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Presented by

Yongjin Kim

Accepted by

Bryan Routledge	**4/20/15**
Chair: Prof. Bryan Routledge	**Date**

Approved by The Dean

Robert M. Dammon	**4/22/15**
Dean Robert M. Dammon	**Date**

DOCTORAL DISSERTATION

Essays on Corporate Investment

by

Yongjin Kim

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Dissertation Committee:

Associate Professor Bryan R. Routledge
Associate Professor Lars-Alexander Kuehn
Professor Richard C. Green
Assistant Professor Brent Glover
Assistant Professor Stephen A. Karolyi

Abstracts

In the first essay (joint work with Bryan Routledge), we calculate the value implications of sub-optimal capital budgeting decisions in an asset-pricing model calibrated to match the standard asset pricing empirical properties – in particular, the time-variation in the equity premium. Specifically, we calculate that an investment policy that ignores the time variation in the equity premium, such as would occur with a cost of capital following the CAPM, incurs a 14.8% value loss. We also document the implications for a firm's asset returns in this context.

The second essay revisits the relation between firms' choices of debt maturity and their investment in a dynamic world. Prior research, including Myers (1977), suggests that financing with short-term debt resolves the underinvestment problem caused by debt financing. In contrast, I establish that short-term debt can reduce the incentive to invest due to larger exposure to default risk from more frequent debt rollovers. Long-term debt, however, is more subject to illiquidity costs, so firms find optimal maturity by balancing these opposing forces. For the firm with average investment and financing, the agency cost arising from the underinvestment is 0.77% of firm value. This suggests that previous studies overestimate the cost by ignoring firms' flexibility in choosing maturity. I also measure firm-specific agency costs using likelihood-based structural estimation. The measured agency costs show significant cross-sectional variation due to heterogeneity in firm characteristics and convexity of the agency costs. The economy-wide average of the costs is 7.28%, which is considerably higher than the cost for the average firm.

In the third essay, I empirically test whether firms' investment decisions take time-varying risk into account. I construct the firm-specific risk premium implicit in option prices. Specifically, individual equity options and historical equity returns are used to infer the joint distribution of the stochastic discount factor and equity return, and the joint distribution determines the risk premium of the equity. The result is that firms' actual investments correctly respond to the time-varying risk premium lagged by 3 to 5 quarters. However, firm-by-firm analyses show that some firms do not adapt appropriately to the risk as theory predicts, demonstrating a room for improvement in capital budgeting practice for those firms.

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Does Macro-Asset Pricing Matter for Corporate Finance?

1.1 Introduction

In business practice and education, the Net Present Value (NPV) rule is widely used for a firm's capital budgeting decision. The wide spread use of NPV is one of the great successes of business education. Evaluation of the NPV, of course, entails discounting future cash flows at an appropriate cost of capital. The work-horse model used in both the classroom and industry is the Capital Asset Pricing Model. While there is much ad-hoc adjustment in use (e.g., CFO's tend to round up the cost of capital), the CAPM is the de facto standard way to determine the risk-adjusted discount rate. Graham and Harvey (2001) survey companies throughout the U.S. and Canada and find that 74.9% of respondents use the NPV for capital budgeting, and 73.5% use the CAPM.

The CAPM is, of course, a static model and is agnostic about the dynamic properties of the equity premium. Thus, the use of the CAPM in practice implies that the discount rate is constant across time or economic-state. Typically, people use a number like 5% or 6% (Welch (2000) and subsequent update). In contrast, the central feature of research in macro-asset pricing for at least the last decade has been focussed on not just explaining the level of the equity premium ("the equity premium puzzle"), but in understanding its dynamic properties. Cochrane (2011), for example, points out that the time-variation in the equity premium is on the same order of magnitude as the level. That is; the equity premium swings between 1% and 11%. Given this fluctuation, using a constant CAPM-inspired discount rate is sub-optimal. In this paper, we quantify the value loss caused by an investment policy that ignores the time-variation in risk premium.

To measure the quantitative implications, it is necessary to construct a model that is reasonably well calibrated. With this objective in mind, we build a model for the underlying economic environment and firm-level investment. In order to capture in a tractable way how a firm managers' characterization of risks influences its investment decision, we assume and calibrate a standard endowment economy. We tune the model to have time-varying risk or, counterfactually, not. Specifically, the economic environment is based on the long-run risk models of Bansal and Yaron (2004). We use the version from Backus, Routledge, and Zin (2010), where the economy is described by two state Markov process; the two states are the expected growth of endowment and the volatility of the growth. Here, the time-variation in risk premium arises from the stochastic volatility of endowment growth.

Given the setup of economic environment, we model firm-level investment as follows. On each date, a firm receives an opportunity to invest in a new project. The investment project is exposed to systematic risk in that its future cash flows are correlated to aggregate endowment. The firm evaluates the NPV of the project based on the perceived state of economy as well as project-specific characteristics. If the evaluated NPV turns out positive, the firm invests. If the firm does not invest, the opportunity vanishes. This resembles the now-or-never options in Berk, Green, and Naik (1999). Projects have a finite life. Hence, assets in place evolve as new projects come in and old ones retire. Given the firm operation, the firm value consists of the value of the existing projects as well as the value of future opportunities. Since how firm managers invest will depend on their model of the economy – is the price of risk time varying? – firm value, both assets in place and growth options, will also depend on their model. If the firm fails to model the price of risk or discount rates correctly, it will incur a value loss as a result of sub-optimal investment decisions. Here, we quantify the size of this loss.

The basic idea is to consider two economies. One economy will have constant equity premium (from a constant volatility assumption) and the other economy will feature time-variation in the equity premium. While both of these calibrations will match the usual moments of aggregate asset returns, only second economy generates a dynamic equity premium. In each of these two economies, we will consider two representative firms and their investment policies. One firm - Type 1 - will act as if the equity premium is constant. The other - Type 2 - will act as if the equity premium is dynamic. This will let us consider the optimal investment behavior (Type 1 in the constant-volatility economy and Type 2 in the stochastic-volatility economy) as well as measure the cost of a sub-optimal policy. Thus we can measure the cost of acting as a Type 1 firm (a CAPM-like cost of capital) in a world with a dynamic equity premium. We can also measure the counter-factual cost of a Type 2 firm that happens to live in a world with a static equity premium. In addition, we also look at the returns produced by firms in each of these settings.

The estimate of value loss and return differentials also depend on project-specific characteristics. We calibrate these characteristics so that the average of the book-to-market ratio in a simulated firm-panel replicates its empirical counterpart. Within the calibrated economies, the estimated value loss is as follows. In a world with dynamic equity premium, the sub-optimally investing Type 1 firm has the present value of growth options 14.8% lower than the Type 2. In contrast, if the world features constant equity premium, as implied in the CAPM, the Type 2 firm incurs only 0.8% loss in growth option by its sup-optimal investments. The asymmetry in the value loss is largely driven by the timing of sub-optimal investment. In the world with dynamic equity premium, the Type 1 firm overinvests most at the state of highest uncertainty in growth, the exact state where the marginal rate of substitution is the highest. On the contrary, the Type 2 firm in the economy with constant equity premium is not exposed to such coordination between erroneous investment and the marginal rate of substitution, thus having a lower value loss. In addition to quantification of value losses, we also document the firm types yield statistically significant return differences in each economy. The returns on firms with sub-optimal investment policy are higher than those with correct investment in both economies.

The paper is organized as follows. In the next section, we describe the economic environment and firm-level investments. In section 1.3, we provide the valuations of projects and then derive the investment rule and the resulting firm values. In section 1.4, we calibrate the model and examine differences in firm values and returns on firm caused by the firm type and investment rule. Section 5 concludes.

1.2 Model

We model the pricing kernel that results from a standard Bansal and Yaron (2004) endowment economy. Characterizing the pricing kernel as the product of preferences and consumption growth facilitates calibration (and comparison to other well-known models). In addition, we model investment projects that deliver cash flows exposed to consumption growth risk. This set-up gives the projects their "systematic" risk. Our model is only partial-equilibrium since we do not connect the sum of all projects in the economy back to aggregate consumption.

1.2.1 Economic Environments

Preferences of the representative agent are recursive as in Epstein and Zin (1989), and Weil (1989). The decision interval is one month. Preferences at date t are given by

$$U_t = [(1-\beta)c_t^{\rho} + \beta\mu\,(U_{t+1})^{\rho}]^{1/\rho} \tag{1.1}$$

where μ is the certainty equivalent, i.e., $\mu(U_{t+1}) = E_t\left[U_{t+1}^{\alpha}\right]^{1/\alpha}$. The marginal rate of substitution – the pricing kernel – is

$$m_{t+1} \quad = \quad \beta(c_{t+1}/c_t)^{\rho-1}\left[U_{t+1}/\mu(U_{t+1})\right]^{\alpha-\rho}$$

where $(c_{t+1}/c_t)^{\rho-1}$ accounts for the short-run consumption growth risk, and the next term $(U_{t+1}/\mu_t(U_{t+1}))^{\alpha-\rho}$ captures the effect of the agent's expectation on future utility. The derivation of the pricing kernel is provided in appendix A.1 through A.3. (It is algebra similar to Backus et al. (2010)).

