation from Section 2.3 and show precisely under what conditions high interest rate volatility is necessarily associated with high pricing kernel volatility.

[67]This is implied by the UIP evidence if (i) $b = -1$, (ii) UIP holds unconditionally, and (iii) the unconditional mean of the interest rate differential is zero, $E(i_t - i^*_t) = 0$. For many currency pairs all of these conditions are empirically plausible (Engel (2011) provides some up-to-date evidence and his survey paper, Engel (1996), is a standard reference for a more exhaustive survey). For those for which they are not, our story is basically unchanged once we subtract out an innocuous mean.

The Duffie and Kan (1996) class of lognormal, affine pricing kernel models can be specified as follows. Uncertainty in the domestic country is described by the k-dimensional vector of state variables z that follows a square-root model:

$$z_{t+1} = (I - \Phi)\theta + \Phi z_t + \Sigma(z_t)^{1/2}\epsilon_{t+1},$$

where $\{\epsilon\} \sim$ NID(0,1), $\Sigma(z_t)$ is a $k \times k$ diagonal matrix with a typical element given by $\sigma_i(z_t) = \alpha_i + \beta_i^\top z_t$, where β_i has nonnegative elements, and Φ is a $k \times k$ stable matrix with positive diagonal elements. The process for z requires that the volatility functions, $\sigma_i(z)$, be positive, which places additional restrictions on the parameters. The pricing kernel is

$$-\log m_{t+1} = \delta + \gamma^\top z_t + \lambda^\top \Sigma(z_t)^{1/2}\epsilon_{t+1}, \tag{3.1}$$

where the $k \times 1$ vector γ is referred to as the "factor loadings" for the pricing kernel, and the $k \times 1$ vector λ is referred to as the "price of risk" vector.

Using Eq. (2.1), together with the dynamics of the pricing kernel from Eq. (3.1), the interest rate is

$$i_{t+1} = \left(\delta - \frac{1}{2}\sum_{j=1}^{k} \lambda_j^2 \alpha_j\right) + \left(\gamma^\top - \frac{1}{2}\sum_{j=1}^{k} \lambda_j^2 \beta_j^\top\right) z_{t+1}.$$

The conditional variance of the home pricing kernel is

$$\begin{aligned} Var_t(m_{t+1}) &= \left(\lambda^\top \Sigma(z_t)^{1/2}\right) Var_t(\epsilon_{t+1}) \left(\lambda^\top \Sigma(z_t)^{1/2}\right)^\top \\ &= \lambda^\top \Sigma(z_t)\lambda = \sum_{j=1}^{k} \lambda_j^2 \sigma_j(z_t), \end{aligned}$$

and the conditional variance of the home interest rate is

$$\begin{aligned} Var_t(i_{t+1}) &= \left(\gamma^\top - \frac{1}{2}\sum_{j=1}^{k} \lambda_j^2 \beta_j^\top\right) \Sigma(z_t) \left(\gamma^\top - \frac{1}{2}\sum_{j=1}^{k} \lambda_j^2 \beta_j^\top\right)^\top \\ &= \sum_{n=1}^{k} \left(\gamma_n - \frac{1}{2}\sum_{j=1}^{k} \lambda_j^2 \beta_{n,j}\right)^2 \sigma_n(z_t). \end{aligned}$$

Similarly to the domestic country, the foreign country is described by a k-dimensional vector z^*. Foreign parameters are denoted with an asterisk. It is straightforward to derive the expressions for the foreign pricing kernel, m^* and the foreign interest rate, i^*.

Under what conditions can we say that

$$Var_t(m_{t+1}) > Var_t(m^*_{t+1}) \Leftrightarrow Var_t(i_{t+1}) > Var_t(i^*_{t+1})? \tag{3.2}$$

The validity — or not — of result (3.2) is in general parameter dependent. However, for two special cases which are ubiquitous in the term structure and the currency risk premium literature, it turns out that we can say a fair bit.

Theorem 1 (Symmetric Coefficients). *Let Ω (Ω^*) denote the vector containing the home (foreign) parameters. Let $\Omega = \Omega^*$, $\lambda \neq 0$ and $\left(\gamma^\top - \frac{1}{2}\sum_{j=1}^{k} \lambda_j^2 \beta_j^\top\right) \neq 0$. Then $(\Sigma(z_t) - \Sigma(z_t^*))$ is positive definite if and only if $\mathrm{Var}_t(m_{t+1}) > \mathrm{Var}_t(m^*_{t+1})$, if and only if $\mathrm{Var}_t(i_{t+1}) > \mathrm{Var}_t(i^*_{t+1})$.*

The proof is in Appendix A. Theorem 1 says that, with symmetric coefficients, a relatively large conditional variance of the home state variables is associated with a relatively large conditional variance of the home pricing kernel and a relatively large conditional variance of the home interest rate. A given ranking in the conditional variance of the state variables in each country is associated with the same ranking in the conditional variance of the pricing kernels and the conditional variance of the interest rates.

As an example, consider the single-factor (per-country) case, $k = 1$. Theorem 1 simplifies to

$$\begin{aligned} Var_t(z_{t+1}) > Var_t(z^*_{t+1}) &\Leftrightarrow Var_t(m_{t+1}) > Var_t(m^*_{t+1}) \\ &\Leftrightarrow Var_t(i_{t+1}) > Var_t(i^*_{t+1}), \end{aligned}$$

which can be read as "high conditional variance at home means high conditional variance in the home kernel and high conditional variance in the home interest rate."

Theorem 2 (Common Factors). *Consider the case of common factors, $z_t = z_t^*$. Assume that $\gamma = \gamma^*$, $\beta = \beta^*$, and that there exists a strong enough 'precautionary savings motive' associated with both the domestic and foreign pricing kernels, so that*

$$\gamma^\top - \frac{1}{2}\sum_{j=1}^{k} \lambda_j^2 \beta_j^\top < 0 \quad \textit{and} \quad \gamma^\top - \frac{1}{2}\sum_{j=1}^{k} (\lambda_j^*)^2 \beta_j^\top < 0.$$

Suppose that the prices of risk λ and λ^* can be ordered*, so that either $\lambda > \lambda^*$ or $\lambda < \lambda^*$. Then, $|\lambda| > |\lambda^*|$ if and only if $\mathrm{Var}_t(m_{t+1}) > \mathrm{Var}_t(m^*_{t+1})$ if and only if $\mathrm{Var}_t(i_{t+1}) > \mathrm{Var}_t(i^*_{t+1})$.*

The proof is in Appendix A. Theorem 2 says that, with common factors, a sufficient condition for the conditional variance of the domestic pricing kernel to be larger than the conditional variance of the foreign pricing kernel is that the price of risk of each of the home state variables is at least as large (in absolute value) than the price of risk of each of the foreign state variables. When this is the case, a strong enough precautionary saving demand in each country delivers a larger conditional variance of the home interest rate, relative to the conditional variance of the foreign interest rate. For the theorem to hold, symmetry in the factor loadings and in the sensitivity of the conditional variances to the state variables is required.

The condition that $\gamma^\top - \frac{1}{2}\sum_{j=1}^{k}\lambda_j^2\beta_j^\top < 0$ (and its foreign counterpart) is usually referred to in the literature as a *strong enough precautionary savings motive.* Lustig, Roussanov, and Verdelhan (2011) is an example of Theorem 2 at work. They show that an affine model with a common factor, common coefficients across countries with the exception of the price of risk, and a precautionary saving motive that is strong enough in each country delivers interest rate and exchange rate dynamics that are consistent with the carry trade facts. Hence, the assumptions of Theorem 2 are not stringent at all. They *must* be satisfied for an affine model of the Duffie-Kan class to fit the data.

As an example, consider the single factor case, $k = 1$. Here, a stronger result is available. Under the conditions of Theorem 2

$$|\lambda| > |\lambda^*| \Leftrightarrow Var_t(m_{t+1}) > Var_t(m^*_{t+1}) \Leftrightarrow Var_t(i_{t+1}) > Var_t(i^*_{t+1}),$$

where now λ and λ^* are scalars. In words, in a world with one single common factor, when all other coefficients are symmetric, having a relatively large home price of risk (in absolute value) means having a relatively high domestic kernel volatility and a relatively high interest rate volatility.

4 DATA AND ESTIMATION

For our main analysis, we use data for 114 countries on 3-month treasury bill yields, forward and spot exchange rates from *Global Financial Data* (GFD) for the period 1950-2009. As a comparison, we also use eurocurrency interest rates for 27 countries from the *Financial Times/ICAP* (FT/ICAP), 1975-2009. Appendix B provides details. The number of countries is extensive, but, obviously, we do not have data for all countries and time periods, and the relevance of many of the countries for our question is questionable. We therefore report results for a number of different subsamples of time and country. Our main result is not sensitive to either.

