Carnegie Mellon
Tepper
SCHOOL OF BUSINESS

DISSERTATION

Submitted in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY
INDUSTRIAL ADMINISTRATION
(MANAGEMENT OF MANUFACTURING & AUTOMATION)

Titled

"ESSAYS ON CORPORATE GOVERNANCE, MANAGERIAL INCENTIVES, AND CROWDFUNDING"

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Essays on Corporate Governance, Managerial Incentives, and Crowdfunding

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April 2016

CARNEGIE MELLON UNIVERSITY

Essays on Corporate Governance, Managerial Incentives, and Crowdfunding

A dissertation

submitted to the Tepper School of Business

in partial fulfillment of the requirements for the degree of

Doctor of Philosophy

Field of Industrial Administration

by

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April 2016

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Abstract

Corporate governance attracted much attention after the corporate failures of Enron, WorldCom, and others, and was again at the center of debate after the financial crisis in 2008. In response, regulations such as Sarbanes-Oxley (SOX) Act and Dodd-Frank Wall Street Reform and Consumer Protection Act were enacted to protect investors by mandating requirements for board composition and executive compensation. On the other hand, to alleviate financing constraints for business development and expand investment opportunities to retail investors, the Jumpstart Our Business Startups (JOBS) Act legalizes crowdfunding by authorizing U.S. Securities and Exchange Commission (SEC)-approved portals for companies to seek funding from anyone. My dissertation aims at studying the incentive issues, especially how regulations might affect the strategic interaction between managers, directors, and shareholders (including crowdfunders).

In the first chapter, titled "Nominal versus Real Board Independence: The Impact of Director Tenure," (joint work with Jonathan Glover and Carlos Corona), we investigate whether a regulation that mandates a greater proportion of outside directors on the board results in a more independent board. We find that the higher the nominal independence level of the board, that is, the higher fraction of outside directors, the more reluctant the CEO is to replace the existing directors. The resultant longer tenures of outside directors make the CEO even less willing to replace them, which causes lower real independence. Regulations that mandate higher nominal independence can have the unintended consequence that they lower both the real independence and the expertise of the board of directors in the long run.

In the second chapter, titled "Compensation Duration, Shareholder Governance, and Managerial Short-termism," I investigate the interaction between the duration of executive compensation and shareholder governance. I show that short-term compensation can elicit shareholder intervention and thus enhance firm value. The central mechanism is that the use of short-term incentives enables informed incumbent shareholders to commit to using their private information to intervene (voice) instead of selling their shares (exit). Without a commitment to voice, incumbent shareholders might find, ex post, that exit is more appealing than voice if they privately observe that a firm's type is bad. Short-term incentives encourage a good firm to take actions that reveal its type early on. This, in turn, reduces the information advantage of the incumbent shareholders and their ability to profit from exit. Effectively, short-term compensation serves as a commitment device for value-enhancing intervention.

In the third chapter, titled "A Crowdfunding Model for Green Energy Investment," (joint work with Ying Xu, Nilanjan Chakraborty, and Katia Sycara), we study a new renewable energy investment model through crowdfunding, which is motivated by emerging community solar farms. In this paper we develop a sequential game theory model to capture the interactions among crowdfunders, the solar farm owner, and an electricity company who purchases renewable energy generated by the solar farm in a multi-period framework. By characterizing the sub-game perfect equilibrium, and comparing it with a benchmark model without crowdfunding, we find that under crowdfunding although the farm owner lowers its investment, the overall green energy investment level is increased due to the contribution of crowdfunders. We also find that crowdfunding can increase the penetration of green energy in consumption and thus reduce the energy procurement cost of the electricity company. Finally, the numerical results based on real data indicates crowdfunding is a simple but effective way to boost green generation.

Acknowledgments

I am greatly indebted to Jonathan Glover and Katia Sycara for their guidance and encouragement. It was Katia who brought me to Carnegie Mellon, and it was Jon who brought me to the area of information economics and accounting.

I am grateful to my committee members Carlos Corona, Fabrizio Ferri, and Pierre Liang for their valuable advice and comments.

I also thank Tim Baldenius, Andrew Bird, Nilanjan Chakraborty, Tinglong Dai, Xin Fang, Stephen Karolyi, Volker Laux, Chen Li, Jing Li, Yi Liang, Jieming Liu, Xiao Liu, Thomas Ruchti, Jack Stecher, Austin Sudbury, Ying Xu, Jessie Jiaxu Wang, Hao Xue, Gaoqing Zhang, Ran Zhao, and Helen Yangfang Zhou.

I have benefited from discussions and interactions with my fellow students Cyrus Aghamolla, Patrick Choi, Francisco Cisternas, Lee Lili Gao, Ryan Kim, Musab Kurnaz, Jiayi Li, Nan Li, Guoyu Lin, Eric Siyu Lu, Carlos Ramirez, Ethan Rouen, Lufei Ruan, June Zijun Shi, Nan Xiong, Wenjie Xue, Andreas Veit, Xin Wang, Shunyuan Zhang, and Yuan Zou.

I also would like to thank Lawrence Rapp and Alan Scheller-Wolf for making life easier for me at Carnegie Mellon .

Finally, I want to dedicate the thesis to my parents, my sister, and my niece, for always believing and supporting me.

Contents

[1]This chapter is based on a joint work with Carlos Corona and Jonathan Glover.

[2]This chapter is based on my job market paper. I am greatly indebted to Jonathan Glover and Katia Sycara for their guidance and encouragement. I am grateful to Carlos Corona, Fabrizio Ferri, and Pierre Liang for valuable advice and comments. I also thank Tim Baldenius, Jing Li, Jack Stecher, Austin Sudbury, Hao Xue, and workshop participants at Carnegie Mellon University, The Chinese University of Hong Kong, The University of Hong Kong, Purdue University, National University of Singapore, Singapore Management University, The University of Texas at Austin, Tilburg University. This paper has also benefited from my discussion with my fellow students Ryan Kim, Nan Li, Ethan Rouen, Lufei Ruan, and Yuan Zou. All errors are my own.

[3]This chapter is based on a joint work with Ying Xu, Nilanjan Chakraborty, and Katia Sycara. A previous version of this work without the proofs appeared as a conference proceeding of the 24th International Conference on Artificial Intelligence (Zheng et al., 2015).

Chapter 1

Nominal versus Real Board Independence: The Impact of Director Tenure[1]

Abstract

In this paper, we investigate whether a regulation that mandates a greater proportion of outside directors on a corporate board results in a more independent board. Instead of taking the fraction of outside directors directly as the measure of board independence, we define it as the nominal independence level. The real independence level of the board is determined by both the nominal independence level and the length of the relationship between board members and the CEO. We assume that the real independence between a board member and the CEO decreases over time as long as the board member stays on the board. In our dynamic model, the board both monitors and advises the CEO, and the CEO decides whether to replace one of the directors in each period. The CEO's tradeoff is between the possibly higher board expertise introduced by new directors versus the lower board real

[1]This chapter is based on a joint work with Carlos Corona and Jonathan Glover.

independence obtained by retaining the same directors. In our model, the higher the nominal independence level of the board, the more reluctant the CEO is in replacing existing directors. The resultant longer tenure of outside directors makes the CEO even less willing to replace them. Regulations that mandate higher nominal independence can have the unintended consequence that they lower both the real independence and the expertise of the board of directors in the long-run.

Key Words: Board Independence, Corporate Governance, Director Tenure

1.1 Introduction

Corporate governance, especially the independence of the board of directors, attracted great attention after the corporate failures of Enron, WorldCom, Tyco, ImClone, and others. In response, regulations were enacted to protect investors by mandating requirements for board composition. In 2002, Congress passed the Sarbanes-Oxley Ac.t, which requires the board's audit committee to consist of a majority of independent members (also sometimes referred to as outside directors). The independent members are directors who have no affiliation with the company and are, therefore, supposedly less inclined to favor insiders when taking decisions and more inclined to protect shareholder interests. Both the New York Stock Exchange (NYSE) and NASDAQ amended their rules to require boards to have a majority of independent directors.[2] The fraction of independent directors is often used as the measure of board independence (e.g., Linck et al., 2008; Faleye et al., 2011; Ferreira et al., 2011).

Conceptually, independence is "the ability of an individual to maintain perspective or judgment that is unbiased by a relationship with others"(Larcker and Tayan, 2013). Unfortunately, in practice, it is often difficult to determine whether a director on the board is really independent. Both the NYSE and the NASDAQ have had to provide guidance on

[2]See NYSE rule 303A.01 and NASDAQ rule 5605(b)(1).

what "independent director" means. According to NYSE rule 303A.02(a)(i):

> No director qualifies as independent unless the board of directors affirmatively determines that the director has no material relationship with the listed company (either directly or as a partner, shareholder or officer of an organization that has a relationship with the company).[3]

The NASDAQ has a similar rule. According to NASDAQ rule 5605(a)(2):

> Independent Director means a person other than an Executive Officer or employee of the Company or any other individual having a relationship which, in the opinion of the Company's board of directors, would interfere with the exercise of independent judgment in carrying out the responsibilities of a director.[4]

Following the spirit of the rules by the securities exchanges, we refer to directors who care only about the shareholders' interest when they newly join the board as outside directors. We define the fraction of outside directors on the board as the board's "nominal independence." Under the current rules, one reason "real independence" may differ from "nominal independence" is the duration of the relationship between outside directors and the CEO. As suggested by the Council of Institutional Investors (whose members consist of pension funds with more than $3 trillion), long tenure can affect a director's "unbiased judgment," and "[e]xtended tenure can lead an outside director to start to think more like an insider." According to the EY Center for Board Matters (2015), "[s]ome investors believe that, after a certain point, a director's ties to the company are deep and long-standing enough to potentially impair the director's independence." In other words, extended board tenure can create a culture of undue deference to management, which weakens the monitoring role of the board of directors. This can be a natural sociopsychological effect justified on behavioral grounds (Fichman and Levinthal, 1991) or

[3]The NYSE rules are quoted from the NYSE listed company manual, which is available online at http://nysemanual.nyse.com/. The quotes are as of March 26 2014.

[4]The NASDAQ rules are quoted from the NASDAQ stock market equity rules, which is available online at http://nasdaq.cchwallstreet.com/. The quotes are as of March 26 2014.

a purely rational consequence of reputation concerns (in the sense that a director is reluctant to publicly overturn her previous judgements on management to avoid a reputation cost) (Corona and Randhawa, 2010).[5] Meanwhile, increased firm-specific investments over time also make a director less willing to jeopardize her board seat by challenging management. Indeed, Huang (2013) documents evidence of high average tenure of outside board members being negatively associated with proxies for director oversight and company value. Jia (2015) also finds that extended director tenure is associated with lower innovation productivity and quality.

Adopting tenure-related guidelines or restrictions for independent directors seems a natural response, as has been done in some economies (e.g., United Kingdom, France, and Hong Kong). Most countries adopt the "recommend and explain" approach. For example, under the UK Corporate Governance Code, a board should explain the reason for classifying a director with tenure longer than 9 years as independent in its annual disclosure. France is the only country with a mandatory regime under which directors who serve on the board for more than 12 years cannot be deemed independent. In the United States, there are no formal laws, rules, or regulations under which a long tenure would prevent a director from qualifying as independent. However, in September 2013, the Council of Institutional Investors revised its best-practices corporate governance policies to include tenure as a factor boards should consider when determining whether a director is independent. Also, one of the major proxy advisory firms, Institutional Shareholder Services (ISS), has added tenure to the checklist of factors used in its rating system QuickScore but still opposes a specific "narrow rule" for director tenure or term limits. One reason against a formal regulation on director tenure is that there is no ideal term limit which can be applied to all directors, because one size does not fit all. Due to a similar concern, Australia repealed a recent move toward a recommended director term limit.

[5] Similar arguments are frequently made in discussing auditor-client relationships. The claim is that the auditor becomes less independent from the client over time, which is used to argue in favor of auditor rotation.

In our model, the "real independence" level of the board of directors is determined by both the nominal independence level and the length of relationship between outside directors and the CEO. The real independence between an inside director and the CEO is zero, while the real independence between an outside director and the CEO decreases over time if the board member stays on the board. The real independence of the board is determined by the average real independence of the board members. The question we address is: does a regulation that mandates higher nominal independence result in higher or lower real independence?

Board tenure varies across different firms due to the variation in the board succession plans. One important reason for replacing an existing board member and hiring a new director is to bring new expertise into the existing board to deal with changes in the business environment.[6] In this paper, we assume the firm demands the expertise of new outside directors to adapt to such changes in the business environment (e.g, transforming into new product strategies or expansion to new markets). Therefore, replacing existing outside directors with new outside directors brings new expertise to the board as well as increases the "real independence" of the board.

Previous research has recognized the influence of incumbent CEOs on director selection (e.g., Hermalin and Weisbach, 1998; Shivdasani and Yermack, 1999; Withers et al., 2012). For simplicity, in our main model, we assume that the incumbent CEO chooses whether or not to replace existing outside directors. There is a conflict between the CEO's and shareholders' preferences over projects, as in Adams and Ferreira (2007). By choosing no replacement and keeping the same board of directors, the CEO can take advantage of the decreased real independence which leads to a smaller conflict over project choice between the board and the CEO and thereby to less monitoring of the CEO by the board. Lower monitoring enables the CEO to have a higher chance of retaining control over project choice.

[6]In discussing director succession, the Stuart Spencer Board Index 2014(Stuart, 2014) states "Globalization and advancements in technology are giving rise to a slew of competitive and business threats and opportunities. At a time when growth and innovation are top priorities for most organizations, companies are transforming themselves through new product strategies, different product mixes, and expansion into new markets and geographies. In an ideal world, outside directors with relevant experience can serve as valuable advisers to the board and management about the company's market, geographic and product directions."

The board of directors not only monitors the CEO but also provides managerial advice to the CEO (Adams and Ferreira, 2007; Baldenius et al., 2014; Linck et al., 2008). The CEO, hoping to obtain better advice from the board of directors, has incentives to replace existing outside directors with new directors who might provide better advice. The CEO faces a trade-off between lower monitoring and possible better advising when deciding whether to replace existing outside directors with new directors.

To make the distinction between nominal and real independence meaningful while focusing on director tenure, a multi-period model is needed. A three period model suffices to make our point. To illustrate the intuition for our results, consider two boards that are both formed at the beginning of Period 0 (which is a dummy initialization period since there is no replacement decision at the start of Period 0) and are otherwise the same except that the number of outsiders on the first board is determined by Regulation R and the number of outsiders on the second board is determined by Regulation R', where R' requires a greater proportion of outsiders than R. Since there is no replacement decision in Period 0, the second board has both higher nominal and higher real independence at the start of the first period. From the perspective of the CEO, higher real board independence not only increases monitoring but also makes the project chosen by the board less desirable. Consequently, the marginal cost of higher real board independence is increasing in real independence. However, the marginal benefit of higher expertise is constant. (This assumption is relaxed later in the paper.) Since the second board has higher real independence at the start of the first period, the CEO is less willing to replace outside directors. As a result of increased retention of directors in the second board, the two boards will be closer in their real independence at the start of the second period. In the second period, a new force emerges that makes the CEO of the second board less willing to replace outside directors. When the CEO of the first board has replaced a larger fraction of directors than the CEO of second board in the first period, the longer tenure of the directors on the second board results in every replacement causing a larger increase in real independence. This new force driven by tenure makes the CEO of the second board less willing to replace outside directors than the first board, even if the second board has the same (or lower) real independence than the first board has. Hence, the second board can actually end up with a lower real independence level. That is, an unintended consequence of regulation that requires greater nominal board independence is that both real independence and advising expertise may be reduced in the long-run, because of the effect of regulation on board turnover.

Consistent with our results, outside board member turnover has trended down over the past decades according to the 2013 Spencer Stuart U.S. Board Index.[7] Our results may also help explain why many studies (see Bhagat and Black, 1999 and the survey by Adams et al., 2010) fail to find an association between performance and the fraction of independent directors on the board, as the fraction of independent directors is not necessarily a good measure of the independence level of the board. Recent studies using exogenous shocks find a casual effect of board independence on firm value (e.g., Black and Kim, 2012; Knyazeva et al., 2013). From this perspective, another implication of our results is that while there is little cross-sectional variation in nominal independence after the regulations on board compositions, the variation in real independence still contributes to the variation in firm performance.

We have designed our model to highlight conditions under which mandating (by regulation) higher nominal independence leads to lower real independence. The analysis focuses on interior solutions, which is somewhat restrictive. When the comparison is between two boards with similar proportions of outsiders, the sufficient conditions for an interior solution are not particularly demanding. However, when the difference in the proportions of outside directors is large, the solution is interior (for both boards) only for a narrow set of parameters. In the latter case, our result should be interpreted as an

[7]According to the 2013 Stuart Board Index (p.6), the boards of S&P 500 companies elected 339 new independent board members this past proxy season, down 11 percent from five years ago and 14 percent from 10 years ago.

existence result. In Section 1.7, we present extensions of our model in which mandated higher nominal independence results in either lower or higher real independence. For example, if the contribution in expertise of new directors is sufficiently decreasing in the number of new directors hired, higher nominal independence will result in higher real independence. Also, if the board of directors instead of the CEO makes the replacement decision and real independence decreases slowly enough, higher nominal independence again results in higher real independence.

The paper proceeds as follows. Section 1.2 provides a review of related literature. Section 2.3 presents the model setup and the sequence of events. Section 1.4 states the board's optimal monitoring problem and solves it. Section 1.5 presents the CEO's trade-off in deciding on board turnover in each period and analyzes the impact of the proportion of outside directors on the board on the CEO's decision. Section 1.6 provides conditions under which higher nominal board independence results in lower real board independence. Section 1.7 studies extensions, including allowing the board to make the replacement decision, and Section 2.7 concludes. All proofs are included in the appendix.

1.2 Related Literature

The effect of board independence has been examined in both empirical and theoretical studies on corporate governance. Most empirical studies of corporate boards (e.g., Linck et al., 2008; Faleye et al., 2011; Ferreira et al., 2011) consider the fraction of outsiders on the board of directors as being equivalent to the independence of the board of directors. However, the empirical evidence on the impact of board independence on firm performance is ambiguous (see the review paper by Adams et al., 2010). There are two possible theoretical explanations.

First, increasing board independence may not always be beneficial for the firm or its shareholders. Arya et al. (1998), in discussing substitutes for the role of earnings management in their model, argue that a board that is fully independent from the CEO can be more active than is efficient ex ante, due to the limited commitment of the board in making CEO replacement decisions. Laux (2008) models and refines the argument of Arya et al. (1998). Drymiotes (2007) models the monitoring role of the board as improving the precision of performance measurement and shows that putting insiders whose interests are partially aligned with the CEO on the board can serve as a commitment device that facilitates effective monitoring by the board. Kumar and Sivaramakrishnan (2008) models the monitoring role as generating independent information on the firm's economic prospects used in a CEO's compensation contract and shows that a more dependent director increases her monitoring effort ex ante to offset the wealth cost from her poorer contract choice ex post. Adams and Ferreira (2007) build a model in which both the board's monitoring and advising roles depend on the CEO's information. An overly independent board inhibits the communication between the CEO and the board, since the CEO may withhold information to avoid stricter monitoring. Harris and Raviv (2008) shows that insider-controlled boards may be optimal in some cases, since insider-control better exploits insiders' information.[8] Baldenius et al. (2014) shows that shareholders may prefer an advisor-heavy (monitor-light) board to prevent CEOs from entrenching themselves by choosing "complex" projects. In this paper, we also consider the dual advising and monitoring roles of the board of directors. However, departing from these papers, our focus is on whether regulation requiring a higher proportion of outside directors on the board (nominal independence) leads to increased real independence.

Second, as this paper argues, the fraction of outsiders on the board may not be a good measure of real independence, which depends on board tenure and is endogenously determined. There are other papers on the endogenous determination of the composition of the board of directors, although none of them study the effect of director tenure on board independence. For example, Hermalin and Weisbach (1998) consider the determination of the composition of the board of directors as the outcome of a bargaining game between the

[8] Harris and Raviv (2008) builds on Dessein (2002)'s model of delegation as an alternative to cheap talk communication in which there is a tradeoff between "*loss of control* under delegation and *loss of information* under communication."

CEO and the board of directors. Schmeiser (2012) models the composition of the board of directors as determined by director voting and examines the impact of majority rule on shareholder value.

1.3 The Model

Consider a three period model of a firm consisting of a CEO and a board of directors. The size of the board (the number of directors) is denoted by N and is a constant. Of the N directors, O directors are outsiders. We measure the nominal independence level of the board as the proportion of outside directors, $o = \frac{O}{N}$, which is also constant. We distinguish the nominal independence level of the board from its real independence. The real independence of the board is the extent to which the board cares about shareholders' interests as opposed to the CEO's interests. We assume that the real independence level of the board of directors depends not only on the proportion of outside directors but also on how long these outside directors stay on the board. The real independence level of an outside director decreases over time as long as the outside director stays on the board. In particular, we assume the real independence level of any insider is 0, while the real independence level of an outsider who has been on the board for τ periods (or has a tenure of τ) is η^τ, where $\eta \in (0, 1)$. The real independence of the board of directors in period t, i_t, is defined as the average real independence of all board members, which is given by:

$$\begin{aligned} i_t &= 0 \cdot (1 - o) + \sum_{k=1}^{O} \frac{\eta^{\tau_k}}{N} \\ &= \frac{\sum_{k=1}^{O} \eta^{\tau_k}}{N}, \end{aligned} \tag{1.1}$$

where τ_k is the number of periods that an outsider k has stayed on the board.

Each period $t = 0, 1, 2$, the firm's profits depend on the outcome of a business decision, $s_t \in \mathbb{R}$, taken by the CEO. In our model, the board of directors can affect how the CEO takes these periodic decisions in two different ways. First, the board can provide expertise to improve the efficiency of the periodic decisions. Second, the board can monitor the CEO to ensure that he maximizes the board's preferences as opposed to his own.

The expertise of the board depends on the expertise of its members. In particular, each individual director j has an expertise $\epsilon_{j,t} \in (0,1)$ in period t. The expertise of the board of directors in period t, e_t, is then defined as the average expertise of all board members, which is given by:

$$e_t = \sum_{j=1}^{N} \frac{\epsilon_{j,t}}{N}. \tag{1.2}$$

The board's expertise level, e_t, measures the probability with which the board can learn a parameter, $\tilde{p}_t$, which affects the efficiency of the business decision in period t. We assume that as long as $\tilde{p}_t$ is learned by the board, it is also learned by the CEO. This is a simplifying assumption used to avoid having to introduce communication between the board and the CEO but can also be justified as describing practice, since CEOs almost always serve on the board themselves. We assume that the prior distribution of $\tilde{p}_t$ is a normal distribution with mean μ_p and variance σ_p^2, i.e.,

$$\tilde{p}_t \sim N\left(\mu_p, \sigma_p^2\right). \tag{1.3}$$

The optimal decision s_t for both the shareholders and the CEO depends on the preference parameter, $\tilde{p}_t$. In particular, the utility of the shareholders is given by:

$$u_t^S = -\left(s_t - \tilde{p}_t\right)^2, \tag{1.4}$$

while the CEO's preferences are given by:

$$u_t^M = -\left[s_t - \left(\tilde{p}_t + b\right)\right]^2, \tag{1.5}$$

where $b > 0$ is common knowledge and measures the CEO's bias relative to the preference of shareholders.

The board of directors not only provides advice to the CEO but also monitors the CEO's strategic decision. The purpose of the board's monitoring is to influence the decision s_t. Specifically, the board monitors the CEO with intensity $m_t \in [0,1]$ in period t, incurring a monitoring cost $c(m_t)$ and obtains the right to determine the decision s_t if the monitoring succeeds, which happens with probability m_t. With probability $1 - m_t$, the monitoring fails and the CEO retains the right to decide s_t. The cost associated with the monitoring intensity satisfies $c'(m_t) \geq 0$, $c''(m_t) > 0$.

Following the existing literature (e.g., Adams and Ferreira, 2007; Drymiotes, 2007; Kumar and Sivaramakrishnan, 2008; Laux, 2008), we represent the preferences of the board by considering the board of directors as an average agent. To capture the influence of real independence, i_t, on the preferences of the board, we assume the board's utility function is a weighed average of the manager's utility with weight $(1 - i_t)$ and the shareholder's utility with the weight i_t. Thus, the utility function of a board with real independence level i_t is:

$$u_t^B = (1 - i_t)\, u^M + i_t u_t^S - c(m_t)\,. \tag{1.6}$$

Plugging in the expressions for u_t^M and u_t^S, the utility of the board can be written as:

$$u_t^B = -\left\{s_t - \left[\tilde{p}_t + (1 - i_t)\, b\right]\right\}^2 - c(m_t) - i_t\,(1 - i_t)\,(2\tilde{p}_t + b)^2\,.$$

Since the last term does not depend on the decision s_t, the objective function of the board can be normalized to:

$$u_t^B = -\left\{s_t - \left[\tilde{p}_t + (1 - i_t)\, b\right]\right\}^2 - c(m_t)\,. \tag{1.7}$$

Both the CEO and the shareholders, and therefore the board, have inter-temporally additive utility and discount future utility with a discount factor β.

Previous research has recognized the influence of incumbent CEOs on director selection (e.g, Hermalin and Weisbach, 1998; Shivdasani and Yermack, 1999; Withers et al., 2012). In Periods 1 and 2, we assume the CEO can choose the fraction of directors to be replaced with new outside directors with relevant experience who can provide valuable advice about a strategic decision. The assumption that the CEO makes replacement decision is a simplifying assumption intended to capture the influence of the CEO on the board's decision on director selection. In Section 1.7, we also present an extension of our model in which the board of directors instead of the CEO makes the replacement decision. The board composition is taken as given at the beginning of Period 0. The replacement decisions are denoted by the variables r_1 and r_2, which represent the proportion of board directors that are replaced in Periods 1 and 2 respectively. Here, we assume the CEO never replaces an insider because of the dependence of insiders on the CEO as well as the importance of some unmodeled firm specific expertise of inside directors, which implies $r_t \in [0, o]$, $t = 1, 2$.

Replacement decisions affect both the real independence and the expertise level of the board of directors. Specifically, the influence of the replacement decision r_t on the real independence i_t is given by the expression:

$$\begin{aligned} i_t\left(i_{t-1}, r_t; o\right) &= \left(o - r_t\right)\eta\frac{i_{t-1}}{o} + r_t \cdot 1 + (1-o)\cdot 0 \\ &= i_{t-1}\eta + r_t\left(1 - \eta\frac{i_{t-1}}{o}\right), \end{aligned} \tag{1.8}$$

where $\frac{\eta i_{t-1}}{o}$ is the average real independence of existing outside directors, 1 is the real independence of new outside directors, and 0 is the real independence of inside directors. Also, the level of expertise of the board in period t after the replacement decision r_t, is given by

$$e_t\left(r_t\right) = e_o\left(1 - r_t\right) + r_t\epsilon = e_o + r_t\Delta, \tag{1.9}$$

where ϵ is the expertise of new outside directors and e_o is the expertise of existing board members before replacement, with $e_o < \epsilon$ and $\Delta = \epsilon - e_o$. We assume that all incumbent directors have a generalist level of expertise e_o that is applicable to all decisions s_t. However, the CEO can obtain a higher level expertise that is decision specific in a given period by replacing current directors with new outside directors of expertise $\epsilon > e_o$. In subsequent periods, all directors that remain on the board revert back to the generalist expertise level e_o. This is a simplifying assumption intended to capture the idea that the board periodically needs the new skills of new directors to deal with new business strategies, which can be justified based on the criticism that the longer-tenured directors "can no longer keep current with respect to industrial or technological developments,"[9] which suggests a persistent demand for outside directors with new skills and experience to guide the company in changing economic environments. In explaining the association between director turnover and firm value, Anderson and Chun (2014) argue "new directors bring fresh perspectives and new skills, and they may be more likely than established members to challenge orthodoxy and raise previously unasked questions." We discuss the implications of expertise persistence in Section 1.7.

Overall, the sequence of events in Period $t = 1, 2$, is as shown in Figure 2.1. (Period 0 is a dummy period since there is no replacement decision at the start of Period 0.) At the beginning of Period t, $t = 1, 2$, the CEO chooses the fraction of directors to be replaced with new outside directors with relevant experience on the new strategic decision. Then, the board chooses the monitoring intensity, m_1. After that, the board learns the private information $\tilde{p_1}$ with probability e_1. Next, with probability m_1, the monitoring succeeds and the board chooses the project. With probability $1 - m_1$, monitoring fails, and the CEO chooses the project. After the project is chosen, the payoffs of the CEO, the board of directors and the shareholders are realized, and the real independence of outside directors on the board

[9] In discussing activists' criticism on long director tenure, Katz and McIntosh (2014) states "critics posit that older directors—who are typically the longer tenured directors—can no longer keep current with respect to industrial or technological developments and are unable to offer new insights into corporate issues; they fear that these directors may hold fossilized positions that are no longer relevant in the changing economic and business environment."

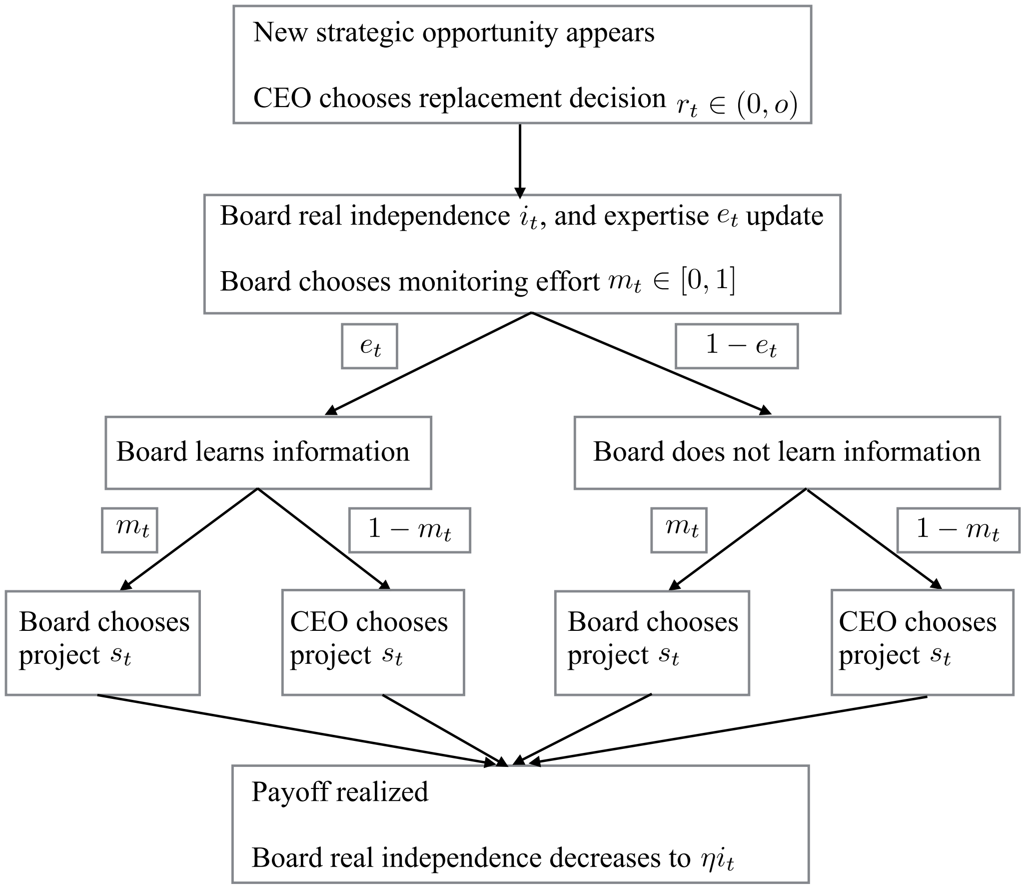

Figure 1.1: Time-line of Period t

decreases.

1.4 Board's Problem

In Period t, given the resultant real independence level of the board, i_t, and the resultant expertise level of the board, e_t, the monitoring intensity of the board affects only the payoffs of the CEO and shareholders in Period t. Moreover, the board's utility is a weighted average of the CEO's and the shareholders' utility. Therefore, the board's optimal monitoring intensity is a short-term decision that only maximizes the board's utility in Period t, u_t^B.

Within each period, we proceed by backward induction in solving the board's optimal monitoring problem. If monitoring is successful, the board has full control over project choice. The project choice depends on whether the board successfully acquires the information about the true optimal project. So, if the board observes the realization of $\tilde{p}_t$, it faces the maximization problem:

$$\max_{s_t} -\{s_t - [\tilde{p}_t + (1 - i_t)b]\}^2 - c(m_t). \tag{1.10}$$

Otherwise, if the board does not know $\tilde{p}_t$, it solves:

$$\max_{s_t} -E[s_t - [\tilde{p}_t + (1 - i_t)b]]^2 - c(m_t). \tag{1.11}$$

Therefore, the board chooses the optimal project according to:

$$s_t^B = \begin{cases} \tilde{p}_t + (1 - i_t)\, b & \text{if the board learns } \tilde{p}_t \\ \mathbb{E}\left[\tilde{p}_t\right] + (1 - i_t)\, b = \mu_p + (1 - i_t)\, b & \text{if the board does not learn } \tilde{p}_t. \end{cases} \tag{1.12}$$

This implies that, if the board learns the true optimal project, the board chooses the project with a bias $(1 - i_t)b$ away from the optimal project $\tilde{p}_t$ from the shareholder's perspective, and, therefore, obtains utility $u_t^B = -c\,(m_t)$. Otherwise, if the board does not learn the true optimal project, the board chooses the project with a bias $(1 - i_t)b$ from the expected optimal project $\mathbb{E}[\tilde{p}_t]$ and, therefore, earns a utility $u_t^B = -\left\{\tilde{p}_t - \mathbb{E}\left[\tilde{p}_t\right]\right\}^2 - c\,(m_t)$.

If monitoring is not successful, the CEO retains control over the project choice decision, s_t. The CEO's project choice also depends on whether the board successfully learns the true optimal project. Recall that, if the board learns what the optimal project is, so does the CEO. Thus, the CEO chooses:

$$s_t^M = \begin{cases} \tilde{p}_t + b & \text{if board learns } \tilde{p}_t \\ \mathbb{E}\left[\tilde{p}_t\right] + b = \mu_p + b & \text{if board does not learn } \tilde{p}_t. \end{cases} \tag{1.13}$$

If the board learns $\tilde{p}_t$, the CEO chooses the project with a bias b from $\tilde{p}_t$. Otherwise, the

CEO chooses the project with a bias b from the expected optimal project $\mathbb{E}[\tilde{p}_t]$. The utility of the board resulting from the CEO's choice is $u_t^B = -i_t^2 b_t^2 - c(m_t)$ if the CEO learns the project type, and $u_t^B = -\{\mathbb{E}[\tilde{p}_t] - \tilde{p}_t + i_t b\}^2 - c(m_t)$ if the CEO does not learn the project type.

Anticipating the effects of its monitoring effort, the board chooses the optimal monitoring intensity to maximize its expected utility by solving:

$$\begin{aligned} & \max_{m_t \in [0,1]} m_t \left[e_t \cdot 0 - (1-e_t)\,\sigma_p^2\right] + (1-m_t)\left[-e_t i_t^2 b^2 - (1-e_t)\left(\sigma_p^2 + i_t^2 b^2\right)\right] - c(m_t) \\ = & \max_{m_t \in [0,1]} -(1-e_t)\,\sigma_p^2 - (1-m_t)\, i_t^2 b^2 - c(m_t). \end{aligned} \tag{1.14}$$

We further simplify the problem by assuming that the cost of the monitoring effort is quadratic, i.e., $c(m_t) = k\frac{m_t^2}{2}$, for $m_t \in [0,1]$. Solving the above program for the board, we obtain the optimal monitoring effort and state it in the following lemma:

Lemma 1.1. *The optimal monitoring effort by the board of directors is* $m_t^*(i_t, e_t) = \frac{i_t^2 b^2}{k}$. *The monitoring effort is increasing and convex in the real independence level, i.e.,* $\frac{\partial m_t^*(i_t,e_t)}{\partial i_t} = \frac{2 i_t b^2}{k} > 0$.

Proof. All proofs are in the appendix. □

Lemma 1.1 states that the optimal monitoring effort of the board is increasing in its real independence level, i_t. Indeed, the higher the real independence level of the board is, the larger is the conflict over the project choice between the board and the CEO and, thus, the more the board values its control over the project choice.

Given the optimal project choices by the CEO and the board and the optimal monitoring intensity chosen by the board, we can derive the expected payoff for shareholders in each period. The expression for the shareholders' expected payoff is:

$$\mathbb{E}\left[u_t^S(i_t, e_t)\right] = -\sigma_p^2 - b^2 + \underbrace{e_t \sigma_p^2}_{\text{advisory benefits}} + \underbrace{\left(2i_t - i_t^2\right) b^2 m_t^*(i_t, e_t)}_{\text{monitoring benefits}}. \tag{1.15}$$

In this expression, the term $e_t\sigma_p^2$ measures the advisory benefits shareholders derive from board expertise, and the term $(2i_t - i_t^2)\, b^2 m_t^* (i_t, e_t)$ measures the monitoring benefits that shareholders derive from board monitoring.