We specify an exogenous stochastic process for consumption growth. The consumption growth from date $t-1$ to t, denoted by $g_t = c_t/c_{t-1}$, is described with the underlying state variable, x_t, a vector of arbitrary dimension. Specifically, the logarithm of consumption growth is assumed to be $\log g_t = g + e^T x_t$, where e is a constant vector. The dynamics of the state variable x_t features AR(1) with a stochastic volatility:

$$\begin{aligned} x_{t+1} &= Ax_t + v_t^{1/2}Bw_{t+1} \\ v_{t+1} &= (1-\varphi)v + \varphi v_t + bw_{t+1} \end{aligned} \tag{1.2}$$

where v is the unconditional mean of v_t, $\{w_t\} \sim NID(0, I)$, and $Bb^T = 0$. With this representation, x_t and v_t controls the conditional mean and volatility of consumption growth in future, respectively, and they summarize the state of economy.[1] The stochastic volatility in the growth is the source of creating time-variation in equity premium. Therefore, we can represent the different types of economy or firm - having the time-varying equity premium or not - by turning on or off the stochastic volatility channel while keeping others equal. Thus, for the description of the economy with constant equity premium, we impose that the volatility of consumption growth is constant.

With these agent preferences and consumption dynamics, we can derive the pricing kernel. The logarithm of the pricing kernel is

$$\log m_{t+1} \quad = \quad \delta_0 + \delta_x^T x_t + \delta_v v_t + \lambda_x^T w_{t+1} + \lambda_v^T w_{t+1} \tag{1.3}$$

where δ_0, δ_x, δ_v, λ_x, and λ_v are known functions of preference and consumption dynamics parameters. The derivation of the pricing kernel is provided in appendix A.2 and A.3. As equation (1.3) shows, the pricing kernel changes across the states of the economy, i.e., the conditional mean and volatility of the consumption growth, and innovations. If a firm manager does not perceive correctly the underlying economy and risks in growth, the manager's pricing kernel will be different from the other with correct understanding of risks. This discrepancy leads to different evaluations for the same cash flows, so they would invest differently from each other.

1.2.2 Firms

Firms operate with an infinite horizon. Individual projects are finite-lived. At each date, a new opportunity becomes available to a firm. The firm decides whether to undertake the new project or not. If a new project is not undertaken, that opportunity is gone (now-or-never option), and the firm will receive a new opportunity next date. If the firm decides to invest in a new project, the initial cash flow is negative (investment) followed by subsequent positive cash flows. Project termination is deterministic in our setting.

To undertake a project at date t requires upfront investment of Id_t, where d_t represents the size

[1] If $e = \begin{bmatrix} 1 & 0 \end{bmatrix}$, $A = \begin{bmatrix} \phi & \theta \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} \sigma_e & 0 \\ \sigma_e & 0 \end{bmatrix}$ and $b = \begin{bmatrix} 0 & \sigma_w \end{bmatrix}$, the dynamics is an approximation of the dynamics in Bansal and Yaron (2004) with stochastic volatility. The Guassian shock to volatility is an approximation, obviously. It is straightforward to relax but makes the algebra less transparent.

of cash flows from the project. Once undertaken, the project delivers cash flows of which growth is correlated to consumption growth, and the positive cash flows start at date $t+1$. Let d_{t+s} denote cash flow at date $t+s$ from the project. The growth in cash flow at date $t+s$ is given by

$$\frac{d_{t+s}}{d_{t+s-1}} = \exp\left(g + e^T(Ax_{t+s-1} + \beta_t v_{t+s-1}^{1/2} B w_{t+s}) - \frac{\beta_t^2 v_{t+s-1}}{2} e^T B B^T e\right). \tag{1.4}$$

where β_t controls the covariance between the cash flows and consumption, thereby capturing systematic risk of the project. The basic idea of the expression for cash flow is that the mean growth and volatility of the project-level cash flows are influenced by the economic state x_t and v_t, respectively, which describe consumption growth. The project generates these cash flows during lifetime N and becomes obsolete N periods after the starting date. Note that the systematic risk β_t is project-specific. The β_t is drawn from a distribution and known at the date of investment decision and the realized β_t is constant for the life of the project. For simplicity, we assume that systematic risk β_t before realization is uniformly distributed over $[0, \beta_{max}]$.

This specification of project might seem to imply too strong tie between a project payout and aggregate consumption. However, in the firm-level, the specification still enables an imperfect correlation between the firm payout and consumption, as in Bansal and Yaron (2004). This is because the firm-level payout on a date is a collection of cash flows of projects that have idiosyncratic covariance with consumption. The idiosyncracy produces a loose link between the firm payout and consumption, while the exposure to consumption growth risk captures the systematic risk.

1.3 Valuation

The projects we consider have cash flows across time, so we begin the valuation by pricing elementary assets that deliver cash flow at a single date. With the values of these elementary assets, we can evaluate the project and analyze the firm's investment decision. Also we evaluate the value of resulting assets in place and the value of future investment opportunities or growth options.

1.3.1 The Valuation of Project Payout

Consider an elementary asset that delivers risky cash flow d_{t+s} with the systematic risk β_j at a single date $t+s$. Let q_t^s denote the date t-price-payout ratio of the asset. The price at t of the asset that pays at date $t+1$ is determined by

$$q_t^1 = E_t\left[m_{t+1}\frac{d_{t+1}}{d_t}\right]. \tag{1.5}$$

Similar to the prices of "zero-coupon equity" in Lettau and Wachter (2007), the price is an exponential affine function of the state variables as follows:

$$q_t^1 = \exp\left(\delta_0 + g + \frac{\lambda_v^T\lambda_v}{2} + (\delta_x^T + e^T A)x_t + \left(\delta_v + \frac{\lambda_x^T\lambda_x^T}{2} + \beta_j e^T B\lambda_x\right)v_t\right). \tag{1.6}$$

where δ_0, δ_x, δ_v, λ_x, and λ_v are known functions of parameters for preferences and consumption dynamics. The price-payout ratio of the asset with maturity $s > 1$ is

$$q_t^s = \exp\left(D_{0,s} + D_{x,s}x_t + D_{v,s}v_t\right) \tag{1.7}$$

where $D_{0,s}$, $D_{x,s}$, and $D_{v,s}$ are the constants which are recursively related with the constants for the asset with maturity of $s-1$ in the following way:

$$\begin{aligned}
D_{0,s} &= \delta_0 + g + D_{0,s-1} + D_{v,s-1}(1-\varphi)v + (1/2)\left(\lambda_v^T + D_{v,s-1}b\right)\left(\lambda_v^T + D_{v,s-1}b\right)^T \\
D_{x,s} &= \delta_x^T + (e^T + D_{x,s-1})A \\
D_{v,s} &= \delta_v + D_{v,s-1}\phi_v - (1/2)\beta^2 e^T BB^T e + (1/2)(\lambda_x^T + \beta e^T B + D_{x,s-1}B)(\lambda_x^T + \beta e^T B + D_{x,s-1}B)^T.
\end{aligned} \tag{1.8}$$

The derivation is presented in appendix A.5.

1.3.2 The Valuation of Assets in Place

A firm has the two sources of the value - assets in place and growth options. Assets in place refer to a collection of existing projects coming from past investment decisions. Growth options denote the value of future investment opportunities.

The NPV of the date-t project, say P_t, is

$$P_t = E_t\left[\sum_{s=1}^{N} m_{t,t+s}d_{t+s}\right] - Id_t \tag{1.9}$$

where $m_{t,t+s}$ is the marginal rate of substitution between consumption at t and at $t+s$, which is given by $m_{t,t+s} = \prod_{k=1}^{s} m_{t+k}$. The NPV normalized by payout, p_t, is

$$p_t = E_t \left[\sum_{s=1}^{N} m_{t,t+s} \frac{d_{t+s}}{d_t} \right] - I = \sum_{s=1}^{N} q^s(x_t, v_t, \beta_t) - I \tag{1.10}$$

where the value is expressed with the prices of the elementary assets in section 1.3.1. As the investment opportunity is a now-or-never option, the firm undertakes the project whenever its NPV, p_t, is positive. Thus the firm's investment decision at date t also depends on both the project-specific shock, β_t, and the state of the economy, (x_t, v_t). Finally, the firm's perception of the economy and the pricing kernel may well influence investment policy.

We represent the investment decision at date j with an indicator χ_j such that $\chi_j = 1$ if the firm invests or 0 otherwise. Then the value of assets in place, denoted by K_t, is

$$K_t = \sum_{j=t-N+1}^{t} \sum_{s=1}^{N-t+j} \chi_j q^s(x_t, v_t, \beta_j).$$

The expression simply means that the assets in place include past projects which were undertaken at date $t - N + 1$ or afterwards, because projects become obsolete N periods after the inception date.

1.3.3 The Valuation of Growth Options

To value the firm's growth options, we consider a single investment opportunity that will arrive at $t+1$. Because the firm will take on the project only if its NPV turns out to be positive, the payoff of the investment opportunity is similar to that of a financial option. When both firm-specific shock and economic states are realized at date $t+1$, the option value is $\max(p_{t+1}, 0)$. Let $f(x_{t+1}, v_{t+1})$ denote the value of the option conditional on states (x_{t+1}, v_{t+1}) and prior to realization of project-specific risk β_{t+1}. The option value is expressed as follows:

$$f\left(x_{t+1}, v_{t+1}\right) = \int_0^{\overline{\beta}} p(x_{t+1}, v_{t+1}, \beta) \frac{1}{\beta_{\max}} d\beta \tag{1.11}$$

where $\overline{\beta}$ is the investment threshold for systematic risk such that $p(x_{t+1}, v_{t+1}, \overline{\beta}) = 0$.[2]

[2] Of course, $\overline{\beta}$ is a function of the state, (x_{t+1}, v_{t+1}), but we omit the argument for simplicity.