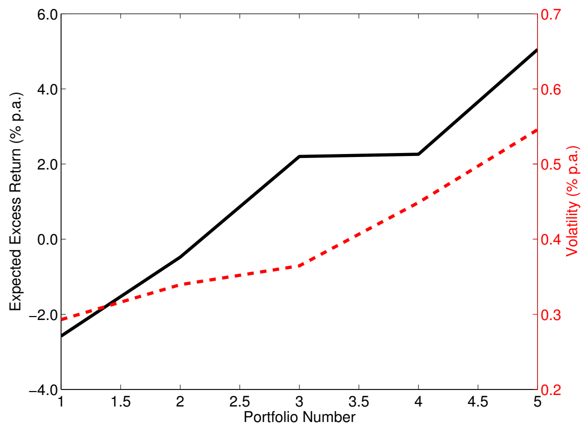
We estimate a GARCH model for interest rate volatility. Details on the estimation procedure are given in Appendix B. We report results for both bilateral currency pairs and portfolios of currencies. The latter are formed in a manner identical to Lustig, Roussanov, and Verdelhan (2011), by sorting on interest rate levels at a monthly frequency.

5 RESULTS

In the introduction we used Figure 24 to illustrate our main question: is pricing kernel volatility decreasing in the interest rate differential, as lognormal theory predicts it should? Figure 25 answers our question. We conduct the exact same exercise as Lustig, Roussanov, and Verdelhan (2011). We sort currencies by interest rate and, each month, form 5 portfolios. Figure 25 plots the time-averaged excess return on each portfolio, and the time-averaged interest rate volatility. Both are increasing as we move from low to high interest rate portfolios. The answer to the question is no.

Figure 25: High Interest Rates are Associated with High Interest Rate Volatility

The solid black line (left axis) is the same as that in Figure 24 from the introduction. It is the sample mean of the excess return on 5 interest-rate sorted currency portfolios, similar to that reported in Lustig, Roussanov, and Verdelhan (2011). The red dashed line (right axis) is, the time-averaged interest rate volatility on each of the portfolios. The class of lognormal models described in Section 3 restrict the red line to be decreasing if the black line is increasing. The data are, therefore, inconsistent with lognormal models in this dimension.



The remainder of our empirical work simply consists of further substantiating the point of Figure 25 by considering different samples and data sources. Figure 27 shows average volatility for the currency portfolios for a number of different subsamples, starting from 1950, 1975, 1987 and 1995. Figure 28 reports analogous results for bilateral currency pairs. Figure 29 compares results across the GFD and FT/ICAP data sources. In all cases, we see robust evidence of high interest rates differentials being associated with high interest rate volatility differentials.

6 CONCLUSION

We have shown that Gaussian models of currency risk face an empirical challenge. A high interest rate differential seems to be associated with a high interest-rate *volatility* differential. In a broad class of Gaussian models of the pricing kernel, this is inconsistent with (i) the empirical observation that high interest rates are associated with high excess expected returns, and (ii) the theoretical restriction that high excess expected returns accrue to the *low* volatility pricing kernel. This inconsistency derives from some conditions under which interest rate volatility and pricing kernel volatility are positively related. These conditions are not exhaustive. But they are satisfied by some models that are prominent in the literature. This suggests that we either enrich these models, or consider non-Gaussian alternatives.

We've couched our analysis in terms of trying to observe *directly* the difference in the moments of pricing kernels. This stands in contrast to much existing work, which tends to be based on moments of the difference in pricing kernels (*e.g.*, the variance of the exchange rate). We find our approach useful in that it emphasizes something fundamental about what currency risk *is*, while at the same time providing some links to other asset pricing models and results such as those from the literature on the term structure of interest rates. Nevertheless, we admit that ours does have the flavor of many previous papers that have gone searching for the unicorn by trying to identify the magical pricing kernel using very few assumptions and very little data (*i.e.*, asset return data only). An alternative interpretation, therefore, is as follows.

Ever since Bilson (1981), Fama (1984) and Tryon (1979) discovered the existence of excess expected returns in currency markets, many models have been developed to account for this behavior. The first models focused almost exclusively on the UIP regression coefficient. For subsequent models the bar has been raised higher. Other moments of the joint distribution of exchange rates, interest rates, consumption and so on have been emphasized. Sharpe ratios on traded currency *portfolios* have been emphasized. Our results can be viewed as simply suggesting one more moment: the interest-rate volatility differential. There is clear evidence on this, and it places very binding restrictions on state-of-the-art models. Moreover, it seems (to us) to be a particularly important moment. There is a clear link to theory and this link might be pointing us to non-Gaussian behavior, something that has been strongly emphasized in the recent literature on crashes, disasters and the like. We like this moment.

Finally, we close with a broad, interpretive point. Currency risk is a particular type of "change of units risk." In Gaussian models, an inescapable characteristic of this is that high risk is associated with low volatility. If the units that I care about are subject to *relatively* volatile shocks, then financial securities with payoffs denominated in these units will pay a *negative* risk premium (relative to the other units in question). Our goal has been to ask if this characteristic fits the facts. To find out, one must first take a stand on what are "the units that I care about." The obvious answer (to an economist) is "real marginal utility units," where the word "real" makes clear that this is "marginal utility per unit of *goods*." We have circumvented this, focusing instead on *nominal* marginal utility units: "utils per dollar." We've done this because most financial securities are denominated in dollars, not in goods. We must therefore be silent on whether or not Gaussian models of *real* exchange rates fit the facts. But we can say something about nominal models, which, one-way-or-another, have been most prevalent in the literature. We find evidence suggesting that nominal exchange rate risk is more than just a Gaussian phenomenon. We also note that, while our data have not addressed them, Gaussian models of real marginal utility have become more and more prevalent in the literature ever since Hansen, Heaton, and Li (2008) developed their linearization of the recursive class of preferences. Much of the long-run risk literature initiated by Bansal and Yaron (2004) features models that are conditionally Gaussian. A prominent example featuring exchange rates is Bansal and Shaliastovich (2013). Our approach is easily applied in this setting, and will feature relative consumption volatility in addition to interest rate volatility. This is work-in-progress.

A APPENDIX: PROOFS OF THEOREMS

Proof of Theorem 1.
Note that, since $(\Sigma(z_t) - \Sigma(z_t^*))$ is diagonal, positive definiteness requires $\sigma_j(z_t) \geq \sigma_j(z_t^*)$, for every j, with at least one strict inequality (i.e. $\exists i$ such that $\sigma_i(z_t) > \sigma_i(z_t^*)$). Now, when $\Omega = \Omega^*$, we have

$$Var_t(m_{t+1}) - Var_t(m_{t+1}^*) = \lambda^\top (\Sigma(z_t) - \Sigma(z_t^*))\, \lambda.$$

Therefore, by the definition of positive definiteness, we have

$$(\Sigma(z_t) - \Sigma(z_t^*)) \text{ is positive definite } \Leftrightarrow Var_t(m_{t+1}) > Var_t(m_{t+1}^*),$$

whenever $\lambda \neq 0$. Similarly,

$$Var_t(i_{t+1}) - Var_t(i_{t+1}^*) = \left(\gamma^\top - \frac{1}{2}\sum_{j=1}^{k} \lambda_j^2 \beta_j^\top\right) (\Sigma(z_t) - \Sigma(z_t^*)) \left(\gamma^\top - \frac{1}{2}\sum_{j=1}^{k} \lambda_j^2 \beta_j^\top\right)^\top.$$

Again, positive definiteness gives

$$(\Sigma(z_t) - \Sigma(z_t^*)) \text{ is positive definite } \Leftrightarrow Var_t(i_{t+1}) > Var_t(i_{t+1}^*),$$

whenever $\left(\gamma^\top - \frac{1}{2}\sum_{j=1}^{k} \lambda_j^2 \beta_j^\top\right) \neq 0$. Last, let $a \neq 0$ be any $k \times 1$ vector and note that the matrix $(\Sigma(z_t) - \Sigma(z_t^*))$ is diagonal. Therefore,

$$a^\top (\Sigma(z_t) - \Sigma(z_t^*))\, a = \sum_{j=1}^{k} a_j^2 (\sigma(z_t) - \sigma(z_t^*)) > 0 \Leftrightarrow \sigma_j(z_t) \geq \sigma_j(z_t^*), \text{ for every } j$$

with at least one strict inequality associated with a non zero element of a. □

Proof of Theorem 2.

$$Var_t(m_{t+1}) - Var_t(m_{t+1}^*) = (\lambda^2 - (\lambda^*)^2)^\top \, diag\{\Sigma(z_t)\}$$

which, under the assumption that λ and λ^* can be ordered, is positive if and only if $\lambda^2 > (\lambda^*)^2$, that is $|\lambda| > |\lambda^*|$. Moreover,

$$Var_t(i_{t+1}) - Var_t(i_{t+1}^*) = \left(\left(\gamma^\top - \frac{1}{2}\sum_{j=1}^{k} \lambda_j^2 \beta_j^\top\right)^2 - \left(\gamma^\top - \frac{1}{2}\sum_{j=1}^{k} (\lambda_j^*)^2 \beta_j^\top\right)^2\right)^\top diag\{\Sigma(z_t)\}$$

which, under the conditions that $\gamma = \gamma^*$, $\beta = \beta^*$, $\gamma^\top - \frac{1}{2}\sum_{j=1}^{k} \lambda_j^2 \beta_j^\top < 0$, and $\gamma^\top - \frac{1}{2}\sum_{j=1}^{k} (\lambda_j^*)^2 \beta_j^\top < 0$ is positive if only if $\lambda^2 > (\lambda^*)^2$, that is $|\lambda| > |\lambda^*|$. □

B APPENDIX: DATA

B.1 *Interest Rates*

Our main analysis is done with 3-month treasury bill yields for 114 countries from *Global Financial Data* (GFD) for the period 1950-2009 as it is the most comprehensive interest rate data available. As a robustness check, we also use eurocurrency interest rates from FT/ICAP through Datastream. FT/ICAP data consists of data for 27 mostly developed countries where for many of them data starts in 1975.