Lemma 1.2. *In Period t, the shareholders' expected payoff is increasing in both the expertise level of the board, e_t, and the real independence level of the board, i_t.*

Lemma 1.2 states the fairly obvious result that shareholders always prefer a board with a higher real independence and a higher expertise. This is true because the advisory benefit is increasing in the expertise of the board, and the monitoring benefit is increasing in the real independence of the board.

1.5 CEO's Problem

In Section 1.4, we investigated the board's optimal monitoring effort given its real independence level and expertise level. In this section, we study the CEO's optimal strategy regarding outside director replacement given that the CEO rationally anticipates the optimal monitoring effort chosen by the board. The outside director replacement decision affects the real independence level and the expertise of the board, so we first examine how the real independence and the expertise of the board determines the CEO's expected payoff in each period.

Let $\mathbb{E}\left[u_t^M (i_t, e_t)\right]$ denote the CEO's expected payoff in Period t when the real independence of the board is i_t and the expertise level of the board is e_t. Given the optimal monitoring effort chosen by the board, we have:

$$\mathbb{E}\left[u_t^M (i_t, e_t)\right] = -\sigma_p^2 + \underbrace{e_t\sigma_p^2}_{\text{advisory benefits}} - \underbrace{i_t^2 b^2 m^* (i_t, e_t)}_{\text{monitoring costs}}, \tag{1.16}$$

where the term $e_t\sigma_p^2$ represents the advisory benefits derived by the CEO from board expertise. That is, this term reflects the improvement in the decision derived from the information about $\tilde{p}_t$ available through board expertise. The term $i_t^2 b^2 m_t^* (i_t, e_t)$ represents the monitoring costs imposed on the CEO by board monitoring. It reflects the divergence of s_t from the CEO's preferred choice enforced by the board's monitoring.

Lemma 1.3. *In each period t, the CEO's expected payoff is increasing in the expertise level of the board, e_t, and decreasing in the real independence level of the board, i_t. Moreover, the monitoring costs imposed on the CEO are increasing and convex in the real independence of the board, i_t.*

Lemma 1.3 states that the CEO prefers a board with a higher expertise level, congruent with the preferences of shareholders, but prefers a board with a lower real independence level, incongruent with shareholders. The intuition is that the advisory benefits derived by the CEO are increasing in board expertise, and the monitoring costs imposed on the CEO are increasing in the real independence level of the board. Due to the conflict with shareholders, when the CEO decides on director replacement, the CEO chooses a composition of the board that maximizes his or her own benefit, which may harm shareholder value. Moreover, the monitoring cost imposed on the CEO is increasing and convex in the real independence of the board, which implies the CEO has a strong preference for a board with a lower real independence level. Indeed, increasing the real independence of the board has two effects on the CEO's payoff. On the one hand, increased real independence increases the gap between the CEO's and the board's preferred projects, making the CEO worse off when the board takes control over project selection. On the other hand, increased real independence also increases the monitoring intensity of the board (according to Lemma 1.1), making the CEO worse off as the board obtains control over the project choice more often.

After analyzing how the real independence level and expertise level of the board affects the CEO's expected payoff in each period, we now investigate the CEO's optimal replacement decision of independent directors on the board at the beginning of Period 1 and Period 2 for the three-date problem. Since it is a sequential decision problem for the CEO, we proceed by backward induction first solving the CEO's optimal replacement decision at the beginning of Period 2. In the following analysis, we focus on the interior solutions of the CEO's optimization programs, since they reflect tradeoff of the incentives faced by the CEO. In footnotes, we provide conditions under which an interior solution exists. At Section 7, we discuss the potential implications of non-interior solutions.

1.5.1 CEO's Optimal Replacement Decision in Period 2

At the beginning of Period 2, considering the effect of outsider replacement on both the real independence and the expertise of the board, the CEO takes the replacement decision maximizing his expected utility in Period 2, which we denote by $\mathbb{E}\left[u_2^M\left(i_2, e_2\right)\right]$. The manager's problem in Period 2 is:

$$\begin{aligned}
\max_{r_2} \mathbb{E}\left[u_2^M\left(i_2, e_2\right)\right] &= -\left(1-e_2\right)\sigma_p^2 - \frac{b^4 i_2^4}{k} \\
\text{s.t. } i_2\left(i_1, r_2; o\right) &= i_1\eta + r_2\left(1 - \eta\frac{i_1}{o}\right) \\
e_2\left(r_2\right) &= e_o + r_2\Delta.
\end{aligned} \tag{1.17}$$

On the one hand, a more intense replacement decision increases the expertise of the board, which is desirable to the CEO. On the other hand, a more intense replacement decision also increases the real independence of the board, which is undesirable to the CEO. The optimal replacement decision is then derived by balancing this tradeoff.

Define $r_2^*(i_1; o)$ and $i_2^*\left(i_1; o\right)$ as the CEO's optimal replacement decision in Period 2 and the resultant real independence of the board respectively.

Lemma 1.4. *Restricting attention to interior solutions,*[10] *the CEO's optimal replacement decision in Period 2 and the resultant real independence of the board,* $r_2^*(i_1; o)$ *and* $i_2^*\left(i_1; o\right)$*, are given by* $i_2^*\left(i_1; o\right) = \left(\frac{\sigma_p^2\Delta}{\frac{4b^4}{k}\left(1-\eta\frac{i_1}{o}\right)}\right)^{\frac{1}{3}}$ *and* $r_2^*\left(i_1; o\right) = \frac{i_2^*(i_1, o) - i_1\eta}{\left(1-\eta\frac{i_1}{o}\right)}$.

[10]Here, for the solution to be interior it suffices to assume a non-extreme improvement in expertise from replacement Δ, i.e., $\Delta \in \left(\frac{4b^4}{k}\left(1-\eta\frac{i_1}{o}\right)\frac{(\eta i_1)^3}{\sigma^2}, \frac{4b^4}{k}\left(1-\eta\frac{i_1}{o}\right)\frac{o^3}{\sigma^2}\right)$.

The marginal benefit of replacement comes from increasing the expertise of the board and has the expression $\sigma_p^2\Delta$, where Δ measures the marginal effect of replacement on board expertise in Period 2 and σ_p^2 measures the marginal effect of board expertise on the manager's utility in Period 2. The marginal cost of replacement is due to an increase in the real independence of the board and reflected in the expression $\frac{4b^4 i_2^3}{k}\left(1-\eta\frac{i_1}{o}\right)$. In this last expression, the term $\left(1-\eta\frac{i_1}{o}\right)$ represents the difference between the real independence of the new and the existing outsiders and measures the marginal effect of replacement on board real independence. The term $\frac{4b^4 i_2^3}{k}$ measures the marginal effect of real independence on the manager's utility in Period 2. By balancing the marginal cost and the marginal benefit, we derive the optimal replacement decision and the resulting real independence in Period 2 as a function of i_1 and o. Perhaps surprisingly, we find that the resultant real independence of the board in Period 2 is increasing in the ratio $\frac{i_1}{o}$.

Corollary 1.1. *The optimal real independence of the board in Period 2, $i_2^*(i_1, o)$, is increasing in the ratio $\frac{i_1}{o}$.*

Note that $\log_\eta\left[\eta\frac{i_1}{o}\right]$ is the average tenure of the existing board members viewed collectively as an "average agent" at the beginning of Period 2 before the replacement decision. Corollary 1.1 states that a higher average board tenure before replacement results in a lower real independence after replacement in Period 2. A longer average tenure of existing board members (i.e., a lower ratio $\frac{i_1}{o}$) increases the real independence gap between the existing outside directors and the new directors, $\left(1-\eta\frac{i_1}{o}\right)$. The marginal cost of the replacement decision due to increased real independence is increasing in both the resultant real independence and the real independence gap, while the marginal benefit of the replacement decision from increasing board expertise is constant. The optimal replacement decision balances the marginal cost and the marginal benefit. Therefore, a longer average tenure of existing board results in a lower real independence of the board.

1.5.2 CEO's Optimal Replacement Decision in Period 1

At the beginning of Period 1, the average real independence level of the board is ηo, and the average expertise level of the board for the strategic decision in Period 1 is e_o. Based on the real independence and expertise of the board, the CEO takes the outside director replacement decision at the beginning of Period 1 anticipating the optimal replacement decision in the following period. The replacement decision in Period 1 affects the real independence and expertise of the board in both Period 1 and Period 2, so it affects the expected utility of the CEO in both periods . We use $U_M(r_1; o)$ to denote the CEO's expected discounted sum of utilities in Period 1 and Period 2 if the CEO's replacement decision at the beginning of Period 1 is r_1 and the nominal board independence is o. Let β be the CEO's discount factor. The CEO's problem regarding the optimal outside director replacement decision at the beginning of Period 1 can be stated as follows:

$$\begin{aligned} \max_{r_1} U_M(r_1; o) &= \mathbb{E}\left[u_1^M(i_1, e_1)\right] + \beta \mathbb{E}\left[u_2^M(i_2, e_2) \,|r_2 = r_2^*(i_1, o)\right] \\ \text{s.t. } i_1(r_1; o) &= \eta o + r_1(1-\eta) \\ e_1(r_1) &= e_o + r_1 \Delta, \end{aligned} \tag{1.18}$$

where $\mathbb{E}\left[u_1^M(i_1, e_1)\right]$ is the expected utility in Period 1, and $\mathbb{E}\left[u_2^M(i_2, e_2) \,|r_2 = r_2^*(i_1; o)\right]$ is the expected utility in Period 2 anticipating the CEO's optimal replacement decision at the beginning of Period 2 given by expression (17).

Lemma 1.5. *Restricting attentions to interior solutions,*[11] *the CEO's optimal replacement*

[11] Here, for the solutions to be interior it suffices to assume that the real independence decays quickly enough with director tenure, i.e., η is small enough, and that the improvement in expertise from replacement Δ is not extreme. A small η guarantees that the real independence decays quickly enough with board tenure so that the CEO is not willing to replace all incumbent outsiders. The assumption of a non-extreme Δ serves to guarantee that the optimal replacement decisions are interior solutions. We show in the appendix that to obtain an interior solution it is sufficient to assume: $\eta < \min\left\{\frac{3}{4}, 1 - \frac{\sqrt{\beta}}{3}\right\}, \Delta \in \left(\frac{4b^4 o^3(1-\eta)}{k\sigma^2} \frac{\eta^3}{1-\beta\frac{\eta}{(1+\eta)}}, \frac{4b^4 o^3(1-\eta)}{\sigma^2 k}\right)$.

decision in Period 1, $i_1^*(o)$, is determined by:

$$\sigma^2\Delta - \left[\frac{4b^4 i_1^3}{k} + \beta\frac{\eta\left(1-\frac{i_2^*(i_1;o)}{o}\right)\Delta}{\left(1-\eta\frac{i_1}{o}\right)^2}\right](1-\eta) = 0. \tag{1.19}$$

The marginal benefit of replacement from increasing the expertise of the board is $\sigma_p^2\Delta_1$, where Δ_1 measures the marginal effect of the replacement decision on board expertise, and σ_p^2 measures the marginal effect of board expertise on the manager's utility. The marginal cost of replacement due to an increase in real independence of the board is $\left[\frac{4b^4 i_1^3}{k} + \beta\frac{\eta\left(1-\frac{i_2^*(i_1;o)}{o}\right)\Delta_2}{\left(1-\eta\frac{i_1}{o}\right)^2}\right](1-\eta)$. Specifically, $(1-\eta)$, which represents the difference in the real independence between the new outsiders and the existing outside directors at the beginning of Period 1, measures the marginal effect of the replacement decision on the board real independence. The marginal effect of the real independence on the manager's utility in Period 1 is captured by $\frac{4b^4 i_1^3}{k}$, and the marginal effect of the real independence in Period 1 on the manager's utility in Period 2 is captured by $\eta\left(1-\frac{i_2^*(i_1;o)}{o}\right)\Delta/\left(1-\eta\frac{i_1}{o}\right)^2$. By balancing the marginal cost and the marginal benefit, the resultant real independence of the board in Period 1 is determined by an implicit function of the nominal independence denoted as $i_1^*(o)$. Interestingly, we find that the ratio of the optimal real independence of the board in Period 1, $i_1^*(o)$, and the nominal independence, o, is decreasing in nominal independence o.

Lemma 1.6. *The ratio $\frac{i_1^*(o)}{o}$ is decreasing in the nominal independence o.*

Lemma 1.6 states that, as the nominal independence of the board increases, the resultant ratio $\frac{i_1^*(o)}{o}$ after the replacement decision of the CEO at the beginning of period 1 decreases, where $i_1^*(o)$ is the interior solution to the CEO's optimal replacement decision in Period 1 defined in Lemma 1.5. Lemma 1.6 implies that a higher nominal board independence results in a longer average tenure of the outside directors after replacement, since the term $\log_\eta\left(\frac{i_1}{o}\right)$ measures the average tenure of existing outside directors after replacement at the beginning of period 1. If, alternatively, the resultant average tenure of the board with higher nominal independence were lower after replacement at the beginning of period 1, it would immediately imply that the real independence in period 1 is higher for the board with higher nominal independence. Moreover, by Corollary 1.1, the board with lower average tenure would end up with a higher real independence in period 2. Therefore, the board with higher nominal independence would have a higher real independence in both periods. Then, the marginal cost of replacement would be higher in both period 1 and period 2 for the board with higher nominal independence. However, this cannot be true in equilibrium because the marginal benefit of the replacement decision is the same regardless of the nominal independence.

1.6 Nominal versus Real Independence

In Section 1.5, we derived the optimal director replacement strategy of the CEO. Given the optimal replacement strategy, we now study how the nominal board independence or the fraction of outside directors affects the CEO's decision about replacing directors and, thus, the real board independence. Recall that the replacement decision made by the CEO does not change the fraction of outside directors on the board, so the nominal independence of the board stays the same.

Proposition 1.1. *With interior solutions for optimal replacement decisions in both periods,*[12] *and regulation R' that mandates O' outside directors and regulation R that mandates O outside directors, $O' > O$, the average real independence level of the board at the end of Period 2 is lower for the board with O' outsiders than for the board with O outsiders.*

Proposition 1.1 states that the more outside directors a regulation requires to be on the board initially (the higher the initial nominal board independence is), the lower the real

[12] The sufficient conditions for the solutions to the CEO's optimal replacement decisions to be interior for both boards with nominal independence $o = \frac{O}{N}$ and $o' = \frac{O'}{N}$ are $\eta < \min\left\{\frac{3}{4}, 1 - \frac{\sqrt{\beta}}{3}\right\}, \Delta \in \left(\frac{4b^4\left(o'\right)^3(1-\eta)}{k\sigma^2}\frac{\eta^3}{1-\beta\frac{\eta}{(1+\eta)}}, \frac{4b^4o^3(1-\eta)}{\sigma^2 k}\right)$. The set of parameters that satisfy these conditions is not empty if $\frac{o}{o'} > \frac{\eta}{\sqrt[3]{1-\beta\frac{\eta}{(1+\eta)}}}$.

independence level of the board is in Period 2. In other words, a higher nominal board independence leads to a lower real independence of the board with time. At the beginning of Period 1 before replacement, the average tenure of the outside directors on the board is η which is the same across different boards with different numbers of outsiders. So, the real independence of the board with O' outsiders is also higher, which makes the CEO more reluctant to replace existing directors on the board with new outsiders whose real independence level is higher than the existing directors. Therefore, a higher initial nominal independence level of the board results in a higher average tenure of the board, hence, at the beginning of Period 2 before replacement. The higher average tenure of the board at the beginning of Period 2 again makes the CEO more reluctant to replace existing directors, since the gap of the real independence between the existing board members and new outsiders is larger. Therefore, a higher initial nominal independence level of the board results in a lower real independence level of the board by the end of Period 2.

The result in Proposition 1.1 relies on the interior solutions. We have shown that the solution is interior if and only if the marginal effect of the replacement decision on board expertise is non-extreme. If we consider the board expertise as innovative expertise, the marginal effect depends on the urgency for innovation of a firm. In other words, the marginal effect is stronger if a firm is more urgent for innovation. In this case, this result implies that the higher nominal independence results in a lower real independence for firms with moderate urgency for innovation.

Although we restricted attention to interior solutions, we must stress that our result can be overturned when the solution is not interior. Essentially, a higher nominal board independence leads to a higher real independence if the CEOs in both firms make the same extreme replacement decisions. We discuss this point further in Section 1.7.4.

1.7 Extensions

In this section, we consider three extensions. In the first extension, we extend the time horizon to infinite periods, because it gives a natural setup to study the relaxed assumption on improvements in board expertise. We also relax the assumption that the improvement in board expertise from replacement is constant in the number of new outside directors hired. In the last extension, we let the board of directors rather than the CEO make the replacement decision. In the last subsection, we discuss some of other assumptions of our analysis and their role in driving our results.

1.7.1 Infinite Horizon Problem of the CEO

In the main model, we consider a two period model to illustrate the key tradeoffs. To check the robustness of our result and also develop conditions under which our results are overturned, we extend the time horizon to infinite periods and look at whether our results continue to hold. Here, all the myopic decisions taken by the board and the CEO (which include the monitoring effort of the board and the project choice of the board and the CEO) in each period from the main model remain the same and, therefore, the results for those decisions from the main model continue to hold. We only change the analysis of the replacement decision by moving to an infinite horizon.

Define $V_M(i;o)$ as the present value of the expected utility of the CEO if the current real board independence is i and the nominal board independence is o. By the definition of the value function, we have:

$$\begin{aligned} V_M(i;o) &= \max_{r\in[0,o]} -(1-e_o-\Delta r)\,\sigma^2 - \frac{(bi')^4}{k} + \beta V_M(i';o)\,, \\ s.t. i' &= \eta i + r\left(1-\eta\frac{i}{o}\right), \end{aligned} \tag{1.20}$$

which can be rewritten as:

$$V_M(i;o) \;=\; \max_{i' \in [\eta i, o]} -\left(1 - e_o - \Delta \frac{i' - \eta i}{1 - \eta \frac{i}{o}}\right)\sigma^2 - \frac{(bi')^4}{k} + \beta V_M(i';o). \tag{1.21}$$

We define $I(i;o)$ as the solution to the problem above, which is also called the policy function. Since it is a deterministic problem and the utility function is continuous and bounded, there is a unique policy function $I(i;o)$. We search for a fixed point of the policy function, $i^*(o)$, which satisfies $I(i^*(o);o) = i^*(o)$, as the stable real independence. If the current real independence is equal to $i^*(o)$, it will remain the same after the CEO's optimal replacement decision in next period. By showing how the fixed point varies with the nominal board independence, o, we are able to show the impact of nominal board independence on the real board independence.

Proposition 1.2. $\forall o, o', s.t., 0 < o < o' < 1$, *a higher nominal independence results in a lower real independence, i.e.,* $i^*(o') < i^*(o)$, *if the improvement in board expertise from replacement is not too large, i.e.,* $\Delta \in \left(0, \frac{b^4}{k\sigma^2} 4o^3(1-\eta)\right)$. *If the improvement in board expertise from replacement is large enough, i.e.,* $\Delta \geq \frac{b^4}{k\sigma^2} 4(o')^3(1-\eta)$, *a higher nominal independence results in a higher real independence, i.e.,* $i^*(o') > i^*(o)$.

Proposition 1.2 confirms Proposition 1.1 can be generalized to an infinite horizon as long as the real independence of outsiders decreases with tenure, i.e., $\eta < 1$, and the improvement in board expertise from replacement, Δ, is not too large.

1.7.2 Assumptions on the Impact of Replacement on Expertise

So far, we have assumed that the marginal improvement in board expertise from replacement is constant. That is, we assumed that the contribution of each new director to the board expertise is the same. Now, we relax this assumption by allowing the marginal improvement in board expertise to be decreasing in the size of the replacement. A possible reason for such a decrease in the expertise contribution could be that new directors have different levels of expertise and are hired prioritizing directors with higher expertise. An alternative reason could be that new directors have overlapping skills and, therefore, it becomes harder to find new directors with different skills the more new directors are hired. Specifically, we assume:

$$e_t = e_0 + G(r), \tag{1.22}$$

where $G'(r) \geq 0$, $G''(r) \leq 0$ and $G'(0) = \Delta$. Our original assumption in the main model is a special case of this assumption, i.e., $G''(r) \equiv 0$. With the new more general assumption, we find that our main results may or may not hold depending on the scale of $\frac{G''(r)}{G'(r)}$. As in our analysis in section 1.7.1, we study an infinite horizon model and look at the fixed point of real independence in order to study the impact of regulating nominal independence on the real board independence.

Proposition 1.3. $\forall o, o', s.t., 0 < o < o' < 1$, *if the real independence of outsiders decreases quickly with tenure, i.e.,* $\eta < \frac{3}{4+\beta}$, *and the improvement in board expertise from replacement is not too large, i.e.,* $G'(r) \in \left(0, \frac{b^4}{k\sigma^2} 4o^3 (1-\eta)\right)$, *we have:*

1. $i^*(o') < i^*(o)$ if $\frac{G''(r)}{G'(r)} > -\left(\frac{1}{1-\eta} + \frac{\beta}{1-\beta\eta}\right)$;
2. $i^*(o') > i^*(o)$ if $\frac{G''(r)}{G'(r)} < -\left(\frac{1-\beta}{1-\eta}\right)$.

Proposition 1.3 states that a higher nominal independence results in a lower real independence if the expertise contribution of new directors does not decrease much with the number of new directors ($\frac{G''(r)}{G'(r)}$ is large enough). However, a higher nominal independence results in a higher real independence if the expertise contribution of new directors decreases significantly with the number of new directors ($\frac{G''(r)}{G'(r)}$ is small enough). The intuition is that if and only if new directors contribute enough to board expertise, the manager of a firm with a board with a lower nominal independence is willing to replace significantly more incumbent board members with new directors, which results in a higher nominal independence. Otherwise, the replacement decision is similar for firms with different nominal board independence levels, and, thus, a higher nominal independence results in a higher real independence. For example, if only one new director is available to provide specialist expertise, the CEO wants to hire that new director anyway as long as the expertise is valuable enough. In such cases, a higher nominal independence would result in a higher real independence.

1.7.3 Replacement Decision by the Board of Directors

In our previous analysis, we assume that the CEO decides whether to replace an existing outside director to reflect the influence of CEO on director succession. We recognize that the hiring and firing decision is taken directly by (the nomination committee of) the board instead of by the CEO. Now, we relax this assumption by allowing the board of directors, viewed as an average agent, to make the replacement decision. The objective of the board of directors is to maximize the present value of its expected future payoffs. As in our analysis in Section 1.7.1, we study an infinite horizon model with the marginal improvement in board expertise from replacement being a constant Δ and solve for the fixed point of real independence in order to study the impact of regulating nominal independence on real board independence.

Define $V_B(i;o)$ as the present value of the board's expected utility if the current real board independence is i and the nominal board independence is o. By the definition of the value function, we have:

$$\begin{aligned} V_B(i;o) &= \max_{r\in[0,o]} -(1-e_o-\Delta r)\,\sigma^2 - \left(1-\frac{(bi')^2}{k}\right)(bi')^2 - \frac{k}{2}\left(\frac{(bi')^2}{k}\right)^2 + \beta V_B(i';o), \\ s.t. i' &= \eta i + r\left(1-\eta\frac{i}{o}\right), \end{aligned} \tag{1.23}$$

which can be rewritten as:

$$V_B(i;o) = \max_{i' \in [\eta i, o]} -\left(1 - e_o - \Delta \frac{i' - \eta i}{1 - \eta \frac{i}{o}}\right)\sigma^2 - \left((bi')^2 - \frac{(bi')^4}{2k}\right) + \beta V_B(i';o). \quad (1.24)$$

As we did in the analysis of the CEO's problem, we define $I(i;o)$ as the solution to the problem above, which is again called the policy function. We also look at how the fixed point varies with the nominal board independence, o, in order to show the impact of nominal board independence on real board independence.

Proposition 1.4. *$\forall o, o', s.t., 0 < o < o' < 1$, if the CEO's bias relative to the preference of shareholders is not too large, i.e., $\frac{b^2}{k} < \frac{1}{3}$:*

1. If the real independence level of directors decreases quickly with tenure, i.e., $\eta < \min\left\{\frac{2}{5}, \frac{1-3\frac{b^2}{k}}{2+\beta-(4+\beta)\frac{b^2}{k}}\right\}$, and the improvement in board expertise from replacement is not too large, i.e., $\Delta \in \left(0, \frac{2b^2}{\sigma^2} o \left(1 - \frac{(bo)^2}{k}\right)(1-\eta)\right)$, then higher nominal independence results in lower real independence, i.e., $i^*(o') < i^*(o)$.

2. If the real independence level of directors decreases slowly with tenure, i.e., $\eta > \frac{1}{2+\beta}$, and the improvement in board expertise from replacement is large enough, i.e., $\Delta \geq \frac{2b^2}{\sigma^2} o' \left(1 - \frac{(bo')^2}{k}\right)(1-\eta)$, then higher nominal independence results in higher real independence, i.e., $i^*(o') > i^*(o)$.

Proposition 1.4 states that when the board of directors makes the replacement decision, a higher nominal independence level of the board results in a lower real independence of the board in the long-term if the real independence level of outsiders decreases quickly with tenure and the improvement in board expertise from replacement is not too large. However, a higher nominal independence level of the board leads to a higher real independence of the board if the real independence level of outsiders does not decrease quickly with tenure and the improvement in board expertise from replacement is large enough. The intuition for this result is that if the real independence level of the directors decreases quickly with tenure (i.e., η is small enough), the incumbent board has a low real independence level and, thus, it is more aligned with the CEO. Therefore, similar to the impact of replacement on the CEO's utility, the marginal cost of replacement for the existing board is higher if either the real independence of the board is higher or the average tenure is longer. Given the same real independence, the board with a higher nominal independence has a longer average tenure. Since the marginal benefit is fixed, in order to satisfy the equilibrium condition, the marginal cost of replacement should be the same across the boards with different nominal independence. Therefore, in the fixed point, a higher nominal independence results in a lower real independence of the board. Alternatively, if the real independence level of outsiders does not decreases quickly with tenure, as long as the improvement in board expertise from replacement is large enough, periodical director replacements will maintain the real independence of the board close to the nominal independence. In this case, a higher nominal independence level of the board leads to a higher real independence of the board.

1.7.4 Discussion of Other Assumptions

In this subsection, we return to our main (three period) model and discuss some of our other assumptions and their role in driving our results.

In the above analysis, we restricted attention to interior solutions. Now, we discuss the conditions under which an interior solution exists and what happens if there is no interior solution. The existence of interior solutions requires only that the real independence decays quickly enough with director tenure, i.e., η is small enough, and that the improvement in expertise from replacement Δ is not extreme.

To illustrate when the restriction to interior solutions is restrictive and when it is not, we now provide a numerical example. The values of the parameters used in the numerical example are summarized in Table 1.1. Given the parameters in Table 1.1, the sufficient condition for interior solutions for a pair of (O, O') can be represented by an interval of the expertise improvement from replacement $\Delta \in (\Delta_{\min}, \Delta_{\max})$. The values of $\Delta_{\min}$ and $\Delta_{\max}$ corresponding to different pairs of (O, O') are summarized in Table 1.2. From Table 1.2, the size of the interval $(\Delta_{\min}, \Delta_{\max})$ is large if O and O' are close to each other, which implies our result is permissive for close O and O'. We can also find that the interval $(\Delta_{\min}, \Delta_{\max})$ is not empty for $O' = N = 10$, i.e., a regulated board with all outsiders can end up with a lower real independence than a regulated board with some insiders. Indeed, if the real independence decays quickly over time (e.g., $\eta = 0.05$), there exists an interval of the expertise improvement from replacement such that the real independence of a regulated board with all outsiders is lower than the real independence of a regulated board with only one outsider at the end of Period 2.[13] In this case, the sufficient conditions seem restrictive–to the point of resembling an existence result.

parameters	meaning	value
b	CEO's bias on project choice	0.2
k	board's monitoring effort tolerance	0.04
N	number of directors on board	10
β	discount factor	0.8
σ_p	ex-ante uncertainty of optimal project	0.2
η	decay rate of real independence	0.3

Table 1.1: The values of the parameters

O	O'	$\Delta_{\min}$	$\Delta_{\max}$
4	5	0.0119	0.1792
4	6	0.0206	0.1792
4	7	0.0327	0.1792
4	8	0.0489	0.1792
4	9	0.0696	0.1792
4	10	0.0954	0.1792

Table 1.2: The interval of the expertise improvement from replacement $\Delta \in (\Delta_{\min}, \Delta_{\max})$ to the satisfy sufficient conditions for interior solutions

If the sufficient conditions are violated, the interior solutions may not exist. On the one hand, if either the real independence does not decay quickly with the director tenure, i.e., η

[13]For example, given the values of the parameters in Table 1.1 except that $\eta = 0.05$ instead of 0.3, the interval is $\Delta \in (0.0005, 0.0038)$.

is large enough, or the improvement in expertise from replacement Δ is too large, the CEO may find it optimal to replace all outside directors regardless of the nominal independence level. On the other hand, if the improvement in expertise from replacement Δ is too small, the CEO may optimally choose to keep all incumbent outsiders no matter what the nominal or real independence is. Although cases without interior solutions are not the focus of our paper, it is important to recognize that our result will sometimes be overturned when the solution is not interior. In particular, a higher nominal board independence leads to a higher real independence if the CEO in the firm with the board with O' outsiders and the CEO in the firm with the board with O outsiders make the same extreme replacement decisions, i.e., either replacing all or no outsiders. Even when only one of the solutions (either O or O') is non-interior, our result is sometimes overturned.

Another important assumption we make in our main model is that the improvement in board expertise is non-persistent, i.e., in each period, all directors that remain on the board revert back to the generalist expertise level e_o. This assumption is reasonable to the extent that new directors are hired to deal with a new business strategy or environment. However, the assumption was made primarily to simplify the analysis, and our main result may not hold if expertise is persistent enough. If, for example, director expertise is perfectly persistent, outside directors with a low expertise level would be gradually replaced by new outsiders. The average expertise level, therefore, should increase over time and converge to a final composition where all existing outside directors have a high expertise level and further replacements are no longer beneficial. In such a scenario, director tenure would keep growing, and outsiders would eventually lose their independence. Consequently, real independence would converge to 0 regardless of the nominal independence level.

1.8 Conclusion

This paper offers a new perspective on board independence by focusing on the impact of director tenure on the "real independence" of the board. Instead of taking the fraction of outside directors directly as the measure of board independence, we define it as the "nominal independence" level of the board. The "real independence" level of the board of directors is determined by both the nominal independence level and the length of the relationship between the board members and the CEO. We assume the real independence of outside directors decreases over time if the outside directors stay on the board (no turnover), while the replacement of existing outside directors with new candidates increases the average real independence level of the board. We study a three period model in which the board both monitors and advises the CEO, and the CEO decides whether to replace some of the outside directors in each period. The CEO balances the trade-off between a higher expertise of the board introduced by new directors and a lower real independence of the board attained by keeping the same board. We find that the higher is the nominal independence level of the board (i.e., the higher fraction of outside directors), the more reluctant the CEO is in replacing existing directors. Therefore, a higher nominal independence may result in a lower real independence and a lower expertise of the board of directors in the long-run. In the extension section, we show our result can be generalized to an infinite horizon and relax some assumptions to investigate the conditions under which our result holds and discuss the conditions under which our result is overturned.

Although we do not directly study the optimal regulation of board composition, our results contribute some insights. The regulation on board composition is intended to protect shareholders by improving board independence. By requiring a higher proportion of outside directors on the board, it is likely to weaken the advisory role of the board due to the superior (or at least different) firm-specific expertise of inside directors. Presumably, optimal regulations on board composition would tradeoff the benefits of higher board independence and the costs of weakening the advising role of the board. However, as shown in this paper, a regulation that requires a higher fraction of outside directors on the board can result in lower board turnover and, hence, decreases the independence of the board. In this case, there are only costs (no benefits) to increasing nominal independence. Building a richer model to study the optimal regulation of tenure is a natural next step.

A related issue that may be interesting to explore in future research is the impact of board regulation on the CEO's incentive to come up with new business strategies. In our setting, the new business strategy is exogenous. If the new business strategy is endogenous and depends on the CEO's effort, will regulations that mandate higher nominal independence discourage the CEO from coming up with new business strategies?

APPENDIX

1.A Proof

Proof of Lemma 1.1:

Proof. By the first order condition, we have

$$\begin{aligned} & km_t^* - i_t^2 b^2 = 0 \\ \Rightarrow \quad & m_t^*\left(i_t, e_t\right) = \frac{i_t^2 b^2}{k}. \end{aligned} \tag{1.A.1}$$

Therefore, we have

$$\frac{\partial m_t^*\left(i_t, e_t\right)}{\partial i_t} = \frac{2 i_t b^2}{k} > 0. \tag{1.A.2}$$

□

Proof of Lemma1.2:

Proof. The expected payoff of the shareholder given the optimal monitoring intensity of the board is,

$$\begin{aligned} E\left[u_t^S\left(i_t, e_t\right)\right] \quad = \quad & -m_t^*\left(i_t, e_t\right)\left[e_t\left(1-i_t\right)^2 b^2 + \left(1-e_t\right)\left(\sigma_p^2 + \left(1-i_t\right)^2 b^2\right)\right] \\ & -\left(1-m_t^*\left(i_t, e_t\right)\right)\left[e_t b^2 + \left(1-e_t\right)\left(\sigma_p^2 + b^2\right)\right], \end{aligned} \tag{1.A.3}$$

which can be simplified to,

$$\begin{aligned} E\left[u_t^S\left(i_t, e_t\right)\right] \quad & = \quad -\left(1-e_t\right)\sigma_p^2 - \left(1-m_t^*\left(i_t, e_t\right)\right) b^2 - m_t^*\left(i_t, e_t\right)\left(1-i_t\right)^2 b^2 \\ & = \quad -\sigma_p^2 - b^2 + e_t \sigma_p^2 + m_t^*\left(i_t, e_t\right)\left(2 i_t - i_t^2\right) b^2 \\ & = \quad -\sigma_p^2 - b^2 + e_t \sigma_p^2 + \frac{b^4}{k}\left(2 i_t^3 - i_t^4\right). \end{aligned} \tag{1.A.4}$$

Therefore, we have

$$\begin{aligned} \frac{\partial E\left[u^S\left(i_t, e_t\right)\right]}{\partial i_t} &= 2\left(3 - 2i_t\right)\frac{i_t^2 b^4}{k} \\ &> 0. \end{aligned} \tag{1.A.5}$$

Moreover,

$$\frac{\partial E\left[u^S\left(i_t, e_t\right)\right]}{\partial e_t} = \sigma_p^2 > 0. \tag{1.A.6}$$

$\square$

Proof of Lemma 1.3:

Proof. The expected payoff of the CEO given the optimal monitoring intensity of the board is

$$\begin{aligned} & E\left[u_t^M\left(i_t, e_t\right)\right] \\ = & -m_t^*\left(i_t, e_t\right)\left[e_t i_t^2 b^2 + \left(1 - e_t\right)\left(\sigma_p^2 + i_t^2 b^2\right)\right] \\ & -\left(1 - m_t^*\left(i_t, e_t\right)\right)\left(1 - e_t\right)\sigma_p^2 \\ = & -\left(1 - e_t\right)\sigma_p^2 - m_t^*\left(i_t, e_t\right) i_t^2 b^2 \\ = & -\left(1 - e_t\right)\sigma_p^2 - \frac{i_t^4 b^4}{k}. \end{aligned} \tag{1.A.7}$$

Then, we have

$$\frac{\partial E\left[u_t^M\left(i_t, e_t\right)\right]}{\partial i_t} = -\frac{4 i_t^3 b^4}{k} < 0 \tag{1.A.8}$$

$$\frac{\partial E\left[u_t^M\left(i_t, e_t\right)\right]}{\partial e_t} = \sigma_p^2 > 0 \tag{1.A.9}$$

$$\frac{\partial^2 E\left[u_t^M\left(i_t, e_t\right)\right]}{\partial i_t \partial e_t} = 0 \tag{1.A.10}$$

$$\frac{\partial^2 E\left[u_t^M\left(i_t, e_t\right)\right]}{\partial i_t^2} = -\frac{12 i_t^2 b^4}{k} < 0 \tag{1.A.11}$$

$$\frac{\partial^2 E\left[u_t^M\left(i_t, e_t\right)\right]}{\partial e_t^2} = 0, \tag{1.A.12}$$

which implies $E\left[u_t^M\left(i_t, e_t\right)\right]$ decreases in i_t and increases in e_t, and $\frac{\partial^2 E\left[u_t^M\left(i_t, e_t\right)\right]}{\partial i_t \partial e_t} = 0$, $\frac{\partial^2 E\left[u_t^M\left(i_t, e_t\right)\right]}{\partial i_t^2} < 0$. □

Proof of Lemma 1.4:

Proof. The manager's Problem in Period 2 is:

$$\max_{r_2} E\left[u_2^M\left(i_2, e_2\right)\right] = -\left(1 - e_2\right)\sigma^2 - \frac{b^4 i_2^4}{k} \tag{1.A.13}$$

$$\text{s.t. } i_2\left(i_1, r_2; o\right) = i_1 \eta + r_2\left(1 - \eta \frac{i_1}{o}\right)$$

$$e_2\left(r_2\right) = e_o + r_2 \Delta.$$

By the first order condition, we have:

$$\frac{\partial E\left[u_2^M\left(i_2, e_2\right)\right]}{\partial r_2} = \sigma^2 \Delta - \frac{4 b^4 i_2^3}{k}\left(1 - \eta \frac{i_1}{o}\right) = 0. \tag{1.A.14}$$

The interior solution determined by the first order condition is as follows:

$$i_2^*(i_1,o) = \left(\frac{\sigma^2\Delta}{\frac{4b^4}{k}\left(1-\eta\frac{i_1}{o}\right)}\right)^{\frac{1}{3}} \tag{1.A.15}$$

$$r_2^*(i_1,o) = \frac{i_2^*(i_1,o) - i_1\eta}{\left(1-\eta\frac{i_1}{o}\right)}. \tag{1.A.16}$$

In order to guarantee that the interior solution is valid, i.e., $r_2^*(i_1,o) \in [0,o]$ or equivalently, $i_2^*(i_1,o) \in [\eta i_1, o]$, we need to assume that the condition for interior solution is satisfied:

$$\begin{aligned}
\eta i_1 < & \left(\frac{\sigma^2\Delta}{\frac{4b^4}{k}\left(1-\eta\frac{i_1}{o}\right)}\right)^{\frac{1}{3}} && < o \\
\eta^3 i_1^3\left(1-\eta\frac{i_1}{o}\right) < & \frac{\sigma^2\Delta}{\frac{4b^4}{k}} && < o^3\left(1-\eta\frac{i_1}{o}\right) \\
\frac{4b^4}{k}\left(\eta\frac{i_1}{o}\right)^3\left(1-\eta\frac{i_1}{o}\right) < & \frac{\sigma^2\Delta}{o^3} && < \frac{4b^4}{k}\left(1-\eta\frac{i_1}{o}\right),
\end{aligned} \tag{1.A.17}$$

which implies Δ should be intermediate, i.e., $\Delta \in \left(\frac{4b^4}{k}\left(1-\eta\frac{i_1}{o}\right)\frac{(\eta i_1)^3}{\sigma^2}, \frac{4b^4}{k}\left(1-\eta\frac{i_1}{o}\right)\frac{o^3}{\sigma^2}\right)$.