The firm has a series of investment options which will become available from $t+1$ onwards. The date-t present value of growth options, say $S(x_t, v_t)$, can be expressed in a recursive way:[3]

$$
\begin{aligned}
S(x_t, v_t) &= E_t\left[\sum_{s=1}^{\infty} \frac{m_{t,t+s}}{d_t} \max(P_{t+s}, 0)\right] \\
&= E_t\left[m_{t,t+1} E_{\beta_{t+1}}\left[\max\left(p_{t+1}, 0\right)\right] + m_{t,t+1}\frac{d_{t+1}}{d_t} E_{t+1}\left[\sum_{s=1}^{\infty} \frac{m_{t+1,t+1+s}}{d_{t+1}} \max(P_{t+1+s}, 0)\right]\right] \\
&= E_t\left[m_{t,t+1} f\left(x_{t+1}, v_{t+1}\right) + m_{t,t+1}\frac{d_{t+1}}{d_t} S\left(x_{t+1}, v_{t+1}\right)\right]. \qquad (1.12)
\end{aligned}
$$

In the derivation, we use the law of iterated expectation to value the investment option available at date $t+1$. From the recursive structure, the present value of growth options is solved numerically as a function of the state variables.[4]

With the expression of growth options, we can analyze the result of using an incorrect pricing kernel. Suppose a firm has an incorrect model of the risk – say it ignores the dynamic properties of the risk premium – then investment policy is sub-optimal. Moreover, the degree of the sub-optimality can be state-dependent. For example, if the firm tends to invest particularly badly when when the marginal rate of substitution is high, the value loss will be particularly large.

1.4 Quantitative Analysis

We are interested in the quantitative implications of a firm's investment policy. It is important to ask this question in a sensibly calibrated model. Here, we look at two economies - the one with constant risk premium and the other with dynamic risk premium. Both are set to match features of aggregate consumption and aggregate asset returns including the equity premium. After the economic environment is quantitatively specified, the project characteristics are chosen to reproduce the empirical book-to-market ratios. With the calibrated model both in aggregate level and firm-level, we analyze firms' investment policies and value implications of incorrect investment rules. Also, we study return differential on firms' assets by simulating firm panels in each economy.

[3] $E_{\beta_{t+1}}$ denotes the expectation over the distribution of systematic risk β_{t+1}.

[4] For the computation, we use a finite support of state variables by discretizing the support of the state based on the method of Tauchen (1985). We then solve for growth options with the value function iteration.

Table 1.1: Calibration Results

		Data	Dynamic Equity Premium	Constant Equity Premium
State variables			x_t, v_t	x_t
			Preference Parameters	
Risk parameter	α		-7	-7.5
IES parameter	ρ		0.9	0.85
			Endowment Parameters	
AR(1)	ϕ		0.90	0.90
MA(1)	θ		-0.70	-0.70
Volatility autocorrelation	φ		0.987	1
			Implications for Consumption Dynamics	
$AC(1)$		0.49	0.43	0.43
$AC(2)$		0.15	0.17	0.18
$AC(5)$		-0.08	0.09	0.09
$AC(10)$		0.05	0.09	0.09
			Implications for Asset Returns	
$E[r_f]$		0.86	1.03	1.00
$\sigma(r_f)$		0.97	0.56	0.38
$E[r_e - r_f]$		6.33	5.02	5.02
$\sigma\left(E_t[r_e - r_f]\right)$			1.18	0
$\sigma(r_e)$		19.42	7.82	7.59

The model is calibrated to match the time series of consumption growth and aggregate stock returns. We match the autocorrelations of consumption growth, return statistics of equity and the risk-free bond. Data statistics are from Bansal and Yaron (2004). $AC(i)$ is ith autocorrelation of yearly consumption. r_i and r_f are yearly returns on equity and the risk-free bond, respectively. Other parameter values are $g = 0.0015$, $v = 0.008^2$, $\sigma_e = 1$, $\sigma_w = 0.23 \times 10^{-5}$, and $\kappa_1 = \beta = 0.997$.

1.4.1 Calibration

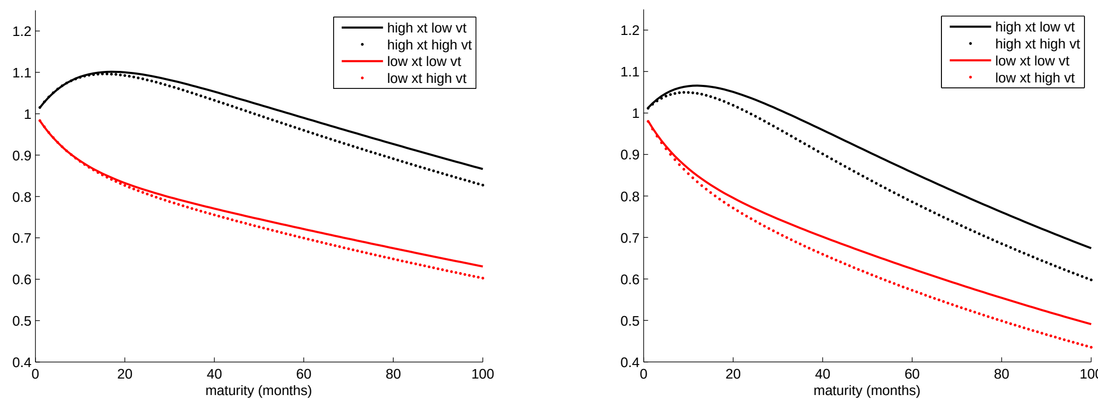
The calibration consists of two steps. First, we calibrate each economic environment to fit the stylized facts of consumption growth and aggregate asset returns, using the empirical moments in Bansal and Yaron (2004). Following their calibration procedure, we simulate 1,000 samples of 840 month-long time series of consumption growth, given a choice of parameters characterizing consumption and preferences. In matching features of aggregate stock returns, we regard the consumption stream as the aggregate equity and compute its monthly returns. Appendix A.4 provides the analytical expression of return on equity in excess of risk-free return. The realized monthly consumption growth and returns are aggregated to annual frequency, in order to compare to corresponding annual empirical moments. Then, we search for the parameters producing the simulated moments close to the empirical counterparts. Note that we calibrate separately the two economies. By making the two economies differ only in terms of the dynamic property of the equity premium and have the same sensible properties otherwise, we can fairly compare one with the other and isolate the effect caused by difference of dynamic or static equity premium.

Table 1.1 shows the calibration results. The Dynamic Equity Premium column refers to the economy featuring the stochastic volatility in growth. This economy is characterized by time-variation both in mean and volatility of growth, x_t and v_t. In contrast, the Constant Equity Premium column is for the economy with constant volatility, where only the mean of growth is time-varying. Thus the economy features state-dependence of cash flow growth but does not have the time-variation in equity premium. This economy is along the line with the CAPM in its risk properties. In the calibration results, both economies match similarly the empirical moments: autocorrelations of consumption growth and the mean and the standard deviation of asset returns. Therefore two types are isomorphic with respect to the matched moments, but a main distinction between the two is the presence of time-variation of the equity premium. As reported in the row of the standard deviation of conditional mean excess return, the Static Equity Premium economy cannot have such time-variation, which a number of recent studies support including Ludvigson and Ng (2007), Welch and Goyal (2008), Jermann (2010), and Cochrane (2011).

What remains is the choice of parameters describing project characteristics: project lifetime N, investment size I, and maximum of systematic risk β_{max}. We find parameter values that lead to the book-to-market ratio close to its empirical counterpart of firms reported in COMPUSTAT. The empirical average of book-to-market ratio for manufacturing firms (SIC 2000-3999) from 1974 to 2012 is 0.637. With the objective of reproducing the empirical average, we generate a 20,000 month-long panel of 500 firms that face monthly realizations of economic state common to all firms and firm-specific project shocks. At each date, each firm makes an investment decision after observing a newly available project as well as the economic state. As time passes, each firm builds its own assets in place as a result of past investment decisions, while old projects expire. By adding the value of growth options to assets in place, we calculate the firm values. In the simulated panel, the book-to-market ratio is defined as the fraction of assets in place in firm value. To ensure for firms to stabilize in their asset composition, we exclude first 500 observations and calculate the average

Figure 1.1: Prices of Elementary Assets in the Economy with Dynamic Equity Premium

(a) Prices of Elementary Assets with $\beta = 1$ (b) Prices of Elementary Assets with $\beta = 3$



The figures plot the prices of the elementary assets in the economy featuring dynamic equity premium. The panel (a) depicts the prices at different states of the economy against delivery date, when the systematic risk is $\beta = 1$. The panel (b) depicts the prices when $\beta = 3$.

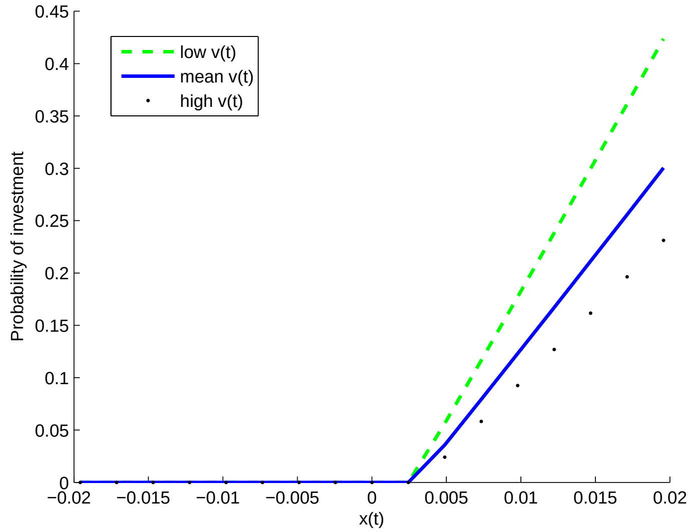
on the panel. The matched book-to-market ratio is 0.686 at $N = 120$, $I = 111.11$, and $\beta_{max} = 5$.