Here we document the availability of various interest rates data from Datastream and why we thought FT/ICAP was the best interbank interest rate data available. In Datastream, the two sources for eurocurrency interest rates are FT/ICAP and ICAP while the British Bankers Association (BBA) gives the London interbank rates (LIBOR).[68] Each have different set of countries and data years.

In 2006, Financial Times (FT) stopped providing euro currency rates to Datastream for the FT/ICAP series (for which Datastream mnemonic all starts with "EC") series. So Datastream continued these same series by splicing them with Intercapital (or ICAP, but formerly Garban Information Services (GS)) data so that starting in 2006, FT/ICAP and ICAP series are identical. For the countries for which GS is not available (e.g. Hong Kong, Singapore, South Africa), Tullett Prebon (TP) data was used instead. The TP data itself starts in 2006. The source column in Datastream's *Navigator* is not very informative. If the name of the series has FT/ICAP/TR, the source column will say "Thomson Reuters" but the source is really FT spliced with ICAP. If the name series has FT/TP, the source column appropriately says Tullett Prebon. The ones that have "dead" in the name have FT as the source and are dead series because neither ICAP nor TP has data on these countries. In Table 26, the countries of these dead series start with Greece and end with Thailand. For all the ICAP series (which start with "GS...."), the source column says "Thomson Reuters" but it is actually all ICAP.

A Datastream documentation file regarding this discontinuation of FT series gives alternative series that can be used instead. This list includes above mentioned ICAP, Tullett Prebon, BBA LIBORs, as well as "locally supplied" rates which are interbank rates from mainly national sources. These rates from national sources can be found under "National Interest Rates." The column "Alt IB" in Table 26 demonstrates the starting dates of the specific series that were listed in this document although there can be more than one interbank type rate under "National Interest Rates".

Datastream has various interest rate data under "National Sources." In the future, our treasury bill yields data from GFD could be potential supplemented with the combination of Eurocurrency rates (mainly, FT/ICAP) and interbank rates from national sources. A Datastream documentation about risk free rates discusses this:

> A risk free interest rate is the internal rate of return that can be obtained by investing in a financial instrument without (or very limited) credit risk. Normally this will relate to a short term investment in a financial instrument backed by the government. These money market securities bear no credit risk and have a limited re-investment risk, when the investment is rolled over for another short term period. In general we recommend the (annualized) yield on 3-month treasury bills, as the best instrument to use for any analysis involving risk free rates. However, for currencies where no liquid treasury bill market exists (or this market is subject to institutional distortions), interbank rates such as LIBOR or EURIBOR rates can be used. These do, however, bear a minimal credit risk inherent to the banks active in the market. Currencies with liquid repo-markets where the general collateral is a risk free long-term government bond, offer another alternative for an interest rate wich comes closest to 'risk-free', but are not available for as many currencies as interbank rates. A good example of this latter altenative is the Japanese 'Gensaki' market. The series we recommend for the main markets are detailed on the following table.

Column titled "Risk Free" in Table 26 demonstrates starting dates for the series specified in the document. Not shown in the table are Datastream's recommendations for risk free rates on Russia, China, Korea, Pakistan, Taiwan, Argentina, Brazil, Chile, Mexico and Venezuela also. Overall, this recommended list is not comprehensive (even few of the series above is

[68] There is also Tullett Prebon through Datastream that provides eurocurrency rates, but their data starts in 2006 only.

suspended) and one would have to manually go through the various interest rates available for each country under “National Interest Rates” and pick the most relevant rate (most likely some interbank rate is the best) in order to supplement GFD data.

Lastly, for the seven (for BBA, it is six) euro legacy countries, since the introduction of the Euro in 1999, GS, EC, and BBA interest rates are identical across these countries. Obviously, we will not have the same problem with GFD treasury yields data.

Table 26: This table compares various interest rates data available from Datastream (the first five columns) with GFD data. “nmiss” stands for the number of missing months.

Market	FT/ICAP	ICAP	BBA	Alt IB	”Risk Free”	GFD		
	Start	Start	Start	Start	Start	Start	End	nmiss
Australia	1997	1988	1986	1987	1977	1950	2009	3
Canada	1975	1995	1990	1992	1981	1950	2009	1
Denmark	1985	1995	2003	1989	1993	1976	2007	29
Hong Kong	1997			1986		1991	2009	1
Japan	1978	1995	1986	1996		1960	2009	1
New Zealand	1997	1988	2003	1987	1989	1978	2009	3
Norway	1997	1995		1986	1986	1984	2009	1
Singapore	1988			1987	1989	1987	2009	1
South Africa	1997			2000		1950	2009	1
Sweden	1997	1995	2006	1993	1994	1955	2009	1
Switzerland	1975	1995	1986	1974	1950	1980	2009	3
United Kingdom	1975	1995	1986	1975	1972	1950	2009	1
United States	1975	1995	1986	1971	1955	1950	2009	1
Other Western European	1999	1999	1998	1999	1999	1984	2009	2
Belgium	1978	1995				1950	2009	2
France	1975	1995	1989			1950	2009	61
Germany	1975	1995	1986			1953	2009	2
Italy	1978	1995	1990			1950	2009	2
Netherlands	1975	1995	1991			1950	2009	2
Portugal	1992	1995	1994			1985	2009	2
Spain	1992	1996	1990			1982	2009	2
Greece	1999					1980	2009	2
India	1999			1999	1994	1993	2009	1
Indonesia	1997			2001	1985	2000	2003	72
Malaysia	1997			1994	1998	1961	2009	1
Philippines	1999					1976	2009	3
Thailand	1997			2005	1995	1977	2009	88
Total	27	18	16	18	15		27	

Table 27: The remaining 87 GFD interest rates (mainly treasury yields). "nmiss" stands for number of missing months.

Market	Start	End	nmiss	Market	Start	End	nmiss
ALBANIA	1994	2009	4	KOREA, REPUBLIC OF	1987	2009	2
ALGERIA	1998	2009	5	KUWAIT	1979	2005	59
ANGOLA	2000	2009	3	KYRGYZSTAN	1994	2009	3
ARGENTINA	2002	2009	16	LATVIA	1994	2008	16
ARMENIA	1995	2009	3	LEBANON	1977	2009	4
AUSTRIA	1960	1990	228	LITHUANIA	1994	2009	4
AZERBAIJAN	1997	2009	9	MACEDONIA	1997	2009	3
BAHAMAS	1971	2009	3	MADAGASCAR	2000	2009	3
BAHRAIN	1987	2009	3	MALTA	1987	2009	3
BANGLADESH	1984	2009	29	MAURITIUS	1996	2009	3
BARBADOS	1966	2009	5	MEXICO	1978	2009	6
BELIZE	1978	2009	4	MOLDOVA, REPUBLIC OF	1995	2009	3
BOLIVIA	1994	2009	6	MONGOLIA	2006	2008	20
BOTSWANA	1996	2009	3	MONTENEGRO	2004	2009	3
BRAZIL	1965	2009	62	MOROCCO	2008	2009	4
BULGARIA	1992	2008	23	MOZAMBIQUE	2000	2009	4
BURUNDI	2001	2009	29	NAMIBIA	1991	2009	8
CAPE VERDE	1998	2009	3	NEPAL	1981	2008	17
CENTRAL AFRICAN REPUBLIC	1996	2004	139	NICARAGUA	2003	2007	24
CHILE	1997	2009	2	NIGERIA	1970	2009	229
CHINA	2002	2009	1	PAKISTAN	1991	2009	1
COLOMBIA	1998	2009	2	POLAND	1991	2009	1
COSTA RICA	1996	2009	3	ROMANIA	1994	2005	51
CROATIA	2000	2009	3	RUSSIAN FEDERATION	1994	2009	18
CYPRUS	1975	2008	21	RWANDA	1999	2009	5
CZECH REPUBLIC	1993	2009	5	SAUDI ARABIA	1991	2008	12
EGYPT	1991	2009	1	SERBIA	1997	2009	34
EL SALVADOR	2001	2005	51	SIERRA LEONE	1965	2008	12
ETHIOPIA	1985	2008	12	SLOVAK REPUBLIC	1993	2007	24
FIJI	1975	2009	4	SLOVENIA	1998	2009	5
GEORGIA	2001	2005	54	SRI LANKA	1981	2009	1
GHANA	1978	2009	9	SWAZILAND	1981	2006	45
GUYANA	1972	2009	6	TAIWAN	1974	2009	3
HAITI	1996	2009	2	TANZANIA, UNITED REPUBLIC OF	1993	2009	5
HONDURAS	1998	2000	108	TRINIDAD AND TOBAGO	1964	2009	7
HUNGARY	1988	2009	3	TUNISIA	1990	2009	2
ICELAND	1987	2009	2	TURKEY	1985	2009	3
IRAQ	2004	2009	3	UGANDA	1980	2009	3
IRELAND	1969	2009	2	URUGUAY	1992	2009	49
ISRAEL	1992	2009	2	VENEZUELA	1996	2003	72
JAMAICA	1953	2009	3	VIET NAM	1997	2009	9
JORDAN	2000	2009	36	ZAMBIA	1978	2009	3
KAZAKHSTAN	1994	2009	2	ZIMBABWE	1962	2009	11
KENYA	1972	2009	8				