□

Proof of Corollary 1.1

Proof. Since $i_2^*(i_1,o) = \left(\frac{\sigma^2\Delta}{\frac{4b^4}{k}\left(1-\eta\frac{i_1}{o}\right)}\right)^{\frac{1}{3}} = \left(\frac{\sigma^2\Delta}{\frac{4b^4}{k}}\right)^{\frac{1}{3}}\left(1-\eta\frac{i_1}{o}\right)^{-\frac{1}{3}}$, we have

$$\begin{aligned}
\frac{\partial i_2^*(i_1,o)}{\partial\frac{i_1}{o}} &= -\frac{1}{3}\left(1-\eta\frac{i_1}{o}\right)^{-\frac{4}{3}}(-\eta)\left(\frac{\sigma^2\Delta}{\frac{4b^4}{k}}\right)^{\frac{1}{3}} \\
&= \frac{1}{3}\left(\frac{\sigma^2\Delta}{\frac{4b^4}{k}}\right)^{\frac{1}{3}}\left(1-\eta\frac{i_1}{o}\right)^{-\frac{4}{3}}\eta \\
&= \frac{1}{3}\eta\frac{i_2^*(i_1,o)}{\left(1-\eta\frac{i_1}{o}\right)} \\
&> 0,
\end{aligned} \tag{1.A.18}$$

which implies $i_2^*(i_1,o)$ is increasing in $\frac{i_1}{o}$. □

Proof of Lemma 1.5:

Proof. By the Envelope Theorem, we have:

$$\begin{aligned}
&\frac{\partial E\left[u_2^M(i_2,e_2)\,|r_2=r_2^*(i_1,o)\right]}{\partial i_1}\\
=&\frac{\partial E\left[u_2^M(i_2(i_1,r_2;o),e_2(r_2))\right]}{\partial i_2}\frac{\partial i_2(i_1,r_2;o)}{\partial i_1}|_{r_2=r_2^*(i_1,o)}\\
=&-\frac{4b^4\left[i_2(i_1,r_2;o)\right]^3}{k}\eta\big(1-\tfrac{r_2}{o}\big)\,|_{r_2=r_2^*(i_1,o)}. \qquad (1.A.19)
\end{aligned}$$

Substituting $r_2^*(i_1,o)=\frac{i_2^*(i_1,o)-i_1\eta}{\left(1-\eta\frac{i_1}{o}\right)}$ and using the result in Lemma 1.4 that $\sigma^2\Delta-\frac{4b^4\left[i_2^*(i_1,o)\right]^3}{k}\left(1-\eta\frac{i_1}{o}\right)=0$, we can obtain:

$$\begin{aligned}
&\frac{\partial E\left[u_2^M(i_2,e_2)\,|r_2=r_2^*(i_1,o)\right]}{\partial i_1}\\
=&-\frac{4b^4\left[i_2^*(i_1,o)\right]^3}{k}\eta\left(1-\frac{\frac{i_2^*(i_1,o)-i_1\eta}{\left(1-\eta\frac{i_1}{o}\right)}}{o}\right)\\
=&-\frac{\sigma^2\Delta}{\left(1-\eta\frac{i_1}{o}\right)}\eta\frac{1-\frac{i_2^*(i_1,o)}{o}}{\left(1-\eta\frac{i_1}{o}\right)}\\
=&-\frac{\sigma^2\Delta\eta\left(1-\frac{i_2^*(i_1,o)}{o}\right)}{\left(1-\eta\frac{i_1}{o}\right)^2}\\
<&0. \qquad (1.A.20)
\end{aligned}$$

Therefore, we have the first order condition to the optimal replacement decision problem at the beginning of Period 1 as follows:

$$
\begin{aligned}
\frac{\partial U_M}{\partial r_1} &= \sigma^2 \frac{de_1\left(r_1\right)}{dr_1} - \frac{4b^4 i_1^3}{k}\frac{di_1\left(r_1;o\right)}{dr_1} + \beta \frac{\partial E\left[u_2^M\left(i_2,e_2\right)|r_2 = r_2^*\left(i_1,o\right)\right]}{\partial i_1}\frac{di_1\left(r_1;o\right)}{dr_1} \\
&= \sigma^2\Delta - \left[\frac{4b^4 i_1^3}{k} + \beta\frac{\sigma^2\Delta\eta\left(1-\frac{i_2^*(i_1,o)}{o}\right)}{\left(1-\eta\frac{i_1}{o}\right)^2}\right]\left(1-\eta\right) \\
&= 0. \qquad (1.A.21)
\end{aligned}
$$

Meanwhile, the second order condition below should also be valid.

$$
\begin{aligned}
\frac{\partial^2 U_M}{\partial r_1^2} &= \frac{\partial\left(\frac{\partial U_M}{\partial r_1}\right)}{\partial r_1} \\
&= \frac{\partial}{\partial i_1}\left(\sigma^2\Delta - \left[\frac{4b^4 i_1^3}{k} + \beta\frac{\sigma^2\Delta\eta\left(1-\frac{i_2^*(i_1,o)}{o}\right)}{\left(1-\eta\frac{i_1}{o}\right)^2}\right]\left(1-\eta\right)\right)\frac{di_1\left(r_1\right)}{dr_1} \\
&= \left(1-\eta\right)\frac{\partial}{\partial i_1}\left(\sigma^2\Delta - \left[\frac{4b^4 i_1^3}{k} + \beta\frac{\sigma^2\Delta\eta\left(1-\frac{i_2^*(i_1,o)}{o}\right)}{\left(1-\eta\frac{i_1}{o}\right)^2}\right]\left(1-\eta\right)\right) \\
&= -\left(1-\eta\right)^2\sigma^2\Delta\left(\frac{12b^4 i_1^2}{k\sigma^2\Delta} + \beta\eta\frac{\frac{\eta}{o}\left(2-\frac{7}{3}\frac{i_2^*(i_1,o)}{o}\right)}{\left(1-\eta\frac{i_1}{o}\right)^3}\right) \\
&< 0. \qquad (1.A.22)
\end{aligned}
$$

One sufficient condition for second order condition is: $\eta < 1 - \frac{\sqrt{\beta}}{3}$, which can be proved as follows.

Since $\eta i_1 < i_2^*(i_1, o) = \left(\frac{\sigma^2\Delta}{\frac{4b^4}{k}\left(1-\eta\frac{i_1}{o}\right)}\right)^{\frac{1}{3}} < o$ and $i_1^*(o) \in [\eta o, o]$, we have

$$\begin{aligned}
\frac{12b^4 i_1^2}{k\sigma^2\Delta} + \beta\eta\frac{\frac{\eta}{o}\left(2-\frac{7}{3}\frac{i_2^*(i_1,o)}{o}\right)}{\left(1-\eta\frac{i_1}{o}\right)^3} &= \frac{3i_1^2}{\left[i_2^*(i_1,o)\right]^3\left(1-\eta\frac{i_1}{o}\right)} + \beta\eta\frac{\frac{\eta}{o}\left(2-\frac{7}{3}\frac{i_2^*(i_1,o)}{o}\right)}{\left(1-\eta\frac{i_1}{o}\right)^3} \\
&> \frac{3i_1^2}{o^3\left(1-\eta\frac{i_1}{o}\right)} + \beta\eta\frac{\frac{\eta}{o}\left(2-\frac{7}{3}\right)}{\left(1-\eta\frac{i_1}{o}\right)^3} \\
&= \frac{3}{o\left(1-\eta\frac{i_1}{o}\right)}\left(\frac{i_1^2}{o^2} - \frac{1}{9}\beta\eta^2\frac{1}{\left(1-\eta\frac{i_1}{o}\right)^2}\right) \\
&> \frac{3}{o\left(1-\eta\frac{i_1}{o}\right)}\left(\eta^2 - \frac{1}{9}\beta\eta^2\frac{1}{(1-\eta)^2}\right) \\
&= \frac{3\eta^2 o^2}{o^3\left(1-\eta\frac{i_1}{o}\right)}\left(1-\frac{\beta}{9(1-\eta)^2}\right), \qquad (1.A.23)
\end{aligned}$$

which is positive if $\eta < 1 - \frac{\sqrt{\beta}}{3}$.

The second order condition implies $\sigma^2\Delta - \left[\frac{4b^4 i_1^3}{k} + \beta\frac{\sigma^2\Delta\eta\left(1-\frac{i_2^*(i_1,o)}{o}\right)}{\left(1-\eta\frac{i_1}{o}\right)^2}\right](1-\eta)$ is decreasing in i_1. Meanwhile, to guarantee that the interior solution exists i.e., $i_1^*(o) \in [\eta o, o]$, we should have the following two conditions:

1. $\frac{\partial U_M}{\partial r_1}$ is negative at $i_1 = o$, which can be stated as:

$$\begin{aligned}
\sigma^2\Delta - \left[\frac{4b^4 o^3}{k} + \beta\frac{\sigma^2\Delta\eta\left(1-\frac{i_2^*(o,o)}{o}\right)}{(1-\eta)^2}\right](1-\eta) &< 0 \\
\sigma^2\Delta\left(1-\beta\frac{\eta\left(1-\frac{1}{o}\left(\frac{\sigma^2\Delta}{\frac{4b^4}{k}(1-\eta)}\right)^{\frac{1}{3}}\right)}{(1-\eta)}\right) - \frac{4b^4 o^3(1-\eta)}{k} &< 0. \qquad (1.A.24)
\end{aligned}$$

A sufficient condition to guarantee the above inequality hold is as follows:

$$\begin{aligned}
\sigma^2\Delta - \frac{4b^4 o^3(1-\eta)}{k} &< 0 \\
\Delta &< \frac{4b^4 o^3(1-\eta)}{\sigma^2 k}. \qquad (1.A.25)
\end{aligned}$$

2. $\frac{\partial U_M}{\partial r_1}$ is positive at $i_1 = \eta o$, which can be stated as:

$$\begin{aligned}
\sigma^2\Delta - \left[\frac{4b^4(\eta o)^3}{k} + \beta\frac{\sigma^2\Delta\eta\left(1-\frac{i_2^*(\eta o,o)}{o}\right)}{(1-\eta^2)^2}\right](1-\eta) &\geq 0 \\
\sigma^2\Delta\left(1-\beta\frac{\eta\left(1-\frac{1}{o}\left(\frac{\sigma^2\Delta}{\frac{4b^4}{k}(1-\eta^2)}\right)^{\frac{1}{3}}\right)}{(1-\eta^2)(1+\eta)}\right) - \frac{4b^4(\eta o)^3(1-\eta)}{k} &\geq 0. \qquad (1.A.26)
\end{aligned}$$

A sufficient condition to guarantee the above inequality hold is as follows:

$$\begin{aligned}
\sigma^2\Delta\left(1-\beta\frac{\eta}{(1+\eta)}\right) - \frac{4b^4(\eta o)^3(1-\eta)}{k} &\geq 0 \\
\Delta &\geq \frac{4b^4 o^3(1-\eta)}{k\sigma^2}\frac{\eta^3}{1-\beta\frac{\eta}{(1+\eta)}}. \qquad (1.A.27)
\end{aligned}$$

Therefore, a sufficient condition for the second order condition is $\Delta \in \left(\frac{4b^4o^3(1-\eta)}{k\sigma^2}\frac{\eta^3}{1-\beta\frac{\eta}{(1+\eta)}}, \frac{4b^4o^3(1-\eta)}{\sigma^2 k}\right)$, which is nonempty if $\eta < 1 - \frac{\sqrt{\beta}}{3}$.

In Lemma 1.4, we have shown that to guarantee that an interior solution in Period 2, the expertise improvement should satisfy: $\Delta \in \left(\frac{4b^4}{k}\left(1-\eta\frac{i_1}{o}\right)\frac{(\eta i_1)^3}{\sigma^2}, \frac{4b^4}{k}\left(1-\eta\frac{i_1}{o}\right)\frac{o^3}{\sigma^2}\right)$. A sufficient condition to guarantee $\Delta \in \left(\frac{4b^4}{k}\left(1-\eta\frac{i_1}{o}\right)\frac{(\eta i_1)^3}{\sigma^2}, \frac{4b^4}{k}\left(1-\eta\frac{i_1}{o}\right)\frac{o^3}{\sigma^2}\right)$, which does not depend on the value of $i_1 \in (\eta o, o)$, is $\eta < \frac{3}{4}, \Delta \in \left(\frac{4b^4}{k}(1-\eta)\frac{(\eta o)^3}{\sigma^2}, \frac{4b^4}{k}(1-\eta^2)\frac{o^3}{\sigma^2}\right)$.

Therefore, an overall sufficient condition to guarantee an interior solution in both periods is $\eta < \min\left\{\frac{3}{4}, 1-\frac{\sqrt{\beta}}{3}\right\}, \Delta \in \left(\frac{4b^4o^3(1-\eta)}{k\sigma^2}\frac{\eta^3}{1-\beta\frac{\eta}{(1+\eta)}}, \frac{4b^4o^3(1-\eta)}{\sigma^2 k}\right)$.

□

Proof of Lemma 1.6:

Proof. We show that $\frac{i_1^*(o)}{o}$ is decreasing in o by showing contradiction. In equilibrium, $i_1^*(o)$

is determined by the first order condition:

$$\sigma^2\Delta - \left[\frac{4b^4\left(i_1\right)^3}{k} + \beta\frac{\sigma^2\Delta\eta\left(1-\frac{i_2^*(i_1,o)}{o}\right)}{\left(1-\eta\frac{i_1}{o}\right)^2}\right](1-\eta)=0 \tag{1.A.28}$$

in which the LHS is decreasing in i_1 due to the second order condition. If $\exists o' > o$ and $\frac{i_1^*(o')}{o'} \geq \frac{i_1^*(o)}{o}$, denote $\hat{i}_1\left(o'\right) = \frac{i_1^*(o)}{o}o'$, we have

$$i_1^*(o') \geq \frac{i_1^*(o)}{o}o' = \hat{i}_1\left(o'\right) > i_1^*(o) \tag{1.A.29}$$

Therefore, we obtain

$$\begin{aligned}
0 =&\sigma^2\Delta - \left[\frac{4b^4\left(i_1^*\left(o\right)\right)^3}{k} + \beta\frac{\sigma^2\Delta\eta\left(1-\frac{i_2^*\left(i_1^*(o),o\right)}{o}\right)}{\left(1-\eta\frac{i_1^*(o)}{o}\right)^2}\right](1-\eta)\\
=&\sigma^2\Delta - \left[\frac{4b^4\left(i_1^*\left(o\right)\right)^3}{k} + \beta\frac{\sigma^2\Delta\eta\left(1-\frac{i_2^*\left(\hat{i}_1(o'),o'\right)}{o}\right)}{\left(1-\eta\frac{\hat{i}_1(o')}{o'}\right)^2}\right](1-\eta)\\
>&\sigma^2\Delta - \left[\frac{4b^4\left(\hat{i}_1\left(o'\right)\right)^3}{k} + \beta\frac{\sigma^2\Delta\eta\left(1-\frac{i_2^*\left(\hat{i}_1(o'),o'\right)}{o}\right)}{\left(1-\eta\frac{\hat{i}_1(o')}{o'}\right)^2}\right](1-\eta)\\
\geq&\sigma^2\Delta - \left[\frac{4b^4\left(i_1^*\left(o'\right)\right)^3}{k} + \beta\frac{\sigma^2\Delta\eta\left(1-\frac{i_2^*\left(i_1^*(o'),o'\right)}{o}\right)}{\left(1-\eta\frac{i_1^*(o')}{o'}\right)^2}\right](1-\eta)\\
=&0,
\end{aligned} \tag{1.A.30}$$

where the first inequality comes from $\hat{i}_1\left(o'\right) > i_1^*(o)$, and the second inequality comes from the second condition that $LHS\left(1.A.28\right)$ is decreasing in i_1 and $i_1^*\left(o'\right) \geq \hat{i}_1\left(o'\right)$, which is a contradiction. Therefore, we have $\frac{i_1^*(o)}{o}$ is decreasing in o.

$\square$

Proof of Proposition 1.1:

Proof. By Lemma 1.6, we have the ratio $\frac{i_1^*(o)}{o}$ is decreasing in the nominal independence o. Moreover, by Corollary 1.1, we have the resultant real independence in Period 2 $i_2^*(i_1, o)$ is increasing in the ratio $\frac{i_1}{o}$. Therefore, we have the real independence in Period 2 $i_2^*(i_1^*(o), o)$ is decreasing in the nominal independence o.

□

Proof of Proposition 1.2:

Proof. Since the policy function $I(i; o)$is the solution to maximize the value function $V_M(i; o)$, by the first order condition, we have:

$$\frac{1}{1-\eta\frac{i}{o}}\Delta\sigma^2 - \frac{4b^4\left(I(i;o)\right)^3}{k} + \beta\frac{\partial V_M(i;o)}{\partial i}|_{i=I(i;o)} = 0, \tag{1.A.31}$$

and by Envelope Theorem, we have:

$$\frac{\partial V_M(i;o)}{\partial i} = \frac{\eta\left(\frac{I(i;o)}{o}-1\right)}{\left(1-\eta\frac{i}{o}\right)^2}\Delta\sigma^2. \tag{1.A.32}$$

If there is a fixed point, i.e., $i^*(o)$ s.t. $I(i^*(o); o) = i^*(o)$, it should satisfy both the first order condition for the optimal replacement decision to determined the value function and the the first derivative formula of the value function derived using the Envelope Theorem.

Therefore, the fixed point $i^*(o)$ satisfies:

$$\begin{aligned}
\frac{1}{1-\eta\frac{i}{o}}\Delta\sigma^2 - \frac{4b^4 i^3}{k} + \beta\frac{\eta\left(\frac{i}{o}-1\right)}{\left(1-\eta\frac{i}{o}\right)^2}\Delta\sigma^2 &= 0 \\
\left[\frac{\beta(1-\eta)}{\left(1-\eta\frac{i}{o}\right)^2} + \frac{(1-\beta)}{\left(1-\eta\frac{i}{o}\right)}\right]\Delta\sigma^2 - \frac{4b^4 i^3}{k} &= 0 \\
\frac{i^3}{\frac{\beta(1-\eta)}{\left(1-\eta\frac{i}{o}\right)^2} + \frac{(1-\beta)}{\left(1-\eta\frac{i}{o}\right)}} &= \frac{\Delta\sigma^2}{\frac{4b^4}{k}} \\
\frac{i^3\left(1-\eta\frac{i}{o}\right)^2}{(1-\beta\eta)-(1-\beta)\eta\frac{i}{o}} &= \frac{\Delta\sigma^2}{\frac{4b^4}{k}}. \qquad (1.A.33)
\end{aligned}$$

Now, we show that equation (1.A.33) has a unique solution if $\Delta \in \left(0, \frac{b^4}{k\sigma^2}4o^3(1-\eta)\right)$.

In order to prove the unique solution, we discuss two cases, i.e., $\frac{\partial LHS(1.A.33)}{\partial i} \geq 0$ and $\frac{\partial LHS(1.A.33)}{\partial i} < 0$.

First, we show that if $\eta \leq \frac{3}{4+\beta}$, $\frac{\partial LHS(1.A.33)}{\partial i} \geq 0$. Specifically, we have

$$\begin{aligned}
&\frac{\partial LHS(1.A.33)}{\partial i} \\
= &\frac{i^2\left[3\left(1-\eta\frac{i}{o}\right)^2 - 2\eta\frac{i}{o}\left(1-\eta\frac{i}{o}\right)\right]\left[(1-\beta\eta)-(1-\beta)\eta\frac{i}{o}\right] + (1-\beta)\frac{\eta}{o}i^3\left(1-\eta\frac{i}{o}\right)^2}{\left[(1-\beta\eta)-(1-\beta)\eta\frac{i}{o}\right]^2} \\
= &\frac{i^2\left(1-\eta\frac{i}{o}\right)\left[\left(3-5\eta\frac{i}{o}\right)(1-\beta\eta) - 2(1-\beta)\eta\frac{i}{o}\left(1-2\eta\frac{i}{o}\right)\right]}{\left[(1-\beta\eta)-(1-\beta)\eta\frac{i}{o}\right]^2} \\
= &\frac{\frac{4b^4 i^2}{k}\left(1-\eta\frac{i}{o}\right)\left[4(1-\beta)\left(\eta\frac{i}{o}\right)^2 - (7-5\beta\eta-2\beta)\eta\frac{i}{o} + 3(1-\beta\eta)\right]}{\left[(1-\beta\eta)-(1-\beta)\eta\frac{i}{o}\right]^2}, \qquad (1.A.34)
\end{aligned}$$

in which

$$\begin{aligned}
& \frac{\partial}{\partial \eta \frac{i}{o}} \left[4\left(1-\beta\right) \left(\eta \frac{i}{o}\right)^2 - \left(7 - 5\beta\eta - 2\beta\right) \eta \frac{i}{o} + 3\left(1-\beta\eta\right) \right] \\
= \; & 8\left(1-\beta\right)\left(\eta \frac{i}{o}\right) - \left(7-5\beta\eta-2\beta\right) \\
< \; & 8\left(1-\beta\right)\eta - \left(7-5\beta\eta-2\beta\right) \\
= \; & -7+8\eta-3\beta\eta+2\beta \\
< \; & \max\left\{-5+5\eta, -7+8\eta\right\}. \qquad (1.A.35)
\end{aligned}$$

The last term is negative if $\eta < \frac{7}{8}$. Therefore, if $\eta \leq \frac{3}{4+\beta} < \frac{7}{8}$, we have

$$\begin{aligned}
& \left[4\left(1-\beta\right) \left(\eta \frac{i}{o}\right)^2 - \left(7 - 5\beta\eta - 2\beta\right) \eta \frac{i}{o} + 3\left(1-\beta\eta\right) \right] \\
\geq \; & 4\left(1-\beta\right)\eta^2 - \left(7-5\beta\eta-2\beta\right)\eta + 3\left(1-\beta\eta\right) \\
= \; & \left(4+\beta\right)\eta^2 - \left(7+\beta\right)\eta + 3 \\
= \; & \left(\eta-1\right)\left(\left(4+\beta\right)\eta-3\right) \\
\geq \; & 0, \qquad (1.A.36)
\end{aligned}$$

which implies $\frac{\partial LHS(1.A.33)}{\partial i} \geq 0$.

If $\eta \leq \frac{3}{4+\beta}$, and, thus, $\frac{\partial LHS(1.A.33)}{\partial i} \geq 0$, in order to guarantee that the fixed point exists, i.e., $i^*\left(o\right) \in \left(0, o\right)$, we have:

$$\begin{array}{ccccc}
LHS(1.A.33)|_{i=0} & < RHS(1.A.33) < & LHS(1.A.33)|_{i=o} \\
0 & < \Delta < & \dfrac{b^4}{k\sigma^2} 4o^3\left(1-\eta\right).
\end{array} \qquad (1.A.37)$$

Otherwise if $\Delta \geq \frac{b^4}{k\sigma^2} 4o^3\left(1-\eta\right)$, it will be corner solution that $i^*\left(o\right) = o$.

Alternatively, if $\eta > \frac{3}{4+\beta}$, we have $4\left(1-\beta\right)\left(\eta\frac{i}{o}\right)^2 - \left(7-5\beta\eta-2\beta\right)\eta\frac{i}{o} + 3\left(1-\beta\eta\right)$ is positive when $i=0$ and is negative when $i=o$, which implies that $\exists i^{\#}\left(o\right) \in \left(0, o\right)$, s.t.,

$\frac{\partial LHS(1.A.33)}{\partial i}$ is positive for $i \in \left(0, i^{\#}(o)\right)$ and is negative for $i \in \left(i^{\#}(o), o\right)$. Thus, $LHS(1.A.33)$ is a concave function of $i \in (0, o)$. Moreover, it is easy to show that $LHS(1.A.33)|_{i=0} = 0 < LHS(1.A.33)|_{i=o}$. Hence, if $RHS(1.A.33) < LHS(1.A.33)|_{i=o}$ or equivalently, $\Delta < \frac{b^4}{k\sigma^2}4o^3(1-\eta)$, equation (1.A.33) also has a unique solution for $i^*(o)$ with $\frac{\partial LHS(1.A.33)}{\partial i}|_{i=i^*(o)} > 0$. Otherwise if $\Delta \geq \frac{b^4}{k\sigma^2}4o^3(1-\eta)$, we obtain a corner solution with $i^*(o) = o$, or there are two interior solutions which satisfy equation (1.A.33) , and the stable fixed point of real independence which can be achieved in the dynamics is closer to the nominal independence o and satisfies $\frac{\partial LHS(1.A.33)}{\partial i}|_{i=i^*(o)} < 0$.

Moreover,

$$\frac{\partial}{\partial o}\left[\frac{4b^4i^3}{k\left[\frac{\beta(1-\eta)}{\left(1-\eta\frac{i}{o}\right)^2}+\frac{(1-\beta)}{\left(1-\eta\frac{i}{o}\right)}\right]}\right] \;>\; 0. \tag{1.A.38}$$

Therefore, $\forall o, o', s.t., 0 < o < o' < 1$, if $\Delta \in \left(0, \frac{b^4}{k\sigma^2}4o^3(1-\eta)\right)$, we have $i^*(o') < i^*(o)$, which implies higher nominal independence results in lower real independence. Otherwise if $\Delta \geq \frac{b^4}{k\sigma^2}4(o')^3(1-\eta)$, we have $i^*(o') > i^*(o)$, which implies higher nominal independence results in higher real independence

$\square$

Proof of Proposition 1.3:

Proof. Similar to the proof of Proposition 1.2, if there is a fixed point, i.e., $i^*(o)$ s.t. $I(i^*(o); o) = i^*(o)$, it should satisfy:

$$\frac{1}{1-\eta\frac{i}{o}}G'\left(\frac{i-\eta i}{1-\eta\frac{i}{o}}\right)\sigma^2 - \frac{4b^4i^3}{k} + \beta\frac{\eta\left(\frac{i}{o}-1\right)}{\left(1-\eta\frac{i}{o}\right)^2}G'\left(\frac{i-\eta i}{1-\eta\frac{i}{o}}\right)\sigma^2 \;=\; 0,$$

which can be rewritten as:

$$
\begin{aligned}
\frac{(1-\beta\eta)-(1-\beta)\,\eta\frac{i}{o}}{\left(1-\eta\frac{i}{o}\right)^{2}}G'\left(\frac{i-\eta i}{1-\eta\frac{i}{o}}\right)\sigma^{2}-\frac{4b^{4}i^{3}}{k} &= 0\\
\left[\frac{\beta\,(1-\eta)}{\left(1-\eta\frac{i}{o}\right)^{2}}+\frac{(1-\beta)}{\left(1-\eta\frac{i}{o}\right)}\right]G'\left(\frac{i-\eta i}{1-\eta\frac{i}{o}}\right)\sigma^{2}-\frac{4b^{4}i^{3}}{k} &= 0\\
\frac{1}{i^{3}}\left[\frac{\beta\,(1-\eta)}{\left(1-\eta\frac{i}{o}\right)^{2}}+\frac{(1-\beta)}{\left(1-\eta\frac{i}{o}\right)}\right]G'\left(\frac{1-\eta}{\frac{1}{i}-\eta\frac{1}{o}}\right) &= \frac{4b^{4}}{k\sigma^{2}}. \qquad (1.A.39)
\end{aligned}
$$

We look at the sign of $\frac{\partial}{\partial i}LHS\,(1.A.39)$ and $\frac{\partial}{\partial o}LHS\,(1.A.39)$ to determine whether $i^{*}\,(o)$ is increasing or decreasing in o.

On one hand, we have

$$
\begin{aligned}
&\frac{\partial}{\partial i}LHS\,(1.A.39)\\
=\;& \frac{1}{i^{3}}\left[\frac{\beta\,(1-\eta)}{\left(1-\eta\frac{i}{o}\right)^{2}}+\frac{(1-\beta)}{\left(1-\eta\frac{i}{o}\right)}\right]G''\left(\frac{1-\eta}{\frac{1}{i}-\eta\frac{1}{o}}\right)\frac{1}{i^{2}}\frac{1-\eta}{\frac{1}{i}-\eta\frac{1}{o}}\\
&+\frac{\partial}{\partial i}\left\{\frac{1}{i^{3}}\left[\frac{\beta\,(1-\eta)}{\left(1-\eta\frac{i}{o}\right)^{2}}+\frac{(1-\beta)}{\left(1-\eta\frac{i}{o}\right)}\right]\right\}G'\left(\frac{1-\eta}{\frac{1}{i}-\eta\frac{1}{o}}\right)\\
=\;& \frac{1}{i^{3}}\left[\frac{\beta\,(1-\eta)}{\left(1-\eta\frac{i}{o}\right)^{2}}+\frac{(1-\beta)}{\left(1-\eta\frac{i}{o}\right)}\right]G''\left(\frac{1-\eta}{\frac{1}{i}-\eta\frac{1}{o}}\right)\frac{1}{i^{2}}\frac{1-\eta}{\frac{1}{i}-\eta\frac{1}{o}}\\
&+\left\{-3\frac{1}{i^{4}}\left[\frac{\beta\,(1-\eta)}{\left(1-\eta\frac{i}{o}\right)^{2}}+\frac{(1-\beta)}{\left(1-\eta\frac{i}{o}\right)}\right]+\frac{1}{i^{3}}\eta\frac{1}{o}\left[2\frac{\beta\,(1-\eta)}{\left(1-\eta\frac{i}{o}\right)^{3}}+\frac{(1-\beta)}{\left(1-\eta\frac{i}{o}\right)^{2}}\right]\right\}G'\left(\frac{1-\eta}{\frac{1}{i}-\eta\frac{1}{o}}\right)\\
=\;& \frac{1-\eta}{i^{4}\left(1-\eta\frac{i}{o}\right)^{2}}\left[\frac{\beta\,(1-\eta)}{\left(1-\eta\frac{i}{o}\right)}+(1-\beta)\right]G''\left(\frac{1-\eta}{\frac{1}{i}-\eta\frac{1}{o}}\right)\\
&+\frac{1}{i^{4}\left(1-\eta\frac{i}{o}\right)}\left\{2\frac{\beta\,(1-\eta)}{\left(1-\eta\frac{i}{o}\right)^{2}}+\frac{(1-\beta)-5\beta\,(1-\eta)}{\left(1-\eta\frac{i}{o}\right)}-4\,(1-\beta)\right\}G'\left(\frac{1-\eta}{\frac{1}{i}-\eta\frac{1}{o}}\right) \qquad (1.A.40)
\end{aligned}
$$

If $\eta < \frac{3}{4+\beta}$, we have

$$\begin{aligned}
& 2\frac{\beta(1-\eta)}{\left(1-\eta\frac{i}{o}\right)^2} + \frac{(1-\beta)-5\beta(1-\eta)}{\left(1-\eta\frac{i}{o}\right)} - 4(1-\beta) \\
\leq\ & \max\left\{2\frac{\beta(1-\eta)}{(1-\eta)^2} + \frac{(1-\beta)-5\beta(1-\eta)}{(1-\eta)} - 4(1-\beta), 2\beta(1-\eta) - 5\beta(1-\eta) - 3(1-\beta)\right\} \\
=\ & \max\left\{\frac{-3+(4+\beta)\eta}{(1-\eta)}, -3\left[\beta(1-\eta)+(1-\beta)\right]\right\} \qquad (1.A.41) \\
<\ & 0.
\end{aligned}$$

Therefore, if $\eta < \frac{3}{4+\beta}$, we have $\frac{\partial}{\partial i}LHS\,(1.A.39) < 0$. Meanwhile, in order to guarantee that $\exists i \in (0,o)$ s.t. $LHS\,(1.A.39) = \frac{4b^4}{k\sigma^2}$, we should have $LHS\,(1.A.39)\,|_{i=0} > \frac{4b^4}{k\sigma^2}$ and $LHS\,(1.A.39)\,|_{i=o} < \frac{4b^4}{k\sigma^2}$, which implies $G'(r) \in \left(0, \frac{b^4}{k\sigma^2}4o^3(1-\eta)\right)$.

On the other hand, we have

$$\begin{aligned}
& \frac{\partial}{\partial o}LHS\,(1.A.39) \\
=\ & -\frac{1}{i^3}\left[\frac{\beta(1-\eta)}{\left(1-\eta\frac{i}{o}\right)^2} + \frac{(1-\beta)}{\left(1-\eta\frac{i}{o}\right)}\right] G''\left(\frac{1-\eta}{\frac{1}{i}-\eta\frac{1}{o}}\right)\eta\frac{1}{o^2}\frac{1-\eta}{\frac{1}{i}-\eta\frac{1}{o}} \\
& + \frac{\partial}{\partial o}\left\{\frac{1}{i^3}\left[\frac{\beta(1-\eta)}{\left(1-\eta\frac{i}{o}\right)^2} + \frac{(1-\beta)}{\left(1-\eta\frac{i}{o}\right)}\right]\right\} G'\left(\frac{1-\eta}{\frac{1}{i}-\eta\frac{1}{o}}\right) \\
=\ & -\frac{\eta\frac{i}{o^2}}{i^3\left(1-\eta\frac{i}{o}\right)^2}\{(1-\eta)\left[\frac{\beta(1-\eta)}{\left(1-\eta\frac{i}{o}\right)} + (1-\beta)\right] G''\left(\frac{1-\eta}{\frac{1}{i}-\eta\frac{1}{o}}\right) \\
& + \left[2\frac{\beta(1-\eta)}{\left(1-\eta\frac{i}{o}\right)} + (1-\beta)\right] G'\left(\frac{1-\eta}{\frac{1}{i}-\eta\frac{1}{o}}\right)\}. \qquad (1.A.42)
\end{aligned}$$

If $\frac{G''(r)}{G'(r)} < -\frac{1+\beta}{1-\eta}$, since $\frac{1}{1+\frac{(1-\beta)\left(1-\eta\frac{i}{o}\right)}{\beta(1-\eta)}} \leq \frac{1}{1+\frac{(1-\beta)(1-\eta\frac{o}{o})}{\beta(1-\eta)}} = \beta$, we have

$$
\begin{aligned}
\frac{G''\left(\frac{1-\eta}{\frac{1}{i}-\eta\frac{1}{o}}\right)}{G'\left(\frac{1-\eta}{\frac{1}{i}-\eta\frac{1}{o}}\right)} &< -\frac{1+\beta}{1-\eta} \\
&\leq -\left(\frac{1}{1-\eta} + \frac{1}{(1-\eta)\left[1+\frac{(1-\beta)\left(1-\eta\frac{i}{o}\right)}{\beta(1-\eta)}\right]}\right) \\
&= -\frac{2\frac{\beta(1-\eta)}{\left(1-\eta\frac{i}{o}\right)} + (1-\beta)}{(1-\eta)\left[\frac{\beta(1-\eta)}{\left(1-\eta\frac{i}{o}\right)} + (1-\beta)\right]},
\end{aligned}
\tag{1.A.43}
$$

which implies $(1-\eta)\left[\frac{\beta(1-\eta)}{\left(1-\eta\frac{i}{o}\right)} + (1-\beta)\right] G''\left(\frac{1-\eta}{\frac{1}{i}-\eta\frac{1}{o}}\right) + \left[2\frac{\beta(1-\eta)}{\left(1-\eta\frac{i}{o}\right)} + (1-\beta)\right] G'\left(\frac{1-\eta}{\frac{1}{i}-\eta\frac{1}{o}}\right) < 0$ and, thus, $\frac{\partial}{\partial o} LHS\,(1.A.39) > 0$.