1.4.2 Project Values and Firm-level Investment

In the model, the project is a collection of the elementary assets delivering single risky cash flow. Thus examining the state dependence of the prices of the elementary assets helps understand how project value and investment rule should change across the states. Figure 1.1 plots the prices of the elementary assets against their maturity in the economy with dynamic equity premium. Generally, the price decreases with maturity of the asset - the date at which the asset pays. The price of the asset also changes across the states characterizing economic growth: the conditional mean and the volatility of the growth. To illustrate the price changes, the figure plots the prices at high and low state for each state variable by one standard deviation. If the economy is expected to have a high mean growth (high x_t), a large payoff is expected to be delivered by the assets. Therefore, the asset price increases with the expected growth; at enough high x_t, some strips with particularly short maturities have more value than a unit in spite of the time preference.

The dependence of the asset prices on the conditional volatility is two-fold. A rise in the volatility enlarges a negative covariance between the pricing kernel and the asset payoff, magnifying

Figure 1.2: Investment Probability in the Economy with Dynamic Equity Premium



The figure plots the probability of investment before project-specific systematic risk is realized in the economy with dynamic equity premium. The project characteristics are $N = 120$, $I = 111.11$, and $\beta_{max} = 5$.

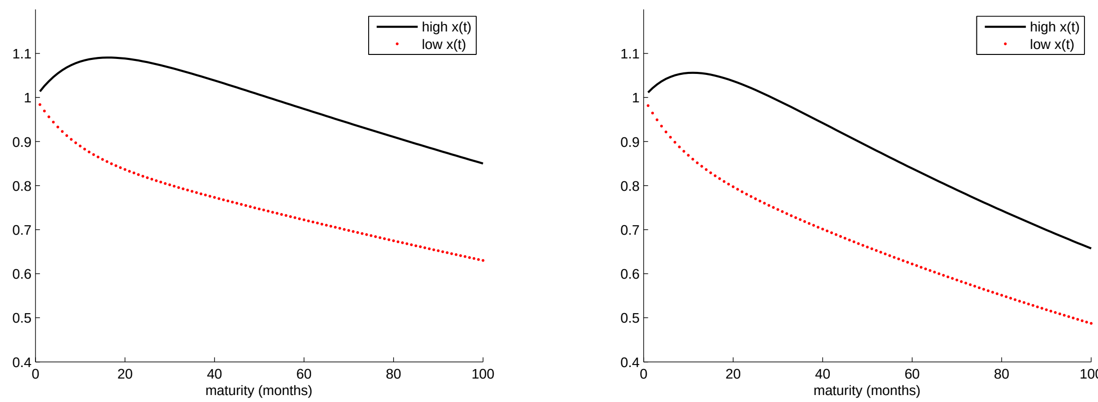
the systematic risk. At the same time, the agent values future payoff more at the high volatility, due to the penalty for risk. The trade-off between two opposing forces depends on the delivery date of the assets. For short-term assets, the second effect dominates, so the price increases with the volatility. However, as the delivery date becomes farther from now, the first effect takes place to a greater extent, lowering the asset value with the volatility.

In addition to the economic states, the asset's exposure to systematic risk is another determinant of the prices. Of course, the prices of assets with greater systematic risk are lower: in Figure 1.1, the prices of assets of $\beta = 3$ are lower than those of assets of $\beta = 1$ with corresponding maturity, irrespective of economic state.

Next, we examine the firm's investment in the economy with dynamic equity premium. We characterize dependence of investment behavior on economic state by looking at the investment probability before the project-specific shock is realized. Figure 1.2 plots the probability that $\beta_t \leq \overline{\beta}(x_t, v_t)$, where $\overline{\beta}(x_t, v_t)$ is the investment threshold. The investment threshold changes across different economic states. For example, the firm is more likely to invest at state of a large expected growth. This is intuitive because at such a state, the project is expected to generate large cash flows, raising the ex-ante value of the project prior to realization of project-specific systematic risk.

Figure 1.3: Prices of Elementary Assets in the Economy with Constant Equity Premium

(a) Prices of Elementary Assets with $\beta = 1$ (b) Prices of Elementary Assets with $\beta = 3$

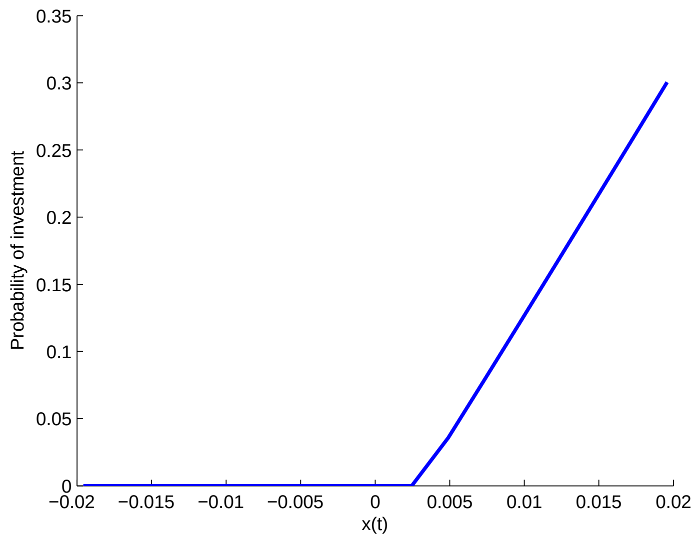


The figures plot the prices of the elementary assets in the economy featuring constant equity premium. The panel (a) depicts the prices at different states of the economy against delivery date, when the systematic risk is $\beta = 1$. The panel (b) depicts the prices when $\beta = 3$.

The volatility of the growth also influences the investment policy. Specifically, the firm is more likely to invest when it faces lower uncertainty of economic growth. This negative association arises from the state-dependence of prices of the elementary assets. A higher volatility magnifies the negative covariance between the pricing kernel and payouts, thus lowering the ex-ante value of project prior to realization of project-specific risk. As a result, the firm tightens investment policy and the probability of investment falls. This relation between investment and the volatility has the same direction as the prediction of the real option theory as in Dixit and Pindyck (1993). The mechanism here, however, is different from their argument. In the real option theory, the option value of waiting is higher when the underlying project value is more volatile, so a firm invests less at high volatility. In our model, in contrast, the investment is now-or-never option, so there is no value of waiting. Instead, the dependence on the volatility comes from its impact on the pricing kernel and amplifying the exposure to systematic risk. Another point worth mentioning is that if we interpret the state of high x_t and low v_t as a boom of economy and the state of low x_t and high v_t as a recession, our model replicates the procyclical behavior of aggregate investment, a stylized fact in the business cycle literature such as King and Rebelo (1999).

We now turn to the asset prices and investment behavior in the economy with static equity

Figure 1.4: Investment Probability in the Economy with Constant Equity Premium



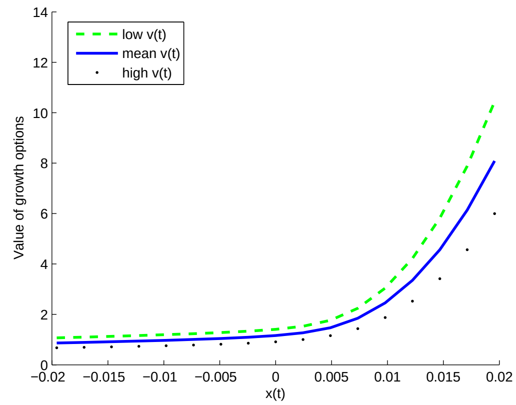
The figure plots the probability of investment before project-specific systematic risk is realized in the economy with constant equity premium. The project characteristics are $N = 120$, $I = 111.11$, and $\beta_{max} = 5$.

premium. Figure 1.3 demonstrates the asset prices. Dependence of the prices on x_t is similar to the economy with dynamic equity premium: the prices are high at state of a large expected growth. The main difference from the economy with dynamic equity premium is that there is no price-dependence on v_t, obviously because the volatility is assumed to be constant. Hence, the investment probability in Figure 1.4 only responds to changes in the expected growth.

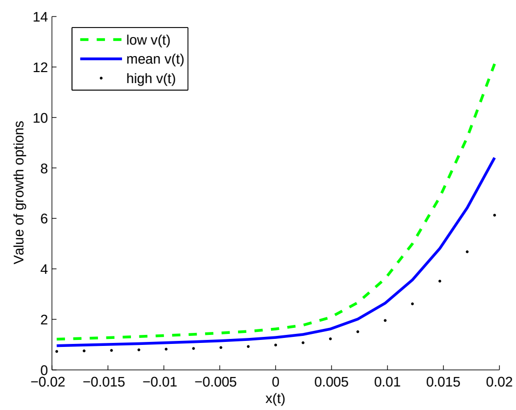
By comparing investment probabilities in Figure 1.2 and 1.4, we can expect consequences when the firm perceives the risk premium differently from the reality. If the real economy features the stochastic volatility and a dynamic risk premium but a firm considers the risk premium as constant, the firm invests with the rule of Figure 1.4, even though the correct decision should be based on Figure 1.2. As a result, the firm may underinvest or overinvest, because it fails to adjust the valuation according to changes in risk premium. Another sub-optimal investment rises in the opposite case that a firm considers the uncertainty of the economy as time-varying, while there is actually no such a time variation. These sub-optimal investments result in value loss and return differentials, which will be measured in the following section.