B.2 *Exchange Rate Data*

Our main exchange rate data is from GFD for 172 countries for the period 1950-2009. This dataset is most comprehensive compared to three different data sources of forward and spot exchange rates available through Datastream: Barclays Bank PLC (BBI), Tenfore, and WM/Reuters (WMR). For each of these sources, full sample of countries and data years vary and are a lot more limited compared to GFD as shown in the table belows.

Table 28: Exchange rate data sources from Datastream

	WM/R	Tenfore	Barclays		WM/R	Tenfore	Barclays
Argentine Peso	2004			Latvian Lat	2004		
Australian Dollar	1996	1990	1984	Lithuanian Lita	2004		
Austrian Schilling	1996			Malaysian Ringgit	1996		
Belgian Franc	1996			Maltese Lira	2004		
Brazilian Real	2004			Mexican Peso	1996		
Bulgarian Lev	2004			Moroccan Dirham	2004		
Canadian Dollar	1996	1990	1984	New Zealand Dollar	1996	1990	1984
Chilean Peso	2004			Norwegian Krone	1996	1990	1984
Chinese Yuan Renminbi	2002			Omani Rial	2004		
Colombian Peso	2004			Pakistani Rupee	2004		
Croatian Kuna	2004			Peruvian Nuevo Sol	2004		
Cyprian Pound	2004			Philippine Peso	1996	2006	
Czech Koruna	1996	1996		Polish Zloty	2002	1996	
Danish Krone	1996	1990	1984	Portuguese Escudo	1996		
Dutch Guilder	1996			Qatari Riyal	2004		
Egyptian Pound	2004			Romanian Leu	2004	2008	
Estonian Kroon	2004			Russian Federation Rouble	2004		
Euro	1998	1990	1999	Saudi Arabian Riyal	1996	1990	
Finnish Markka	1996			Singaporean Dollar	1996	1990	1984
French Franc	1996			Slovak Koruna	2002		
German Mark	1996			Slovenian Tolar	2004		
Greek Drachma	1996			South African Rand	1996	1990	1983
Hong Kong Dollar	1996	1990	1983	Spanish Peseta	1996		
Hungarian Forint	1997			Swedish Krona	1996	1990	1984
Icelandic Krona	2004	2006		Swiss Franc	1996	1990	1983
Indian Rupee	1997			Taiwanese Dollar	1996		
Indonesian Rupiah	1996			Thai Baht	1996	1995	
Irish Punt or Pound	1996			Tunisian Dinar	2004		
Israeli Sheqel	2004	2006		Turkish Lira	1996	2006	
Italian Lira	1996			Ukrainian Hryvnia	2004	2008	
Japanese Yen	1996	1990	1983	United Arab Emirates Dirham	1996	1995	
Jordanian Dinar	2004			United Kingdom Pound	1996	1990	1983
Kazakh Tenge	2004			Venezuelan Bolivar	2004		
Kenyan Shilling	2004						
Korean Won	2002			Total	69	25	13
Kuwaiti Dinar	1996	1990					

Table 29: The countries that get included in the various data period subsamples. One means it is included, zero otherwise. For example, countries with ones in the column for 1975 means these were the countries that had non-missing interest rate data in Jan 1975

	all	1975	1987	1995		all	1975	1987	1995
ALB	1	0	0	1	KOR	1	0	1	1
DZA	1	0	0	0	KWT	1	0	1	1
AGO	1	0	0	0	KGZ	1	0	0	1
ARG	1	0	0	0	LVA	1	0	0	1
ARM	1	0	0	0	LBN	1	0	1	1
AUS	1	1	1	1	LTU	1	0	0	1
AUT	1	1	1	0	MKD	1	0	0	0
AZE	1	0	0	0	MDG	1	0	0	0
BHS	1	1	1	1	MYS	1	1	1	1
BHR	1	0	0	1	MLT	1	0	0	1
BGD	1	0	1	1	MUS	1	0	0	0
BRB	1	1	1	1	MEX	1	0	1	1
BEL	1	1	1	1	MDA	1	0	0	0
BLZ	1	0	1	1	MNG	1	0	0	0
BOL	1	0	0	1	MNE	1	0	0	0
BWA	1	0	0	0	MAR	1	0	0	0
BRA	1	1	1	1	MOZ	1	0	0	0
BGR	1	0	0	1	NAM	1	0	0	1
BDI	1	0	0	0	NPL	1	0	1	1
CAN	1	1	1	1	NLD	1	1	1	1
CPV	1	0	0	0	NZL	1	0	1	1
CAF	1	0	0	0	NIC	1	0	0	0
CHL	1	0	0	0	NGA	1	0	0	1
CHN	1	0	0	0	NOR	1	0	1	1
COL	1	0	0	0	PAK	1	0	0	1
CRI	1	0	0	0	PHL	1	0	1	1
HRV	1	0	0	0	POL	1	0	0	1
CYP	1	1	1	1	PRT	1	0	1	1
CZE	1	0	0	1	ROU	1	0	0	1
DNK	1	0	1	1	RUS	1	0	0	1
EGY	1	0	0	1	RWA	1	0	0	0
SLV	1	0	0	0	SAU	1	0	0	1
ETH	1	0	1	1	SRB	1	0	0	0
EUR	1	0	1	1	SLE	1	1	1	1
FJI	1	1	1	1	SGP	1	0	0	1
FRA	1	1	1	1	SVK	1	0	0	1
GEO	1	0	0	0	SVN	1	0	0	0
DEU	1	1	1	1	ZAF	1	1	1	1
GHA	1	0	1	1	ESP	1	0	1	1
GRC	1	0	1	1	LKA	1	0	1	1
GUY	1	1	1	1	SWZ	1	0	1	1
HTI	1	0	0	0	SWE	1	1	1	1
HND	1	0	0	0	CHE	1	0	1	1
HKG	1	0	0	1	TWN	1	1	1	1
HUN	1	0	0	1	TZA	1	0	0	1
ISL	1	0	0	1	THA	1	0	1	0
IND	1	0	0	1	TTO	1	1	1	1
IDN	1	0	0	0	TUN	1	0	0	1
IRQ	1	0	0	0	TUR	1	0	1	1
IRL	1	1	1	1	UGA	1	0	1	1
ISR	1	0	0	1	GBR	1	1	1	1
ITA	1	1	1	1	USA	1	1	1	1
JAM	1	1	1	1	URY	1	0	0	1
JPN	1	1	1	1	VEN	1	0	0	0
JOR	1	0	0	0	VNM	1	0	0	0
KAZ	1	0	0	1	ZMB	1	0	1	1
KEN	1	1	1	1	ZWE	1	1	1	1

Table 30: Descriptive statistics of GFD interest rates and the computed volatility. The volatility is based on the entire interest rate data available for that country. The last column (nobs) shows the number of consecutive months with non-missing data.