If $\frac{G''(r)}{G'(r)} > -\left(\frac{1}{1-\eta} + \frac{\beta}{1-\beta\eta}\right)$, since $\frac{1}{1+\frac{(1-\beta)\left(1-\eta\frac{i}{o}\right)}{\beta(1-\eta)}} \geq \frac{1}{1+\frac{(1-\beta)}{\beta(1-\eta)}} = \frac{\beta(1-\eta)}{1-\beta\eta}$, we have

$$
\begin{aligned}
\frac{G''\left(\frac{1-\eta}{\frac{1}{i}-\eta\frac{1}{o}}\right)}{G'\left(\frac{1-\eta}{\frac{1}{i}-\eta\frac{1}{o}}\right)} &> -\left(\frac{1}{1-\eta} + \frac{\beta}{1-\beta\eta}\right) \\
&\geq -\left(\frac{1}{1-\eta} + \frac{1}{(1-\eta)\left[1+\frac{(1-\beta)\left(1-\eta\frac{i}{o}\right)}{\beta(1-\eta)}\right]}\right) \\
&= -\frac{2\frac{\beta(1-\eta)}{\left(1-\eta\frac{i}{o}\right)} + (1-\beta)}{(1-\eta)\left[\frac{\beta(1-\eta)}{\left(1-\eta\frac{i}{o}\right)} + (1-\beta)\right]},
\end{aligned}
\tag{1.A.44}
$$

which implies $(1-\eta)\left[\frac{\beta(1-\eta)}{\left(1-\eta\frac{i}{o}\right)} + (1-\beta)\right] G''\left(\frac{1-\eta}{\frac{1}{i}-\eta\frac{1}{o}}\right) + \left[2\frac{\beta(1-\eta)}{\left(1-\eta\frac{i}{o}\right)} + (1-\beta)\right] G'\left(\frac{1-\eta}{\frac{1}{i}-\eta\frac{1}{o}}\right) > 0$ and, thus, $\frac{\partial}{\partial o} LHS\,(1.A.39) < 0$.

Therefore, $\forall o, o', s.t., 0 < o < o' < 1$, if $\eta < \frac{3}{4+\beta}$, $G'(r) \in \left(0, \frac{b^4}{k\sigma^2} 4o^3 (1-\eta)\right)$, we have $i^*(o') < i^*(o)$ if $\frac{G''(r)}{G'(r)} > -\left(\frac{1}{1-\eta} + \frac{\beta}{1-\beta\eta}\right)$, and $i^*(o') > i^*(o)$ if $\frac{G''(r)}{G'(r)} < -\left(\frac{1-\beta}{1-\eta}\right)$. Note that if $G''\left(\frac{1-\eta}{\frac{1}{i}-\eta\frac{1}{o}}\right) = 0$, we have the original result in which the formula is decreasing in both i

and o, so a higher o results in a lower $i^*(o)$. Otherwise if $G''\left(\frac{1-\eta}{\frac{1}{i}-\eta\frac{1}{o}}\right)$ is small enough, the formula is decreasing in i but increasing in o, so a higher o results in a higher $i^*(o)$.

$\square$

Proof of Proposition 1.4:

Proof. By the first order condition of the value function, we have:

$$\frac{1}{1-\eta\frac{i}{o}}\Delta\sigma^2 - 2b^2 i'\left(1-\frac{(bi')^2}{k}\right) + \beta\frac{\partial V(i;o)}{\partial i}|_{i=I(i;o)} = 0, \tag{1.A.45}$$

and by Envelope Theorem, we have:

$$\frac{\partial V(i;o)}{\partial i} = \frac{\eta\left(\frac{I(i;o)}{o}-1\right)}{\left(1-\eta\frac{i}{o}\right)^2}\Delta\sigma^2. \tag{1.A.46}$$

If there is a fixed point, i.e., $i^*(o)$ s.t. $I(i^*(o);o) = i^*(o)$, it satisfy both the first order condition for optimal replacement decision to determined the value function and the the first derivative formula of the value function derived using the Envelope Theorem. Therefore, the fixed point $i^*(o)$ should make the following equation hold:

$$\begin{aligned}
\frac{1}{1-\eta\frac{i}{o}}\Delta\sigma^2 - 2b^2 i\left(1-\frac{(bi)^2}{k}\right) + \beta\frac{\eta\left(\frac{i}{o}-1\right)}{\left(1-\eta\frac{i}{o}\right)^2}\Delta\sigma^2 &= 0\\
\frac{(1-\beta\eta)-(1-\beta)\eta\frac{i}{o}}{\left(1-\eta\frac{i}{o}\right)^2}\Delta\sigma^2 - 2b^2 i\left(1-\frac{(bi)^2}{k}\right) &= 0\\
\left[\frac{\beta(1-\eta)}{\left(1-\eta\frac{i}{o}\right)^2}+\frac{(1-\beta)}{\left(1-\eta\frac{i}{o}\right)}\right]\Delta\sigma^2 - 2b^2 i\left(1-\frac{(bi)^2}{k}\right) &= 0\\
\frac{i\left(1-\frac{(bi)^2}{k}\right)\left(1-\eta\frac{i}{o}\right)^2}{(1-\beta\eta)-(1-\beta)\eta\frac{i}{o}} &= \frac{\Delta\sigma^2}{2b^2}.
\end{aligned} \tag{1.A.47}$$

We will prove that equation (1.A.47) has a unique solution if $\frac{b^2}{k} < \frac{1}{3}$, $\eta < \min\left\{\frac{2}{5}, \frac{1-3\frac{b^2}{k}}{2+\beta-(4+\beta)\frac{b^2}{k}}\right\}$, and $\Delta \in \left(0, \frac{2b^2}{\sigma^2}o\left(1-\frac{(bo)^2}{k}\right)(1-\eta)\right)$.

First, we prove that if $\frac{b^2}{k} < \frac{1}{3}$, $\eta < \min\left\{\frac{2}{5}, \frac{1-3\frac{b^2}{k}}{2+\beta-(4+\beta)\frac{b^2}{k}}\right\}$, we have $\frac{\partial}{\partial i}\left[\frac{i\left(1-\frac{(bi)^2}{k}\right)\left(1-\eta\frac{i}{o}\right)^2}{(1-\beta\eta)-(1-\beta)\eta\frac{i}{o}}\right] > 0$. Specifically, we obtain:

$$
\begin{aligned}
&\frac{\partial}{\partial i} LHS\,(1.A.47) \\
= \; & \frac{\left[\left(1-\frac{3(bi)^2}{k}\right)\left(1-\eta\frac{i}{o}\right)^2 - i\left(1-\frac{(bi)^2}{k}\right)2\eta\frac{1}{o}\left(1-\eta\frac{i}{o}\right)\right]\left[(1-\beta\eta)-(1-\beta)\eta\frac{i}{o}\right]}{\left[(1-\beta\eta)-(1-\beta)\eta\frac{i}{o}\right]^2} \\
& + \frac{(1-\beta)\frac{\eta}{o}i\left(1-\frac{(bi)^2}{k}\right)\left(1-\eta\frac{i}{o}\right)^2}{\left[(1-\beta\eta)-(1-\beta)\eta\frac{i}{o}\right]^2} \\
= \; & \left(1-\eta\frac{i}{o}\right)\frac{\left(1-3\eta\frac{i}{o}\right)(1-\beta\eta)+2(1-\beta)\left(\eta\frac{i}{o}\right)^2}{\left[(1-\beta\eta)-(1-\beta)\eta\frac{i}{o}\right]^2} \\
& + \left(1-\eta\frac{i}{o}\right)\frac{\frac{(bi)^2}{k}\left[-3(1-\beta\eta)+(7-5\beta\eta-2\beta)\eta\frac{i}{o}-4(1-\beta)\left(\eta\frac{i}{o}\right)^2\right]}{\left[(1-\beta\eta)-(1-\beta)\eta\frac{i}{o}\right]^2}, \quad (1.A.48)
\end{aligned}
$$

where

$$
\begin{aligned}
\frac{\partial}{\partial i}\left[\left(1-3\eta\frac{i}{o}\right)(1-\beta\eta)+2(1-\beta)\left(\eta\frac{i}{o}\right)^2\right] &= \eta\frac{1}{o}\left[-3(1-\beta\eta)+4(1-\beta)\left(\eta\frac{i}{o}\right)\right] \\
&< \eta\frac{1}{o}\left[-3(1-\beta\eta)+4(1-\beta)\eta\right] \\
&= \eta\frac{1}{o}\left[-3-\beta\eta+4\eta\right], \quad (1.A.49)
\end{aligned}
$$

which is negative if $\eta < \frac{3}{4-\beta}$. Therefore, if $\eta < \frac{3}{4-\beta}$, we have:

$$
\begin{aligned}
\left(1-3\eta\frac{i}{o}\right)(1-\beta\eta)+2(1-\beta)\left(\eta\frac{i}{o}\right)^2 &\geq (1-3\eta)(1-\beta\eta)+2(1-\beta)\eta^2 \\
&= 1-(3+\beta)\eta+(2+\beta)\eta^2 \\
&= (1-(2+\beta)\eta)(1-\eta), \quad (1.A.50)
\end{aligned}
$$

and

$$
\begin{aligned}
& \frac{\partial}{\partial i}\left\{i^2\left[-3\left(1-\beta\eta\right)+\left(7-5\beta\eta-2\beta\right)\eta\frac{i}{o}-4\left(1-\beta\right)\left(\eta\frac{i}{o}\right)^2\right]\right\} \\
= \;& i\left[-6\left(1-\beta\eta\right)+3\left(7-5\beta\eta-2\beta\right)\eta\frac{i}{o}-16\left(1-\beta\right)\left(\eta\frac{i}{o}\right)^2\right] \\
= \;& \left(1-\beta\right)i\left[-6\frac{\left(1-\beta\eta\right)}{\left(1-\beta\right)}+3\left(5\frac{\left(1-\beta\eta\right)}{\left(1-\beta\right)}+2\right)\eta\frac{i}{o}-16\left(\eta\frac{i}{o}\right)^2\right]. \qquad (1.A.51)
\end{aligned}
$$

$\frac{3\left(5\frac{(1-\beta\eta)}{(1-\beta)}+2\right)}{32} > \frac{3(5+2)}{32} > \frac{2}{5} > \eta$ implies $-6\frac{(1-\beta\eta)}{(1-\beta)}+3\left(5\frac{(1-\beta\eta)}{(1-\beta)}+2\right)\eta\frac{i}{o}-16\left(\eta\frac{i}{o}\right)^2$ is increasing in $(0,o)$. So we have

$$
\begin{aligned}
& \frac{\partial}{\partial i}\left\{i^2\left[-3\left(1-\beta\eta\right)+\left(7-5\beta\eta-2\beta\right)\eta\frac{i}{o}-4\left(1-\beta\right)\left(\eta\frac{i}{o}\right)^2\right]\right\} \\
\leq \;& \left(1-\beta\right)i\left[-6\frac{\left(1-\beta\eta\right)}{\left(1-\beta\right)}+3\left(5\frac{\left(1-\beta\eta\right)}{\left(1-\beta\right)}+2\right)\eta-16\eta^2\right] \\
= \;& i\left[-6\left(1-\beta\eta\right)+3\left(7-5\beta\eta-2\beta\right)\eta-16\left(1-\beta\right)\eta^2\right] \\
= \;& i\left[-6+21\eta-16\eta^2+\beta\eta^2\right] \\
= \;& i\left[-\left(1-\eta\right)\left(6-15\eta\right)-\left(1-\beta\right)\eta^2\right], \qquad (1.A.52)
\end{aligned}
$$

which is negative if $\eta < \frac{2}{5}$. Therefore, if $\eta < \frac{2}{5}$, we have

$$
\begin{aligned}
& \frac{(bi)^2}{k}\left[-3\left(1-\beta\eta\right)+\left(7-5\beta\eta-2\beta\right)\eta\frac{i}{o}-4\left(1-\beta\right)\left(\eta\frac{i}{o}\right)^2\right] \\
\geq \;& \frac{(bo)^2}{k}\left[-3\left(1-\beta\eta\right)+\left(7-5\beta\eta-2\beta\right)\eta-4\left(1-\beta\right)\eta^2\right] \\
= \;& \frac{(bo)^2}{k}\left[-3+\left(7+\beta\right)\eta-\left(4+\beta\right)\eta^2\right] \\
= \;& -\frac{(bo)^2}{k}\left(1-\eta\right)\left(3-\left(4+\beta\right)\eta\right). \qquad (1.A.53)
\end{aligned}
$$

Overall, we have

$$
\begin{aligned}
& \frac{\partial}{\partial i} LHS\,(1.A.47) \\
\geq \;& \frac{\left(1-\eta\frac{i}{o}\right)\left\{(1-(2+\beta)\,\eta)\,(1-\eta)-\frac{(bo)^2}{k}\,(1-\eta)\,(3-(4+\beta)\,\eta)\right\}}{\left[(1-\beta\eta)-(1-\beta)\,\eta\frac{i}{o}\right]^2} \\
= \;& \frac{\left(1-\eta\frac{i}{o}\right)\left\{(1-\eta)\left[(1-(2+\beta)\,\eta)-\frac{(bo)^2}{k}\,(3-(4+\beta)\,\eta)\right]\right\}}{\left[(1-\beta\eta)-(1-\beta)\,\eta\frac{i}{o}\right]^2} \\
> \;& \frac{\left(1-\eta\frac{i}{o}\right)\left\{(1-\eta)\left[(1-(2+\beta)\,\eta)-\frac{b^2}{k}\,(3-(4+\beta)\,\eta)\right]\right\}}{\left[(1-\beta\eta)-(1-\beta)\,\eta\frac{i}{o}\right]^2} \\
= \;& \frac{\left(1-\eta\frac{i}{o}\right)\left\{(1-\eta)\left[1-3\frac{b^2}{k}-\left(2+\beta-(4+\beta)\,\frac{b^2}{k}\right)\eta\right]\right\}}{\left[(1-\beta\eta)-(1-\beta)\,\eta\frac{i}{o}\right]^2},
\end{aligned}
\tag{1.A.54}
$$

which is positive if $\frac{b^2}{k} < \frac{1}{3}$, and $\eta < \frac{1-3\frac{b^2}{k}}{2+\beta-(4+\beta)\frac{b^2}{k}}$. In order to guarantee that the fixed point exists, i.e., $i^*(o) \in (0, o)$, we have:

$$
\begin{aligned}
LHS(1.A.33)|_{i=0} \quad & < RHS(1.A.33) < \quad LHS(1.A.33)|_{i=o} \\
0 \quad & < \Delta < \quad \frac{2b^2}{\sigma^2} o \left(1-\frac{(bo)^2}{k}\right)(1-\eta)\,.
\end{aligned}
$$

Second, it is easy to show that $\frac{\partial}{\partial o} LHS\,(1.A.47) = \frac{\partial}{\partial o}\left[\frac{i\left(1-\frac{(bi)^2}{k}\right)}{\frac{\beta(1-\eta)}{\left(1-\eta\frac{i}{o}\right)^2}+\frac{(1-\beta)}{\left(1-\eta\frac{i}{o}\right)}}\right] > 0$.

Therefore, as $i^*(o)$ is determined by equation (1.A.47), we can conclude that $\forall o, o', s.t., 0 < o < o' < 1$, if $\frac{b^2}{k} < \frac{1}{3}$, $\eta < \min\left\{\frac{2}{5}, \frac{1-3\frac{b^2}{k}}{2+\beta-(4+\beta)\frac{b^2}{k}}\right\}$, and $\Delta \in \left(0, \frac{2b^2}{\sigma^2} o\left(1-\frac{(bo)^2}{k}\right)(1-\eta)\right)$, we have $i^*(o') < i^*(o)$, which implies higher nominal independence results in lower real independence.

Now we derive the conditions under which our main result is overturned. First, we observe that:

$$
\begin{aligned}
& \frac{\partial}{\partial i} LHS\,(1.A.47)\,|_{i=o} \\
= \quad & \frac{(1-\eta)\left\{(1-(2+\beta)\,\eta)\,(1-\eta) - \frac{(bo)^2}{k}\,(1-\eta)\,(3-(4+\beta)\,\eta)\right\}}{\left[(1-\beta\eta)-(1-\beta)\,\eta\right]^2} \\
= \quad & (1-(2+\beta)\,\eta) - \frac{(bo)^2}{k}\,(3-(4+\beta)\,\eta)\,, \qquad (1.A.55)
\end{aligned}
$$

which is negative if $\frac{b^2}{k} < \frac{1}{3}$, $\eta > \frac{1}{2+\beta}$. Since $\frac{\partial}{\partial i} LHS\,(1.A.47)$ is continuous function of i, it means that $\exists i_B^{\#} \in (0, o)$ s.t. $\frac{\partial}{\partial i} LHS\,(1.A.47)$ is negative in $i \in \left(i_B^{\#}, o\right)$. Therefore, if $\frac{b^2}{k} < \frac{1}{3}$, $\eta > \frac{1}{2+\beta}$ and $\Delta \geq \frac{2b^2}{\sigma^2} o \left(1 - \frac{(bo)^2}{k}\right)(1-\eta)$, the solution to equation (1.A.47) will be either corner solution that $i^*(o) = o$, or there are multiple interior solutions which satisfy equation (1.A.33) , and the stable fixed point of real independence which can be achieved in dynamics is closest to the nominal independence o and satisfies $\frac{\partial LHS(1.A.33)}{\partial i}|_{i=i^*(o)} < 0$. Again, combining the above result with the fact that $\frac{\partial}{\partial o} LHS\,(1.A.47) > 0$, we can conclude that $\forall o, o', s.t., 0 < o < o' < 1$, if $\frac{b^2}{k} < \frac{1}{3}$, $\eta > \frac{1}{2+\beta}$, and $\Delta \geq \frac{2b^2}{\sigma^2} o' \left(1 - \frac{(bo)^2}{k}\right)(1-\eta)$, we have $i^*(o') > i^*(o)$, which implies higher nominal independence results in higher real independence.

$\square$

Chapter 2

Compensation Duration, Shareholder Governance, and Managerial Short-Termism[1]

Abstract

In this paper, I investigate the interaction between the duration of executive compensation and shareholder governance. I show that short-term compensation can elicit shareholder intervention and thus enhance firm value. The central mechanism is that the use of short-term incentives enables informed incumbent shareholders to commit to using their private information to intervene (voice) instead of selling their shares (exit). Without a commitment to voice, incumbent shareholders might find, ex post, that exit is more appealing than voice if they privately observe that a firm's type is bad. Short-term incentives encourage a good

[1]This chapter is based on my job market paper. I am greatly indebted to Jonathan Glover and Katia Sycara for their guidance and encouragement. I am grateful to Carlos Corona, Fabrizio Ferri, and Pierre Liang for valuable advice and comments. I also thank Tim Baldenius, Jing Li, Jack Stecher, Austin Sudbury, Hao Xue, and workshop participants at Carnegie Mellon University, The Chinese University of Hong Kong, The University of Hong Kong, Purdue University, National University of Singapore, Singapore Management University, The University of Texas at Austin, Tilburg University. This paper has also benefited from my discussion with my fellow students Ryan Kim, Nan Li, Ethan Rouen, Lufei Ruan, and Yuan Zou. All errors are my own.

firm to take actions that reveal its type early on. This, in turn, reduces the information advantage of the incumbent shareholders and their ability to profit from exit. Effectively, short-term compensation serves as a commitment device for value-enhancing intervention.

Key Words: Executive Compensation, Corporate Governance, Shareholder Engagement

2.1 Introduction

Managerial short-termism has attracted much attention in academia and the popular press. The concern is that managers might focus on increasing short-term performance at the expense of long-term value. Large shareholders with short horizons are sometimes blamed for fostering managerial short-termism.[2] Large incumbent shareholders such as venture capitalists and private equity investors have been shown to provide managers with short-term incentives through compensation plans (Cadman and Sunder, 2014).[3] To combat short-termism, many commentators have proposed reforms on executive compensation that intend to extend the duration of managerial compensation (e.g., Bebchuk and Fried, 2010; Posner, 2009).

However, the demand for short-term performance can expose problems early on, which facilitates external corrective intervention to get them fixed.[4] External intervention from large shareholders, otherwise known as voice, plays an important role in creating value, for example, by spurring innovation (Brav et al., 2014) and by shaping capital expenditures (Klein and Zur, 2009; Matanova, 2015). Nonetheless, voice is not the only channel through which large shareholders engage in governance. They can also sell a firm's shares if it

[2]On March 31, 2015, Larry Fink, chairman and CEO of BlackRock, sent a letter to S&P 500 CEOs stating, "[m]ore and more corporate leaders have responded with actions that can deliver immediate returns to shareholders ... while underinvesting in innovation, skilled workforces or essential capital expenditures necessary to sustain long-term growth."

[3]According to a survey by PricewaterhouseCoopers (2013), private equity portfolio companies often tie vesting of stock options to short-term metrics such as annual financial targets and exit-based performance.

[4]In a recent article in The Economists titled "The Tyranny of the Long Term: Let's not Get Carried Away in Bashing Short-Termism", the author states, "[s]hort-term demands ... can force problems out in the open, the quicker to get them fixed," and "[long-termism] is a recipe for failure in such businesses as social media, where firms are constantly forced to abandon their plans and 'pivot' to a new strategy."

underperforms, otherwise known as exit (Edmans, 2009; Admati and Pfleiderer, 2009).[5]

In this paper, I show that short-term compensation can be optimal when the interaction between the duration of executive compensation and shareholder governance through voice or exit is taken into account. Short-term compensation acts as a substitute for commitment by large incumbent shareholders to voice instead of exit if they privately learn that a firm's type turns out to be bad. A short-term compensation plan encourages a good firm to take actions that reveal its type early on that in turn reduces incumbent investors' information advantage. As a result, the trading profit from exit decreases, making voice more appealing ex post, which enhances value. Contrary to the view that short-term incentives imposed by incumbent shareholders hurt the long-term value of the firm, I show that they can enhance value by inducing intervention. From this perspective, regulation and other actions that aim to lengthen compensation duration might distort large incumbent shareholders' incentives to intervene and, hence, reduce the firm's value.

To elaborate, my model features a firm initially funded by an incumbent shareholder (e.g., a private equity investor) and managed by a manager. The incumbent shareholder first chooses a managerial compensation scheme and then sells an initial stake to uninformed investors.[6] At a later stage, the incumbent shareholder privately observes the firm's type and chooses between voice and exit if the firm turns out to be a bad firm without a profitable investment opportunity. Central to my argument is the idea that short-term managerial compensation serves as a commitment device for value-enhancing intervention, allowing the incumbent shareholder to sell the initial stake at a higher price. The price of the initial stake set by the uninformed investors incorporates the impact of the incumbent shareholder's subsequent choice between voice and exit. The uninformed investors benefit from the enhanced value triggered by voice but suffer trading losses

[5] See Edmans (2014) for a comprehensive review of both the theoretical and empirical literature.

[6] According to a report of PricewaterhouseCoopers (Steve and Aaron, 2012), a partial IPO exit is a common strategy among private equity firms. In a Fortune article on Jan. 25, 2011, Dan Primack states, "[p]rivate equity firms usually sell few of their shares via IPO, instead slowly bleeding out over subsequent months and years (once lock-up provisions expire)."

caused by exit. Hence, the price of the initial stake is higher if uninformed investors expect that voice is more likely than exit. This conjecture about the incumbent shareholder's choice is based on the publicly observed managerial compensation plan. A compensation plan with a larger weight on short-term incentives encourages a good firm (with many profitable investment opportunities) to differentiate itself from a bad firm by cutting intangible investment to boost short-term earnings at the expense of long-term value. The resultant enhanced earnings informativeness about the firm's type reduces the incumbent shareholder's information advantage, which lowers the trading profit from exit. Consequently, the incumbent shareholder chooses instead to engage in a higher level of value-enhancing intervention. In equilibrium, the correct conjecture about the choice between voice and exit is incorporated into the price of the initial stake. Therefore, the use of short-term incentives allows the incumbent shareholder to commit to intervention and to sell a stake at a higher price ex ante.

Before learning the firm's type, the incumbent shareholder determines the optimal compensation duration, trading the cost of underinvestment by a good firm for the benefit of a higher level of value-enhancing intervention on a bad firm (which results in a higher price for the initial stake). Thus, the model predicts how compensation duration varies across firms. I find compensation duration to be longer in firms with more growth opportunities, greater R&D intensity, and better recent stock performance. These results are consistent with the empirical evidence in Gopalan et al. (2014) and Edmans et al. (2015). Moreover, I find that compensation duration decreases weakly with the value of intervention in a bad firm. Hence, a shorter compensation duration could imply a higher value of the intervention by an incumbent shareholder instead of a more severe agency problem between an incumbent shareholder and other investors. I also find that compensation duration is non-monotonic in the cost of intervention. When the cost of intervention is low, voice dominates exit, and there is no need to use short-term compensation as a substitute for commitment. When the cost of intervention is high, voice is dominated by exit, and short-term compensation is no longer an effective commitment device. A short-term compensation scheme is effective at an intermediate cost of intervention.

In the base model, I assume the incumbent shareholder is the only informed investor in order to highlight the use of short-term incentives as a commitment device. In the extensions, I first introduce a sophisticated institutional investor (such as a mutual fund) who buys part of the initial shares sold by the incumbent shareholder. The institutional investor competes on exit with the incumbent shareholder. This competition reduces the trading profit from exit, making voice more appealing to the incumbent shareholder. Exit by the institutional investor can (in place of short-term compensation) serve as a substitute for the commitment to a value-enhancing intervention by the incumbent shareholder. In other words, compensation duration is lengthened because of the presence of the institutional investor, even if the institutional investor has no say on compensation design. This result is consistent with the empirical evidence in Cadman and Sunder (2014), who show that institutional ownership mitigates the use of short-term incentives by a venture capitalist after an initial public offering. The mechanism illustrated here is different from the existing explanation that institutional investors strengthen corporate governance.

In another extension, I consider two groups of shareholders with different investment horizons and different governance mechanisms. Specifically, I modify the model to explain the use of short-term incentives in the presence of hedge fund activists. Institutional investors are sometimes criticized for merely talking publicly about long-term value but being reluctant to push for long-term growth in response to hedge fund activism (Pozen, 2015).[7] I show that short-term compensation can be optimal when long-term investors (such as index funds), who influence managerial compensation through large voting blocs, rely on intervention by a short-term blockholder or large shareholder (such as a hedge fund) to enhance firm value.

[7]Generally, institutional ownership has been shown to increase the use of equity incentives (e.g., Hartzell and Starks 2003; Fernandes et al. 2013), which might induce myopia if combined with short vesting periods. Gopalan et al. (2014) show that managers typically have significant equity vesting in the short term.

In other words, the long-term investors design a short-term incentive plan to induce value-enhancing intervention by the short-term blockholder.[8]

Overall, this paper contributes to the literature in three ways. First, it provides a new rationale for the use of short-term incentives in executive compensation. A short-term incentive plan serves as a commitment device for value-enhancing intervention. Second, this paper contributes to the debate on the costs and benefits of shareholder governance.[9] Specifically, the result from the base model reconciles the contradictory roles played by private equity firms and venture capitalists (i.e., value-enhancing monitoring versus short-term orientation). The extensions provide new implications for the impact of mutual fund trading and hedge fund activism on managerial short-termism. Third, while numerous studies have investigated executive compensation and shareholder governance through voice and exit independently, little is known about how executive compensation and shareholder governance interact with each other. This paper fills the gap.

The rest of the paper proceeds as follows. Section 2.2 reviews the related literature. Section 2.3 presents the model setup and the sequence of events. Section 2.4 studies two benchmarks. Section 2.5 investigates the optimal compensation duration. Section 2.6 studies extensions with multiple informed investors and Section 2.7 concludes. All proofs are provided in an appendix.

2.2 Related Literature

This paper is not the first paper to rationalize the use of short-term incentives in CEO compensation. The research studies the problem from the perspective of an optimal compensation contract in the presence of moral hazard. Specifically, the optimal compensation contract might depend on the short-term market price, because of the

[8]The result is consistent with the evidence that a higher level of shareholder rights is associated with less pronounced lengthening of incentive horizon (intended to address short-termism) (Dikolli et al., 2013).

[9]As mentioned in a speech by SEC Chair Mary Jo White on March 19, 2015, "an intense debate is taking place in the business, legal and academic communities as to whether activism by hedge funds and others is a positive or negative force for U.S. companies and the economy."

following scenarios: the additional information content generated in the capital market (Bushman and Indjejikian, 1993; Holmström and Tirole, 1993; Dutta and Reichelstein, 2005), the speculative motive of the firm's controlling shareholders (Bolton et al., 2006), market attention inducing managers to truthfully disclose soft information (Almazan et al., 2008), improved decisions on project abandonment from allowing CEOs to time their stock option exercises (Laux, 2010), and more efficient CEO replacement based on early feedback about managerial ability from short-term investment (Laux, 2012). In contrast, in this paper, the short-term incentive is used as a commitment device for value-enhancing intervention in a bad firm by an incumbent shareholder.

The paper is related to the long literature and debate on the costs and benefits of shareholder governance. The early theoretical papers that were motivated by the takeover wave in the 1980s illustrate some unanticipated costs from shareholder intervention, such as discouraging ex-ante information acquisition by managers (Shleifer and Vishny, 1986) and managerial myopia (Stein, 1988). The finance literature (e.g., Maug 1998; Kahn and Winton 1998; Back et al. 2013) focuses on the influences of liquidity and ownership structure on the effectiveness of shareholder governance and reaches different conclusions by considering different underlying forces behind voice and exit.[10] The literature also debates the governance role of venture capital (VC) and private equity (PE) investors. On one hand, the presence of PE investors can have a value-enhancing effect, for example, by providing financial and managerial support (Barry et al., 1990), enhancing earnings quality (Katz, 2009), and improving corporate governance and investment policies (Krishnan et al., 2011; Matanova, 2015). On the other hand, the literature also criticizes PE investors for having a short-term orientation because of the pressure to obtain fast results from the limited partnerships investors (Arthurs et al., 2008). The short-termism of the VC and PE

[10]More recent literature focuses on "low cost" shareholder activism (Ferri, 2012) such as shareholder proposal and voting. Levit and Malenko (2011) find that a strongly biased activist might make the non-binding voting for shareholder proposals more informative. Matsusaka and Ozbas (2014) distinguish the shareholder's right to propose from the right to vote and find proposal right is more effective but might result in managerial accommodation in favor of a biased activist.

investors might lead to short-term managerial incentives (Cadman and Sunder, 2014) and less timely bad news disclosure (Ertimur et al., 2014). This paper reconciles this contradictory evidence by showing that the short-term incentives imposed by blockholders such as VC or PE investors can encourage value-enhancing monitoring and intervention. Regarding the literature on exit, Edmans (2009) shows that the exits of blockholders that are driven by self-interest can improve price and investment efficiencies. Admati and Pfleiderer (2009) show that exit can also be effective in overcoming the moral hazard problem. Building on Edmans (2009), Song (2014) investigates the impact of the activists' reputation concerns on the effectiveness of exit and voice. This paper also builds on Edmans (2009) but focuses instead on the choice of the compensation horizon.

This paper is also related to a small stream of literature that studies the interaction between different governance mechanisms. Cohn and Rajan (2013) find that the internal governance by a board, and the external governance by an activist, can be complementary or substitute for each other depending on the strength of the external governance. In Levit (2014a,b), an active shareholder who has superior information to that of a manager communicates with the manager to change that manager's action. Levit investigates how exit and ex-post intervention impact the effectiveness of communication accordingly. The focus of my paper is different in that it examines the interaction between executive compensation and shareholder governance through voice and exit.

2.3 Base Model

The base model adapts the framework in Edmans (2009), Edmans et al. (2013), and Song (2014) to incorporate endogenous compensation duration. There are four players in the model: a manager (referred to as "he"), an incumbent shareholder (referred to as "she"), uninformed investors, and the market maker. All players are assumed to be risk neutral. Figure 2.1 provides an overview of the event sequence.

At time 0, the firm is initially funded by the incumbent shareholder (e.g., a PE investor) and managed by the manager. The total number of share units is normalized to 1. The incumbent shareholder is endowed with a stake of $1 - \mu$, and the remaining μ shares are equity incentives to the manager. The incumbent shareholder first chooses a compensation scheme w, that determines the structure of the equity incentives (to be specified later). Then, the incumbent shareholder keeps a stake of α and sells a stake of $1 - \alpha - \mu$ to competitive uninformed investors at price P_0, where $\alpha \in (0, 1 - \mu)$ is exogenous for liquidity reasons.

At time 1, the manager privately observes the firm type θ and determines the normalized investment policy $K \in [0, 1]$. The firm has two possible types, $\theta \in \Theta \equiv \{B, G\}$. The prior distribution of θ is common knowledge in that $\theta = G$ with probability $Pr\,(\theta = G) = \varphi$. Type B (G) corresponds to a bad (good) quality firm. A firm of type θ is referred to as a "θ-firm," and its manager is referred to as a "θ-manager."

At time 2, the firm's interim signal $s \in \{s_L, s_H\}$, such as an earnings announcement, is realized and publicly disclosed. The interim signal serves as an imperfect but hard (unmanipulatable) signal for the firm. I refer to $s = s_H$ as a high signal and $s = s_L$ as a low signal (also referred to as "low earnings").

At time 3, the incumbent shareholder chooses between voice and exit based on private information on firm type θ. The incumbent shareholder first decides whether to intervene and then decides how many shares to trade. Meanwhile, the uninformed investors suffer a liquidity shock and provide liquidity demand. The market maker clears the market and sets the market price at P as in Kyle (1985).

At time 4, firm value V becomes publicly known, and the payoffs of all players are realized.

Having completed the timeline, I specify more details of the elements in the model as follows:

Compensation Scheme: The endogenous incentive structure is the key new feature introduced in this model. The incumbent shareholder designs compensation scheme to

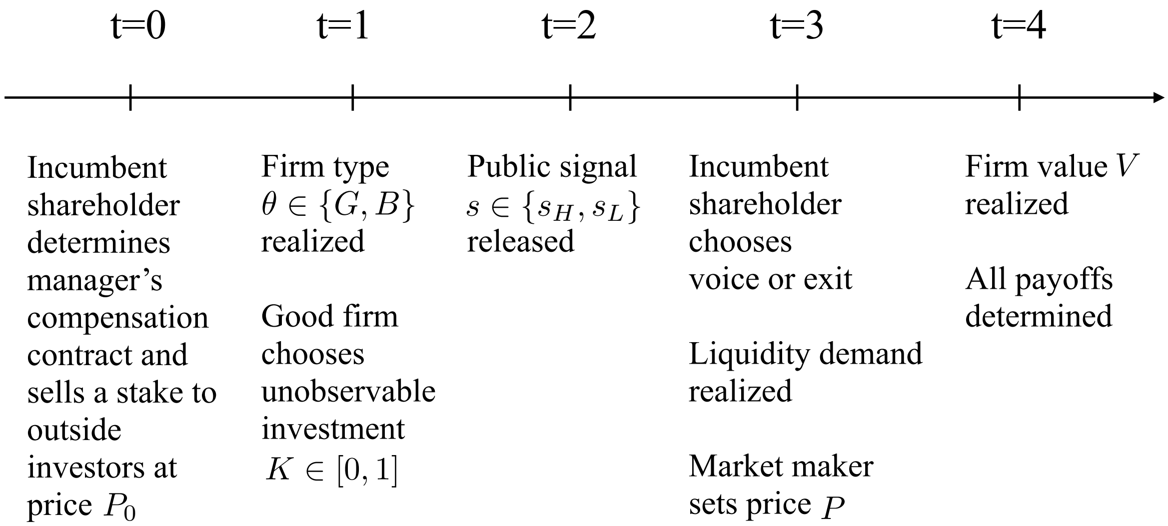

Figure 2.1: Time-line.

balance short-term equity incentives, that is, interim market price P,[11] with long-term incentives, that is, realized cash flow or firm value V. An incentive structure $w \in \mathbf{W} \equiv [\underline{w}, \bar{w}]$, $0 \leq \underline{w} \leq \bar{w} \leq 1$, allows the manager to sell $w\mu$ shares at time 3 and to hold the remaining $(1-w)\mu$ shares until time 4. I assume the manager sells his shares once allowed for liquidity reasons, which can also be justified by the empirical evidence in Edmans et al. (2015) who show that equity vesting significantly predicts equity sales. In this paper, I focus on the optimal compensation duration (Cadman and Sunder, 2014) or pay duration (Gopalan et al., 2014) rather than the optimal contracting problem for the manager.[12] The restriction $[\underline{w}, \bar{w}]$ reflects other forces for managerial short-termism that are not explicitly modeled, such as a takeover threat (Stein, 1988) or a concern for managerial reputation (Narayanan, 1985). The compensation scheme is publicly

[11] In a discussion about short-termism and executive compensation, Donald A. Norman, a director of economic studies at the Manufacturers Alliance for Productivity and Innovation (MAPI), states that "[t]otal shareholder return is not the best measure of performance, especially from a company's long-term perspective. The pay and performance rule risks incentivizing companies to focus on short-term gains (say, by cutting costs and delaying investments) at the expense of long-term investments which can, in the immediate term, reduce total shareholder return. That is, the rule risks contributing to short-termism."