Figure 1.5: Growth Options of Firms in the Economy with Dynamic Equity Premium

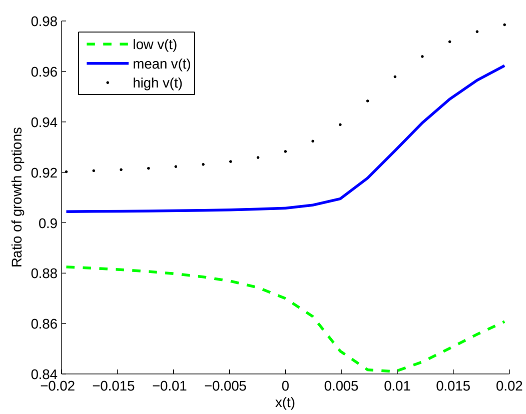
(a) Type 1 Firm



(b) Type 2 Firm



(c) Type 1 / Type 2



The top two figures depict the values of the growth options of the two firms at different states in the economy with dynamic equity premium. The bottom figure shows the ratio of growth options of type 1 firm to those of type 2 to highlight value loss of type 1.

1.4.3 Firm Types and Growth Options

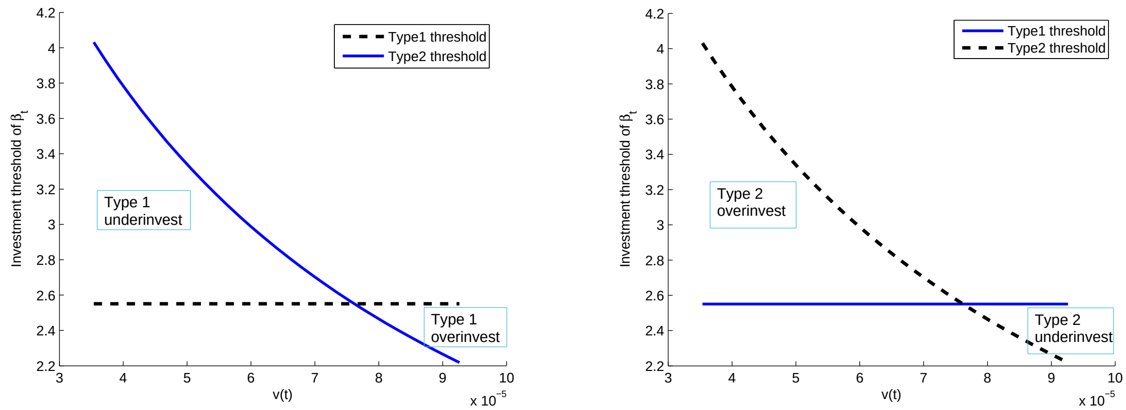
Consider the two firms, Type 1 that acts as if the equity premium is constant and Type 2 that acts as if the equity premium is dynamic. We compare these two firms in the two economies - economy with dynamic equity premium and economy with constant equity premium. A mismatch between the real economy and firm's perception of the economy, for example Type 1 firm in the economy with dynamic equity premium, leads to incorrect valuations of projects and possibility of wrong investment decisions. The value loss caused by the sub-optimal investment is especially evident in the present value of growth options, since the main determinant of growth option is the investment policy itself. In contrast, the value of assets in place depends largely on past realization of project-specific shocks as well as investment policy. Thus we focus our analysis on growth options.

Figure 1.5 compares the present value of growth options of the two firms in the economy with dynamic equity premium. Before comparing the two firms, the state-dependence of the Type 2's growth options, which discounts correctly, is worth mentioning. The value of growth options depends on both the investment policy and the project value at each state. When the economy expects a larger growth (high x_t) or lower uncertainty (low v_t), the value of growth options is high. This is because at such a state, projects are of higher value on average, so the probability of investment is higher.

The Type 1 firm does not consider the fluctuating uncertainty in the growth and ends up with incorrect investment rule. The sub-optimal investment is graphically illustrated in Figure 1.6. The top panel in the figure plots the investment threshold of project-specific systematic risk at different levels of the volatility, when x_t is fixed at 0.0343 as an example. When the realized systematic risk is lower than the threshold of each firm, the firm's evaluation of NPV is positive, so the firm invests. The correctly discounting Type 2 adjusts the threshold to changes in volatility, while Type 1 does not. As a result of ignoring the time-variation in volatility, the Type 1 firm underinvests at low volatility and overinvests at high volatility compared to the Type 2. Since the value of growth options represents the option value associated with the investment policy, the sub-optimal investment leads to the value of the Type 1 lower than that of the Type 2 at all economic states, as the bottom panel in Figure 1.5 shows. In evaluating growth options of Type 1, we assume the erroneous investment behavior is evaluated based on perspectives of Type 2, the correct perspectives

Figure 1.6: Investment Rules of the Two Firm Types

(a) Economy with Dynamic Equity Premium (b) Economy with Constant Equity Premium



The figure plots the threshold of project-specific shock for investment of the two firm types in each economy. The solid line depicts the threshold of the correct investment policy, while the dotted line depicts the threshold of the incorrect policy.

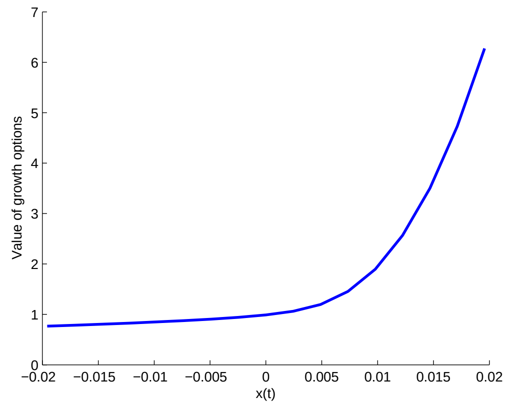
of the economy. The value loss to the Type 1 appears large at states of low volatility. This is because the Type 1 misses some of profitable projects that the Type 2 invests with the correct investment rule, and also because the average value of those missed projects are higher compared to other states.

Given the state-by-state value loss, we measure the time-average of the loss to the Type 1 over the simulated time-series of economic states. On average, the Type 1 firm incurs the loss of 14.8% of the firm value. This quantity represents the value loss if a firm discounts future cash flows along the line with the CAPM, when the underlying economy has the dynamic equity premium as macro-asset pricing literature finds.

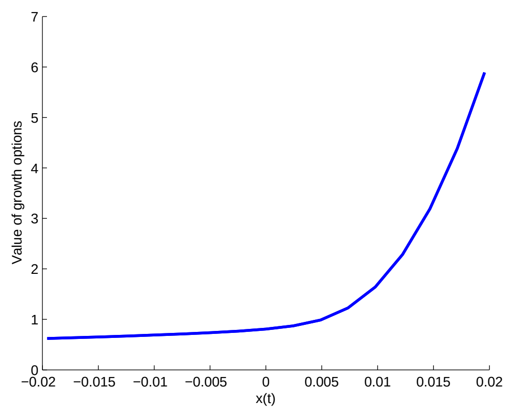
Next, we turn to the economy with static equity premium and compare the two firms in Figure 1.7. Contrary to the previous economy, now the Type 1 firm evaluates projects correctly. The Type 2 firm acts as if the volatility of growth is time-varying, even though the economy actually features constant volatility. Hence the Type 2 firm has an incorrect investment rule, as shown in the bottom panel of Figure 1.6. This results in a value loss to Type 2, as shown in Figure 1.7. However, the magnitude of the loss of the Type 2 is much smaller than that of the mismatching counterpart, the Type 1 firm in the economy with dynamic equity premium. The Type 2 firm in the economy with constant equity premium is exposed to the average loss of only 0.8% of growth

Figure 1.7: Growth Options of Firms in Constant-Volatility Economy

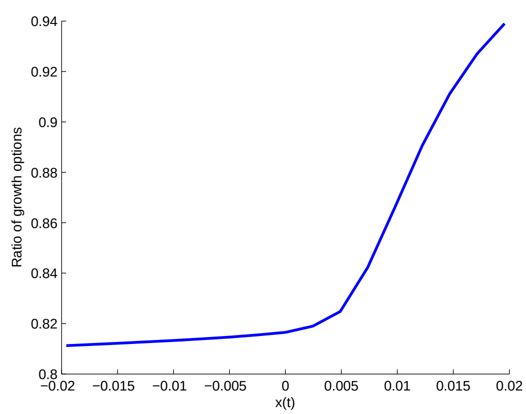
(a) Type 1 Firm



(b) Type 2 Firm

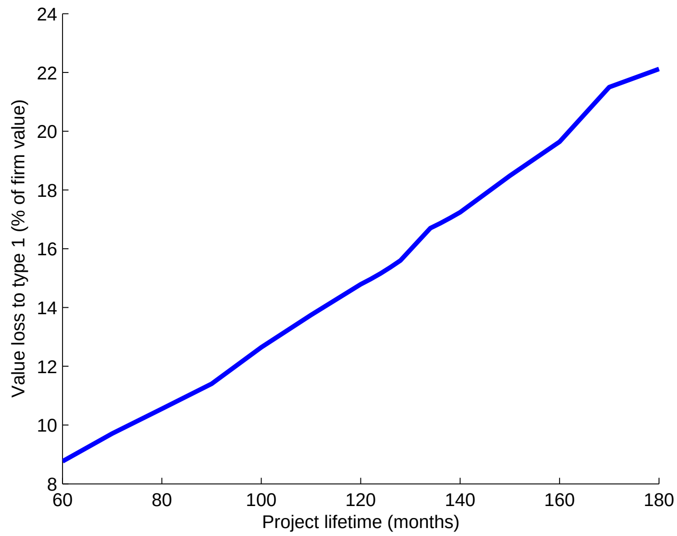


(c) Type 2 / Type 1



The top two figures depict the values of the growth options of the two firms at different states in the economy with constant equity premium. The bottom figure shows the ratio of growth options of type 2 firm to those of type 1 to highlight value loss of type 2.