	rate				vol				
	min	max	mean	std	min	max	mean	std	nobs
ALB	5.05	38.23	12.32	8.32	0.19	4.34	0.70	0.68	183
DZA	0.09	10.13	3.72	3.75	0.42	1.70	0.57	0.26	135
AGO	2.80	134.00	55.13	47.86	1.04	25.02	6.10	5.76	110
ARG	1.10	59.77	10.02	8.81	-	-	-	-	22
ARM	3.24	80.42	22.85	20.02	0.42	17.20	3.57	3.85	170
AUS	0.75	19.40	6.30	4.19	0.08	2.72	0.34	0.41	717
AUT	3.61	10.38	6.63	1.66	0.10	0.81	0.14	0.06	373
AZE	3.92	22.10	11.76	4.37	0.97	3.03	1.54	0.46	144
BHS	0.06	9.90	4.44	2.56	0.28	1.73	0.55	0.26	463
BHR	0.69	9.98	4.73	2.18	0.14	0.52	0.24	0.08	269
BGD	1.86	11.50	8.03	1.80	0.10	1.07	0.19	0.17	205
BRB	0.24	16.02	5.88	2.42	0.17	1.75	0.37	0.25	513
BEL	0.34	14.03	6.18	2.83	0.12	1.67	0.25	0.16	718
BLZ	3.22	14.46	6.38	3.06	0.03	2.27	0.18	0.32	370
BOL	0.75	26.60	11.69	5.69	0.85	3.87	1.18	0.45	187
BWA	8.16	14.31	12.39	0.98	0.19	0.79	0.28	0.11	165
BRA	8.65	933.60	68.71	115.38	1.42	185.95	10.15	21.50	295
BGR	2.12	1232.75	48.10	131.98	6.08	1006.04	24.15	82.10	194
BDI	6.41	19.84	9.85	3.05	0.11	1.17	0.47	0.30	66
CAN	0.20	20.90	5.69	3.73	0.11	1.77	0.38	0.28	719
CPV	2.00	11.08	5.69	2.29	0.14	1.51	0.43	0.32	142
CAF	2.08	3.73	2.63	0.57	-	-	-	-	12
CHL	0.46	19.17	6.73	4.49	0.24	4.55	0.86	0.77	149
CHN	1.21	4.50	2.63	0.91	0.31	1.28	0.39	0.16	96
COL	4.35	52.64	13.75	9.33	0.26	6.38	1.33	1.55	108
CRI	3.33	24.50	15.44	4.90	1.10	5.50	1.37	0.56	163
HRV	1.90	7.60	4.27	1.62	0.31	1.03	0.50	0.17	107
CYP	2.46	6.23	5.30	0.84	0.02	1.04	0.07	0.11	400
CZE	1.66	15.54	5.75	3.71	0.16	1.52	0.31	0.22	193
DNK	2.00	20.70	9.94	5.90	0.40	4.62	0.60	0.48	380
EGY	5.26	19.40	10.53	3.26	0.03	3.79	0.44	0.53	228
SLV	2.82	6.99	3.77	0.95	-	-	-	-	56
ETH	0.04	12.00	3.83	3.71	0.02	6.74	0.35	0.71	284
EUR	0.34	11.75	5.81	3.03	0.07	1.10	0.27	0.19	311
FJI	0.07	18.65	4.06	2.74	0.05	5.47	0.67	0.81	417
FRA	0.37	18.92	6.31	3.71	0.08	1.77	0.36	0.26	600
GEO	9.95	58.44	31.79	14.05	-	-	-	-	55
DEU	0.34	12.05	4.39	2.02	0.15	1.55	0.32	0.17	683
GHA	9.38	46.75	22.22	10.27	0.53	7.69	1.05	0.93	376
GRC	0.72	25.50	11.44	6.13	0.18	6.55	0.44	0.60	359
GUY	2.84	33.75	10.67	6.91	0.04	12.99	0.44	0.95	420
HTI	4.00	27.83	16.42	7.36	1.53	6.26	2.07	0.90	157
HND	13.97	18.00	14.71	1.31	-	-	-	-	26
HKG	-0.08	12.24	3.53	2.27	0.27	3.14	0.56	0.45	223
HUN	5.55	35.30	16.81	8.95	0.52	2.90	1.01	0.47	251
ISL	4.44	34.30	11.46	6.53	0.27	5.01	1.01	0.93	270
IND	3.39	14.00	8.19	2.53	0.57	1.71	0.81	0.25	204
IDN	3.50	14.50	8.97	2.42	-	-	-	-	48
IRQ	1.20	22.00	11.63	6.32	2.16	3.91	2.56	0.46	70
IRL	0.31	39.94	8.07	4.60	0.32	15.05	0.88	1.27	480
ISR	0.29	17.96	8.73	4.54	0.23	1.58	0.58	0.28	215
ITA	0.31	22.08	7.48	5.19	0.05	2.44	0.43	0.42	718
JAM	1.75	51.98	12.81	10.16	0.23	6.48	0.97	1.11	682
JPN	0.00	8.27	3.78	2.50	0.03	0.64	0.14	0.13	600
JOR	2.05	6.88	4.93	1.71	0.23	0.80	0.35	0.12	71
KAZ	2.09	318.78	28.70	60.71	0.17	29.70	2.48	4.87	188
KEN	0.11	70.64	11.98	9.39	0.43	8.72	1.39	1.36	405

Table 31: Descriptive statistics of GFD interest rates and the computed volatility. The volatility is based on the entire interest rate data available for that country. The last column (nobs) shows the number of consecutive months with non-missing data.

	rate				volatility				
	min	max	mean	std	min	max	mean	std	nobs
KOR	2.48	19.20	9.78	4.78	0.21	1.56	0.61	0.33	275
KWT	0.60	8.87	6.12	1.78	0.03	1.19	0.18	0.19	311
KGZ	3.47	216.50	28.94	36.62	1.79	33.82	6.19	5.79	190
LVA	2.30	33.98	8.11	8.06	0.50	4.67	0.94	0.74	173
LBN	2.54	34.18	12.34	5.72	0.12	7.86	0.66	0.95	382
LTU	1.96	37.00	8.81	8.46	0.21	1.09	0.42	0.25	115
MKD	4.66	18.00	8.63	2.72	0.33	2.47	0.67	0.43	143
MDG	3.92	24.04	12.49	5.01	0.53	3.64	0.85	0.57	111
MYS	1.82	9.98	4.39	1.35	0.04	1.48	0.18	0.18	588
MLT	1.46	5.49	4.26	0.81	0.04	0.88	0.10	0.11	264
MUS	3.68	12.91	8.50	2.32	0.26	1.71	0.51	0.24	155
MEX	4.56	153.91	29.03	25.98	0.46	13.42	2.39	2.35	251
MDA	1.21	74.30	19.73	13.39	1.99	6.98	2.94	1.25	176
MNG	5.56	7.91	6.93	0.77	-	-	-	-	27
MNE	0.45	10.80	2.65	3.59	0.17	2.65	0.65	0.58	64
MAR	3.24	3.70	3.46	0.16	-	-	-	-	21
MOZ	7.15	31.65	16.70	6.53	1.53	8.99	1.93	0.93	117
NAM	6.66	21.68	11.60	3.22	0.35	1.79	0.54	0.23	217
NPL	0.62	12.88	5.48	2.62	0.03	1.76	0.50	0.40	332
NLD	0.23	13.80	4.36	2.63	0.10	1.61	0.34	0.25	718
NZL	2.62	27.20	9.51	4.74	0.11	3.92	0.63	0.67	380
NIC	1.02	6.50	3.91	1.83	-	-	-	-	55
NGA	2.00	27.50	12.95	5.42	0.00	8.69	0.79	0.94	220
NOR	0.10	15.75	7.09	3.85	0.30	9.93	0.70	0.99	312
PAK	1.21	17.42	10.05	3.80	0.21	1.79	0.59	0.30	226
PHL	2.92	43.39	12.50	6.57	0.26	5.91	1.05	0.96	399
POL	3.90	49.02	16.57	12.10	0.16	7.64	1.06	1.31	224
PRT	0.31	22.19	7.54	5.07	0.15	1.16	0.38	0.22	292
ROU	7.82	179.94	48.65	32.35	5.11	39.99	9.25	6.77	140
RUS	0.26	355.80	33.46	59.80	1.53	3.00	1.70	0.21	114
RWA	5.24	12.85	9.57	1.80	0.24	2.21	0.57	0.35	128
SAU	1.12	7.16	4.41	1.65	0.16	0.75	0.29	0.12	207
SRB	4.20	99.25	17.52	12.85	0.78	5.92	1.57	0.97	108
SLE	3.80	95.20	16.56	14.98	0.34	23.59	1.46	2.26	519
SGP	0.20	4.90	1.90	1.17	0.16	1.30	0.46	0.22	265
SVK	1.95	26.00	9.36	5.90	0.15	5.81	1.17	1.12	180
SVN	0.55	12.70	6.37	3.30	0.08	1.47	0.47	0.27	96
ZAF	1.00	22.15	8.06	5.11	0.08	1.91	0.34	0.31	719
ESP	0.31	15.27	7.39	4.40	0.15	1.45	0.36	0.24	329
LKA	6.54	21.30	13.08	3.20	0.40	3.07	0.94	0.49	345
SWZ	4.93	19.50	10.83	3.27	0.53	1.90	0.77	0.26	293
SWE	0.13	18.00	6.46	3.78	0.25	3.70	0.53	0.39	660
CHE	0.00	9.30	3.30	2.43	0.09	1.05	0.36	0.22	358
TWN	0.17	14.99	5.46	3.18	0.08	2.52	0.27	0.32	428
TZA	2.60	62.30	13.25	11.01	0.69	11.80	2.36	1.90	189
THA	1.02	19.32	6.19	4.12	0.10	3.28	0.40	0.59	156
TTO	2.30	12.11	5.92	2.52	0.04	1.44	0.18	0.22	535
TUN	4.02	11.62	7.38	2.17	0.45	0.93	0.46	0.05	239
TUR	7.92	159.44	56.71	32.63	1.07	25.30	7.21	5.85	170
UGA	2.97	43.50	16.74	11.90	0.86	10.88	1.73	1.34	358
GBR	0.42	16.27	6.77	3.58	0.18	1.29	0.48	0.28	719
USA	0.01	15.52	4.77	2.87	0.13	1.97	0.37	0.29	719
URY	2.26	146.47	26.98	25.14	1.12	9.09	2.96	2.20	87
VEN	8.89	57.05	21.98	10.40	2.58	13.77	3.60	1.70	86
VNM	3.34	15.60	7.06	2.67	0.21	4.49	0.54	0.62	147
ZMB	4.38	181.78	26.51	26.56	1.14	36.66	3.10	4.38	382
ZWE	3.05	525.00	34.97	78.92	1.60	259.00	5.55	18.79	566