[12] In Gopalan et al. (2014), "pay duration" is a measure of managerial horizon incentives computed as "the weighted average of the vesting periods of the different components of executive pay." Cadman and Sunder (2014) concurrently develop a similar measure that they refer to as "compensation duration."

disclosed.[13]

Investment: The investment might affect the realization of the long-term value V and the interim signal s depending on firm type θ. In terms of long-term value V, the investment unambiguously increases the long-term value for a G-firm. Investment of $K \in [0,1]$ boosts G-firm's value to $V_G = X + gK$, where X represents the fundamental value of the asset in place of a good firm, and the parameter g captures the extent to which the investment increases long-term value. The investment does not add value for a B-firm, and a B-firm is worth $V_B = 0$. In terms of the interim signal s, a G-firm generates signal $s = s_H$ with probability $\frac{1-\gamma K^2}{2}$ and signal $s = s_L$ with probability $\frac{1+\gamma K^2}{2}$, where $0 < \gamma \leq 1$. The primary interpretation is that the investment increases the probability of a good firm delivering low short-term earnings. A B-firm generates signal $s = s_L$ with probability 1. While signal s is public, the firm type θ and the investment K are observable only to the manager and the incumbent shareholder, but are unobservable to the uninformed investors or market maker.

Voice: Intervention in a bad firm is public and value-enhancing so that the value of the bad firm is improved from 0 to X (which is equal to the value of a good firm without intangible investment). The intervention is costly, and the private cost to the incumbent shareholder is $\rho\tau X$ where $\rho > 0$ represents the scale of cost relative to the benefit of intervention X, and τ is a random variable uniformly distributed on $\mathbf{C} \equiv [0,1]$. The distribution of τ is common knowledge but τ is unknown ex ante and learned by the incumbent shareholder at time 3. After learning τ, the incumbent shareholder decides whether to intervene, denoted by $a \in \mathbf{A} \equiv \{0,1\}$, where $a = 0$ represents not intervening, and $a = 1$ represents intervening.

Exit: By the model setup, if $s = s_H$, then the market maker knows the firm is good. If $s = s_L$ and the incumbent shareholder publicly intervenes (i.e., $a = 1$), then the market maker knows the firm is bad, and the firm value improves to X. In either case, the incumbent shareholder cannot gain any further profit by trading, so her trading is

[13] To meet the Item 402 disclosure requirements of Regulation S-K, companies planning on going public are required to provide executive compensation disclosure in their registration statement on Form S-1, and public companies are required to provide executive compensation disclosure in the annual proxy statement or 10-K form.

irrelevant. Hence, I focus on the incumbent shareholder's trading strategy when $s = s_L$ and there is no intervention (i.e., $a = 0$), which I refer to as exit. The incumbent shareholder might sell a fraction of her shares, $\beta \in [0, \alpha]$, at time 3. I assume $\beta \leq \alpha$ to preclude short selling.

Market Price: At time 3, the uninformed investors demand u due to a liquidity shock, where u is a random variable with density function $f(u)$ that is defined as $f(u) = \lambda \exp(-\lambda u)$ if $u > 0$ and $f(u) = 0$ if $u \leq 0$. The market maker observes only the aggregate order $d = u - \beta$ but not the individual orders and sets the market price at $P = E[V|w, a, s, d]$. At time 0, the uninformed investors demand the price of the initial stake P_0 to break even, because they anticipates the liquidity shock at time 3.

The manager's objective function is:

$$U_C = \mu (wP + (1 - w) V). \tag{2.1}$$

The manager's objective depends on the short-term market price P, which might potentially be affected by the incumbent shareholder's intervention and trading strategy.

The incumbent shareholder's objective is to maximize the expected profit. Specifically, the incumbent shareholder's profit is:

$$U_B = (1 - \alpha - \mu) P_0 + \beta P + (\alpha - \beta) V - a\rho\tau X, \tag{2.2}$$

where $(1 - \alpha - \mu) P_0$ results from selling the stake of $1 - \alpha - \mu$ to the uninformed investors at time 0, βP is the trading profit from selling β shares at time 3, $(\alpha - \beta) V$ is the value of the shares she holds until the end of the game, and $\rho\tau X$ is the intervention cost.

Before the analysis of the equilibrium, I discuss some assumptions on the technology and information structure. First, I assume the investment is unobservable to outsiders. This specification reflects the fact that long-term investment, such as employee training or investment in organizational capital, is often intangible. Thus, the intangible investment is difficult to separate from other corporate expenses. In other words, the investors cannot tell whether high corporate expenses are due to the desired investment (a G-firm choosing a high K) or bad firm quality (a B-firm).[14]

Second, the intangible investment increases the probability of the firm delivering low short-term earnings (although the intangible investment increases the long-term value of a G-firm). The primary interpretation is that intangible investment is typically expensed and, thus, lowers earnings, but is difficult to distinguish from a loss due to low firm quality. Alternatively, the investment decision K can be interpreted as a decision of capital allocation (Laux, 2012) or project choice (Gigler et al., 2014) between a short-term project and a long-term project. Compared with a long-term project, a short-term project is more likely to generate high cash flow in the short-term but low cumulative cash flow in the long-term. Hence, a higher investment K on the long-term project instead of the short-term project lowers the cash flow in the short-term.

Third, intervention is public and value-enhancing but costly to the incumbent shareholder. I assume the intervention is public to simplify the analysis, which can also be justified by the Regulation Fair Disclosure rule, which mandates that all publicly traded companies must disclose material information to all investors at the same time. The value-enhancing intervention by the incumbent shareholder is consistent with the empirical evidence that finds a positive association between the firm's performance and the active post-IPO involvement of VC and PE investors (Barry et al., 1990; Katz, 2009; Krishnan et al., 2011; Matanova, 2015). The cost comes from the time and resources the incumbent shareholder devotes to the intervention.

I aim to find a perfect Bayesian equilibrium that is defined formally as followings:

Definition 2.1. The incumbent shareholder's compensation structure strategy $w \in \mathbf{W}$, the

[14]Indeed, even if the investment is listed as a separate item in either the income statement (intangible investment such as R&D and advertising) or the cash flow statement (tangible investment such as capital expenditure), it is difficult to distinguish the quality of the investment. The benefit of the investment is uncertain in the long run and can vary enormously from firm to firm. Cohen et al. (2013) find that "the stock market appears unable to distinguish between 'good' and 'bad' R&D investment."

G-manager's investment strategy $K : \mathbf{W} \to [0, 1]$, the incumbent shareholder's intervention strategy $a : \Theta \times \mathbf{W} \times \{s_L, s_H\} \times \mathbf{C} \to \mathbf{A}$ and trading strategy $\beta : \Theta \times \mathbf{W} \times \{s_L, s_H\} \times \mathbf{C} \to [0, \alpha]$, the market maker's pricing strategy $P : \mathbf{W} \times \mathbb{R} \times \mathbf{A} \times \{s_L, s_H\} \to \mathbb{R}$, the market maker's belief μ about manager's type $\theta = G$ without observing intervention, and the market maker's conjecture about the G-manager's investment level $\hat{K}$ constitute a perfect Bayesian equilibrium if and only if:

1. Given μ and $\hat{K}$, P causes the market maker to break even for any $w \in \mathbf{W}$, $a \in \mathbf{A}$, $s \in \{s_L, s_H\}$, and $d \in \mathbb{R}$;

2. Given $\hat{K}$ and P, a and β jointly maximize the incumbent shareholder's expected payoff for any $\theta \in \mathbf{\Theta}$, $w \in \mathbf{W}$, $s \in \{s_L, s_H\}$, and $\tau \in \mathbf{C}$;

3. Given a, β, and P, K maximizes the G-manager's expected payoff for any $w \in \mathbf{W}$;

4. Given a, β,P, and $\hat{K}$, P_0 causes the uninformed investors to break even for any $w \in \mathbf{W}$;

5. Given a, β,P, $\hat{K}$, and P_0, w maximizes the incumbent shareholder's expected payoff;

6. The beliefs are consistent with the equilibrium strategies.

2.4 Benchmarks

Before analyzing the base model, I study two benchmarks in which the incumbent shareholder can exert governance through either exit or voice only.

2.4.1 Exit Only

First, I look at a benchmark in which the incumbent shareholder cannot exert governance through voice in a bad firm that might be due to either a lack of expertise or the enormous cost of intervention. In such a case, the incumbent shareholder chooses to sell her remaining shares after privately observing that a firm is bad. Nonetheless, the option to exit is neither detrimental nor profitable to the incumbent shareholder ex ante because the ex-post trading profit from exit offsets the ex-ante price premium demanded by the uninformed investors. The trading profits from exit come from the trading losses of the uninformed investors. The uninformed investors anticipate the expected trading losses, which are reflected in the price discount of the initial stake. Because exit is irrelevant ex ante, the incumbent shareholder aims to maximize the expected share value $(1-\mu)V$ when designing the compensation scheme. A compensation plan with more long-term incentives is optimal because it encourages a good manager to invest in long-term growth.

Lemma 2.1. *If there is no value-enhancing intervention by the incumbent shareholder, the incumbent shareholder always chooses exit upon negative private information and chooses a long compensation duration with* $w^{ExitOnly} = \underline{w}$.

Here, the exit option is beneficial because it causes prices to reflect fundamental value rather than current earnings. Exit implicitly lengthens the manager's incentive horizon and encourage a good firm to invest in long-term growth, as suggested in Edmans (2009).

2.4.2 Voice Only

Now, I move to the other benchmark in which the incumbent shareholder cannot exert governance via exit. This scenario might be because of the lock-up period after an initial public offering or a lack of liquidity. In either case, the incumbent shareholder can only exert governance through intervention if she privately observes that a firm is bad. The incumbent shareholder chooses to intervene if and only if the benefit from the increased value of the remaining shares αX is higher than the intervention cost $\rho\tau X$, that is, $\tau < \tau^{VoiceOnly} \equiv \min\left\{1, \frac{\alpha}{\rho}\right\}$. The uninformed investors expect the intervention strategy and demand a price for the initial stake accordingly. To maximize the price of the initial stake and the value of the remaining shares, the incumbent shareholder provides the manager with long-term incentives to invest in long-term growth.

Lemma 2.2. *If there is no exit option by the incumbent shareholder, the incumbent shareholder chooses to intervene if and only if* $\tau < \tau^{VoiceOnly} \equiv \min\left\{1, \frac{\alpha}{\rho}\right\}$ *and chooses a long compensation duration with* $w^{VoiceOnly} = \underline{w}$.

A free-rider problem arises here: the incumbent shareholder bears all the cost of intervention but only enjoys a fraction of the benefit. Because the size of remaining shares (α) is smaller than that of the initial holdings $(1 - \mu)$, it exacerbates the free-rider problem. If the incumbent shareholder is allowed to commit to an intervention strategy ex ante with her initial holdings, there is more intervention. A pre-committed strategy is to intervene if and only if the benefit of her initial holding from intervention $(1 - \mu) X$ is higher than the intervention cost $\rho\tau X$, that is, $\tau < \tau^{Commit} \equiv \min\left\{1, \frac{1-\mu}{\rho}\right\}$. I will illustrate how the presence of exit further exacerbates the free-rider problem.

Voice not only improves the value of a bad firm but also encourages a good firm to invest in long-term growth. The prospect of public intervention in a bad firm makes low short-term earnings (due to long-term investment) less detrimental to the CEO's stock compensation. Specifically, the public intervention reveals a firm that receives intervention is a bad firm. Hence, voice helps to differentiate a good firm with low earnings from a bad firm that always delivers low earnings.

2.5 Analysis

In this section, I analyze the equilibrium of the game in the base model. I solve the game by backward induction. First, I look at the incumbent shareholder's intervention and trading strategy. Second, I move to the investment strategy of the manager in a good firm. Third, I turn to the incumbent shareholder's choice of managerial compensation.

2.5.1 Intervention and Trading

I first look at the incumbent shareholder's intervention and trading strategy given the market maker's conjectures. The market maker's conjecture on the investment by a good firm is denoted by $\hat{K}$. I represent the market maker's conjecture on the incumbent shareholder's intervention and trading strategy by $\hat{\tau}, \hat{\beta}$, which implies the incumbent shareholder intervenes if and only if $\tau < \hat{\tau}$ and sells $\hat{\beta}$ shares if she privately observes $\theta = B$ but does not intervene.

2.5.1.1 Trading Strategy

I focus on exit, that is, the incumbent shareholder's trading strategy when $s = s_L$ and there is no intervention (i.e., $a = 0$). Upon observing signal s_L with no intervention, if the market maker observes a net supply (i.e., $d < 0$), the market maker knows that the incumbent shareholder is selling her shares. It immediately implies that the incumbent shareholder privately observes that the firm is a bad type (i.e., $\theta = B$). The market maker sets the market price accordingly, which means $P = 0$ if $d < 0$. Otherwise, if the market maker observes nonnegative net demand (i.e., $d \geq 0$), the firm can be either a good firm with a noise demand being $u = d$ (with probability $\varphi \frac{\left(1+\gamma \hat{K}^2\right)}{2} \exp\left(-\lambda d\right)$), or a bad firm with the incumbent shareholder selling $\hat{\beta}$ shares with a noise demand of $\mu = d + \hat{\beta}$ (with probability $\left(1-\varphi\right)\left(1-\hat{\tau}\right)\exp\left(-\lambda\left(d+\hat{\beta}\right)\right)$). In this case, the market maker sets the price according to the posterior belief of the firm being a good firm using the Bayes rule. Defining $\Psi = \frac{\varphi}{1-\varphi}$, I have the following lemma.

Lemma 2.3. *Without intervention, after observing signal s_L and net demand d, the market maker decides the price by:*

$$\begin{cases} P = \pi \hat{V}_G & \text{if } d \geq 0 \\ P = 0 & \text{if } d < 0 \end{cases}, \tag{2.3}$$

where $\hat{V}_G$ is the conjectured value of a good firm, and π is the posterior of a firm being a G-firm given signal s_L, nonnegative net demand $d \geq 0$ and no intervention $a = 0$. The $\hat{V}_G$

and π are specified as follows:

$$\hat{V}_G = X + g\hat{K}, \tag{2.4}$$

$$\begin{aligned} \pi &\equiv \Pr\left(\theta = G | s_L, d \geq 0, a = 0\right) \\ &= \frac{\Psi\left(\frac{1+\gamma\hat{K}^2}{2}\right)}{\Psi\left(\frac{1+\gamma\hat{K}^2}{2}\right) + (1-\hat{\tau})\exp\left(-\lambda\hat{\beta}\right)}. \end{aligned} \tag{2.5}$$

According to Lemma 2.3, without intervention, the market price of the firm with low earnings (or signal s_L) and a nonnegative net demand $d \geq 0$ is set at $\pi\hat{V}_G$. That market price $\pi\hat{V}_G$ increases with the market maker's conjecture on the investment by a good firm $\hat{K}$. There are two reasons why a higher investment by a good firm results in a higher market price given a nonnegative net demand $d \geq 0$. First, the higher the investment made by a good firm, the higher the probability that a good firm will deliver low earnings. Consequently, after observing low earnings and a nonnegative net demand $d \geq 0$, the market maker is more likely to believe the firm is a good firm. Hence, the posterior of a firm being a G-firm increases, that is, the posterior $\pi \equiv \Pr\left(\theta = G | s_L, d \geq 0, a = 0\right)$ increases with the investment $\hat{K}$. Second, the higher the conjectured investment level by a good firm, then the higher the conjectured value of a good firm $\hat{V}_G$ is.

Anticipating how the market maker determines the market price, the incumbent shareholder who does not intervene decides her exit strategy. Specifically, after observing signal s_L and privately learning the firm type θ, the incumbent shareholder chooses how many shares to sell so she can maximize her trading profit. If the incumbent shareholder privately learns the firm is a G-firm, she does not sell any shares because a good firm with low earnings is underpriced. Otherwise, if the incumbent shareholder privately learns the firm is a B-firm, she can increase her trading profit of $\pi\hat{V}_G$ per share if the market maker overprices the bad firm. The overpricing occurs if and only if the market maker observes a nonnegative net demand $d \geq 0$. The net trading demand is $d = u - \beta$ when the incumbent shareholder sells β shares. The probability that the market maker observes a nonnegative net demand $d \geq 0$ and overprices a B-firm is $\Pr(d \geq 0) = \Pr(u \geq \beta) = \exp(-\lambda\beta)$. The incumbent shareholder chooses the amount of shares to sell to maximize her expected trading profit:

$$\beta^* = \arg\max_{\beta} \beta \exp(-\lambda\beta) \pi \hat{V}_G. \tag{2.6}$$

Lemma 2.4. *After privately observing a bad firm (i.e., $\theta = B$), if the incumbent shareholder does not intervene, then the incumbent shareholder sells β^* shares, which is determined by:*

$$\beta^* = \min\left\{\alpha, \frac{1}{\lambda}\right\}, \tag{2.7}$$

with trading profit $\beta^* \exp(-\lambda\beta^*) \pi \hat{V}_G$.

The trading profit of the incumbent shareholder increases with the market maker's conjecture on the investment by a good firm $\hat{K}$. The trading profit depends on how much the market maker overprice a bad firm when there is a nonnegative net demand $d \geq 0$. By Lemma 2.3, without observing intervention, the higher the conjectured investment by a good firm $\hat{K}$, then the higher is the market price of a firm with low earnings and a nonnegative net demand $d \geq 0$, $\pi\hat{V}_G$ is. The higher market price immediately implies that the trading profit of the incumbent shareholder is also higher.

2.5.1.2 Intervention Strategy

In time 3, after observing the cost of intervention, $\rho\tau X$, the incumbent shareholder decides whether to publicly intervene or keep silent. On the one hand, if the incumbent shareholder chooses public intervention (i.e., $a = 1$), then the firm value improves to X. The market price of the firm is set to X. The incumbent shareholder cannot gain any further profit by trading. The net benefit of the incumbent shareholder is calculated by deducting the intervention cost $\rho\tau X$ from the benefit from the increased value of the remaining shares αX, which is equal to $(\alpha - \rho\tau) X$. On the other hand, if the incumbent shareholder does not intervene (i.e., $a = 0$), the incumbent shareholder chooses the optimal trading strategy to maximize her trading benefit. The trading profit is $\beta^* \exp(-\lambda\beta^*) \pi \hat{V}_G$ as shown in Lemma 2.4. Therefore, the incumbent shareholder chooses to intervene if and only if the net benefit of intervention is higher than the trading profit. The higher profit from intervention is true if and only if the cost of intervention is not too large, that is, τ is small enough. I denote the intervention cutoff as τ^*, which is the indifference point at which the incumbent shareholder switches to public intervention rather than keeping silent and selling her shares. In other words, the incumbent shareholder chooses to intervene if and only if the intervention cost is lower than $\rho\tau^* X$.

Lemma 2.5. *Given the conjecture on the investment by a good firm $\hat{K}$ and the intervention cutoff $\hat{\tau}$, the incumbent shareholder chooses public intervention, $a = 1$, if and only if the cost is lower than $\rho\tau^* X$, which is equivalent to $\tau < \tau^*$, where τ^* is determined by:*

$$(\alpha - \rho\tau^*) X = \beta^* \exp(-\lambda\beta^*) \pi \hat{V}_G. \tag{2.8}$$

Comparing the result in Lemma 2.5 with that in Lemma 2.2, the intervention cutoff τ^* is lower than that in the benchmark with the voice only, that is, $\tau^* \leq \tau^{VoiceOnly} = \min\left\{1, \frac{\alpha}{\rho}\right\}$. The lower intervention cutoff implies that the presence of an exit option exacerbates the free-rider problem.

The intervention cutoff τ^* decreases with the conjectured investment by a good firm $\hat{K}$. The reason is that the higher the investment by a good firm $\hat{K}$ is, then the higher the trading profit by keeping silent and choosing exit is. A higher trading profit makes keeping silent and exiting more attractive, so the incumbent shareholder is less willing to intervene. Consequently, the incumbent shareholder chooses a lower threshold of intervention cost below which the incumbent shareholder is willing to intervene.

2.5.2 Investment Stage

I now move to the investment strategy of the manager in a good firm. On the one hand, by investing K in intangible assets, the long-term value of the firm increases to $X + gK$. On the other hand, the intangible investment lowers the short-term earnings and, thus, affects the short-term market price. Specifically, if the earnings are high (with probability $\frac{1-\gamma K^2}{2}$), then the market price is $P = \hat{V}_G$. Otherwise, if the earnings are low (with probability $\frac{1+\gamma K^2}{2}$), then the market price is determined by $P = \pi \hat{V}_G$, which is the market price of a firm without intervention, and with low earnings (i.e., $s = s_L$) and a nonnegative net demand $d \geq 0$. There is a nonnegative net demand $d \geq 0$ for a good firm since the incumbent shareholder does not sell any shares of that type of firm. However, the market maker still cannot differentiate between that firm and a bad firm without intervention, and with low earnings and net demand $d \geq 0$. Hence, the market maker sets the market price for a good firm with low earnings at $P = \pi \hat{V}_G$. Therefore, the manager of a good firm decides the optimal investment level to maximize his expected payoff:

$$\max_K w \left(\left(\frac{1 - \gamma K^2}{2} \right) \hat{V}_G + \frac{1 + \gamma K^2}{2} \pi \hat{V}_G \right) + (1 - w)(X + gK). \tag{2.9}$$

The optimal investment level is determined by balancing the trade-off between the benefit of a higher cash flow in the long-term and the cost of a lower market price in the short-term as the investment level increases. Specifically, the marginal benefit of higher investment in the long-term value of a good firm is g, and the marginal effect of higher investment in the expected short-term market price is $\gamma K^* (1 - \pi) \hat{V}_G$. I define $\Omega \equiv \frac{w}{1-w}$, which represents the relative weight of the short-term incentives versus the long-term incentives of the manager.

Lemma 2.6. *Given a weight on the short-term incentives w, the conjectured investment by a good firm $\hat{K}$ and the conjecture on the intervention cutoff $\hat{\tau}$, the optimal investment level* *of the G-manager is determined by:*

$$\Omega\gamma K^* (1-\pi) \hat{V}_G = g. \tag{2.10}$$

The optimal investment level K^* determined by the manager increases with the conjectured intervention cutoff $\hat{\tau}$. A higher cutoff means a bad firm is more likely to receive public intervention. Consequently, without observing intervention, the market maker believes a firm with low earnings and net demand $d \geq 0$ is more likely a good firm, which increases the posterior of a firm being a good firm, that is, $\pi \equiv \Pr(\theta = G|s_L, d \geq 0, a = 0)$ is higher. Hence, the market price of a good firm with low earnings, $\pi\hat{V}_G$, is also higher, which reduces the gap between the market price of a good firm with low earnings ($\pi\hat{V}_G$) and the market price a good firm with high earnings ($\hat{V}_G$). The reduced gap lowers the cost of delivering low earnings by a good firm. As a consequence, the marginal cost of the investment because of a higher probability of delivering low earnings decreases. The marginal benefit of the investment on the long-term value does not change. Therefore, intensive public intervention by the incumbent shareholder in a bad firm increases the investment by a good firm.

In equilibrium, the conjecture on the investment by a good firm and the intervention cutoff of the incumbent shareholder should be consistent with the strategy profiles of the good firm and the incumbent shareholder respectively. In other words, the belief should be correct in equilibrium, that is, $K^* = \hat{K}$ and $\tau^* = \hat{\tau}$. From Lemma 2.5 and Lemma 2.6, I have the following proposition to jointly determine the optimal investment decision of the manager in a good firm and the optimal intervention strategy of the incumbent shareholder, given a public compensation scheme.

Proposition 2.1. *Given a compensation structure $\Omega \equiv \frac{w}{1-w}$, the optimal investment level of G-manager $K^*(\Omega)$ and the unique cutoff $\tau^*(\Omega)$ below which the incumbent shareholder*

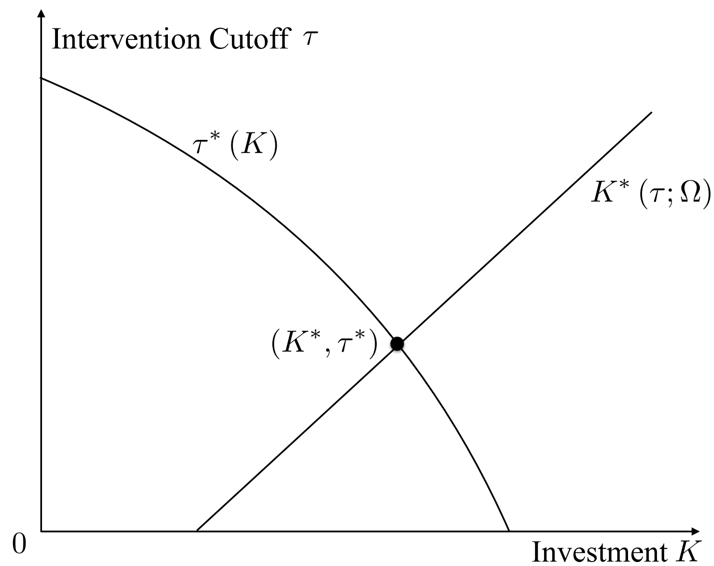

Figure 2.2: Illustration of how the investment and intervention cutoff are jointly determined given the compensation structure

chooses to intervene are jointly determined by:

$$\alpha - \rho\tau = \beta^* \frac{\Psi\left(1+\gamma K^2\right)\left(1+\frac{g}{X}K\right)}{\exp\left(\lambda\beta^*\right)\Psi\left(1+\gamma K^2\right)+2\left(1-\tau\right)} \tag{2.11}$$

$$\gamma K = \frac{1}{\Omega}\frac{g}{X+gK}\frac{\exp\left(\lambda\beta^*\right)\Psi\left(1+\gamma K^2\right)+2\left(1-\tau\right)}{2\left(1-\tau\right)}, \tag{2.12}$$

where $\Psi \equiv \frac{\varphi}{1-\varphi}$, $\beta^* = \min\left\{\frac{1}{\lambda}, \alpha\right\}$.

The unique pair of equilibrium intervention cutoff of the incumbent shareholder and equilibrium investment by a good firm $\left(K^*\left(\Omega\right), \tau^*\left(\Omega\right)\right)$ is determined by the Equations (2.11) and (2.12). By Lemma 2.5, the optimal intervention cutoff τ^* decreases in response to an increase in the investment by a G-firm, K, according to Equation (2.11). The basic reasoning is that the higher investment by a G-firm increases the trading profit of the incumbent shareholder, which makes exit more attractive. Thus, the incumbent shareholder has a weaker incentive to intervene and reduces the intervention cutoff. By Lemma 2.6, the optimal investment level K^* increases in response to an increase in the intervention cutoff of the incumbent shareholder, τ, according to Equation (2.12). The reason is that a higher intervention cutoff implies more intervention by the incumbent shareholder, so a bad firm is more likely to receive an intervention. As a response, the market price is higher for a firm that has low earnings but does not receive an intervention, which decreases the cost of delivering lower earnings that is associated with higher investment by a good firm. Therefore, a unique pair of $(K^*(\Omega), \tau^*(\Omega))$ is determined by the unique crosspoint that satisfies both equations (see Figure 2.2).

Corollary 2.1. *A higher weight on the short-term incentives decreases the investment by a good firm and increases the value-enhancing intervention in bad firms, that is,* $\frac{\partial K^*}{\partial \Omega} < 0$ *and* $\frac{\partial \tau^*}{\partial \Omega} > 0$.

Corollary 2.1 shows that a compensation structure with more short-term incentives decreases the investment by a good firm and increases the value-enhancing intervention a bad firm receives (see Figure 2.3). A higher weight on the short-term incentives makes the manager focus more on the short-term market price of the firm. However, in a good firm, a higher intangible investment increases the probability of delivering low short-term earnings, which results in a lower short-term market price. Therefore, in response to a higher weight on short-term incentives, the manager of a good firm reduces the intangible investment to maximize his expected compensation. A lower investment by a good firm in turn decreases the trading benefit of the incumbent shareholder, which makes exit less appealing. Consequently, the incumbent shareholder is more willing to intervene and, thus, increases the intervention cutoff below which the incumbent shareholder is willing to intervene.

2.5.3 Compensation Structure

I now turn to the choice of the compensation structure, which is the focus of the paper. First, I need to derive how the compensation structure affects the expected payoff of the incumbent shareholder. The incumbent shareholder's payoff depends on both the price of the initial

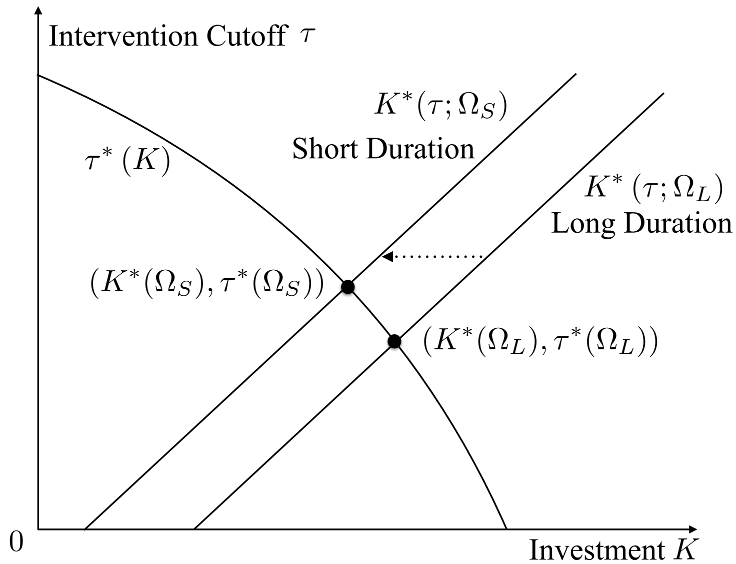

Figure 2.3: Illustration of how the investment and intervention cutoff changes with the weight on short-term managerial incentives

stake and the value of the remaining shares with an exit option. The trading profits from exit of the incumbent shareholder come from the trading losses of the uninformed investors. The uninformed investors anticipate the expected trading losses when buying the initial shares. To offset the trading losses, the uninformed investors demand a price discount when buying the initial shares. Therefore, although the incumbent shareholder earns trading profits from exit ex post, the exit option does not improve her ex-ante expected payoff.

Moreover, the exit option prohibits the incumbent shareholder from committing to value-enhancing intervention in a bad firm ex post, which lowers the price of the initial stake. The uninformed investors demand the price of the initial stake to break even, according to their conjecture on the incumbent shareholder's choice between voice and exit. That conjecture is based on the publicly observed compensation scheme. A longer compensation duration induces higher investment by a good firm, that increases the trading profit from exit. As a result, the incumbent shareholder is more likely to chooses exit over voice. A bad firm is less likely to receive intervention that enhances the firm's value. Hence, a longer compensation duration might not necessarily make the uninformed investors evaluate the initial stake at a higher price. When designing the managerial compensation plan, the incumbent shareholder considers its impact not only on motivating the manager but also on eliciting more voice than exit.

Overall, from the incumbent shareholder's perspective, the compensation structure should be set up to maximize her ex-ante expected payoff:

$$\max_{\Omega} (1-\mu)\left[\varphi\left(X+gK^*(\Omega)\right)+(1-\varphi)\,\tau^*(\Omega)\,X\right]-(1-\varphi)\,\frac{1}{2}\rho\left(\tau^*(\Omega)\right)^2 X \tag{2.13}$$

where the first term $(1-\mu)\left[\varphi\left(X+gK^*(\Omega)\right)+(1-\varphi)\,\tau^*(\Omega)\,X\right]$ is the expected value of the initial stake held by the incumbent shareholder, and the second term $(1-\varphi)\,\frac{1}{2}\rho\left(\tau^*(\Omega)\right)^2 X$ is the expected intervention cost.

As shown in Corollary 2.1, a compensation structure with more short-term incentives decreases the investment by a good firm and increases the intervention a bad firm receives. The incumbent shareholder strategically chooses a managerial compensation plan by balancing its role in encouraging long-term investment and in eliciting more voice than exit from the incumbent shareholder.

Proposition 2.2. *If the value of intervention in a bad firm by an incumbent shareholder is significant enough compared with the profitability of the long-term investment by a good firm and the ex-ante probability of the firm being bad is not low, that is,* $\frac{X}{g}$ *and* φ *are not small, then the incumbent shareholder chooses a short compensation duration with* $w^* > \underline{w}$*. Otherwise, the incumbent shareholder chooses a long compensation duration with* $w^* = \underline{w}$*.*

Proposition 2.2 provides the conditions under which the optimal choice for the incumbent shareholder is a short duration for the managerial compensation. The purpose of using short-term compensation (i.e., placing a higher weight on the short-term incentives, w) is as a substitute of commitment for the incumbent shareholder to undertake more value-enhancing intervention in a bad firm. The incumbent shareholder only desires the use of a short-term compensation plan if and only if the value of intervention in a bad firm is significant enough and the probability that a firm is bad and in need of intervention is high.

Proposition 2.3. *If the incumbent shareholder chooses $w^* > \underline{w}$, the expected firm value* ex-ante *with $w = w^*$ is higher than that with $w = \underline{w}$.*

Corollary 2.3 states that the expected firm value is higher under the optimal short compensation duration with $w = w^*$ chosen by the incumbent shareholder than under a long compensation duration with $w = \underline{w}$. A shorter compensation duration elicits more value-enhancing intervention in a bad firm from the incumbent shareholder, which suggests the cost of intervention is higher under a shorter compensation duration. The cost of intervention under the optimal short compensation duration with $w = w^*$ is higher than that under a long compensation duration with $w = \underline{w}$. Moreover, the optimal short compensation duration with $w = w^*$ maximizes the expected value of the incumbent shareholder's initial holding after deducting the cost of intervention. Therefore, the expected value of the initial holding under the optimal short compensation duration with $w = w^*$ must be higher than that under a long compensation duration with $w = \underline{w}$. The higher value immediately implies that the expected firm value is higher under the short compensation duration with $w = w^*$.

The optimal compensation duration is determined by trading the cost of decreased long-term intangible investment by a good firm for the benefit of increased value-enhancing intervention in a bad firm. Thus, the model predicts how the compensation duration varies across firms. Because Ω is the ratio of the weight on short-term market price to the weight on long-term cash flow, I interpret $\frac{1}{\Omega}$ as a measure of compensation duration. Proposition 2.4 gives the global comparative statics.

Proposition 2.4. *(Comparative statics):*

1. Both investment K^* and compensation duration $\frac{1}{\Omega^*}$ weakly increase with the profitability of investment g.

2. Both investment K^* and compensation duration $\frac{1}{\Omega^*}$ are weakly increase with the probability of the firm's type being good φ.

3. Both investment K^* and compensation duration $\frac{1}{\Omega^*}$ weakly increase with the value of intervention on a bad firm by an incumbent shareholder X.

4. Both investment K^* and compensation duration $\frac{1}{\Omega^*}$ first weakly decrease and then weakly increase with the scale of intervention cost ρ.

5. In equilibrium, the compensation duration $\frac{1}{\Omega^*}$ and the investment K^* are positively associated with each other.

Proposition 2.4 implies that compensation duration is longer in firms with more growth opportunities (a higher profitability of investment g), greater R&D intensity (higher investment K^*), and better recent stock performance (a higher probability of the firm's type being goodφ). These results are consistent with the empirical evidence in Gopalan et al. (2014) and Edmans et al. (2015).

I also find that the compensation duration weakly decreases with the value of intervention in a bad firm by an incumbent shareholder. This finding implies that a shorter compensation duration imposed by the incumbent shareholder might indicate a higher value from the intervention instead of the typical criticism that the incumbent shareholder is more short-term oriented.

In addition, I find the compensation duration is non-monotonic on the cost of intervention. When the cost of intervention is small, intervention dominates exit, and there is no need to use short-term compensation as a substitute for commitment. When the cost of intervention is large, intervention is dominated by exit, and short-term compensation is no longer an effective commitment device. For intermediate intervention cost, a short-term incentive plan is an effective commitment device and, thus, desirable.

2.6 Extensions

In the main model, I assume the incumbent shareholder is the only informed investor. In the extensions, I consider multiple informed investors and investigate the interaction between them. In the first extension, I introduce a sophisticated (informed) institutional investor who buys part of the initial shares offered by the incumbent shareholder. In the second extension, I consider two groups of shareholders with different investment horizons and different governance mechanisms.

2.6.1 The Presence of Sophisticated Institutional Investor

In the base model, I assumed all investors who buy shares from the incumbent shareholder are uninformed. Now I relax the assumption and assume that there is a sophisticated institutional investor (such as a mutual fund) who can also privately observe the firm type. Specifically, I assume the institutional investor buys α_{new} initial shares at $t = 0$ and chooses to sell β_{new} shares at $t = 3$. In this setting, I denote the optimal weight on the short-term incentives determined by the incumbent shareholder as $w^{*,I}$.

Proposition 2.5. *The optimal managerial compensation for the incumbent shareholder comprises (weakly) fewer short-term incentives in the presence of an informed institutional investor, that is, $w^{*,I} \leq w^*$.*

Proposition 2.6 implies that the compensation duration is lengthened in the presence of the informed institutional investor. This result is consistent with the empirical evidence in Cadman and Sunder (2014) who find that institutional ownership mitigates the use of short-term incentives by a venture capitalist after an initial public offering. Indeed, informed trading of the institutional investor reduces the trading profit of the incumbent shareholder when the firm turns out to be a bad firm. This reduction implies that in the presence of the institutional investor, compared with exit, the intervention becomes more attractive for the incumbent shareholder, which mitigate the limited commitment problem of the incumbent shareholder. Hence, the incumbent shareholder does not have to design the compensation scheme to have as much weight on short-term incentives, because there is less need for a commitment to intervene in a bad firm. The institutional investor (in place of short-term compensation) serves as the substitute for commitment to more value enhancing intervention. In other words, the compensation duration is lengthened in the presence of the institutional investor. The lengthened compensation duration is not due to the direct demand of the institutional investor but is an indirect consequence of the speculative trading by the institutional investor.