Figure 1.8: Project Lifetime and Value Loss to Type 1 in Economy with Dynamic Equity Premium



The figure depicts the value loss to Type 1 in the economy with dynamic equity premium across different lifetimes of project, when $I = 111.11$. The value loss is defined as 1-(value of growth options of Type 1)/(value of growth options of Type 2).

options. The asymmetry in the value loss of the two mismatch cases comes from the timing of sub-optimal investment. In the economy with dynamic equity premium, the Type 1 overinvests most at the highest level of the volatility, the exact state when the marginal rate of substitution is high. On the other hand, the Type 2's incorrect investment decisions do not have such coordination with the sub-optimal investment and the pricing kernel. As a result, the failure to reflect correctly the risk property causes a larger value loss in the economy with dynamic equity premium.

As to our main question, whether the time-variation in equity premium matters to a firm's capital budgeting, our answer is yes. If the economy features the time-varying equity premium, whether to consider the variation or not results in a sizable difference in growth options of 14.8%.

1.4.4 Project Lifetime and Sub-Optimal Investment

Obviously, the value loss due to the sub-optimal investment depends on the project characteristics - lifetime of projects, N, and size of required initial investment, I. In this section, we focus on the economy with dynamic equity premium and study how the value loss to the Type 1 changes across

Table 1.2: **Return Differences between Type 1 and Type 2**

Economy		Dynamic Equity Premium	Constant Equity Premium
r_t (type 1) $- r_t$ (type 2)	mean	0.24%	-0.01%
	t-stat	6.36	-14.72
$r_{G,t}$ (type 1) $- r_{G,t}$ (type 2)	mean	0.79%	0.0009%
	t-stat	37.27	2.83
$r_{A,t}$ (type 1) $- r_{A,t}$ (type 2)	mean	0.02%	0.01%
	t-stat	1.81	4.30

The table reports return differences between the two portfolios, one consisting of Type 1 firms and the other consisting of Type 2 firms in each underlying economy. r_t denotes realized rate of returns on firms, $r_{G,t}$ does returns on growth options, and $r_{A,t}$ returns on assets in place in the simulated panel.

different lifetimes of project. To control for the effect of investment probability on the value loss, we adjust the amount of initial investment so that the investment probability is the same across different lifetimes. We can interpret this sensitivity analysis as a benchmark for industry-specific value implications of investment policy; industries are different in terms of average project duration.

Figure 1.8 plots value loss to growth options of Type 1 against project lifetime. We observe that value loss increases considerably from near 8% to 22%, as the project lifetime increases from 5 to 15 years, indicating that when Type 1 sub-optimally invests in longer-term projects, Type 1 incurs a larger value loss. This is because for later cash flows in a longer-term project, there is a greater uncertainty due to a longer time interval between now and delivery date of the cash flow. Thus, a precision of characterizing risk of cash flows becomes more important, and Type 1's ignoring dynamic risk premium leads to a larger value loss.

1.4.5 Firm Types and Returns

In this section, we look at the model's implication on the relation between firm types and returns on firms. Specifically, we study how using the correct or incorrect investment rule influences the returns in each economy. As the firm value, assets in place in particular, depends on a long history of past investment decisions, it is difficult to study the returns analytically so we use a simulation. The basic idea of the simulation is that we have a group of firms with the optimal policy for investment, for example, Type 2 in the economy with dynamic equity premium, and the other group of firms with sub-optimal policy, Type 1 in the example. Then we allow firms to build their project portfolios over time by employing the given investment rules and investigate effects on the firm values and returns.

The simulation procedure is as follows. For each underlying economy, we generate 200,000 month-long history of economic states. We then let 500 Type 1 firms and 500 Type 2 firms operate in the economy. At the beginning, all firms have no project in place. As time passes, each firm faces the economic state and the investment opportunity with project-specific systematic risk, and each decides whether to invest or not in the new project, based on its evaluation of NPV. To control the effect of different realization of project shocks, we assume that each firm from Type 1 has the counterpart from Type 2 with the identical history of project shocks. With this setting, any difference in collective firm assets between the two groups is attributable to their investment policies. In this firm panel, we compute realized returns on the portfolios - one consisting of Type 1 firms and the other consisting of Type 2 firms.

Table 1.2 reports the return differences between the firm types in each economy. In both economies, firm's risk characterization and the resulting investment rule generate statistically significant return differences. In economy with dynamic equity premium, the monthly return on Type 1 portfolio is on average 0.24% higher than that on Type 2 portfolio. In order to understand what drives such return differences, we decompose the firm value into growth options, $V_{G,t}$, and assets in place, $V_{A,t}$, plus cash, H_t. Then, returns on firms can be decomposed as follows:

$$\begin{aligned} R_{t+1} &= \frac{V_{t+1}+H_{t+1}}{V_t} \\ &= \frac{V_{G,t+1}/d_{t+1}}{V_t/d_t}\frac{d_{t+1}}{d_t} + \frac{V_{A,t+1}/d_{t+1}+H_{t+1}/d_{t+1}}{V_t/d_t}\frac{d_{t+1}}{d_t} \\ &= \frac{S_{t+1}}{S_t}\frac{d_{t+1}}{d_t}(1-\mathrm{BM}_t) + \frac{K_{t+1}+h_{t+1}}{K_t}\frac{d_{t+1}}{d_t}\mathrm{BM}_t \\ &= R_{G,t+1}\frac{d_{t+1}}{d_t}(1-\mathrm{BM}_t) + R_{A,t+1}\frac{d_{t+1}}{d_t}\mathrm{BM}_t \end{aligned} \tag{1.13}$$

where S_t denotes growth options normalized by current payout, K_t denotes normalized assets in place, h_t normalized cash, $R_{G,t+1}$ return on growth options, $R_{A,t+1}$ return on assets in place, and $BM_t(=K_t/(S_t+K_t))$ a pseudo book-to-market ratio - the fraction of assets in place in the firm value. The expression shows that return on firm is a weighted average of return on growth options and return on assets in place, where the weight is the book-to-market ratio. Given this decomposition, we study the return differences by examining how the two firm types differ in each component of the returns.

The second row of Table 1.2 shows that in the economy with dynamic equity premium, returns on growth options of firms with incorrect investment rule, Type 1, are 0.79% higher than those returns of optimally investing Type 2. We can identify the source of this return differences by looking at how the magnitude of value loss in Type 1's growth options changes across the states. Figure 1.5 shows the Type 1's value loss is particularly large when the expected growth is low and the loss decreases as the expected growth approaches to the mean level. Combined with the mean-reverting property of the economic state, the state dependence of value loss implies that the expected return on growth options of Type 1 is large at the state of low expected growth, surpassing the return of Type 2. The panel (a) in Figure 1.9 depicts the differences in returns on growth options and documents difference of as large as 3.5% when the economy is expected to have a low mean and a low volatility in growth.

Type 1 also has higher returns on assets in place, but the difference is not statistically significant. This is not surprising in that assets in place is largely determined by history of project-specific shocks, so its dependence on economic state is not as clear as we observe for growth options. As a result of the weighted average of these two returns, returns on Type 1 firms are statistically significantly higher than those on Type 2, mainly due to higher return on growth options.

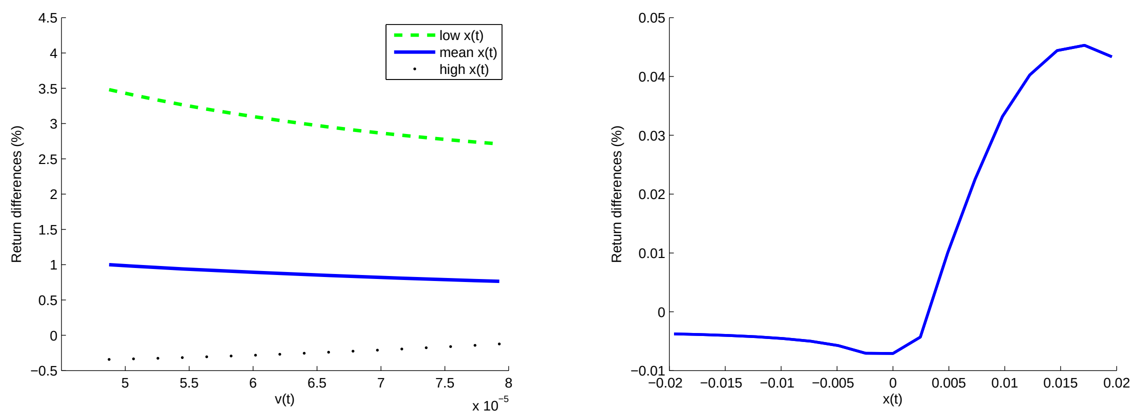
In economy with constant equity premium, the sub-optimally investing Type 2 portfolio has higher returns than Type 1 by 0.01%. The panel (b) of Figure 1.9 compares return on growth options between the two firm types. Although Type 2 firm has higher return than Type 1 in some states of high expected growth, Type 2 firm has lower return in other states including states near mean, which firms frequently face. Consequently, Type 2's average return on growth options is lower by 0.0009%, as documented in Table 1.2. Quantitatively, the major determinant of return differentials in this economy is the covariance between return on assets in place and the book-to-market ratio. Type 2's returns on assets in place are higher than Type 1 at states of high mean growth, when the book-to-market ratio appears high, thus imposing more weight on returns on assets in place. As a result of the coordination effect, Type 2 has higher return on firm, as Table 1.2 reports, even though both of the expected return components are lower for Type 2.