Table 32: The whole sample of countries. This table shows for each portfolio 1 through 5 the average 3-month change in log spot exchange rates (i.e. the appreciation of the foreign currency) Δs, the average interest rate relative to USD $i^* - i$, the average log 3 month excess return rx, and the average volatility of interest rates $\sigma(i^*)$. All moments are annualized and in percentage points. In brackets are standard deviations. The average excess return and volatility of interest rates are plotted in the left panels of figure 27.

	1	2	3	4	5	Total
Spot Change: Δs	-1.028	-0.184	-1.845	-3.797	-2.935	-1.955
	(13.03)	(13.83)	(14.32)	(13.85)	(46.44)	(24.15)
$i^* - i$	-1.594	0.0685	1.724	3.823	9.282	2.660
	(2.004)	(1.882)	(2.231)	(3.326)	(8.661)	(5.820)
Excess Return: rx	-2.713	-0.143	-0.207	-0.0346	6.667	0.716
	(13.70)	(14.03)	(14.33)	(13.68)	(46.56)	(24.48)
Volatility: $\sigma(i^*)$	0.301	0.308	0.426	0.548	1.079	0.533
	(0.134)	(0.146)	(0.212)	(0.337)	(0.979)	(0.560)

Table 33: The sample of countries that had interest rate data on January 1975. This table shows for each portfolio 1 through 5 the average 3-month change in log spot exchange rates (i.e. the appreciation of the foreign currency) Δs, the average interest rate relative to USD $i^* - i$, the average log 3 month excess return rx, and the average volatility of interest rates $\sigma(i^*)$. All moments are annualized and in percentage points. In brackets are standard deviations. The average excess return and volatility of interest rates are plotted in the left panels of figure 27.

	1	2	3	4	5	Total
Spot Change: Δs	-0.805	0.627	-0.776	-5.063	-9.395	-3.082
	(11.85)	(14.42)	(19.64)	(19.66)	(21.48)	(18.15)
$i^* - i$	-2.292	-0.570	1.082	3.438	11.60	2.653
	(2.416)	(2.198)	(2.024)	(2.446)	(7.739)	(6.304)
Excess Return: rx	-3.241	0.0177	0.244	-1.677	2.506	-0.430
	(12.47)	(14.80)	(19.86)	(19.62)	(21.24)	(18.00)
Volatility: $\sigma(i^*)$	0.303	0.305	0.415	0.511	1.372	0.581
	(0.158)	(0.142)	(0.260)	(0.383)	(0.927)	(0.621)

Table 34: The sample of countries that had interest rate data on January 1987. This table shows for each portfolio 1 through 5 the average 3-month change in log spot exchange rates (i.e. the appreciation of the foreign currency) Δs, the average interest rate relative to USD $i^* - i$, the average log 3 month excess return rx, and the average volatility of interest rates $\sigma(i^*)$. All moments are annualized and in percentage points. In brackets are standard deviations. The average excess return and volatility of interest rates are plotted in the left panels of figure 27.

	1	2	3	4	5	Total
Spot Change: Δs	-0.443	1.060	0.0293	-4.578	-6.897	-2.166
	(11.75)	(17.22)	(16.03)	(13.65)	(54.49)	(27.87)
$i^* - i$	-0.964	0.725	2.394	5.551	17.18	4.977
	(1.609)	(1.764)	(2.165)	(2.630)	(7.755)	(7.571)
Excess Return: rx	-1.555	1.753	2.385	0.926	10.66	2.834
	(12.11)	(17.26)	(15.92)	(13.35)	(53.12)	(27.47)
Volatility: $\sigma(i^*)$	0.321	0.308	0.401	0.684	1.912	0.725
	(0.139)	(0.131)	(0.225)	(0.397)	(1.050)	(0.800)

Table 35: The sample countries that had interest rate data on January 1995. This table shows for each portfolio 1 through 5 the average 3-month change in log spot exchange rates (i.e. the appreciation of the foreign currency) Δs, the average interest rate relative to USD $i^* - i$, the average log 3 month excess return rx, and the average volatility of interest rates $\sigma(i^*)$. All moments are annualized and in percentage points. In brackets are standard deviations. The average excess return and volatility of interest rates are plotted in the left panels of figure 27.

	1	2	3	4	5	Total
Spot Change: Δs	-0.861	1.839	-0.740	-4.033	-0.234	-0.806
	(11.84)	(14.77)	(13.07)	(13.27)	(41.09)	(21.91)
$i^* - i$	-1.144	0.408	2.171	5.311	13.56	4.062
	(1.172)	(1.040)	(1.229)	(1.928)	(4.855)	(5.781)
Excess Return: rx	-2.152	2.220	1.373	1.218	13.80	3.292
	(12.14)	(14.80)	(12.92)	(12.82)	(40.06)	(22.10)
Volatility: $\sigma(i^*)$	0.278	0.288	0.433	0.645	1.371	0.603
	(0.0753)	(0.0918)	(0.119)	(0.187)	(0.419)	(0.461)

Figure 26: U.S. and Australian Interest Rates: Volatility and Spread

GARCH estimates of U.S. and Australian interest rate volatility appear in the upper left panel. Interest rates appear in the upper-right panel. U.S. data are the blue dashed-lines and Australian data are the green solid lines. The lower panel plots the differences, U.S. minus Australia from the two top panels. The red dashed line is the volatility difference and the the black solid line is the interest rate differential, divided by 5. Lognormal models predict that, in the lower panel, the lines appear on opposite sides of zero, and are negatively correlated. By-and-large, the opposite seems to be true. The correlation is 0.52. Data source: 3-month interbank deposits from Global Financial Data (www.globalfinancialdata.com).