2.6.2 Multiple Shareholders with Different Governance Mechanisms

Now, I modify the base model to address whether short-term compensation is also desirable when there are multiple shareholders with different governance mechanisms. Specifically, I consider a setting in which passive long-term shareholders (such as index funds) who influence the managerial compensation through large voting blocs rely on the intervention by an active investor (such as a hedge fund) to enhance value. The passive shareholders exert influences on the design of the managerial compensation through "say on pay" or shareholder proposals. The value-enhancing intervention by an active investor is consistent with the empirical evidence that finds a positive association between firm performance and the entrepreneurial shareholder activism (Brav et al., 2008, 2014; Klein and Zur, 2009).

In the modified model, the key difference is that the passive shareholders set the managerial compensation scheme instead of the informed incumbent shareholder, and there is no initial trading stage at time 0. The active investor who holds α shares privately observes the firm's type and plays a similar role to the informed incumbent shareholder in choosing exit versus voice at time 3. A group of liquidity traders provides the liquidity supply μ, which has the same distribution as the liquidity supply of the uninformed investors in the base model. The analysis of the trading stage, the intervention strategy, and the investment stage is the same as the base model. Now, I study the optimal compensation design strategy of the passive investors. I denote the optimal weight on the short-term incentives determined by the passive investors as $w^{*,II}$. The objective of the passive investors is to maximize the expected value of the firm. In this case, the optimal compensation horizon Ω is determined by:

$$\max_{\Omega} \left[\varphi \left(X + gK\right) + \left(1 - \varphi\right) \tau X\right] \tag{2.14}$$

$$\text{s.t. } \alpha - \rho\tau = \beta \frac{\Psi \left(1 + \gamma K^2\right) \left(1 + \frac{g}{X} K\right)}{\exp\left(\lambda\beta\right) \Psi \left(1 + \gamma K^2\right) + 2\left(1 - \tau\right)} \tag{2.15}$$

$$\gamma K = \frac{1}{\Omega} \frac{\frac{g}{X}}{1 + \frac{g}{X} K} \frac{\exp\left(\lambda\beta\right) \Psi \left(1 + \gamma K^2\right) + 2\left(1 - \tau\right)}{2\left(1 - \tau\right)} \tag{2.16}$$

Proposition 2.6. *The optimal managerial compensation scheme for the institutional investors comprises (weakly) more short-term incentives, that is, $w^{*,II} \geq \underline{w}$, where the inequality is strict if $\frac{X}{g}$ and φ is large enough, that is, the fundamental value is important enough compared with the return on the intangible investment and the ex-ante probability that the firm is bad is high enough.*

Proposition 2.6 provides the conditions under which the passive institutional investors choose a short duration of managerial compensation in equilibrium. The resultant short compensation duration appears to exacerbate managerial myopia, which lowers the long-term investment by a good firm. A lower investment lowers the probability of a good firm delivering low earnings. The lower investment by a good firm improves the informativeness of low earnings on firm type and, thus, reduces the active investor's information advantage. Trading profits from exit decrease for the active investor, making exit less attractive than voice. Consequently, the active shareholder is more likely to intervene in the operations of a bad firm. The passive institutional shareholders trade off the cost of lower investment by a good firm with the benefit of more value-enhancing intervention in a bad firm by the active shareholder. The benefit from more intervention dominates the cost of lower investment if and only if the firm is likely to be bad (i.e., φ is large enough), and the value of intervention (i.e., X) is high enough. When those conditions hold, it is optimal to choose a short compensation duration.

The result of Proposition 2.6 explains the use of short-term incentives in the presence of hedge fund activism. Short-term compensation can be optimal when long-term institutional investors who influence managerial compensation through large voting blocs rely on intervention by a short-term blockholder such as a hedge fund to enhance firm value.

2.6.3 Alternative Managerial Decisions

In this extension, I provide more details about how the model framework can fit the alternative managerial decisions rather than the intangible investment decision.

First, as mentioned in the base model setup, instead of investment in the intangibles, the decision K can be interpreted as capital allocation (Laux, 2012) or project choice (Gigler et al., 2014) between a short-term project and a long-term project. A long-term project is more likely to generate high cumulative cash flow in the long term, while a short-term project is more likely to generate high cash flow in the short term. Hence, higher investment K in the long-term project instead of the short-term project lowers the short-term cash flow but increases the cumulative cash flow in the long run. In this case, the assumptions on the technology and information structure can be justified as follows. If the firm is a bad firm, both the short-term cash flow and cumulative cash flow in the long run is 0. Otherwise, if the firm is a good firm and invests K on the long-term project and $(1-K)$ on the short-term project, the short-term cash flow s is $s = s_H$ with probability $\frac{1-\gamma K^2}{2}$ and $s = s_L = 0$ with probability $\frac{1+\gamma K^2}{2}$, where $0 < \gamma \leq 1$, and the expected cumulative cash flow in the long run is $V_G = X + gK$. While the short-term cash flow is publicly reported, the firm type θ and the investment allocation between short-term and long-term projects, K, are observable only to the manager and the incumbent shareholder, but are unobservable to the uninformed investors or market maker.

The managerial decision can also be considered as voluntary disclosure. Specifically, a good firm can disclose good news early on to boost the short-term market price. Nonetheless, the disclosure of good news results in a proprietary cost because the competitors can learn from the disclosure. The long-term value is lower if the good firm discloses good news in the short run. I model the voluntary disclosure as imperfect communication between the firm and the market and denote the vagueness of the voluntary disclosure by K. The market always perceives a bad signal from a bad firm, and the value of a bad firm is 0. For a good firm, the market perceives a good signal with probability $\frac{1-\gamma K^2}{2}$ and signal $s = s_L$ with probability $\frac{1+\gamma K^2}{2}$, where $0 < \gamma \leq 1$, which implies more vague disclosure results in a lower probability that the market perceives a good signal. On the other hand, the G-firm's value in the long run is $V_G = X + gK$, which can be interpreted as the benefit of lowered competition due to more vague disclosure.

2.7 Conclusion

In this paper, I investigate the interaction between compensation duration and shareholder governance. The paper proposes that a short managerial compensation duration can serve as a substitute of commitment for an incumbent shareholder to undertake value-enhancing intervention instead of exit. The short-term managerial compensation induces a good firm to reveal its type early on. This in turn reduces information asymmetry between the informed incumbent shareholder and the uninformed outside investors. Thus, the trading profit from exit is lower for the incumbent shareholder. Consequently, voice becomes more appealing to the incumbent shareholder, and a bad firm is more likely to receive value-enhancing intervention. Contrary to the view that short-term incentives imposed by incumbent shareholders reduce firm value, I show that they can improve firm value by inducing intervention. From this perspective, regulation and other actions aiming to extend compensation duration might distort incumbent shareholders' incentives to pursue value-enhancing activities.

This paper has abstracted from some interesting issues. In the initial stake's selling stage, I assume complete information and an exogenous ownership structure so I can focus on the compensation duration. An investigation on how the incumbent shareholder jointly determines share retention and the executive compensation structure, in the presence of information asymmetry before selling the initial stake might be an interesting extension of this paper. Both share retention and the compensation scheme can be used as tools to signal the firm's quality and to commit to future value-enhancing involvement. The extension could provide more implications for the PE-backed initial public offering. Another possible extension is to consider an alternative means of voice. While this paper focuses on direct intervention, investigating soft communication as studied in Levit (2014b) also might be interesting.

Future research could explore a related issue of the interaction between the managerial incentives and the board's incentives, instead of the interaction between managerial incentives and the shareholder governance studied in this paper. The managerial decisions that need the approval of the board of directors are often long-term decisions such as new business strategies or major asset purchases. Can the board use short-term managerial incentives to get early feedback about the long-term decision made by the board?

APPENDIX

2.A Proof

Proof of Lemma 2.1: The proof is omitted.

Proof of Lemma 2.2: The incumbent shareholder chooses public intervention if and only if the net profit from voice is positive:

$$(\alpha - \rho\tau) X > 0 \Leftrightarrow \tau < \tau^{VoiceOnly} \equiv \min\left\{1, \frac{\alpha}{\rho}\right\}. \tag{2.A.1}$$

Proof of Lemma 2.3: If the incumbent shareholder does not intervene and chooses to sell β shares, the net trading demand is

$$d = u - \beta, u \sim Exp(\lambda). \tag{2.A.2}$$

The probability that there is net demand, i.e., $d > 0$ is:

$$\Pr(d > 0) = \Pr(u > \beta) = \exp(-\lambda\beta). \tag{2.A.3}$$

Given the market maker's conjecture on the investment by the good firm, $\hat{K}$, the intervention cutoff for the incumbent shareholder, $\hat{\tau}$, and the trading amount of the incumbent shareholder $\hat{\beta}$ by Bayesian rule, the posterior belief that a firms good given no intervention, low earnings and net demand $d \geq 0$, which is denoted by π, is determined by:

$$\pi = \frac{\varphi^{\frac{\left(1+\gamma\hat{K}^2\right)}{2}}}{\varphi^{\frac{\left(1+\gamma\hat{K}^2\right)}{2}} + (1-\varphi)(1-\hat{\tau})\exp\left(-\lambda\hat{\beta}\right)}, \tag{2.A.4}$$

and the value of a good firm is given by $\hat{V}_G = X + g\hat{K}$. Therefore, the market maker sets the market price according to:

$$\begin{cases} P = \pi \hat{V}_G & \text{if } d \geq 0 \\ P = 0 & \text{if } d < 0 \end{cases}. \tag{2.A.5}$$

Proof of Lemma 2.4: The incumbent shareholder's trading strategy is determined by:

$$\beta^* = \arg\max_{\beta \in [0,\alpha]} \beta \exp(-\lambda\beta)\, \pi \hat{V}_G. \tag{2.A.6}$$

From the first order condition, I obtain

$$(1 - \lambda\beta) \exp(-\lambda\beta)\, \pi \hat{V}_G = 0, \tag{2.A.7}$$

which has a unique solution $\beta = \frac{1}{\lambda}$. However, because the no short selling constraint requires $\beta \in [0, \alpha]$, $\frac{1}{\lambda}$ might not be achieved if $\frac{1}{\lambda} > \alpha$. In that case, the optimal choice of β is $\beta = \alpha$, because the marginal profit of selling an additional share (which is the LHS in the above equation) is positive for any $\beta \in \left(0, \frac{1}{\lambda}\right)$. Overall, the optimal number of shares is given by:

$$\beta^* = \min\left\{\alpha, \frac{1}{\lambda}\right\}. \tag{2.A.8}$$

Proof of Lemma 2.5: The net profit of the incumbent shareholder from intervention is $(\alpha - \rho\tau) X$. The trading profit from exit is $\beta^* \exp(-\lambda\beta^*)\, \pi \hat{V}_G$. Therefore, the incumbent shareholder chooses public intervention if and only if the net profit from voice is higher than the trading profit from exit. The net profit from voice is higher when the intervention cost is lower than $\rho\tau^* X$, in which τ^* is determined by:

$$\begin{aligned} (\alpha - \rho\tau^*) X &= \beta^* \exp(-\lambda\beta^*) \frac{\varphi\left(1 + \gamma\hat{K}^2\right)\left(X + g\hat{K}\right)}{\varphi\left(1 + \gamma\hat{K}^2\right) + 2(1-\varphi)(1-\hat{\tau})\exp\left(-\lambda\hat{\beta}\right)} \\ &= \beta^* \exp(-\lambda\beta^*)\, \pi \hat{V}_G. \end{aligned} \tag{2.A.9}$$

Proof of Lemma 2.6: The CEO of a good firm decides the optimal investment level that maximizes his expected payoff:

$$K^{*}=\arg\max_{K} w\left(\frac{1-\gamma K^{2}}{2}\hat{V}_{G}+\frac{1+\gamma K^{2}}{2}\pi\hat{V}_{G}\right)+(1-w)\left(X+gK\right). \tag{2.A.10}$$

From the first order condition, I get:

$$\begin{aligned} w\left(-\gamma K\hat{V}_{G}+\gamma K\pi\hat{V}_{G}\right)+(1-w)\,g &= 0 \\ w\gamma K\left(1-\pi\right)\hat{V}_{G} &= (1-w)\,g. \end{aligned} \tag{2.A.11}$$

Let $\Omega=\frac{w}{1-w}$, so the equation above can be rewritten as:

$$\Omega\gamma K^{*}\left(1-\pi\right)\hat{V}_{G}=g. \tag{2.A.12}$$

Proof of Proposition 2.1: In equilibrium, $\hat{\tau}=\tau^{*},\hat{\beta}=\beta^{*},\hat{K}=K^{*}$, plugging $\hat{V}_{G}=X+g\hat{K}$ and $\pi=\frac{\Psi\left(\frac{1+\gamma\hat{K}^{2}}{2}\right)}{\Psi\left(\frac{1+\gamma\hat{K}^{2}}{2}\right)+(1-\hat{\tau})\exp\left(-\lambda\hat{\beta}\right)}$ into the equations in Lemma 2.5 and 2.6, the equilibrium investment by a good firm $K^{*}(\Omega)$ and the equilibrium intervention cutoff of the incumbent shareholder $\tau^{*}(\Omega)$ given a compensation scheme Ω are jointly determined by:

$$\alpha-\rho\tau = \beta\frac{\Psi\left(1+\gamma K^{2}\right)\left(1+\frac{g}{X}K\right)}{\exp\left(\lambda\beta\right)\Psi\left(1+\gamma K^{2}\right)+2\left(1-\tau\right)}, \tag{2.A.13}$$

$$\gamma K = \frac{1}{\Omega}\frac{\frac{g}{X}}{1+\frac{g}{X}K}\frac{\exp\left(\lambda\beta\right)\Psi\left(1+\gamma K^{2}\right)+2\left(1-\tau\right)}{2\left(1-\tau\right)}. \tag{2.A.14}$$

Equation (2.A.14) can be rewritten as:

$$\begin{aligned} \Omega\left(K+\frac{X}{g}\right)\gamma K-\frac{\exp\left(\lambda\beta\right)\Psi}{2\left(1-\tau\right)}\left(1+\gamma K^{2}\right)-1 &= 0 \\ \left(\Omega-\frac{\exp\left(\lambda\beta\right)\Psi}{2\left(1-\tau\right)}\right)\gamma K^{2}+\Omega\frac{X}{g}\gamma K-1-\frac{\exp\left(\lambda\beta\right)\Psi}{2\left(1-\tau\right)} &= 0. \end{aligned} \tag{2.A.15}$$

Therefore, I obtain the investment by a good firm as a function of Ω and τ:

$$
\begin{aligned}
K^{*}(\tau;\Omega) &= \frac{-\Omega\frac{X}{g}\gamma+\sqrt{\left(\Omega\frac{X}{g}\gamma\right)^{2}+4\left(\Omega-\frac{\exp(\lambda\beta)\Psi}{2(1-\tau)}\right)\gamma\left(1+\frac{\exp(\lambda\beta)\Psi}{2(1-\tau)}\right)}}{2\gamma\left(\Omega-\frac{\exp(\lambda\beta)\Psi}{2(1-\tau)}\right)} \\
&= \frac{2\left(1+\frac{\exp(\lambda\beta)\Psi}{2(1-\tau)}\right)}{\Omega\frac{X}{g}\gamma+\sqrt{\left(\Omega\frac{X}{g}\gamma\right)^{2}+4\left(\Omega-\frac{\exp(\lambda\beta)\Psi}{2(1-\tau)}\right)\gamma\left(1+\frac{\exp(\lambda\beta)\Psi}{2(1-\tau)}\right)}}. \qquad (2.A.16)
\end{aligned}
$$

Consequently, $\frac{\partial K^{*}}{\partial\Omega}<0$, which implies that a heavier weight on the short-term market price reduces the intangible investment.

Let $\Gamma=\frac{\exp(\lambda\beta^{*})\Psi}{2(1-\tau)}$, and $K^{*}(\tau;\Omega)$ in Equation (2.A.16) can be written as a function of Γ:

$$
K=\frac{2(1+\Gamma)}{\Omega\frac{X}{g}\gamma+\sqrt{\left(\Omega\frac{X}{g}\gamma\right)^{2}+4(\Omega-\Gamma)\gamma(1+\Gamma)}}, \qquad (2.A.17)
$$

and I also obtain:

$$
\begin{aligned}
\frac{\partial K}{\partial\Gamma} &= \frac{2\left(\Omega\frac{X}{g}\gamma+\sqrt{\left(\Omega\frac{X}{g}\gamma\right)^{2}+4(\Omega-\Gamma)\gamma(1+\Gamma)}\right)-\frac{(1+\Gamma)4\gamma(\Omega-1-2\Gamma)}{\sqrt{\left(\Omega\frac{X}{g}\gamma\right)^{2}+4(\Omega-\Gamma)\gamma(1+\Gamma)}}}{\left(\Omega\frac{X}{g}\gamma+\sqrt{\left(\Omega\frac{X}{g}\gamma\right)^{2}+4(\Omega-\Gamma)\gamma(1+\Gamma)}\right)^{2}} \\
&= \frac{2\Omega\frac{X}{g}\gamma+\frac{\left(\Omega\frac{X}{g}\gamma\right)^{2}+4(\Omega-\Gamma)\gamma(1+\Gamma)-(1+\Gamma)4\gamma(\Omega-1-2\Gamma)}{\sqrt{\left(\Omega\frac{X}{g}\right)^{2}+4(\Omega-\Gamma)(1+\Gamma)}}}{\left(\Omega\frac{X}{g}\gamma+\sqrt{\left(\Omega\frac{X}{g}\gamma\right)^{2}+4(\Omega-\Gamma)\gamma(1+\Gamma)}\right)^{2}} \\
&= \frac{2\Omega\frac{X}{g}\gamma+\frac{\left(\Omega\frac{X}{g}\gamma\right)^{2}+4\gamma(1+\Gamma)^{2}}{\sqrt{\left(\Omega\frac{X}{g}\gamma\right)^{2}+4(\Omega-\Gamma)\gamma(1+\Gamma)}}}{\left(\Omega\frac{X}{g}\gamma+\sqrt{\left(\Omega\frac{X}{g}\gamma\right)^{2}+4(\Omega-\Gamma)\gamma(1+\Gamma)}\right)^{2}}. \qquad (2.A.18)
\end{aligned}
$$

Thus, $\frac{\partial\Gamma}{\partial\tau}>0$, so I have $\frac{\partial K^{*}(\tau;\Omega)}{\partial\tau}=\frac{\partial K}{\partial\Gamma}\frac{\partial\Gamma}{\partial\tau}>0$, which implies that more intervention in bad firms increases the intangible investment of good firms.

Equation (2.A.13) can be rewritten as:

$$\left(1+\frac{g}{X}K\right) \quad = \quad \frac{\alpha-\rho\tau}{\beta^*}\left(\exp\left(\lambda\beta^*\right)+\frac{2\left(1-\tau\right)}{\Psi\left(1+\gamma K^2\right)}\right). \tag{2.A.19}$$

I denote the cutoff determined by the above equation with $\tau^*(K)$. Because the LHS increases with K, and the RHS decreases with both K and τ, I have $\frac{\partial\tau^*(K)}{\partial K} < 0$, which implies that more intangible investment by the good firms rules out the value-enhancing intervention in bad firms. More precisely, I define $T(\tau, K)$ as below:

$$T(\tau,K) = 1+\frac{g}{X}K-\frac{\alpha-\rho\tau}{\beta^*}\left(\exp\left(\lambda\beta^*\right)+\frac{2\left(1-\tau\right)}{\Psi\left(1+\gamma K^2\right)}\right). \tag{2.A.20}$$

Equation (2.A.13) is equivalent to $T(\tau,K) = 0$. As long as the interior solution exist, the conditions that $\alpha - \rho\tau > 0$ and $\tau \leq 1$ should be valid. Therefore, I can obtain

$$\frac{\partial T\left(\tau,K\right)}{\partial K} = \frac{g}{X}+\frac{\left(\alpha-\rho\tau\right)\left(1-\tau\right)}{\beta^*}\frac{4K}{\Psi\left(1+\gamma K^2\right)^2} > 0, \tag{2.A.21}$$

$$\frac{\partial T\left(\tau,K\right)}{\partial \tau} = \frac{\rho}{\beta^*}\exp\left(\lambda\beta^*\right)+\frac{\alpha-\rho\tau+\rho\left(1-\tau\right)}{\beta^*}\frac{2}{\Psi\left(1+\gamma K^2\right)} > 0. \tag{2.A.22}$$

Thus, I have

$$\frac{d\tau^*\left(K\right)}{dK} = -\frac{\frac{\partial T(\tau,K)}{\partial K}}{\frac{\partial T(\tau,K)}{\partial \tau}} < 0, \tag{2.A.23}$$

which implies a higher investment by a good firm decreases the intervention cutoff of the incumbent shareholder.

Now, I prove that there is a unique pair of (K^*, τ^*) that satisfies the above equation system given Ω. I have proved that the function of $\tau^*(K)$ that is determined by the Equation (2.A.13) decreases with K. I also have proved that the function of $K^*(\tau;\Omega)$ that is determined by the Equation (2.A.14) increases with τ and decreases with Ω. Therefore, there is a unique pair of $(K^*(\Omega), \tau^*(\Omega))$ that satisfies the above equation system given Ω.

Proof of Corollary 2.1: In the proof of Proposition 2.1, I show that the function of $\tau^*(K)$ that is determined by the Equation (2.A.13) decreases with K. I also have proved that the function of $K^*(\tau;\Omega)$ that is determined by the Equation (2.A.14) increases with τ and decreases with Ω. Therefore, if Ω increases, then $K^*(\Omega)$ decreases and $\tau^*(\Omega)$ increases. This corollary can also be proved by showing a contradiction. Assuming $K^*(\Omega)$ increases, because $\tau^*(K)$ that is determined by the Equation (2.A.13) decreases with K, the resultant equilibrium $\tau^*(\Omega)$ should decrease. Then, because Ω increases and $K^*(\tau;\Omega)$ increases with τ and decreases with Ω, these conditions imply that $K^*(\Omega)$ decreases and does not increase.

Proof of Proposition 2.2: The incumbent shareholder's problem of determining the optimal w or, equivalently, the optimal Ω is given by:

$$\max_{\Omega}(1-\mu)\left[\varphi\left(X+gK^*(\Omega)\right)+(1-\varphi)\,\tau^*(\Omega)\,X\right]-(1-\varphi)\,\frac{1}{2}\rho\left(\tau^*(\Omega)\right)^2 X,$$

which is equivalent to:

$$\max_{\Omega}\varPsi\left(1+\frac{g}{X}K\right)+\tau-\frac{\rho}{2(1-\mu)}\tau^2 \tag{2.A.24}$$

$$\begin{aligned}
\text{s.t. } \alpha-\rho\tau &= \beta\frac{\varPsi\left(1+\gamma K^2\right)\left(1+\frac{g}{X}K\right)}{\exp(\lambda\beta)\,\varPsi\left(1+\gamma K^2\right)+2(1-\tau)} && \text{(2.A.25)}\\
\gamma K &= \frac{1}{\Omega}\frac{\frac{g}{X}}{1+\frac{g}{X}K}\frac{\exp(\lambda\beta)\,\varPsi\left(1+\gamma K^2\right)+2(1-\tau)}{2(1-\tau)}. && \text{(2.A.26)}
\end{aligned}$$

The unique pair (K,τ) is determined by Ω. Observing that Ω only appears in Equation (2.A.26), it is equivalent to find a (K,τ) that satisfies Equation (2.A.25) and maximizes the objective function (2.A.24). Because $\tau^*(K)$ is a function that is defined by Equation

(2.A.25), the equivalent optimization problem is as follows:

$$\max_K \Psi\left(1+\frac{g}{X}K\right)+\tau^*(K)-\frac{\rho}{2(1-\mu)}\left(\tau^*(K)\right)^2. \tag{2.A.27}$$

Based on the objective function 2.A.24, the marginal benefit of increasing the investment of a good firm K is $\Psi\frac{g}{X}$, and the marginal cost of increasing the investment of a good firm K is $-\left(1-\frac{\rho}{1-\mu}\tau^*(K)\right)\frac{d\tau^*(K)}{dK}$ where $\frac{d\tau^*(K)}{dK}<0$ from Lemma 2.5. The marginal effect of K on the marginal cost is:

$$\begin{aligned}&\frac{\partial\left[-\left(1-\frac{\rho}{1-\mu}\tau^*(K)\right)\frac{d\tau^*(K)}{dK}\right]}{\partial K}\\ =&\frac{\rho}{1-\mu}\left(\frac{d\tau^*(K)}{dK}\right)^2-\left(1-\frac{\rho}{1-\mu}\tau^*(K)\right)\frac{d^2\tau^*(K)}{dK^2},\end{aligned} \tag{2.A.28}$$

which is positive if I can show $\frac{d^2\tau^*(K)}{dK^2}<0$ (Note that $1-\frac{\rho}{1-\mu}\tau^*(K)>0$ because $\alpha-\rho\tau^*(K)>0$ and $1-\mu>\alpha$). Equation (2.A.25) is equivalent to:

$$\frac{\Psi(1+\gamma K^2)}{2}\left[\left(1+\frac{g}{X}K\right)-\frac{\alpha-\rho\tau}{\beta^*}\exp(\lambda\beta^*)\right]-\frac{\alpha-\rho\tau}{\beta^*}(1-\tau)\;=\;0. \tag{2.A.29}$$

Based on the Implicit Function Theorem, I get:

$$\begin{aligned}&\frac{\Psi(1+\gamma K^2)}{2}\left(\frac{g}{X}+\frac{\rho\tau'}{\beta^*}\exp(\lambda\beta^*)\right)\\ &+\Psi\gamma K\left(1+\frac{g}{X}K-\frac{\alpha-\rho\tau}{\beta^*}\exp(\lambda\beta^*)\right)+\frac{\rho\tau'}{\beta^*}(1-\tau)+\frac{\alpha-\rho\tau}{\beta^*}\tau' \;=\; 0,\end{aligned} \tag{2.A.30}$$

and

$$\begin{aligned}&\Psi\gamma\left(1+\frac{g}{X}K-\frac{\alpha-\rho\tau}{\beta^*}\exp(\lambda\beta^*)\right)+2\Psi\gamma K\left(\frac{g}{X}+\frac{\rho\tau'}{\beta^*}\exp(\lambda\beta^*)\right)\\ &+\frac{\Psi(1+\gamma K^2)}{2}\frac{\rho\tau''}{\beta^*}\exp(\lambda\beta^*)+\frac{\rho\tau''}{\beta^*}(1-\tau)-2\frac{\rho\tau'}{\beta^*}\tau'+\frac{\alpha-\rho\tau}{\beta^*}\tau'' \;=\; 0.\end{aligned} \tag{2.A.31}$$

Hence, I obtain:

$$\tau' = -\frac{\frac{\Psi\left(1+\gamma K^2\right)}{2}\frac{g}{X} + \Psi\gamma K\left(1+\frac{g}{X}K - \frac{\alpha-\rho\tau}{\beta^*}\exp\left(\lambda\beta^*\right)\right)}{\frac{\rho}{\beta^*}\left(\frac{\Psi(1+\gamma K^2)}{2}\exp\left(\lambda\beta^*\right)+(1-\tau)\right)+\frac{\alpha-\rho\tau}{\beta^*}} \tag{2.A.32}$$

$$\tau'' = -\frac{\Psi\gamma\left(1+\frac{g}{X}K-\frac{\alpha-\rho\tau}{\beta^*}\exp\left(\lambda\beta^*\right)\right)+2\Psi\gamma K\left(\frac{g}{X}+\frac{\rho\tau'}{\beta^*}\exp\left(\lambda\beta^*\right)\right)-2\frac{\rho\tau'}{\beta^*}\tau'}{\frac{\Psi(1+\gamma K^2)}{2}\frac{\rho}{\beta^*}\exp\left(\lambda\beta^*\right)+\frac{\rho}{\beta^*}(1-\tau)+\frac{\alpha-\rho\tau}{\beta^*}} \tag{2.A.33}$$

In order to prove $\tau'' < 0$, it is equivalent to prove:

$$\Psi\gamma\left(1+\frac{g}{X}K-\frac{\alpha-\rho\tau}{\beta^*}\exp\left(\lambda\beta^*\right)\right)+2\Psi\gamma K\left(\frac{g}{X}+\frac{\rho\tau'}{\beta^*}\exp\left(\lambda\beta^*\right)\right)-2\frac{\rho\tau'}{\beta^*}\tau' > 0 \tag{2.A.34}$$

$$\gamma\left(1+\frac{g}{X}K-\frac{\alpha-\rho\tau}{\beta^*}\exp\left(\lambda\beta^*\right)\right)+2\gamma K\frac{g}{X}+2\frac{\rho\tau'}{\beta^*}\left(\gamma K\exp\left(\lambda\beta^*\right)-\frac{\tau'}{\Psi}\right) > 0, \tag{2.A.35}$$

Observing that:

$$\lim_{\Psi\to 0}\tau'\left(\gamma K\exp\left(\lambda\beta^*\right)-\frac{\tau'}{\Psi}\right)=0, \tag{2.A.36}$$

and $\gamma\left(1+\frac{g}{X}K-\frac{\alpha-\rho\tau}{\beta^*}\exp\left(\lambda\beta^*\right)\right)+2\gamma K\frac{g}{X} > 0$ which does not depend on Ψ, I conclude that $\tau'' < 0$ if Ψ is not too large.

Let $\underline{\Omega} = \frac{\underline{w}}{1-\underline{w}}$. Because $\frac{\partial\left[-\left(1-\frac{\rho}{1-\mu}\tau^*(K)\right)\frac{d\tau^*(K)}{dK}\right]}{\partial K} > 0$, I conclude that if $-\left(1-\frac{\rho}{1-\mu}\tau^*\left(K\right)\right)\frac{d\tau^*(K)}{dK}|_{\Omega=\underline{\Omega}} > \Psi\frac{g}{X}$, the optimal choice for the incumbent shareholder is the short-term compensation scheme with $w^* > \underline{w}$ or equivalently $\Omega^* > \underline{\Omega}$. Plugging in Equation (2.A.32), the inequality $-\left(1-\frac{\rho}{1-\mu}\tau^*\left(K\right)\right)\frac{d\tau^*(K)}{dK}|_{\Omega=\underline{\Omega}} > \Psi\frac{g}{X}$ can be rewritten as:

$$\left(1-\frac{\rho}{1-\mu}\tau^*\left(K\right)\right)\frac{\frac{\left(1+\gamma K^2\right)}{2}+\gamma K\left(\frac{1-\frac{\alpha-\rho\tau^*(K)}{\beta^*}\exp(\lambda\beta^*)}{\frac{g}{X}}+K\right)}{\frac{\rho}{\beta^*}\left(\frac{\Psi(1+\gamma K^2)}{2}\exp\left(\lambda\beta^*\right)+\left(1-\tau^*\left(K\right)\right)\right)+\frac{\alpha-\rho\tau^*(K)}{\beta^*}}|_{\Omega=\underline{\Omega}} > 1 \tag{2.A.37}$$

which is true if $\frac{g}{X}$ and Ψ are not too large.

Proof of Corollary 2.3: When choosing the compensation duration, the equivalent objective of the incumbent shareholder is:

$$\max_{\Omega} (1-\mu) \left[\varphi \left(X + gK^* \left(\Omega \right) \right) + (1-\varphi) \tau^* \left(\Omega \right) X \right] - (1-\varphi) \frac{1}{2} \rho \left(\tau^* \left(\Omega \right) \right)^2 X, \qquad (2.A.38)$$

which is the expected value of $(1-\mu)$ shares deducted by the intervention cost. Because $\Omega = \Omega^*$ maximizes the incumbent shareholder's objective, I obtain:

$$\begin{aligned} &(1-\mu) \left[\varphi \left(X + gK^* \left(\Omega^* \right) \right) + (1-\varphi) \tau^* \left(\Omega^* \right) X \right] - (1-\varphi) \frac{1}{2} \rho \left(\tau^* \left(\Omega^* \right) \right)^2 X \\ &> (1-\mu) \left[\varphi \left(X + gK^* \left(\underline{\Omega} \right) \right) + (1-\varphi) \tau^* \left(\underline{\Omega} \right) X \right] - (1-\varphi) \frac{1}{2} \rho \left(\tau^* \left(\underline{\Omega} \right) \right)^2 X. \end{aligned} \qquad (2.A.39)$$

Moreover, because $\Omega^* > \underline{\Omega}$ and $\tau^* \left(\underline{\Omega} \right)$ increases with Ω, $\tau^* \left(\Omega^* \right) > \tau^* \left(\underline{\Omega} \right)$. Therefore, I get:

$$\varphi \left(X + gK^* \left(\Omega^* \right) \right) + (1-\varphi) \tau^* \left(\Omega^* \right) X > \varphi \left(X + gK^* \left(\underline{\Omega} \right) \right) + (1-\varphi) \tau^* \left(\underline{\Omega} \right) X, \qquad (2.A.40)$$

which implies that the expected firm value under the compensation duration with $\Omega = \Omega^*$ is higher than that under the long compensation duration with $\Omega = \underline{\Omega}$.

Proof of Proposition 2.4: The unique pair of (K, τ) is determined by Ω. Observing that Ω only appears in Equation (2.A.26), it is equivalent to find a (K, τ) that satisfies Equation (2.A.25) and maximizes the objective function (2.A.24). From the first order condition, I have

$$\varPsi \frac{g}{X} - \left(1 - \frac{\rho}{1-\mu} \tau^* \left(K \right) \right) \frac{d\tau^* \left(K \right)}{dK} = 0, \qquad (2.A.41)$$

which can be rewritten as:

$$\left(1 - \frac{\rho}{1-\mu} \tau^* \left(K \right) \right) \frac{\frac{\left(1+\gamma K^2\right)}{2} + \gamma K \left(\frac{1 - \frac{\alpha - \rho \tau^*(K)}{\beta^*} \exp(\lambda \beta^*)}{\frac{g}{X}} + K \right)}{\frac{\rho}{\beta^*} \left(\frac{\varPsi (1+\gamma K^2)}{2} \exp \left(\lambda \beta^* \right) + \left(1 - \tau^* \left(K \right) \right) \right) + \frac{\alpha - \rho \tau^*(K)}{\beta^*}} = 1. \qquad (2.A.42)$$

Let $\mathcal{L}^* = \left(1 - \frac{\rho}{1-\mu}\tau^*(K)\right) \frac{\frac{\left(1+\gamma K^2\right)}{2} + \gamma K \left(\frac{1 - \frac{\alpha - \rho\tau}{\beta^*}\exp(\lambda\beta^*)}{\frac{g}{X}} + K\right)}{\frac{\rho}{\beta^*}\left(\frac{\Psi\left(1+\gamma K^2\right)}{2}\exp(\lambda\beta^*) + (1-\tau)\right) + \frac{\alpha-\rho\tau}{\beta^*}}$. Based on the proof of Proposition 2.2, I already have $\frac{\partial \mathcal{L}^*}{\partial K} > 0$ if Ψ is not too large.

Let $\mathcal{L} = \left(1 - \frac{\rho}{1-\mu}\tau\right) \frac{\frac{\left(1+\gamma K^2\right)}{2} + \gamma K \left(\frac{1 - \frac{\alpha - \rho\tau}{\beta^*}\exp(\lambda\beta^*)}{\frac{g}{X}} + K\right)}{\frac{\rho}{\beta^*}\left(\frac{\Psi\left(1+\gamma K^2\right)}{2}\exp(\lambda\beta^*) + (1-\tau)\right) + \frac{\alpha-\rho\tau}{\beta^*}}$, I obtain

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \tau} &= \left(1 - \frac{\rho}{1-\mu}\tau\right) \frac{\frac{\frac{\rho}{\beta^*}\exp(\lambda\beta^*)\gamma K}{\frac{g}{X}}}{\left(\frac{\rho}{\beta^*}\left(\frac{\Psi(1+\gamma K^2)}{2}\exp\left(\lambda\beta^*\right) + (1-\tau)\right) + \frac{\alpha-\rho\tau}{\beta^*}\right)} \\
&+ \left(1 - \frac{\rho}{1-\mu}\tau\right) \frac{\left(\frac{\left(1+\gamma K^2\right)}{2} + \gamma K \left(\frac{1 - \frac{\alpha-\rho\tau}{\beta^*}\exp(\lambda\beta^*)}{\frac{g}{X}} + K\right)\right)\frac{2\rho}{\beta^*}}{\left(\frac{\rho}{\beta^*}\left(\frac{\Psi(1+\gamma K^2)}{2}\exp\left(\lambda\beta^*\right) + (1-\tau)\right) + \frac{\alpha-\rho\tau}{\beta^*}\right)^2} \\
&- \frac{\rho}{1-\mu} \frac{\frac{\left(1+\gamma K^2\right)}{2} + \gamma K \left(\frac{1 - \frac{\alpha-\rho\tau}{\beta^*}\exp(\lambda\beta^*)}{\frac{g}{X}} + K\right)}{\frac{\rho}{\beta^*}\left(\frac{\Psi(1+\gamma K^2)}{2}\exp\left(\lambda\beta^*\right) + (1-\tau)\right) + \frac{\alpha-\rho\tau}{\beta^*}},
\end{aligned} \tag{2.A.43}$$

which is positive if

$$\begin{aligned}
&\left(1 - \frac{\rho}{1-\mu}\tau\right) \frac{\frac{\rho}{\beta^*}\exp\left(\lambda\beta^*\right)\gamma K}{\frac{g}{X}} - \frac{\rho}{1-\mu}\left(\frac{(1+\gamma K^2)}{2} + \gamma K\left(\frac{1 - \frac{\alpha-\rho\tau}{\beta^*}\exp\left(\lambda\beta^*\right)}{\frac{g}{X}} + K\right)\right) \\
=&\left[\left(1 - \frac{2\rho}{1-\mu}\tau\right)\frac{\rho}{\beta^*}\exp\left(\lambda\beta^*\right) + \frac{\rho}{1-\mu}\left(\frac{\alpha}{\beta^*}\exp\left(\lambda\beta^*\right) - 1\right)\right]\frac{\gamma K}{\frac{g}{X}} - \frac{\rho}{1-\mu}\frac{(1+3\gamma K^2)}{2} \\
>&0.
\end{aligned} \tag{2.A.44}$$

The above inequality is true if $\frac{g}{X}$ is not too large and $1-\mu > 2\alpha$.