In short, whether a firm's discounting correctly reflects the underlying economy or not changes

Figure 1.9: Difference in Returns on Growth Options between Two Firm Types

(a) $E_t\left[r_G(\text{Type }1)\right] - E_t\left[r_G(\text{Type }2)\right]$ in Economy with Dynamic Equity Premium

(b) $E_t\left[r_G(\text{Type }2)\right] - E_t\left[r_G(\text{Type }1)\right]$ in Economy with Constant Equity Premium



The figure plots differences in returns on growth options between the two firm types. The panel (a) depicts the differences in economy with dynamic equity premium. Panel (b) depicts the differences in economy with constant equity premium.

risk characteristics including exposure to sub-optimal investment and sensitivity to the risk measured by the book-to-market ratio. As a result, it produces statically significant return differences, as well as firm value differences.

1.5 Conclusion

Capital investment decisions in practice are typically hard business problems. They involve difficult and long-horizon forecasts, cut across many functional business areas, and are often strategic. The NPV rule and framework is a powerful tool for structuring this complex decision. The framework is, of course, not without many assumptions that do not strictly hold. In this paper, we look at one specific common practice in capital budgeting – discounting cash flows at a constant cost of capital. In our calibrated model, the implication is large. We estimate a 14.8% value loss from this decision. This loss is much larger that the 0.8% loss for the (counter-factual) scenario of a firm that is in a constant risk-premium economy but investing according to a dynamic-risk-premium model. The dramatic difference is that the over-investment in the first case is correlated with bad states of the economy. It is quite possible that firms do take account of time variation in equity premium without direct calculation. Capital budgets are procyclical in part, it seems from casual observations, as capital budgeting projects receive more scrutiny in recessions. One way we might infer if firms are indeed investing without regard for the time-variation in the equity premium is through subsequent return behavior. In our calibrated model, monthly returns on the incorrectly discounting firms are 0.24% higher on average than those on correctly discounting firms. We leave an empirical exploration of this question to future research.

Chapter 2

Debt Maturity Choice and Firms' Investment

2.1 Introduction

In a world with financing frictions, capital structure impacts corporate investment. Debt overhang, first described by Myers (1977), is one of the channels through which this occurs. Debt overhang refers to underinvestment when financing with debt, compared to financing with equity only. In particular, existing debt reduces the incentive for shareholders to invest because they expect that part of the return on new investment will accrue to debtholders. The resulting underinvestment is costly for firms, and previous studies, such as Mello and Parsons (1992), Leland (1998), Moyen (2007), and Titman and Tsyplakov (2007), have tried to measure the value lost due to the agency conflict.

I study the impact that debt maturity choice has on firm investment and debt overhang. Given that debt maturity determines how returns on capital investment will be distributed among stakeholders over time, maturity should influence shareholders' investment decisions. Also, as found in recent studies including He and Xiong (2012), He and Milbradt (2013), and Chen, Xu, and Wang (2013), the maturity choice and the resulting rollover frequency can amplify default risk. Because default results in shareholders' losing all returns on investment, the maturity will again influence investment. Hence, to analyze the debt overhang, we must consider a firm's ability to choose debt maturity when determining its debt amount and investment policy, but prior studies on debt overhang have not considered an endogenous maturity choice. To move beyond this limitation, one of goals of this study is to quantify agency costs, taking the maturity choice into account.

Another goal is to measure the agency cost at an economy level, considering heterogeneity of firms' characteristics. Prior studies have measured the cost for an average firm, a hypothetical firm whose investment and financing resemble the empirical average moments. However, firms are exposed to varying degrees of debt overhang, depending on their characteristics, such as growth opportunities and default risk. Moreover, the dependence is likely to be nonlinear, so the agency cost for the average firm may not reflect the economy-wide average. The importance of considering heterogeneity is discussed in recent papers on structural models, including Strebulaev (2007), David (2008), Bhamra, Kuehn, and Strebulaev (2009), and Glover (2013). In particular, they suggest that a seemingly representative firm may not reflect the cross-sectional average of an economic quantity of interest. Extending the argument to a measurement of agency costs, we must also consider the distribution of agency costs to fairly quantify the economy-wide cost.

To this end, I build a structural model that endogenizes a firm's investment and capital structure choice. Specifically, I embed a financing decision inside the neoclassical framework described by Abel and Eberly (1994). Ideal for the research question, this framework allows investment flexibility; that is, a firm adjusts its investment in capital stock over time in response to fluctuations in productivity and/or demand. Operating income generated by the capital stock is then split between shareholders and debtholders in accordance with debt structure. Thus, debt structure alters investment decisions by shareholders, who have control rights over firm operations. Looking forward, the incentive for shareholders to invest is another determinant of optimal debt structure in addition to the traditional trade-off components of default risk and tax benefits.

Moreover, I assume that optimal debt maturity results from a trade-off between rollover risks and illiquidity costs, following He and Xiong (2012) and He and Milbradt (2013). When refinancing a maturing debt, the firm generates cash flow due to the difference between the amount to repay and the amount it raises by issuing new debt. The cash flow from rollovers is time-varying, depending on the firm's ability to repay debtholders. If the firm uses short-term debt and needs to roll over its debt more frequently, the variability in rollover cash flow increases, making net cash flow more volatile. In turn, default risk rises, making shorter-term debt more costly. At the same time, I assume that there are liquidity or holding costs associated with search frictions in the over-the-counter bond market, which make longer-term debt more costly. By using a reduced-form representation of holding costs, I am able to solve for equity and debt values and investment policy in analytic expressions.

I calibrate the model to represent the average firm that matches the empirical average of observables on firms' investments and financing. The calibrated model generates reasonable predictions for investment-capital ratio, leverage ratio, debt maturity, and default rates. The model serves as a basis for studying the behavior of the average firm.

A key finding of the calibrated model is that financing with extremely short-term debt actually exacerbates the debt overhang problem. This result is in direct conflict with Myers (1977)' prediction that a shorter maturity is preferable to mitigate the debt overhang. In fact, the use of short-term debt results in frequent rollovers and increases default risk, so the firm invests less due to a higher expectation of default. This theoretical result is consistent with the recent empirical findings on debt maturity effects; Almeida, Campello, Laranjeira, and Weisbenner (2011) find firms that had a large amount of maturing debt in 2007 credit crises experienced a large drop in investment. Moreover, Gopalan, Song, and Yerramilli (2011) show that firms with a larger proportion of short term debt is more likely to suffer downgrades in credit rating in the following year.

I also find that debt maturity matters in measuring agency cost. Controlling for leverage, the agency cost displays significant variation from near 0% of firm value to 15%, depending on debt maturity choices. The variation suggests that debt maturity is important in analyzing the agency cost, and a failure to consider the maturity is likely to result in a biased estimate of the agency cost. More importantly, given the endogenous choice of maturity of 3.78 years, which reflects the empirical average, the agency cost is 0.77%. This estimate is far below Moyen (2007)'s estimate of 4.70% or 5.12%, depending on whether the exogenous maturity is long or short-term. Also, the estimate is lower than the 4.6% of Titman and Tsyplakov (2007), where the maturity is fixed at 20 years. The above differences are caused by debt maturity management. By ignoring the firm's flexibility to adjust debt maturity, previous studies overestimate the agency cost.

At the economy-level, however, we cannot simply conclude that the agency cost for the average firm represents the economy-wide cost, considering firm heterogeneity. Agency cost may well depend on operating characteristics, such as the profile of productivity and investment adjustment cost, capital structure. Obviously, interactions among these attributes influence the agency cost, leading to nonlinear dependence of agency costs on these firm attributes. Hence, to quantify the economy-wide cost, I first measure the agency cost firm-by-firm and then study the cross-sectional distribution of the costs. To do so, I estimate firm-level parameters with the model structure. Specifically, I employ a likelihood approach and find the model parameters for each firm that maximize the probability of observing its actual investment and debt structure. The firm-level parameter estimates are used to measure agency costs for each firm.

The resulting cross-sectional distribution of agency costs displays significant variation across firms from near 0% to 72.03%, with a standard deviation of 9.86%. The value-weighted average of the agency costs when the asset values are used as weights is 7.28%; this estimate is significantly larger than the cost of 0.77% for the average firm. This difference arises from firm heterogeneity in both operating characteristics and debt policy. In turn, the convex dependence of agency costs on the attributes increases the cross-sectional average of the costs. This economy-level estimate indicates that ignoring the cross-sectional heterogeneity leads to a considerable bias in quantifying the economy-wide cost. I also examine how the firm-level agency costs relate to firm observables. Firms with higher leverage, shorter debt maturity, or lower earnings-to-asset ratio are associated with larger agency costs. The association is intuitive because firms with these characteristics are more exposed to default risk, thereby incurring a higher degree of agency conflict.