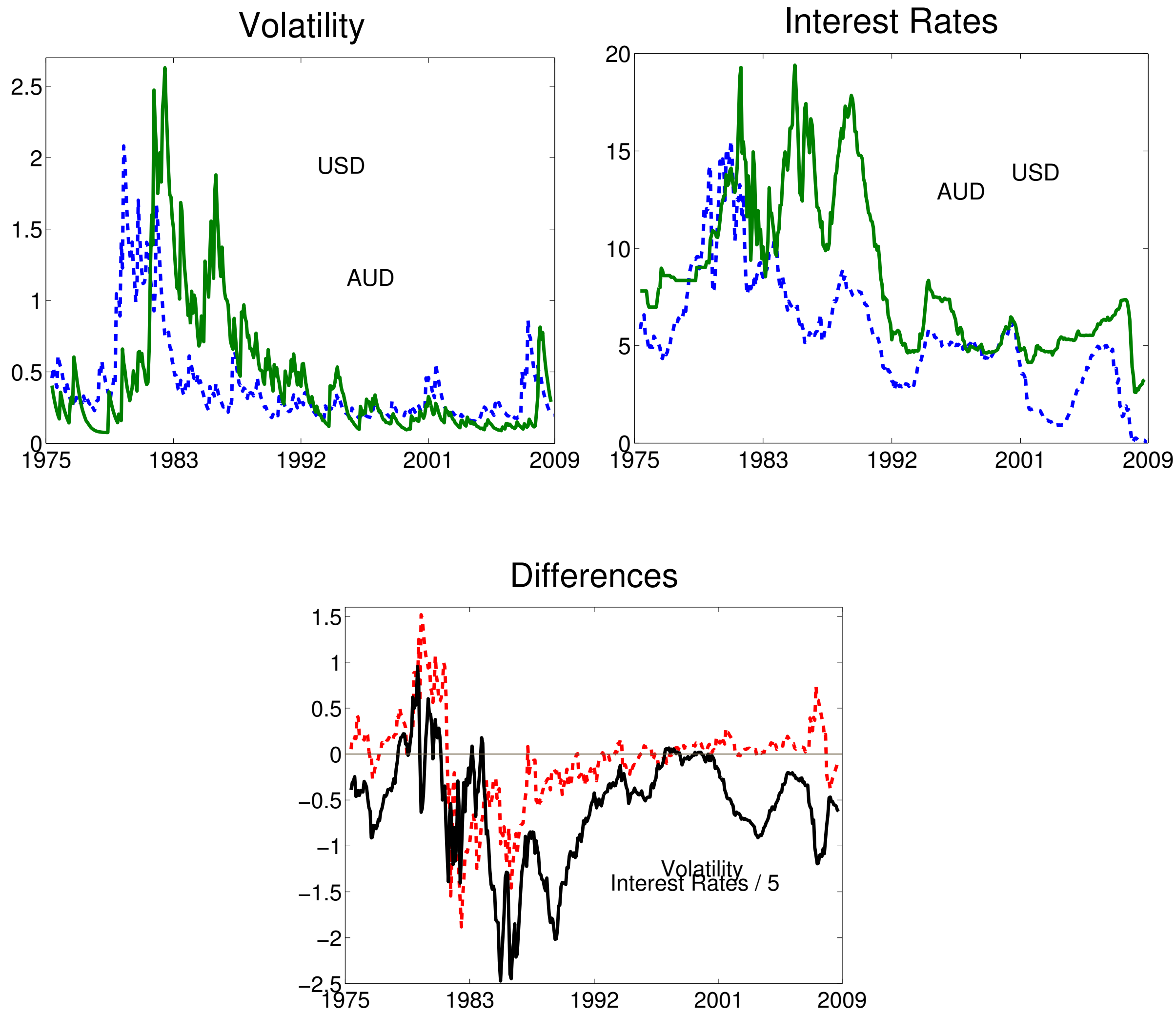
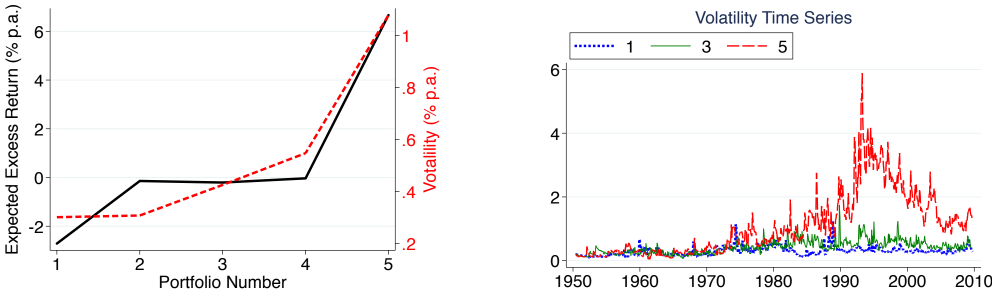
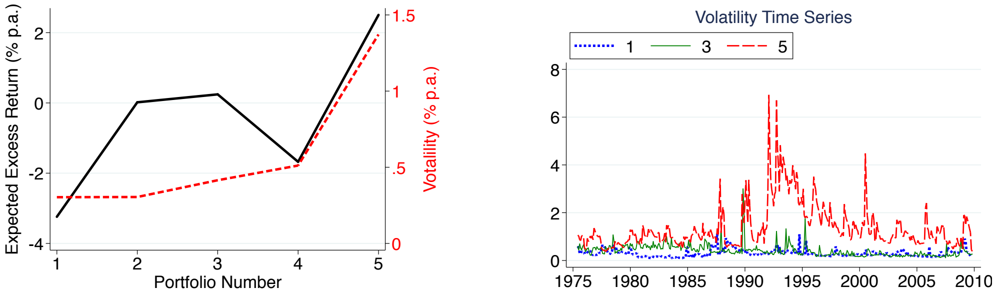


Figure 27: Average Volatility of Currency Portfolios

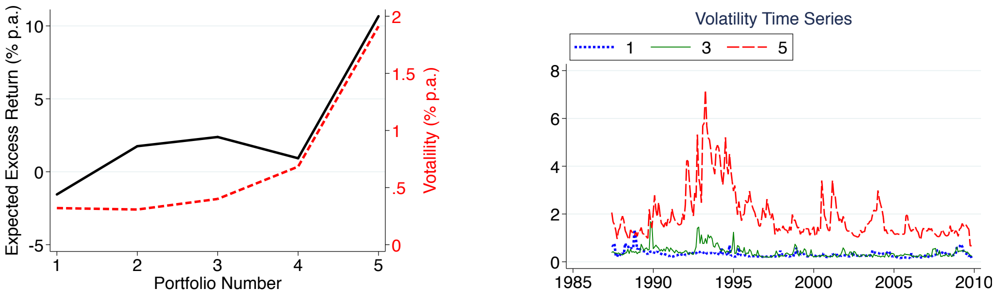
GFD interest rates, the whole sample.



GFD interest rates, only the countries that had non missing data on January 1975. See tables 29, 30, and 31 to see which countries are in this sample.



GFD interest rates, only the countries that had non missing data on January 1987 Jan. See tables 29, 30, and 31 to see which countries are in this sample.



GFD interest rates, only the countries that had non missing data on January 1995 Jan. See tables 29, 30, and 31 to see which countries are in this sample.

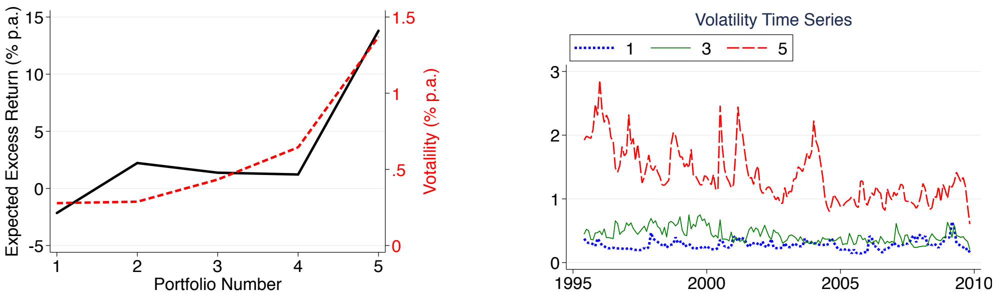


Figure 28: GFD Interest Rates: Bilateral Currency Pairs

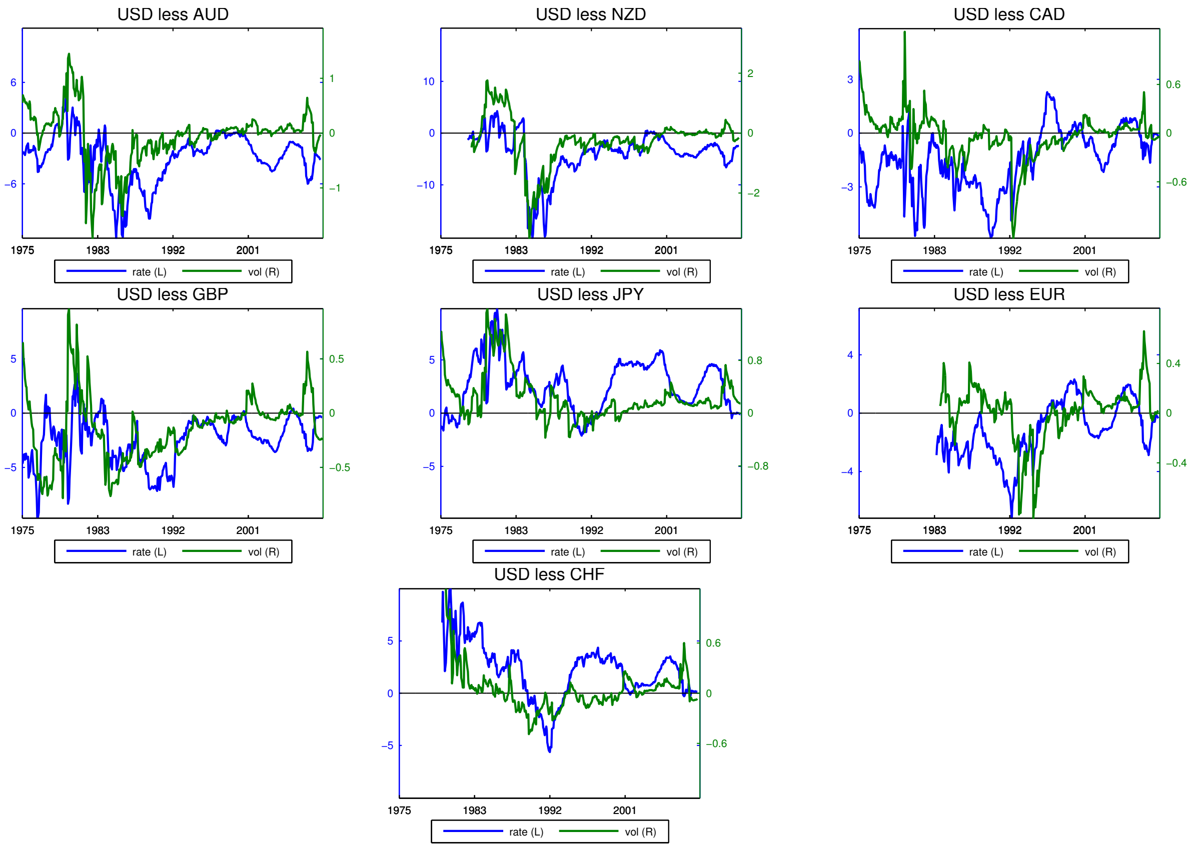
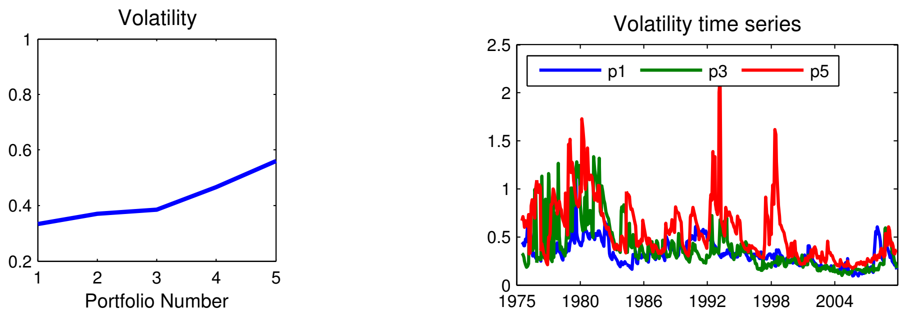
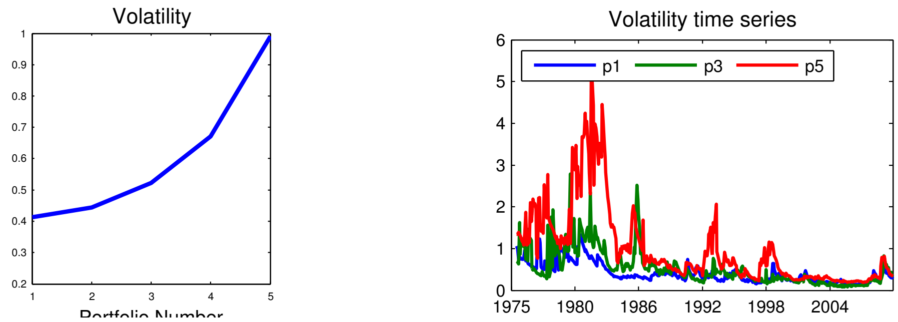