Therefore,

$$\frac{\partial \mathcal{L}}{\partial g} < 0, \tag{2.A.45}$$

$$\frac{\partial \mathcal{L}}{\partial X} > 0, \tag{2.A.46}$$

$$\frac{\partial \mathcal{L}}{\partial \Psi} < 0. \tag{2.A.47}$$

Similar to the proof of Proposition 2.1, I derive that:

$$\frac{\partial T(\tau, K)}{\partial g} = \frac{K}{X} > 0, \tag{2.A.48}$$

$$\frac{\partial T(\tau, K)}{\partial X} = -\frac{gK}{X^2} < 0, \tag{2.A.49}$$

$$\frac{\partial T(\tau, K)}{\partial \Psi} = \frac{\alpha - \rho\tau}{\beta} \frac{2(1-\tau)}{\Psi^2 (1 + \gamma K^2)} > 0, \tag{2.A.50}$$

$$\frac{\partial T(\tau, K)}{\partial \rho} = \frac{\tau}{\beta} \left(\exp(\lambda\beta) + \frac{(1-\tau)}{\Pi(K)} \right) > 0. \tag{2.A.51}$$

Hence, I obtain:

$$\frac{d\tau^*(K)}{dg} = -\frac{\frac{\partial T(\tau,K)}{\partial g}}{\frac{\partial T(\tau,K)}{\partial \tau}} < 0, \tag{2.A.52}$$

$$\frac{d\tau^*(K)}{dX} = -\frac{\frac{\partial T(\tau,K)}{\partial X}}{\frac{\partial T(\tau,K)}{\partial \tau}} > 0, \tag{2.A.53}$$

$$\frac{d\tau^*(K)}{d\Psi} = -\frac{\frac{\partial T(\tau,K)}{\partial \Psi}}{\frac{\partial T(\tau,K)}{\partial \tau}} < 0, \tag{2.A.54}$$

$$\frac{d\tau^*(K)}{d\rho} = -\frac{\frac{\partial T(\tau,K)}{\partial \rho}}{\frac{\partial T(\tau,K)}{\partial \tau}} < 0. \tag{2.A.55}$$

Therefore, I obtain:

$$\frac{\partial \mathcal{L}^*}{\partial g} = \frac{\partial \mathcal{L}}{\partial \tau} \frac{\partial \tau}{\partial g} + \frac{\partial \mathcal{L}}{\partial g} < 0, \tag{2.A.56}$$

$$\frac{\partial \mathcal{L}^*}{\partial X} = \frac{\partial \mathcal{L}}{\partial \tau} \frac{\partial \tau}{\partial X} + \frac{\partial \mathcal{L}}{\partial X} > 0, \tag{2.A.57}$$

$$\frac{\partial \mathcal{L}^*}{\partial \Psi} = \frac{\partial \mathcal{L}}{\partial \tau} \frac{\partial \tau}{\partial \Psi} + \frac{\partial \mathcal{L}}{\partial \Psi} < 0. \tag{2.A.58}$$

Because K^* is determined by $\mathcal{L}^* = 1$, I get:

$$\frac{\partial K^*}{\partial g} > 0, \tag{2.A.59}$$
$$\frac{\partial K^*}{\partial X} < 0, \tag{2.A.60}$$
$$\frac{\partial K^*}{\partial \Psi} > 0. \tag{2.A.61}$$

Moreover, I have

$$\begin{aligned}\Omega &= \frac{\frac{g}{X}}{1+\frac{g}{X}K}\frac{\exp(\lambda\beta)\mathit{\Psi}\left(1+\gamma K^2\right)+2(1-\tau)}{2(1-\tau)\gamma K} \\ &= \frac{\frac{g}{X}}{\frac{\alpha-\rho\tau}{\beta^*}\left(\exp(\lambda\beta^*)+\frac{2(1-\tau)}{\mathit{\Psi}(1+\gamma K^2)}\right)}\frac{\exp(\lambda\beta)\mathit{\Psi}\left(1+\gamma K^2\right)+2(1-\tau)}{2(1-\tau)\gamma K} \\ &= \frac{\frac{g}{X}}{\alpha-\rho\tau}\frac{\beta\mathit{\Psi}\left(1+\gamma K^2\right)}{2(1-\tau)\gamma K}.\end{aligned} \tag{2.A.62}$$

Plugging $1+\frac{g}{X}K = \frac{\alpha-\rho\tau}{\beta^*}\left(\exp(\lambda\beta^*)+\frac{2(1-\tau)}{\mathit{\Psi}(1+\gamma K^2)}\right)$, the above equation can be rewritten as:

$$\begin{aligned}\Omega &= \frac{\frac{g}{X}}{\frac{\alpha-\rho\tau}{\beta^*}\left(\exp(\lambda\beta^*)+\frac{2(1-\tau)}{\mathit{\Psi}(1+\gamma K^2)}\right)}\frac{\exp(\lambda\beta)\mathit{\Psi}\left(1+\gamma K^2\right)+2(1-\tau)}{2(1-\tau)\gamma K} \\ &= \frac{\frac{g}{X}}{\alpha-\rho\tau}\frac{\beta^*\mathit{\Psi}\left(1+\gamma K^2\right)}{2(1-\tau)\gamma K}.\end{aligned} \tag{2.A.63}$$

Moreover, from the first order condition, I have:

$$\begin{aligned}&\left(1-\frac{\rho}{1-\mu}\tau^*(K)\right)\frac{\frac{\left(1+\gamma K^2\right)}{2}+\gamma K\left(\frac{1-\frac{\alpha-\rho\tau^*(K)}{\beta^*}\exp(\lambda\beta^*)}{\frac{g}{X}}+K\right)}{\frac{\rho}{\beta^*}\left(\frac{\mathit{\Psi}(1+\gamma K^2)}{2}\exp(\lambda\beta^*)+(1-\tau^*(K))\right)+\frac{\alpha-\rho\tau^*(K)}{\beta^*}} = 1 \\ &\left(1-\frac{\rho}{1-\mu}\tau^*(K)\right)\frac{\frac{\left(1+\gamma K^2\right)}{2}+\gamma K\frac{\frac{\alpha-\rho\tau}{\beta^*}\frac{2(1-\tau)}{\mathit{\Psi}(1+\gamma K^2)}}{\frac{g}{X}}}{\frac{\rho}{\beta^*}\left(\frac{\mathit{\Psi}(1+\gamma K^2)}{2}\exp(\lambda\beta^*)+(1-\tau^*(K))\right)+\frac{\alpha-\rho\tau^*(K)}{\beta^*}} = 1,\end{aligned} \tag{2.A.64}$$

so the formula of Ω can be rewritten as:

$$\Omega = \frac{1}{\left(\frac{\rho}{\beta^*\left(1-\frac{\rho}{1-\mu}\tau\right)}\exp\left(\lambda\beta^*\right)\mathit{\Psi}-1\right)\frac{\left(1+\gamma K^2\right)}{2}+\frac{\alpha-\rho\tau+\rho(1-\tau)}{\beta^*\left(1-\frac{\rho}{1-\mu}\tau\right)}}, \tag{2.A.65}$$

which decreases with K. Therefore, I obtain:

$$\frac{\partial\Omega^*}{\partial g} < 0, \tag{2.A.66}$$
$$\frac{\partial\Omega^*}{\partial X} > 0, \tag{2.A.67}$$
$$\frac{\partial\Omega^*}{\partial\Psi} < 0. \tag{2.A.68}$$

Moreover, I have

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial\rho} = & \frac{\left(1-\frac{\rho}{1-\mu}\tau\right)\frac{\frac{\tau}{\beta}\exp(\lambda\beta)\gamma K}{\frac{g}{X}}}{\left(\frac{\rho}{\beta^*}\left(\frac{\mathit{\Psi}(1+\gamma K^2)}{2}\exp\left(\lambda\beta^*\right)+(1-\tau)\right)+\frac{\alpha-\rho\tau}{\beta^*}\right)} \\ & -\left(1-\frac{\rho}{1-\mu}\tau\right)\frac{\left(\frac{\left(1+\gamma K^2\right)}{2}+\gamma K\left(\frac{1-\frac{\alpha-\rho\tau}{\beta^*}\exp(\lambda\beta^*)}{\frac{g}{X}}+K\right)\right)\left(\frac{1}{\beta}\exp\left(\lambda\beta\right)\mathit{\Psi}\frac{\left(1+\gamma K^2\right)}{2}+\frac{-\tau+(1-\tau)}{\beta}\right)}{\left(\frac{\rho}{\beta^*}\left(\frac{\mathit{\Psi}(1+\gamma K^2)}{2}\exp\left(\lambda\beta^*\right)+(1-\tau)\right)+\frac{\alpha-\rho\tau}{\beta^*}\right)^2} \\ & -\frac{1}{1-\mu}\tau\frac{\frac{\left(1+\gamma K^2\right)}{2}+\gamma K\left(\frac{1-\frac{\alpha-\rho\tau}{\beta^*}\exp(\lambda\beta^*)}{\frac{g}{X}}+K\right)}{\frac{\rho}{\beta^*}\left(\frac{\mathit{\Psi}(1+\gamma K^2)}{2}\exp\left(\lambda\beta^*\right)+(1-\tau)\right)+\frac{\alpha-\rho\tau}{\beta^*}},\end{aligned} \tag{2.A.69}$$

which is negative if τ is small (ρ is large), and positive if τ is large (ρ is small). Therefore, I obtain:

$$\frac{\partial L^*}{\partial\rho} = \frac{\partial\mathcal{L}}{\partial\tau}\frac{\partial\tau}{\partial\rho}+\frac{\partial\mathcal{L}}{\partial\rho}\begin{cases}<0 & \text{if } \rho \text{ is large} \\ >0 & \text{if } \rho \text{ is small}\end{cases}, \tag{2.A.70}$$

and, thus, I have:

$$\frac{\partial K^*}{\partial \rho}\begin{cases} > 0 & \text{if } \rho \text{ is large} \\ < 0 & \text{if } \rho \text{ is small} \end{cases}, \tag{2.A.71}$$

and

$$\frac{\partial \Omega^*}{\partial \rho}\begin{cases} < 0 & \text{if } \rho \text{ is large} \\ > 0 & \text{if } \rho \text{ is small} \end{cases}. \tag{2.A.72}$$

Proof of Proposition 2.5: By Bayesian rule, the posterior belief that a firm is good given no intervention, low earnings, and nonnegative net demand $d \geq 0$, which is denoted as π, is determined by:

$$\pi = \frac{\varphi^{\frac{\left(1+\gamma \hat{K}^2\right)}{2}}}{\varphi^{\frac{\left(1+\gamma \hat{K}^2\right)}{2}} + (1-\varphi)(1-\hat{\tau})\exp\left(-\lambda\left(\hat{\beta}+\hat{\beta}_{new}\right)\right)}. \tag{2.A.73}$$

The trading strategy is determined by:

$$\begin{aligned} \beta^* &= \arg\max_{\beta \in [0,\alpha]} \beta \exp\left(-\lambda\left(\beta+\beta_{new}\right)\right)\pi \hat{V}_G, && (2.A.74) \\ \beta^*_{new} &= \arg\max_{\beta \in [0,\alpha_{new}]} \beta \exp\left(-\lambda\left(\beta+\beta_{new}\right)\right)\pi \hat{V}_G. && (2.A.75) \end{aligned}$$

From the first order condition, I obtain

$$\begin{aligned} (1-\lambda\beta)\exp\left(-\lambda\left(\beta+\beta_{new}\right)\right)\pi \hat{V}_G &= 0, && (2.A.76) \\ (1-\lambda\beta_{new})\exp\left(-\lambda\left(\beta+\beta_{new}\right)\right)\pi \hat{V}_G &= 0, && (2.A.77) \end{aligned}$$

which has a unique pair of solutions with $\beta = \frac{1}{\lambda}$ and $\beta_{new} = \frac{1}{\lambda}$. However, because of the no short selling constraint, $\frac{1}{\lambda}$ might not be achieved. The optimal solutions are $\beta^* = \min\left\{\alpha, \frac{1}{\lambda}\right\}$ and $\beta^*_{new} = \min\left\{\alpha_{new}, \frac{1}{\lambda}\right\}$. Therefore, the incumbent shareholder's trading profit from exit is $\beta^* \exp\left(-\lambda\left(\beta^*+\beta^*_{new}\right)\right)\pi \hat{V}_G$. In equilibrium, $\hat{\beta} = \beta^*$ and $\hat{\beta}_{new} = \beta^*_{new}$, so the trading profit is:

$$\begin{aligned}
&\beta^* \exp\left(-\lambda\left(\beta^* + \beta^*_{new}\right)\right)\pi \hat{V}_G \\
=&\beta^* \frac{\Psi\left(1+\gamma K^2\right)\left(1+\frac{g}{X}K\right)}{\exp\left(\lambda\left(\beta^*+\beta^*_{new}\right)\right)\Psi\left(1+\gamma K^2\right)+2\left(1-\tau\right)} \\
<&\beta^* \frac{\Psi\left(1+\gamma K^2\right)\left(1+\frac{g}{X}K\right)}{\exp\left(\lambda\beta^*\right)\Psi\left(1+\gamma K^2\right)+2\left(1-\tau\right)},
\end{aligned} \tag{2.A.78}$$

which implies the trading of the institutional investor reduces the trading profit of the incumbent shareholder.

The net profit of the incumbent shareholder by intervention is $(\alpha - \rho\tau) X$. Therefore, the incumbent shareholder chooses public intervention if and only if $\tau < \tau^*$, in which τ^* is determined by:

$$\begin{aligned}
(\alpha - \rho\tau^*) X &= \beta^* \exp\left(-\lambda\left(\beta^* + \beta^*_{new}\right)\right) \frac{\varphi\left(1+\gamma\hat{K}^2\right)\left(X+g\hat{K}\right)}{\varphi\left(1+\gamma\hat{K}^2\right)+2\left(1-\varphi\right)\left(1-\hat{\tau}\right)\exp\left(-\lambda\left(\hat{\beta}+\hat{\beta}_{new}\right)\right)} \\
&= \beta^* \frac{\Psi\left(1+\gamma K^2\right)\left(1+\frac{g}{X}K\right)}{\exp\left(\lambda\left(\beta^*+\beta^*_{new}\right)\right)\Psi\left(1+\gamma K^2\right)+2\left(1-\tau\right)}.
\end{aligned} \tag{2.A.79}$$

Because the trading profit decreases, the intervention cutoff increases, which implies a higher level of intervention by the incumbent shareholder. The informed trading by the institutional investor mitigates the limited commitment problem of the incumbent shareholder. Hence, the incumbent shareholder does not have to provide many short-term managerial incentives to commit to value-enhancing intervention in a bad firm, i.e., $w^{*,I} \leq w^*$.

Proof of Proposition 2.6: From the perspective of passive investors, the marginal benefit of increasing the investment by a good firm K is $\Psi \frac{g}{X}$, and the marginal cost of increasing the investment by a good firm K is $-\frac{d\tau^*(K)}{dK}$ where $\frac{d\tau^*(K)}{dK} < 0$ from Lemma 2.5. From the proof of Proposition 2.2, I have shown $\tau'' < 0$ if Ψ is not too large. I can conclude that if $-\frac{d\tau^*(K)}{dK}|_{\Omega=\underline{\Omega}} > \Psi \frac{g}{X}$, the optimal choice for the incumbent shareholder is the short-term compensation scheme with $w^{*,II} > \underline{w}$ or equivalently $\Omega^{*} > \underline{\Omega}$. Plugging in Equation (2.A.32), the inequality $-\frac{d\tau^{*}(K)}{dK}|_{\Omega=\underline{\Omega}} > \Psi\frac{g}{X}$ can be rewritten as:

$$\frac{\frac{\left(1+\gamma K^{2}\right)}{2} + \gamma K\left(\frac{1-\frac{\alpha-\rho\tau}{\beta^{*}}\exp(\lambda\beta^{*})}{\frac{g}{X}} + K\right)}{\frac{\rho}{\beta^{*}}\left(\frac{\Psi(1+\gamma K^{2})}{2}\exp\left(\lambda\beta^{*}\right) + (1-\tau)\right) + \frac{\alpha-\rho\tau}{\beta^{*}}}|_{\Omega=\underline{\Omega}} > 1, \tag{2.A.80}$$

which is true if $\frac{g}{X}$ and Ψ are not too large.

Chapter 3

A Crowdfunding Model for Green Energy Investment[1]

Abstract

This paper studies a new renewable energy investment model through crowdfunding, which is motivated by emerging community solar farms. In this paper we develop a sequential game theory model to capture the interactions among crowdfunders, the solar farm owner, and an electricity company who purchases renewable energy generated by the solar farm in a multi-period framework. By characterizing the sub-game perfect equilibrium, and comparing it with a benchmark model without crowdfunding, we find that under crowdfunding although the farm owner reduces its investment level, the overall green energy investment level is increased due to the contribution of crowdfunders. We also find that crowdfunding can increase the penetration of green energy in consumption and thus reduce the energy procurement cost of the electricity company. Finally, the numerical results based on real data indicates crowdfunding is a simple but effective way to boost

[1]This chapter is based on a joint work with Ying Xu, Nilanjan Chakraborty, and Katia Sycara. A previous version of this work without the proofs appeared as a conference proceeding of the 24th International Conference on Artificial Intelligence (Zheng et al., 2015).

green generation.

Key Words: Renewable Energy, Crowdfunding, Game Theory

3.1 Introduction

One of the most prevalent ways to boost investment in renewable energy generation is to motivate individuals to participate in green energy investment and generation. However, not all individuals are willing or able to install on-site renewable energy generation at their homes. Therefore, a special *crowdfunding* green energy investment model has been recently introduced in the form of community shared renewable energy projects. For example, Clean Energy Collective (CEC) has launched three community shared solar projects in Colorado (Coughlin et al., 2012). By joining in such a community shared solar project, individual investors can acquire the ownership of solar panels which may be installed on a solar farm remotely located from the individual's home. Due to the distance, the energy generated by the solar panels on a solar farm are not directly consumed by the owners of the panels. Instead, the generated energy are sold by the owner of the solar farm (e.g., the CEC), as an agent for all the individual investors, to an electricity company through a long-term power purchase agreement (PPA). Then, the solar farm owner collects the revenue from selling the generated energy and allocates the payoff to the individual investors depending on the amount of the electricity generated by the panels they invested.

The newly emerging investment pattern in green energy—crowdfunding, can benefit all stakeholders (e.g., individual investors, the solar farm owner who represents traditional investors, electricity company). From the perspective of individual investors who are unable to install green generation system at home due to geometry/physical/time restrictions, the new crowdfunding pattern provides a bridge for them to access the renewable energy investment. For the solar farm owner, crowdfunding helps expand investment participation and hence pools investment risk. Indeed, Gerber and Hui (2013) study the incentive of participants of crowdfunding platforms based on interviews, and find the primary motivation of project creators is raising fund and sharing risk while that of fund providers is collecting the reward. Moreover, the electricity company also benefits from the increased green energy investment level in that the potential increment in green energy penetration can lower its energy procurement cost.

Despite the various merits, to our knowledge few researchers have studied this new business model and thus its impact on green energy investment is unclear. Most papers on renewable energy investment focus on evaluating the effectiveness and efficiency of these renewable energy policies such as feed-in tariffs, tax credits, and certificate systems (Wüstenhagen and Menichetti, 2012). They often conduct the examination through case study (Wiser et al., 1998), literature review (Couture and Gagnon, 2010) or numerical simulation (Palmer and Burtraw, 2005). Zhou et al. (2011) is the only paper that aims to design an incentive policy by taking into account the response of the investors but not in the framework of crowdfunding. Reuter et al. (2012) and Fleten et al. (2007) are among a few exceptions which quantitatively estimate the risk and return associated with renewable energy investment from the perspective of investors, but not crowdfunders. Our paper aims to fill the gap through answering the following questions: First, we are interested in the crowdfunding mechanism; i.e., how should a farm owner allocate the cost and returns among crowdfunders to maximize its own utility? Second, how will the presence of crowdfunding affect the green energy investment of the farm owner? Thirdly, from the perspective of the electricity company, how should the electricity company adjust its procurement strategy as a response to the new investment pattern? Most importantly, how would the emergence of crowdfunding affect the overall green energy investment level and also the penetration of green energy in the total energy consumption?

To answer these questions, inspired by practice examples of community shared solar projects (e.g., CEC in Colorado), we develop a sequential game model to represent the strategic interaction among three groups of players: a group of individual crowdfunders, the owner of a solar farm who initiates the crowdfunding, and an electricity company who purchases green energy generated by the farm through a wholesale contract. The electricity company chooses the wholesale price. Based on the wholesale price, the solar farm chooses its own solar panel investment level and designs the crowdfunding mechanism to raise funds from the crowdfunder group. Accordingly, each crowdfunder makes their individual investment decision. The decision process is characterized by a three-player sequential game, in contrast to current crowdfunding models only involving two parties: crowdfunders and fund-raisers (e.g., Belleflamme et al. (2014), Hu et al. (2014)) or current mechanism design models on green energy (e.g., Vinyals et al. (2014), Robu et al. (2012)). We characterize the subgame perfect equilibrium of the game. Our model applies to any investment structure that involves 3 players whose investment decisions are inter-dependent.

To further study the impact of the crowdfunding model, we also consider a benchmark model without crowdfunding. By comparing the results between the benchmark and the crowdfunding model, we have the following findings:

1. Under crowdfunding, the farm owner gains more utility by taking advantage of crowdfunding to shift the investment risk. In particular, under crowdfunding the farm owner gains the same sales revenue at a lower investment cost;

2. Although crowdfunding reduces the farm owner's investment level, it does increase the overall green energy investment level as well as the penetration of green energy in consumption due to the contribution of crowdfunder;

3. Due to increased green energy penetration, crowdfunding reduces the procurement cost of the electricity company, which also increases the total welfare of energy consumers. The *analytical* results are proved by solving first order conditions for each corresponding optimization problem.

At last, we *numerically* estimate the practical impact of crowdfunding through simulations based on real data and find that crowdfunding is a simple but effective way to increase green generation.

The rest of the paper proceeds as follows. Section 3.2 presents the problem formulation and the sequence of events. Section 3.3 characterizes the sub-game perfect equilibrium and compare it with the benchmark without crowdfunding. Section 3.4 comducts the numerical analysis using real data and compares the performance of crowdfunding with that of other renewable energy policies and technology development. Section 3.5 concludes. All proofs are provided in an appendix.

3.2 Problem Formulation

We consider a solar farm that needs long-term investment in renewable energy. There are multiple players in the model: the owner of the farm, a group of crowdfunders, and the electricity company. Based on the behavior of investors while exposed to uncertainty to attempt to reduce that uncertainty, we assume that both the farm owner and the crowdfunders are risk-averse. As the electricity company is an organization, we assume the electricity company is risk-neutral following most economic literature on industry organization (Tirole, 1988). In practice, the electricity company often has a much stronger bargaining power which allows him to make a wholesale price offer to the green farm. The owner of the farm then decides its own investment and can also attract funding from crowdfunders by offering a crowdfunding contract. Given the contract, the crowdfunders decide how much to invest in the solar farm. As can be seen, the optimal decisions of those players depend on each other's decision. A similar sequence of events is also true for other crowdfunding settings (Hu et al., 2014). Therefore, we model this problem as a sequential game with continuous decision space.

The time horizon is T periods. The electricity company needs to provide energy to a group of consumers with aggregate demand $\widetilde{D}_t$ with mean D_t in period t, $t = 1, 2, \cdots, T$. In each period, in order to fulfill the demand, the electricity company buys electricity from the green farm at a wholesale price w_t and from the electricity market at a market price $\widetilde{q}_t$ with mean q_t. In practice the electricity company may receive energy from various sources such as conventional energy generations. Here we use the electricity market to represent all energy sources other than the renewable ones. The wholesale price $\mathbf{w} \equiv (w_1, w_2, \cdots, w_T) \in \mathbb{R}^T$ is determined in a contract offered by the electricity company, which also states that the electricity company will buy all electricity generated by the green farm. Therefore, the electricity company only purchases electricity from the market to satisfy the demand that cannot be satisfied by green generation. Let $\widetilde{G}_t$ denote the supply of the green farm in period t, then the electricity company's expected total procurement cost is:

$$\mathbb{E}[C] = \mathbb{E}[\sum_{t=1}^{T} \left(w_t \widetilde{G}_t + \widetilde{q}_t \left(\widetilde{D}_t - \widetilde{G}_t \right) \right)]. \tag{3.1}$$

The electricity company's action is choosing the wholesale price of the green generation, $\mathbf{w} \in \mathbb{R}^T$, and its objective is to minimize its expected energy procurement cost.

The total green energy supply $\widetilde{G}_t$ depends on the total investment level of the green farm. The investment level is represented by the total number of solar panels installed on the farm, N. Let $f(N)$ denote the total cost of investing N units of solar panels (including purchasing, installation and maintenance during the period, as well as tax credits reduction). For tractability, we assume that the cost is linear in N, i.e., $f(N) = BN$ where B represents the cost of one unit of solar panel and $b \equiv \frac{B}{T}$ is the allocated cost per period. Each unit of solar panel generates an uncertain amount of green energy supply in each period, which is denoted by $\widetilde{g}_t$, which satisfies normal distribution with mean of μ_g and standard deviation of σ_g. The annual output of one unit panel is uncertain, due to variations in sunshine, manufacturing, installation and maintenance among solar panels. This deviation cannot be ignored given risk aversion of the players. Because the deviation is small, there is no heavy tail for the distribution, so normal distribution is the best a priori assumption in absence of large data samples to fit. The total green energy supply is $\widetilde{G}_t = N\widetilde{g}_t$ in period t. To make the problem nontrivial, we assume that over the whole horizon the average investment cost for per unit green energy generation b/μ_g is lower than the overall average market price $q \equiv \frac{\sum_{t=1}^{T} q_t}{T}$, i.e., $q > b/\mu_g$.

The green farm owner not only invests in the solar panels by itself, but also attracts funding from other investors. Specifically, we consider that the owner offers a crowdfunding contract to risk averse crowdfunders. The investment contract designed by the farm is as follows: crowdfunders pay the farm c for per unit solar panel in the beginning of period 1, and receives r_t for per unit of electricity generation from the invested solar panels in period t. Let N_0 denote the number of solar panels invested by the green farm itself. Let N_C denote the total number of solar panels invested by the crowdfunders. The total green energy investment is the sum of investment by both the farm owner and the crowdfunders, i.e, $N = N_0 + N_C$. Therefore, the total profit of the farm is:

$$\pi_0 = \sum_{t=1}^{T} w_t \widetilde{g}_t N - f(N) + \left(c - \sum_{t=1}^{T} r_t \widetilde{g}_t \right) N_C, \tag{3.2}$$

where $\left(c - \sum_{t=1}^{T} r_t \widetilde{g}_t \right) N_C$ represents the farm owner's payoff from crowdfunding. We assume the utility function of risk-averse farm owner follows Constant Absolute Risk Aversion (CARA) utility. Correspondingly, the utility of the risk-averse farm owner is:

$$u_0(\pi_0) = -e^{-\frac{\pi_0}{\rho}}, \tag{3.3}$$

where $\rho > 0$ is the degree of risk tolerance of the farm owner. The farm owner's action is choosing its own investment $N_0 \in \mathbb{R}^+$ and setting the crowdfunding contract $(c, \mathbf{r}) \in \mathbb{R}^{1+T}$, and its objective is to maximize its expected utility $\mathbb{E}[u_0(\pi_0)]$.

Given the crowdfunding contract offered by the farm owner $(c, \mathbf{r})$, each crowdfunder chooses his/her investment level. The population of crowdfunders is large, while the investment level of an individual crowdfunder is often insignificant compared with that of the farm owner. Therefore, we follow the modeling methodology in the economic literature (Hellwig, 1980) when small individual investors cannot significantly affect the aggregate investment, and use continuous index in [0,1] with the Lebesgue measure for crowdfunders. In other words, the crowdfunders are indexed by $i \in \mathbf{I} = [0, 1]$ with the Lebesgue measure. A crowdfunder i's payoff π_i under investment level n_i given the crowdfunding contract offered by the farm owner $(c, \mathbf{r})$ is:

$$\pi_i = \left(\sum_{t=1}^{T} r_t \widetilde{g}_t - c \right) n_i. \tag{3.4}$$

We also assume the utility function of risk-averse crowdfunders follows Constant Absolute Risk Aversion (CARA) utility. Specifically, each crowd-funder i is uniquely characterized by its degree of risk-tolerance β_i. Therefore, the utility of crowd-funder i is:

$$u_i(\pi_i) = -e^{-\frac{\pi_i}{\beta_i}}. \tag{3.5}$$

Crowd-funder i's action is choosing investment level $n_i \in \mathbb{R}^+$, and its objective is to maximize its expected utility $\mathbb{E}[u_i(\pi_i)]$. For flexibility, we allow the crowdfunders to invest a fraction of a solar panel. By the definition of the aggregate investment level of crowdfunders, N_C, we have $N_C = \sum_{i \in \mathbf{I}} n_i^*$.

The sequence of events (see Figure 3.1) is as below:

1. At t=0:

    (a) The electricity company offers the wholesale price of the green generation, $\mathbf{w} = (w_1, w_2, \cdots, w_T) \in \mathbb{R}^T$;

    (b) The firm offers a contract to the crowdfunders, which specifies the unit panel investment cost c and the return $\mathbf{r} = (r_1, r_2, \cdots, r_T)$ for per unit green energy generation, and also determines its own investment level $N_0 \in \mathbb{R}^+$;

    (c) Based on the contract, the crowdfunders determines their investment levels $n_i \in$

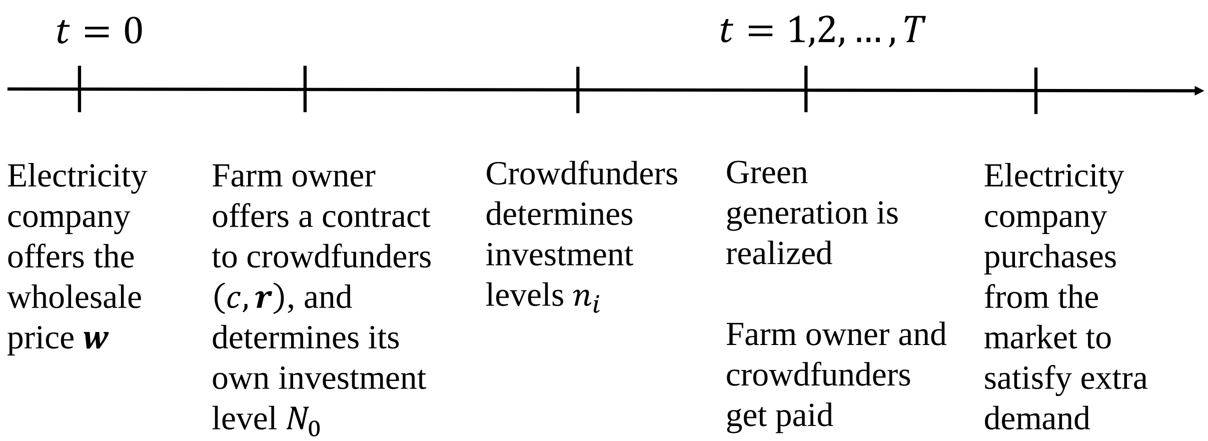

Figure 3.1: The Sequence of Events

$\mathbb{R}^+$;

2. In each period $t \in \{1, 2, \cdots, T\}$:

   (a) The green generation is realized, and the firm owner and the crowdfunders get paid accordingly;

   (b) The electricity company purchases from the market to satisfy the demand that cannot be satisfied by green generation.

3.3 Equilibrium Analysis

In this section, we first consider a benchmark model without crowdfunding, then solve the equilibrium in the crowdfunding model, finally compare the two models to show the effects of the adoption of crowdfunding on the green energy investment and the welfare of the stakeholders. In both models, we solve the sub-game perfect equilibrium through backward induction.

3.3.1 Benchmark: No Crowdfunding

In the benchmark model without crowdfunding, the electricity company determines the wholesale price and then the green farm owner chooses the investment level. Using backward induction, we first characterize the farm's optimal investment decision $N_0^\sharp(\mathbf{w})$ for a given wholesale price $\mathbf{w}$, then derive the optimal wholesale price $\mathbf{w}^\sharp$ set by the electricity company anticipating the optimal investment strategy of the firm owner, $N_0^\sharp(\mathbf{w})$.

3.3.1.1 Green farm owner: investment decision

Without crowdfunding, $N_C = 0$, the total investment level is equivalent to the direct investment by the farm owner, i.e., $N = N_0$. The risk-averse green farm owner chooses its investment level N_0 to maximize its utility. Substituting $N_C = 0$ and $N = N_0$ into (3.2), the total profit of the farm owner is: $\pi_0 = \left(\sum_{t=1}^{T} w_t \widetilde{g}_t - B\right) N_0$. So the farm's expected utility is:

$$\mathbb{E}\left[u\left(\pi_0\right)\right] \equiv -\mathbb{E}\left[e^{-\frac{\pi_0}{\rho}}\right] = e^{-\frac{\mathbb{E}[\pi_0]}{\rho} + \frac{Var[\pi_0]}{2\rho}}. \tag{3.6}$$

Thus, the farm owner's optimal investment problem can be characterized as maximizing the certainty equivalent:

$$N_0^\sharp\left(\mathbf{w}\right) = \arg\max_{N_0} \left(\mu_g \sum_{t=1}^{T} w_t - B\right) N_0 - \frac{\sum_{t=1}^{T} w_t^2 N_0^2 \sigma_g^2}{2\rho}. \tag{3.7}$$

By solving $\frac{\partial CE_0(N_0,\mathbf{w})}{\partial N_0} = 0$, we derive the optimal investment strategy of the farm owner $N_0^\sharp(\mathbf{w})$ in Lemma 3.1.

Lemma 3.1. *Without crowdfunding, the optimal investment strategy of the farm owner given the whole sale price offered by the electricity company w is:*

$$N_0^\sharp\left(\mathbf{w}\right) = \rho \frac{\mu_g \sum_{t=1}^{T} w_t - B}{\sum_{t=1}^{T} w_t^2 \sigma_g^2}. \tag{3.8}$$

3.3.1.2 Electricity company: wholesale price decision

Anticipating the farm owner's optimal investment strategy $N_0^\sharp(w)$ in equation (3.8), the electricity company chooses the wholesale price to minimize the total expected procurement cost defined in equation (3.1), which is:

$$\mathbb{E}\left[C_0(\mathbf{w})\right] = \sum_{t=1}^{T} q_t D_t + \sum_{t=1}^{T} \left(w_t - q_t\right) \mu_g N_0^\sharp\left(\mathbf{w}\right). \tag{3.9}$$

Hence, the optimal wholesale price $\mathbf{w}^\sharp$ is determined by:

$$\mathbf{w}^\sharp = \arg\min_{\mathbf{w}} \mathbb{E}[C_0(\mathbf{w})]. \tag{3.10}$$

Solving the equation system $\nabla_{\mathbf{w}} \mathbb{E}[C_0(\mathbf{w})] = 0$ to derive the optimal wholesale price $\mathbf{w}^\sharp$ and substituting $\mathbf{w}^\sharp$ to the farm owner's optimal investment strategy $N_0^\sharp(\mathbf{w})$, we achieve the subgame perfect equilibrium in the benchmark model without crowdfunding in Proposition 3.1.

Proposition 3.1. *In the traditional business model without crowdfunding, there is a subgame perfect equilibrium, in which the total investment by the green farm is:*

$$N_0^\sharp = \frac{\rho}{\sigma_g^2}\left(\frac{\mu_g^2}{4b} - \frac{b}{4q^2}\right), \tag{3.11}$$

the wholesale price of electricity offered by the electricity company is:

$$w_t^\sharp = w^\sharp \equiv \frac{2}{\frac{1}{b/\mu_g} + \frac{1}{q}}, t = 1, 2, \cdots, T, \tag{3.12}$$

the expected procurement cost is:

$$C^\sharp = \sum_{t=1}^{T} q_t D_t - T\frac{\rho\mu_g}{4bq\sigma_g^2}\left(q\mu_g - b\right)^2. \tag{3.13}$$

Proposition 3.1 has several implications. First, the whole sale price, which is between the average unit cost of renewable energy generated by solar panels b/μ_g and the average market price q, remains the same over the whole horizon. The rationale is that it is optimal for the electricity company not shifting risk to the farm owner as the electricity company is *risk neutral* while the farm owner is *risk averse*.