Figure 29: Data Source Comparison: GFD Versus FT/ICAP

GFD interest rates, the same set of countries (and years) that is in the FT/ICAP interest rate data source in below figure.

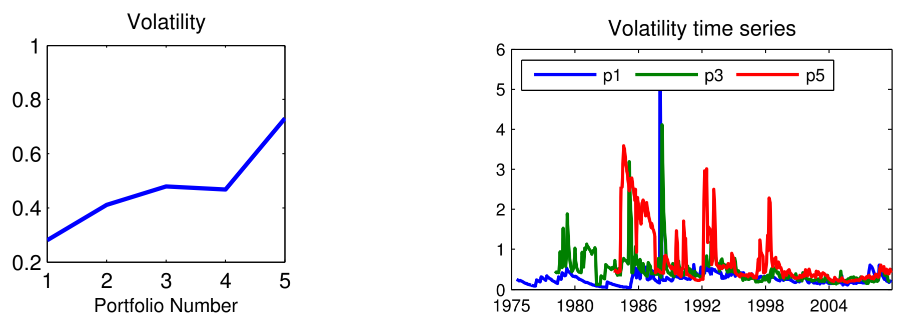


FT/ICAP interest rates, the whole sample.



The difference in volatilities between GFD and FT/ICAP interest rates (especially in early 1980's) is due to how different the GFD and FT/ICAP interest rates look even for the same country and the same date. Good examples were BEL, FRA, and ITA. So in below two figures we re-plot the above two figures without BEL, FRA, and ITA, after which the average volatility figure looks similar (in scale) between GFD and FT/ICAP.

GFD interest rates without BEL, FRA, ITA.



FT/ICAP interest rates without BEL, FRA, ITA.

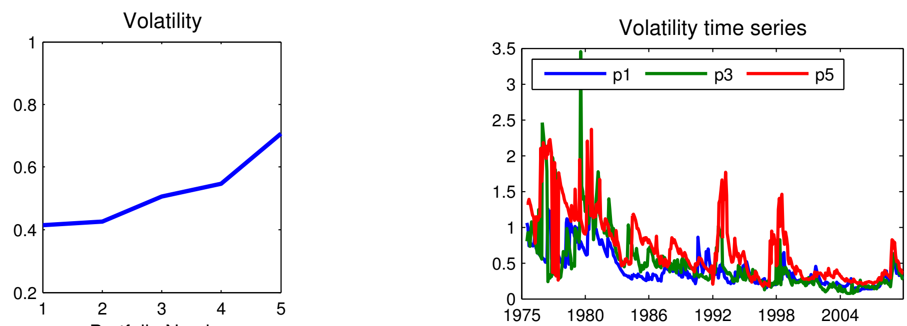
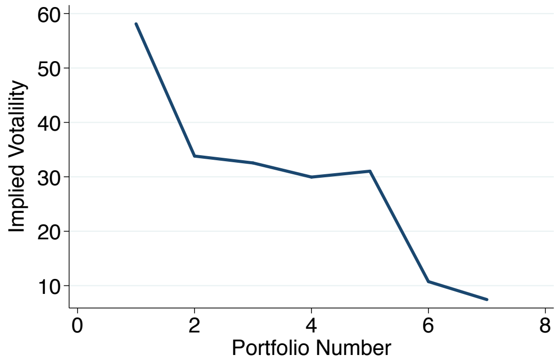
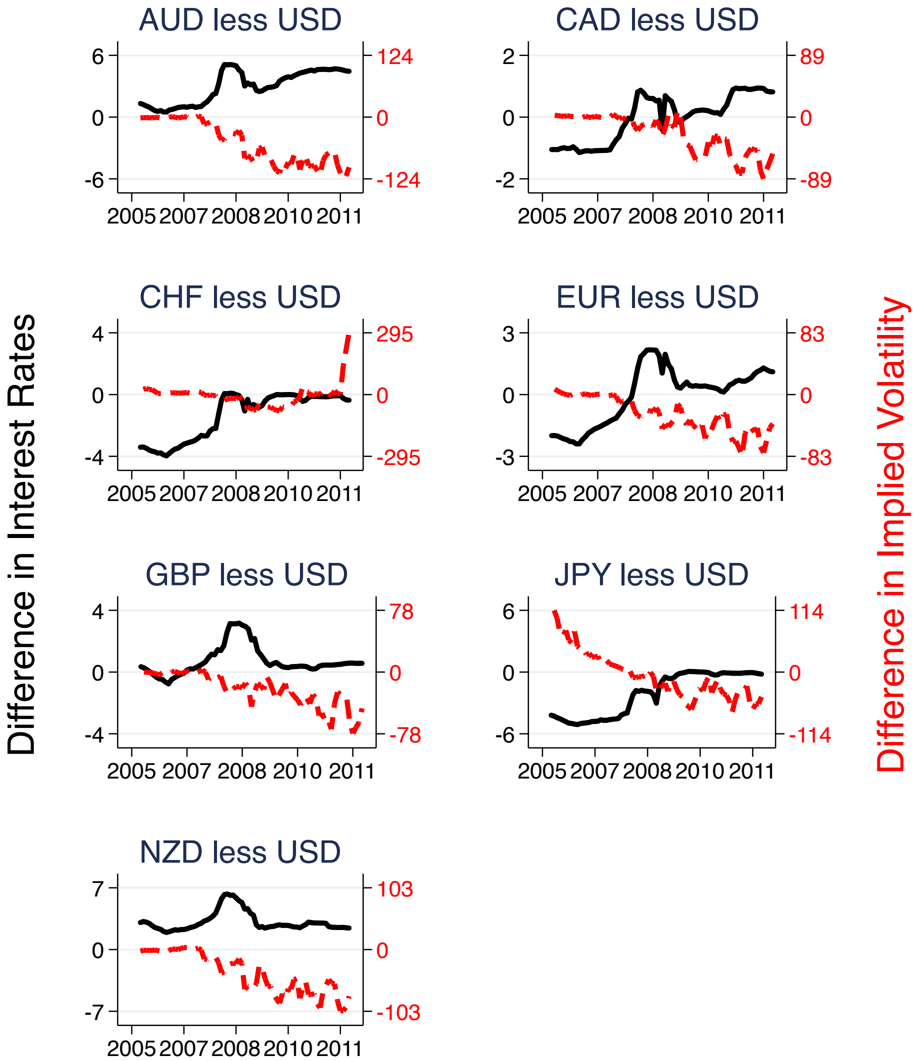


Figure 30: Implied Volatility of Options on Interest Rate Futures





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