Second, from (3.12) we observe that the equilibrium wholesale price increases in (a) the average market price q and (b) the average unit cost of renewable energy generated by solar panels b/μ_g. The rationale is as follows: on one hand, the higher the average market price q is, the higher the wholesale price the electricity company is willing to pay; on the other hand, the higher the average unit cost of renewable energy, the higher the wholesale price the electricity company has to pay in order to encourage the farm owner to invest in solar panels.

Thirdly, Proposition 3.1 indicates the equilibrium solar investment increases in the degree of risk tolerance ρ and the average market price q, but decreases in the generation variance and the average unit cost of renewable energy b/μ_g.

3.3.2 Crowdfunding Model

In the crowdfunding model, in addition to the direct investment by the farm owner, the farm also raises funding from crowdfunders. Using backward induction, we first solve the investment decision of individual small investors $n_i^*(c, \mathbf{r})$ given the contract offered by the farm owner, $(c, \mathbf{r})$, then derive the farm owner's optimal crowdfunding contract $(c^*(\mathbf{w}), \mathbf{r}^*(\mathbf{w}))$ and investment decision $N_0^*(\mathbf{w})$ given the wholesale price offered by the electricity company, $\mathbf{w}$, finally we analyze the electricity company's optimal decision on wholesale price, i.e., $\mathbf{w}^*$.

3.3.2.1 Crowdfunders: investment decision

Given the contract offered by the green farm, $(c, \mathbf{r})$, which claims that crowdfunders pay the farm c for per unit solar panel in the beginning of period 1, and receives r_t for per unit of electricity generation from the invested solar panels in period t, $t \in \{1, 2, \cdots, T\}$, each investor's payoff under investment decision n_i, is: $\pi_i = \left(\sum_{t=1}^{T} r_t \widetilde{g}_t - c\right) n_i$. Therefore, the optimal investment problem for crowd-funder i is equivalent to:

$$\max_{n_i} \mu_g \sum_{t=1}^{T} r_t n_i - c n_i - \frac{1}{2\beta_i} \sum_{t=1}^{T} r_t^2 n_i^2 \sigma_g^2. \tag{3.14}$$

Therefore, the optimal investment level n_i^* is determined by

$$\begin{aligned} &\mu_g \sum_{t=1}^{T} r_t - c - \frac{\sum_{t=1}^{T} r_t^2 n_i \sigma_g^2}{\beta_i} = 0 \\ \Rightarrow & n_i^*(c, \mathbf{r}) = \frac{\beta_i \left(\mu_g \sum_{t=1}^{T} r_t - c\right)}{\sum_{t=1}^{T} r_t^2 \sigma_g^2}. \end{aligned} \tag{3.15}$$

The aggregate investment by crowdfunders is $N_C = \int_0^1 n_i di$, as we assume crowdfunders are indexed by $i \in \mathbf{I} = [0, 1]$ with the Lebesgue measure. From (3.15), the optimal aggregate investment level by crowdfunders given contract $(c, \mathbf{r})$ is:

$$\begin{aligned} N_C^*(c, \mathbf{r}) = &\int_0^1 n_i^* di \\ = &\frac{\left(\mu_g \sum_{t=1}^{T} r_t - c\right) \int_0^1 \beta_i di}{\sum_{t=1}^{T} r_t^2 \sigma_g^2} \\ = &\beta \frac{\mu_g \sum_{t=1}^{T} r_t - c}{\sum_{t=1}^{T} r_t^2 \sigma_g^2}, \end{aligned} \tag{3.16}$$

where $\beta = \int_0^1 \beta_i di$ denotes the mean of the risk-tolerance degree of crowdfunders.

3.3.2.2 The green farm: contract design and investment decision

Anticipating the optimal aggregate investment level by the crowdfunders $N_C^*(c, \mathbf{r})$, the green farm owner decides his own optimal investment and crowdfunding by maximizing the following certainty equivalent:

$$\max_{N_0,c,\mathbf{r}} CE_c\left(N_0,c,\mathbf{r}\right) \equiv \left(\sum_{t=1}^{T} w_t\mu_g - B\right) N_0$$
$$+\left(c - B + \sum_{t=1}^{T}(w_t - r_t)\mu_g\right) N_C^*(c,\mathbf{r})$$
$$-\frac{\sum_{t=1}^{T}\left(w_t N_0 + (w_t - r_t)N_C^*(c,\mathbf{r})\right)^2 \sigma_g^2}{2\rho}. \tag{3.17}$$

Lemma 3.2. *Given the wholesale price* $\mathbf{w}$*, in the optimal contract offered by the green farm, we have:*

$$r_t^*(\mathbf{w}) = \frac{2Bw_t}{B + \sum_{t=1}^{T} w_t\mu_g}, t = 1, 2, \cdots, T \tag{3.18}$$

$$c^*(\mathbf{w}) = B \tag{3.19}$$

under which the optimal number of solar panel invested by the green farm is:

$$N_0^*(\mathbf{w}) = \rho\frac{\mu_g \sum_{t=1}^{T} w_t - B}{\sum_{t=1}^{T} w_t^2\sigma_g^2}\left(1 - \frac{\beta(\mu_g \sum_{t=1}^{T} w_t - B)}{4B\rho}\right);$$

the aggregate investment by the crowdfunders is:

$$N_C^*(\mathbf{w}) = \frac{\beta\left(\left(\sum_{t=1}^{T} w_t\mu_g\right)^2 - B^2\right)}{4B\sum_{t=1}^{T} w_t^2\sigma_g^2}; \tag{3.20}$$

and the overall number of solar panel is:

$$N^*(\mathbf{w}) = \rho\frac{\mu_g \sum_{t=1}^{T} w_t - B}{\sum_{t=1}^{T} w_t^2\sigma_g^2}\left(1 + \frac{\beta}{2\rho}\right). \tag{3.21}$$

Lemma 3.2 states the optimal decision of the farm owner given wholesale prices $\mathbf{w}$ offered by the electricity company. The optimal values, $N_0^*(\mathbf{w}), c^*(\mathbf{w}), \mathbf{r}^*(\mathbf{w})$, are derived by solving the following equation system: $\frac{\partial CE_c(N_0,c,\mathbf{r})}{\partial N_0} = 0, \frac{\partial CE_c(N_0,c,\mathbf{r})}{\partial c} = 0, \nabla_{\mathbf{r}} CE_c(N_0, c, \mathbf{r}) = 0$. These optimal values indicate that for crowd-funders, the unit reward for per unit generation of invested solar panels r_t^* increases in the wholesale price w_t offered by the electricity company, and the unit investment charge c^* is the same as the purchasing and maintenance cost paid by the farm B, irrespective of other parameters.

3.3.2.3 The electricity company: wholesale price

Anticipating the total solar panels investment by the farm and the crowdfunders, the electricity company chooses the wholesale price to minimize its total procurement cost:

$$\mathbb{E}\left[C_c(\mathbf{w})\right] = \sum_{t=1}^{T} \left[q_t D_t + (w_t - q_t)\, \mu_g N^*(\mathbf{w})\right]. \tag{3.22}$$

Solving the equation system $\nabla_{\mathbf{w}} \mathbb{E}[C_c(\mathbf{w})] = 0$ to derive the optimal wholesale price $\mathbf{w}^*$ and substituting $\mathbf{w}^*$ to the farm owner's optimal strategy $N_0^*(\mathbf{w}), c^*(\mathbf{w}), \mathbf{r}^*(\mathbf{w})$, we achieve the sub-game perfect equilibrium in the model with crowdfunding in Proposition 3.2.

Proposition 3.2. *There is a subgame perfect equilibrium, which satisfies:*

(a) The wholesale price of electricity offered by the electricity company is

$$w_t^* = w^* \equiv \frac{2}{\frac{1}{b/\mu_g} + \frac{1}{q}}, t = 1, 2, \cdots, T \tag{3.23}$$

(b) The contract offered by the green farm in equilibrium is:

$$r_t^* = r^* \equiv \frac{4}{\frac{1}{q} + \frac{3}{b/\mu_g}}, t = 1, 2, \cdots, T \tag{3.24}$$

$$c^* = B \tag{3.25}$$

(c) The number of solar panels invested by crowdfunder i is $n_i^* = \frac{\beta_i(q\mu_g - b)(b+3q\mu_g)}{16bq^2\sigma_g^2}$ and by

the whole group is:

$$N_C^* = \frac{\beta\left(q\mu_g - b\right)\left(b + 3q\mu_g\right)}{16bq^2\sigma_g^2}. \tag{3.26}$$

The number of solar panels invested by green farm is:

$$N_0^* = \frac{\rho\left(\mu_g^2 q^2 - b^2\right)}{4bq^2\sigma_g^2}\left(1 - \frac{\beta(q\mu_g - b)}{4\rho(q\mu_g + b)}\right). \tag{3.27}$$

The overall number of solar panel is:

$$N^* = \frac{\rho\left(\mu_g^2 q^2 - b^2\right)}{4bq^2\sigma_g^2}\left(1 + \frac{\beta}{2\rho}\right). \tag{3.28}$$

(d) In the equilibrium, the total expected procurement is:

$$C^* = \sum_{t=1}^{T} q_t D_t - T\left(1 + \frac{\beta}{2\rho}\right)\frac{\rho\mu_g\left(q\mu_g - b\right)^2}{4bq\sigma_g^2}. \tag{3.29}$$

Proposition 3.2(a) shows that the optimal wholesale price determined by the electricity company with crowdfunding, is the same as the optimal wholesale price without crowdfunding. Proposition 3.2(b) shows how the farm designs the crowdfunding contract. First, $c^* = B$, meaning that for per unit solar panel the farm charges the crowdfunders the same as the purchasing and maintenance cost paid by the farm, irrespective of other parameters. In other words, all the investment costs are fairly allocated among investors and there is no extra charge for each investor. Second, under the assumption $q > b/\mu_g$, comparing the crowdfunding return r^* with the wholesale price w^* in each period in equilibrium, we find that $r^* < w^*$, which indicates that the farm owner only returns a portion of investment return to the crowdfunders. The crowdfunding return r^* increases in the average market price q and the average cost of green energy generation b/μ_g.

3.3.3 Comparison

Now we look at the impact of the crowdfunding through comparing the sub-game perfect equilibrium with crowdfunding with the sub-game perfect equilibrium without crowdfunding.

Proposition 3.3. *After introducing crowdfunding, we have:*

(a) the farm owner's sales revenue from green energy generation stays the same, but the investment cost is reduced by $\frac{\beta(q\mu_g - b)}{4\rho(q\mu_g + b)}$*;*

(b) the overall investment in solar panel increases by $\frac{\beta}{2\rho}$ *(i.e.,* $\frac{N^*}{N_0^\sharp} = 1 + \frac{\beta}{2\rho}$*);*

(c) the procurement cost saving from green energy procurement increases by $\frac{\beta}{2\rho}$*.*

Proposition 3.3(a) suggests that by setting the return of crowdfunding lower than the wholesale price, the farm owner achieves the same revenue while reducing his own investment cost in solar panels.

Proposition 3.3(b) shows that the overall investment in solar panels is always larger than that under the traditional investment setting without crowdfunding. This result indicates that crowdfunding does help ramp up the green energy generation. The increase in the overall solar panel investment is mainly contributed by the crowdfunders. Since the ratio increases in the average risk tolerance of crowdfunders β and decreases in the risk tolerance of the farm owner ρ, it implies that the farm owner uses crowdfunding as leverage to shift the investment risk.

Proposition 3.3(c) indicates that the electricity company also benefits from crowdfunding, which is driven by the penetration of green energy increased by crowdfunding. Since the average cost of green energy generation is lower than the average market price, the wholesale price is lower than the market price. Therefore, more green energy supply implies more procurement cost saving for the electricity company, which also increases the welfare of energy consumers.

3.4 Simulation

In the previous, section we have analytically proved that crowdfunding can help increase renewable energy investment. In this section, we aim to show the practical impact through simulations with real data. We aim to compare the effect of government policies and technology improvement in solar panels without crowdfunding with the benefit of crowdfunding in increasing green energy generations.

We use real data to determine the parameters in our model. For the average market price q, we use the daily average price in Southern California of 2013 based on the data provided by U.S. Energy Information Administration (EIA)[2], which is \$48.45 per MWh. According to the report by solar energy industry association, in 2013, the installation cost for solar photovoltaic (PV) with a capacity of 1 Watt is \$2.59 (Solar Energy Industries Association, 2013). Since the majority of manufacturers offer the 25-year standard solar panel warranty, we assume that the lifespan of a solar panel is 25 years, so the annual investment cost for solar photovoltaic (PV) with capacity of 1 Watt is $b = \$0.1036$ per Year. As for the generation parameter of solar farms, we calculate the statistics of solar generation based on a 5-year solar farm's annual average capacity factor during 2008-2012 (see Table 3.1). Capacity factor is defined as the ratio of the system's actual energy output during a fixed period to the potential output if the system ran at full capacity for the entire period (U.S. Department of Energy, 2011). Capacity factor of solar photovoltaic (PV) is primarily determined by solar insolation, so it varies with locations, weather and time of day, etc. Based on the data of capacity factor in Table 3.1, the mean and variance of energy generation of solar panel with capacity of 1 Watt is $\mu_g = 0.0027$ MWh per year and $\sigma_g = 4.9 \times 10^{-5}$ MWh per year.

Table 3.1: Net capacity factor (%) of a solar photovoltaic (PV) farm during 2008-2012

Year	2008	2009	2010	2011	2012
Capacity	31.0	30.5	29.6	30.7	30.9

There are no standard and well accepted values for the risk tolerance degree of the farm

[2]See http://www.eia.gov/electricity/wholesale/

owner ρ and the average risk tolerance degree of the crowdfunders β. According to the values of absolute risk aversion of managers used in Haubrich (1994), we choose the risk tolerance of the farm owner ρ to be 5. Similarly, we let the average risk tolerance of the crowdfunders β to be 20 following Choi et al. (2007).

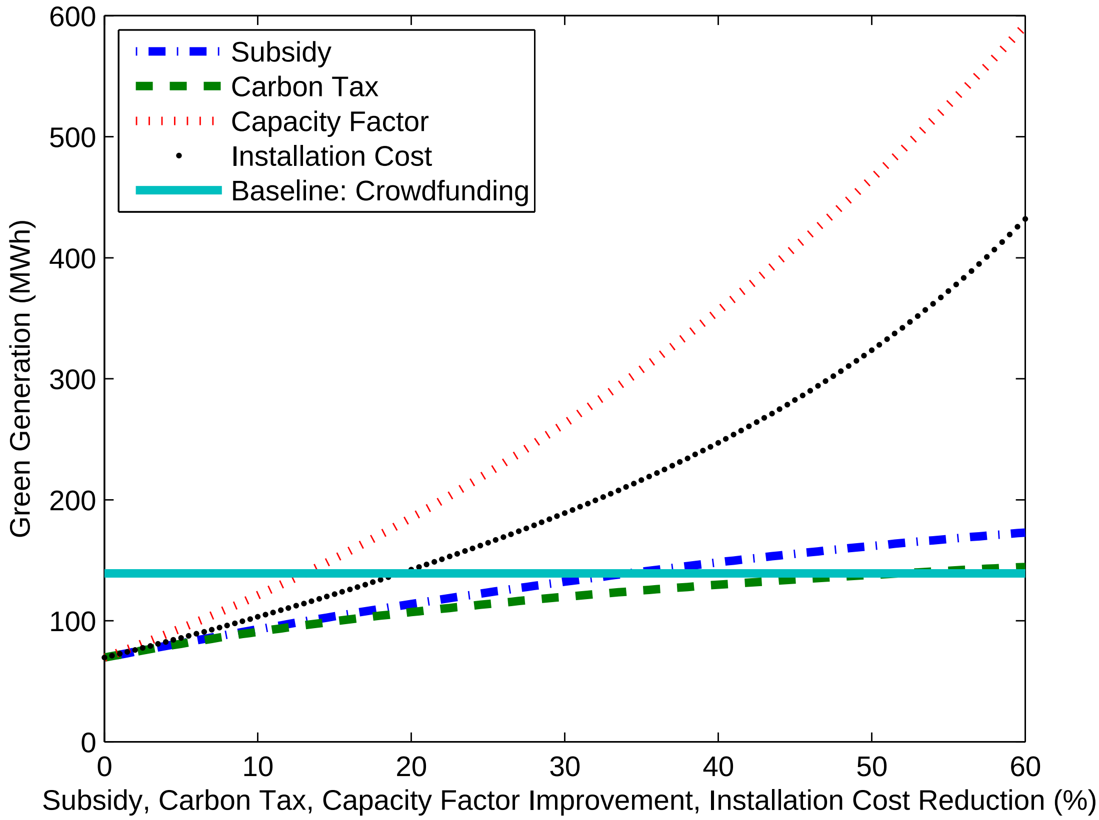

Figure 3.2: Comparison of Green Energy Generation under Subsidy, Carbon Tax, Increased Capacity Factor, Reduced Installation Cost, and Crowdfunding as a Baseline

We compare the impact of crowdfunding with that of government policies and technology improvement aiming to boost green generation. For government policies, we consider government subsidy and carbon tax. Subsidy is characterized by θ, meaning that for \$1 collected by the green farm, \$$\theta$ is from government subsidy, and \$$(1-\theta)$ is from the electricity company. In other words, under the government subsidy of θ, if the wholesale price is w for per unit green energy the green farm still receive w per unit but the electricity company only needs to pay $(1-\theta)w$ per unit. The carbon tax results in an increase in the average market price q. Specifically, a carbon tax τ results in the average market price increasing by τ. For technology improvement, we consider both increasing capacity factor and reducing installation cost. Figure 3.2 plots the green generations in five scenarios:

(a) no crowdfunding with subsidy,

(b) no crowdfunding with carbon tax,

(c) no crowdfunding with increased capacity factor,

(d) no crowdfunding with reduced installation cost,

(e) crowdfunding only.

Figure 3.2 shows that crowdfunding can achieve the same green generation as that under 34% of government subsidy, i.e., $\theta = 34\%$, or under 51% of carbon tax, i.e., $\tau = 51\%$ or with 13% increased in capacity factor, or with 19% decreased in installation cost. The result indicates that crowdfunding is a simple but effective way to boost green generation compared with government policies and technology improvement.

3.5 Conclusion and Future Work

In this paper, we studied an emerging investment pattern in green energy—crowdfunding, which is motivated by emerging community shared renewable energy projects represented by community solar farms. The pattern of crowdfunding investment for renewable energy has various merits but calls little attentions from (quantitative) researchers. Our paper filled the gap and made the following contributions. First, we developed a sequential game model to capture the strategic interactions during crowdfunding and derived the sub-game perfect equilibrium to the three-player sequential game. Second, from the equilibrium we obtained the optimal cost and reward allocation rule in crowdfunding. Thirdly, by comparing with a benchmark model without crowdfunding we analytically showed how crowdfunding benefits the stakeholders and increases the overall renewable energy investment level and hence the the green energy penetration in consumption. Finally, we numerically estimate the potential impact of crowdfunding in practice through simulations based on real data and find that crowdfunding is a simple but effective way to boost green generation compared with government policies and technology improvement.

One possible direction of future work is to consider “consumer-crowdfunders” who can not only contribute to the renewable energy supply, but also adjust their electricity demand to the renewable supply. In this sense, crowdfunding can not only increase the renewable energy generation but also the efficiency in renewable generation utilization.

APPENDIX

3.A Proof

Proof of Lemma 3.1:

Proof. The first-order condition is:

$$\frac{\partial CE_0\left(N_0, \mathbf{w}\right)}{\partial N_0} = 0 \tag{3.A.1}$$

$$\mu_g\left(\sum_{t=1}^{T} w - B\right) - \frac{1}{\rho} N_0 \sigma_g^2 \sum_{t=1}^{T} w_t^2 = 0 \tag{3.A.2}$$

Thus the farm owner's optimal investment level is:

$$N_0^{\#}\left(\mathbf{w}\right) = \rho \frac{\mu_g\left(\sum_{t=1}^{T} w_t - B\right)}{\sigma_g^2 \sum_{t=1}^{T} w_t^2} \tag{3.A.3}$$

□

Proof of Proposition 3.1:

Proof. By the first-order condition, we have

$$\begin{aligned}\frac{\partial E\left(C_0\left(\mathbf{w}\right)\right)}{\partial w_t} &= \mu_g N_0^{\#}\left(\mathbf{w}\right) + \sum_{t=1}^{T}\left(w_t - q_t\right)\mu_g \frac{dN_0^{\#}\left(\mathbf{w}\right)}{dw_t} \\ &= \mu_g N_0^{\#}\left(\mathbf{w}\right) + \sum_{t=1}^{T}\left(w_t - q_t\right)\mu_g \frac{\rho}{\sigma_g^2} \frac{\mu_g \sum_{t=1}^{T} w_t^2 - 2w_t\left(\mu_g \sum_{t=1}^{T} w_t - B\right)}{\left(\sum_{t=1}^{T} w_t^2\right)^2} \\ &= 0 \end{aligned} \tag{3.A.4}$$

which implies:

$$w_t = \frac{\mu_g \sum_{t=1}^{T} w_t^2 - \frac{\mu_g N_0^{\#}(\mathbf{w})\left(\sum_{t=1}^{T} w_t^2\right)^2}{\left(\sum_{t=1}^{T} w_t - \sum_{t=1}^{T} q_t\right)\mu_g \frac{\rho\mu_g}{\sigma_g^2}}}{2\left(\mu_g \sum_{t=1}^{T} w_t - B\right)} \tag{3.A.5}$$

From the symmetric formula of w_t, we have $w_t^{\#} = w^{\#}, t = 1, 2, \cdots, T$, and thus

$$N_0^{\#}(\mathbf{w}) = \rho \frac{\left(\mu_g \sum_{t=1}^{T} w_t - B\right)}{\sigma_g^2 \sum_{t=1}^{T} w_t^2} = \rho \frac{\left(\mu_g w^{\#} - b\right)}{\sigma_g^2 \left(w^{\#}\right)^2} \tag{3.A.6}$$

where $w^{\#}$ satisfies:

$$\mu_g \rho \frac{\mu_g \left(w^{\#} - b\right)}{\sigma_g^2 \left(w^{\#}\right)^2} + \left(T w^{\#} - \sum_{t=1}^{T} q_t\right) \mu_g \frac{\rho}{\sigma_g^2} \frac{\mu_g T \left(w^{\#}\right)^2 - 2w^{\#}\left(\mu_g T w^{\#} - B\right)}{\left(T\left(w^{\#}\right)^2\right)^2} = 0 \tag{3.A.7}$$

$$\left(\mu_g w^{\#} - b\right) + \left(w^{\#} - q\right) \frac{2b - \mu_g w^{\#}}{w^{\#}} = 0 \tag{3.A.8}$$

which implies:

$$w^{\#} = \frac{2}{\frac{1}{b/\mu_g} + \frac{1}{q}} \tag{3.A.9}$$

□

Proof of Lemma 3.2:

Proof. First, since $N_C = \beta \frac{\mu_g \sum_{t=1}^{T} r_t - c}{\sigma_g^2 \sum_{t=1}^{T} r_t^2}$, we have

$$\frac{\partial N_C}{\partial r_\tau} = \beta \frac{\mu_g \sigma_g^2 \sum_{t=1}^{T} r_t^2 - 2\sigma_g^2 r_\tau \left(\mu_g \sum_{t=1}^{T} r_t - c\right)}{\left(\sigma_g^2 \sum_{t=1}^{T} r_t^2\right)^2} = \left[\frac{\mu_g}{\left(\mu_g \sum_{t=1}^{T} r_t - c\right)} - \frac{2r_\tau}{\sum_{t=1}^{T} r_t^2}\right] N_C, \tag{3.A.10}$$

$$\frac{\partial N_C}{\partial c} = -\beta \frac{1}{\sigma_g^2 \sum_{t=1}^{T} r_t^2}. \tag{3.A.11}$$

Therefore, by the first order conditions, the optimal self-invested solar panels N_0 and contract parameters $c, r_\tau, \tau \in 1, 2, 3, \cdots, T$, satisfy the following equations:

$$\begin{aligned}
0 &= \mu_g \sum_{t=1}^{T} w_t - \frac{\sigma_g^2}{\rho} \sum_{t=1}^{T} \left[\left(w_t N_0 + \left(w_t - r_t \right) N_C \right) w_t \right] - B, \\
0 &= \left[\mu_g \sum_{t=1}^{T} \left(w_t - r_t \right) - \frac{\sigma_g^2}{\rho} \sum_{t=1}^{T} \left[\left(w_t N_0 + \left(w_t - r_t \right) N_C \right) \left(w_t - r_t \right) \right] + (c - B) \right] \frac{\partial N_C}{\partial c} + N_C, \\
0 &= \left[\mu_g \sum_{t=1}^{T} \left(w_t - r_t \right) - \frac{\sigma_g^2}{\rho} \sum_{t=1}^{T} \left[\left(w_t N_0 + \left(w_t - r_t \right) N_C \right) \left(w_t - r_t \right) \right] + (c - B) \right] \frac{\partial N_C}{\partial r_\tau} \\
& \quad - \mu_g N_C + \frac{\sigma_g^2}{\rho} \left(w_\tau N_0 + \left(w_\tau - r_\tau \right) N_C \right) N_C.
\end{aligned}$$

By the second and the third equation, we have $\forall \tau \in 1, 2, 3, \cdots, T$

$$\begin{aligned}
-\frac{N_C}{\frac{\partial N_C}{\partial c}} &= \mu_g \sum_{t=1}^{T} \left(w_t - r_t \right) - \frac{\sigma_g^2}{\rho} \sum_{t=1}^{T} \left[\left(w_t N_0 + \left(w_t - r_t \right) N_C \right) \left(w_t - r_t \right) \right] + (c - B) \\
&= \frac{\mu_g N_C - \frac{\sigma_g^2}{\rho} \left(w_\tau N_0 + \left(w_\tau - r_\tau \right) N_C \right) N_C}{\frac{\partial N_C}{\partial r_\tau}}. \qquad (3.A.12)
\end{aligned}$$

Since

$$\begin{aligned}
-\frac{N_C}{\frac{\partial N_C}{\partial c}} &= \frac{\beta \frac{\mu_g \sum_{t=1}^{T} r_t - c}{\sigma_g^2 \sum_{t=1}^{T} r_t^2}}{\beta \frac{1}{\sigma_g^2 \sum_{t=1}^{T} r_t^2}} = \mu_g \sum_{t=1}^{T} r_t - c, \qquad (3.A.13) \\
\frac{\mu_g N_C - \frac{\sigma_g^2}{\rho} \left(w_\tau N_0 + \left(w_\tau - r_\tau \right) N_C \right) N_C}{\frac{\partial N_C}{\partial r_\tau}} &= \frac{\mu_g - \frac{\sigma_g^2}{\rho} \left(w_\tau N_0 + \left(w_\tau - r_\tau \right) N_C \right)}{\frac{\mu_g}{\left(\mu_g \sum_{t=1}^{T} r_t - c \right)} - \frac{2 r_\tau}{\sum_{t=1}^{T} r_t^2}}, \qquad (3.A.14)
\end{aligned}$$

we can obtain that $\forall \tau \in 1, 2, 3, \cdots, T$, we have

$$\begin{aligned}
&\frac{\mu_g - \frac{\sigma_g^2}{\rho}\left(w_\tau N_0 + \left(w_\tau - r_\tau\right) N_C\right)}{\frac{\mu_g}{\left(\mu_g \sum_{t=1}^T r_t - c\right)} - \frac{2r_\tau}{\sum_{t=1}^T r_t^2}} = \mu_g \sum_{t=1}^T r_t - c \\
\Rightarrow &\mu_g - \frac{\sigma_g^2}{\rho}\left(w_\tau N_0 + \left(w_\tau - r_\tau\right) N_C\right) = \mu_g - \frac{2r_\tau\left(\sum_{t=1}^T r_t - c\right)}{\sum_{t=1}^T r_t^2} = \mu_g - \frac{2N_C\sigma_g^2}{\beta} r_\tau \\
\Rightarrow &\mu_g - \frac{\sigma_g^2}{\rho} w_\tau \left(N_0 + N_C\right) + \frac{\sigma_g^2}{\rho} r_\tau N_C = \mu_g - \frac{2N_C\sigma_g^2}{\beta} r_\tau \\
\Rightarrow &r_\tau = \frac{\frac{\sigma_g^2}{\rho} w_\tau \left(N_0 + N_C\right)}{\frac{\sigma_g^2}{\rho} N_C + \frac{2N_C\sigma_g^2}{\beta}} = \frac{\frac{1}{\rho}\left(\frac{N_0}{N_C} + 1\right)}{\frac{1}{\rho} + \frac{2}{\beta}} w_\tau. \qquad (3.A.15)
\end{aligned}$$

Therefore, we have

$$r_\tau = \alpha w_\tau, \qquad (3.A.16)$$

where

$$\alpha = \frac{\frac{1}{\rho}\left(\frac{N_0}{N_C} + 1\right)}{\frac{1}{\rho} + \frac{2}{\beta}}. \qquad (3.A.17)$$

From the second equation of the first order conditions above, we can get:

$$\begin{aligned}
&\left[\mu_g \sum_{t=1}^T \left(w_t - r_t\right) - \frac{\sigma_g^2}{\rho} \sum_{t=1}^T \left[\left(w_t N_0 + \left(w_t - r_t\right) N_C\right)\left(w_t - r_t\right)\right] + (c - B)\right] \frac{\partial N_C}{\partial c} + N_C = 0 \\
\Rightarrow &\mu_g (1 - \alpha) \sum_{t=1}^T w_t - (1 - \alpha)\left(\mu_g \sum_{t=1}^T w_t - B\right) + (c - B) = \mu_g \alpha \sum_{t=1}^T w_t - c \\
\Rightarrow &c = \frac{\alpha}{2}\left(B + \mu_g \sum_{t=1}^T w_t\right), \qquad (3.A.18)
\end{aligned}$$

in which, from the first equation of the first order conditions above:

$$\begin{aligned}
\frac{\sigma_g^2}{\rho} \sum_{t=1}^T \left[\left(w_t N_0 + \left(w_t - r_t\right) N_C\right)\left(w_t - r_t\right)\right] &= (1 - \alpha) \frac{\sigma_g^2}{\rho} \sum_{t=1}^T \left[\left(w_t N_0 + \left(w_t - r_t\right) N_C\right) w_t\right] \\
&= (1 - \alpha)\left(\mu_g \sum_{t=1}^T w_t - B\right). \qquad (3.A.19)
\end{aligned}$$

It immediately implies:

$$\begin{aligned} N_C &= \beta \frac{\mu_g \sum_{t=1}^T r_t - c}{\sigma_g^2 \sum_{t=1}^T r_t^2} = \beta \frac{\mu_g \alpha \sum_{t=1}^T w_t - c}{\sigma_g^2 \alpha^2 \sum_{t=1}^T w_t^2} = \beta \frac{\mu_g \sum_{t=1}^T w_t - B}{2\sigma_g^2 \alpha \sum_{t=1}^T w_t^2}, \quad (3.A.20) \\ N_0 &= \left(\left(1 + \frac{2\rho}{\beta} \right) \alpha - 1 \right) N_C \\ &= \left(1 + \frac{2\rho}{\beta} \right) \beta \frac{\mu_g \sum_{t=1}^T w_t - B}{2\sigma_g^2 \sum_{t=1}^T w_t^2} - \beta \frac{\mu_g \sum_{t=1}^T w_t - B}{2\sigma_g^2 \alpha \sum_{t=1}^T w_t^2}. \quad (3.A.21) \end{aligned}$$

Note that:

$$\begin{aligned} \frac{\partial N_C(\alpha)}{\partial \alpha} &= -\frac{N_C(\alpha)}{\alpha}, \quad (3.A.22) \\ N_0(\alpha) + (1-\alpha) N_C(\alpha) &= \frac{2\rho}{\beta} \alpha N_C(\alpha) \\ &= \rho \frac{\mu_g \sum_{t=1}^T w_t - B}{2\sigma_g^2 \sum_{t=1}^T w_t^2}. \quad (3.A.23) \end{aligned}$$

By writing the original optimization problem as a function of α, we can get:

$$\begin{aligned} & \mu_g \left(\sum_{t=1}^T w_t N_0(\alpha) + (1-\alpha) \sum_{t=1}^T w_t N_C(\alpha) \right) - \frac{\sigma_g^2}{2\rho} \sum_{t=1}^T \left(w_t N_0(\alpha) + (1-\alpha) w_t N_C(\alpha) \right)^2 \\ & -B N_0(\alpha) + (c(\alpha) - B) N_C(\alpha) \\ = \; & \mu_g \left(\sum_{t=1}^T w_t \rho \frac{\mu_g \sum_{t=1}^T w_t - B}{2\sigma_g^2 \sum_{t=1}^T w_t^2} \right) - \frac{\sigma_g^2}{2\rho} \sum_{t=1}^T \left(w_t \rho \frac{\mu_g \sum_{t=1}^T w_t - B}{2\sigma_g^2 \sum_{t=1}^T w_t^2} \right)^2 \\ & -B \left(1 + \frac{2\rho}{\beta} \right) \beta \frac{\mu_g \sum_{t=1}^T w_t - B}{2\sigma_g^2 \sum_{t=1}^T w_t^2} + B\beta \frac{\mu_g \sum_{t=1}^T w_t - B}{2\sigma_g^2 \alpha \sum_{t=1}^T w_t^2} \\ & + \left(\frac{\alpha}{2} \left(B + \mu_g \sum_{t=1}^T w_t \right) - B \right) \beta \frac{\mu_g \sum_{t=1}^T w_t - B}{2\sigma_g^2 \alpha \sum_{t=1}^T w_t^2} \\ = \; & \mu_g \left(\sum_{t=1}^T w_t \rho \frac{\mu_g \sum_{t=1}^T w_t - B}{2\sigma_g^2 \sum_{t=1}^T w_t^2} \right) - \frac{\sigma_g^2}{2\rho} \sum_{t=1}^T \left(w_t \rho \frac{\mu_g \sum_{t=1}^T w_t - B}{2\sigma_g^2 \sum_{t=1}^T w_t^2} \right)^2 \\ & -B \left(1 + \frac{2\rho}{\beta} \right) \beta \frac{\mu_g \sum_{t=1}^T w_t - B}{2\sigma_g^2 \sum_{t=1}^T w_t^2} + \frac{1}{2} \left(B + \mu_g \sum_{t=1}^T w_t \right) \beta \frac{\mu_g \sum_{t=1}^T w_t - B}{2\sigma_g^2 \sum_{t=1}^T w_t^2}, \quad (3.A.24) \end{aligned}$$

which does not depend on α as long as $\alpha > \frac{1}{1+\frac{2\rho}{\beta}}$. Therefore, the solutions is

$$\begin{aligned}
c &= \frac{\alpha}{2}\left(B+\mu_g\sum_{t=1}^{T}w_t\right), && (3.A.25)\\
r_\tau &= \alpha w_\tau, && (3.A.26)\\
N_C &= \beta\frac{\mu_g\sum_{t=1}^{T}w_t-B}{2\sigma_g^2\alpha\sum_{t=1}^{T}w_t^2}, && (3.A.27)\\
N_0 &= \left(1+\frac{2\rho}{\beta}\right)\beta\frac{\mu_g\sum_{t=1}^{T}w_t-B}{2\sigma_g^2\sum_{t=1}^{T}w_t^2}-\beta\frac{\mu_g\sum_{t=1}^{T}w_t-B}{2\sigma_g^2\alpha\sum_{t=1}^{T}w_t^2}, && (3.A.28)\\
N &= \left(1+\frac{2\rho}{\beta}\right)\beta\frac{\mu_g\sum_{t=1}^{T}w_t-B}{2\sigma_g^2\sum_{t=1}^{T}w_t^2}. && (3.A.29)
\end{aligned}$$

Specifically, if we follow the practice and assign $c = B$, we have

$$\begin{aligned}
\alpha &= \frac{2B}{B+\mu_g\sum_{t=1}^{T}w_t}, && (3.A.30)\\
N_C &= \beta\frac{\left(\mu_g\sum_{t=1}^{T}w_t\right)^2-B^2}{4B\sigma_g^2\sum_{t=1}^{T}w_t^2}, && (3.A.31)\\
N_0 &= \frac{\mu_g\sum_{t=1}^{T}w_t-B}{\sigma_g^2\sum_{t=1}^{T}w_t^2}\left(\rho-\frac{\beta\left(\mu_g\sum_{t=1}^{T}w_t-B\right)}{4B}\right), && (3.A.32)\\
N &= \left(1+\frac{2\rho}{\beta}\right)\beta\frac{\mu_g\sum_{t=1}^{T}w_t-B}{2\sigma_g^2\sum_{t=1}^{T}w_t^2}. && (3.A.33)
\end{aligned}$$

Hence, we have the results stated in the proposition. □

Proof of Proposition 3.2:

Proof. Since $N^*(\mathbf{w}) = \left(1+\frac{\beta}{2\rho}\right)N_0^{\#}(\mathbf{w})$, the optimal wholesale price is the same as the case without crowdfunding. □

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