This paper is related to literature that studies interactions between investment and financing decisions in a dynamic environment. Leland (1998), Mauer and Ott (2000), Sundaresan and Wang (2006), Chen and Manso (2010), Hackbarth and Mauer (2012) and Diamond and He (2012) develop dynamic models of investment and study the impact of financing structure on investment. However, their representations of investment have limitations in that firms have only a limited number of investment opportunities. As Moyen (2007) points out, imposing limitations on investment flexibility may underestimate the value loss due to agency conflicts over investment. This paper contributes to the literature by incorporating flexibility, that is, allowing firms to choose the amount of capital expansion at every point in time.

This paper is also related to the body of work that focuses on the debt overhang, including Mauer and Ott (2000), Hennessy (2004), Moyen (2007), and Titman and Tsyplakov (2007). Mauer and Ott (2000) analyzes the impact of debt maturity on investment, but finds a corner solution for maturity choice, which is incompatible with observed corporate practices. Hennessy (2004) shows both theoretically and empirically that the investment is distorted downward by the expected costs of default, but he does not include endogenous choice of financing. Moyen (2007) and Titman and Tsyplakov (2007) provide estimates of the debt overhang costs with calibrated models that match empirical investment and financing, yet they don't consider explicitly endogenous choice of debt maturity. This article attempts to complement those papers by incorporating endogenous choice of debt maturity in quantifying the debt overhang.

Similar to Strebulaev (2007), David (2008), Bhamra et al. (2009) and Glover (2013), I emphasize the importance of cross-sectional heterogeneity in the structural model. They point out that the average of observed behavior or quantity does not reflect the underlying relation between leverage and other firm attributes, term structure of default risk, or the distribution of expected cost of default. In particular, David (2008) demonstrates the convex dependence of credit spreads on leverage. Given that the credit spreads reflect default risk, I also expect both default risk and resulting agency costs to depend on leverage in a convex way. Accordingly, this paper takes into account cross-sectional variation in measuring the agency cost at an economy level.

Finally, this study is related to a growing body of literature on structural estimation in corporate finance, which is surveyed by Strebulaev and Whited (2012). In particular, Morellec, Nikolov, and Schurhoff (2012) use the likelihood approach to estimate a manager's private benefit of control at the firm-level. To focus on the cost arising from agency conflict between manager and shareholders, they assume a fixed process of operating income that is not affected by debt structure. This study, on the contrary, focuses on the conflict over investment between shareholders and debtholders. To do so, I model firm's operation following the neoclassical framework, where investment decisions are influenced by debt structure. In terms of estimation, I use firm-level observations of both debt structure and history of investment, with which the likelihood is calculated, while a history of leverage ratios is observable in Morellec et al. (2012).

The paper is organized as follows. Section 2 presents the model setup and valuations of debt and equity along with investment policies. Financing decisions are also discussed. In Section 3, I calibrate the model to replicate empirical moments that summarize firms' investments and financing, and examine the quantitative implications of the model. In Section 4, I structurally estimate firm-level models with a panel of firms. Section 5 provides cross-sectional distributions of the agency costs and relates the costs to firms' observables. Section 6 concludes.

2.2 Model

The major objects of the analysis are individual firms. I consider a partial equilibrium economy where firms are exposed to systematic risk. To define the risk, first I describe the economic environment including households' preferences and aggregate consumption. Next, I describe firms' operating environments and their investment and financing decisions.

2.2.1 Stochastic Discount Factor

The representative household has time-separable preferences over consumption. At time t, utility flow is described by a power function of current consumption y_t, that is, $u(y_t) = (y_s^{1-\gamma} - 1)/(1-\gamma)$, where γ is the coefficient of relative risk aversion. I denote the rate of time preference by β. The growth in aggregate consumption is identically and independently distributed as

$$\frac{dy_t}{y_t} = gdt + \sigma_y dW_t^y \tag{2.1}$$

where W_t^y is the standard Brownian motion, and g and σ_y are constants. Then, the stochastic discount factor π_t evolves according to

$$\frac{d\pi_t}{\pi_t} = -\beta dt - \gamma g dt + \gamma(\gamma+1)\frac{\sigma_y^2}{2}dt - \gamma\sigma_y dW_t^y. \tag{2.2}$$

The derivation of the stochastic discount factor is provided in Appendix B.1.

It follows that $\exp\left(\int -\frac{\gamma^2\sigma_y^2}{2}dt - \int \gamma\sigma_y dW_t^y\right)$ in the discount factor is the Radon-Nikodym derivative of the risk-neutral probability measure $\mathbb{Q}$ with respect to the physical measure $\mathbb{P}$. Under the risk-neutral measure, the riskless rate r is equal to $\beta+\gamma g-\gamma(\gamma+1)\sigma_y^2/2$. By invoking Girsanov's theorem, the new Brownian motion under the measure $\mathbb{Q}$ is

$$dW_t^Q = dW_t^y + \gamma\sigma_y dt \tag{2.3}$$

This risk-neutral measure and its Brownian motion will be used to determine the value of firms in the following sections.

2.2.2 Production Technology and Investment

The description of firms' operations follows the framework of Abel and Eberly (1994). A firm uses capital for production, employing a linear production technology with respect to capital. This production generates instantaneous earnings before interest and taxes (EBIT) of $A_t K_t$, where A_t is an exogenous state of productivity and/or demand facing the firm.

The firm's productivity is assumed to evolve in accordance with a geometric Brownian motion and to be correlated with consumption growth:

$$\frac{dA_t}{A_t} = \mu^P dt + \sigma \rho dW_t^y + \sigma \sqrt{1-\rho^2} dW_t \tag{2.4}$$

where ρ is the correlation coefficient, and dW_t is an idiosyncratic shock to the firm's productivity. μ^P is the average of productivity growth in the physical measure, while σ controls volatility of the growth. Under the risk-neutral measure, the average growth rate μ^Q is equal to $\mu^P - \sigma\sigma_y\rho\gamma$.

Over the incremental time dt, the capital stock evolves according to

$$dK_t = (I_t - \delta K_t)\, dt \tag{2.5}$$

where δ is a constant depreciation rate, and I_t is an investment at t, which the firm's management chooses. I normalize the price of capital goods to be one and assume that installing capital incurs convex adjustment costs, θI_t^2,[1] in addition to purchase costs. This formulation of adjustment costs enables us to obtain tractable closed-form solutions to the firm's value and investment policy.

After including the costs from capital installment and the tax benefits from capital depreciation, the instantaneous free cash flow is

$$(1-\tau)A_t K_t - I_t - \theta I_t^2 + \tau \delta K_t \tag{2.6}$$

where τ is the corporate income tax rate. The value of firm is the expected present value of the free cash flows, and a closed-form expression of the value is provided in section 2.2.4.

[1] The assumption that adjustment costs are independent of capital is also adopted by Abel (1983) and Caballero (1991).

2.2.3 Financing - Stationary Debt Structure

To capture the effects of debt maturity in a simple way, I employ a stationary debt structure as in Leland (1998), Diamond and He (2012), and Chen et al. (2013). Once the firm issues debt, it commits to maintaining the initial debt structure until it stops operation. Details of debt structure are as follows. The firm has debt with aggregate principal P, and pays interest C to debtholders. The debt does not have an explicitly contracted maturity, but a constant fraction fdt of aggregate amount matures at par over time interval dt at every instant. In this setup, the choice of the repayment rate f is equivalent to the choice of the effective maturity. Specifically, when time interval t elapses after debt issuance, e^{-ft} fraction of debt will remain outstanding. Letting $F(t)$ denote the fraction of debt which has matured by t, I have $F(t) = 1 - e^{-ft}$. Then the effective average of debt maturity, m, is

$$m = \int_0^\infty t dF(t) = \int_0^\infty t f e^{-ft} dt = \frac{1}{f}. \tag{2.7}$$

This expression shows that the average maturity is the inverse of the repayment rate. In what follows, I use this average maturity in place of the repayment rate.

In response to the retirement of the fraction of debt, the firm issues new debt with principal, interest, and maturity identical to the retiring fraction at every instant. With the continuous refinancing, the firm maintains the same debt structure including aggregate principal, coupon rate, and maturity, unless the firm stops operation. However, the rollover generates dynamic cash flows; the firm repays the face value of the retiring fraction but raises the market value of the corresponding amount of debt from the new issuance. While the face value is constant once debt structure is determined, the market value changes over time, reflecting the firm's condition and its ability to repay debtholders. I denote the market value of debt by $D(A_t, K_t)$. As the firm repays $(1/m)dt$ fraction of aggregate debt and refinances to receive proceeds of the market value, the firm generates rollover cash flow $\left(\left(D(A_t, K_t) - P\right)/m\right) dt$ during dt. If the firm ceases to operate and liquidates capital, absolute priority is obeyed; in this case, debtholders take the lesser of face value of debt and the liquidated value of capital. If the liquidated value is lower than the face value, the firm defaults on debt.

Once corporate bonds are issued, they are traded in the over-the-counter market. I assume that investors are subject to idiosyncratic liquidity shocks, and if a shock occurs, they prefer to sell their bond holdings. However, search frictions in the corporate bond market prevent investors from trading immediately when they become liquidity-constrained, so the frictions create holding costs to the bond investors. Following Chen et al. (2013), I represent the holding costs per unit market value of debt and per unit time in the following reduced form:

$$h(m) = \kappa \left(e^{\eta m} - 1\right) \tag{2.8}$$

where $\kappa > 0$ and $\eta > 0$. A main property of the expression of holding costs is that it increases with debt maturity. This property captures illiquidity costs associated with investing in long-term debt, which are consistent with results in theoretical models including Duffie, Garleanu, and Pedersen (2005) and He and Milbradt (2013) as well as the empirical study by Longstaff, Mithal, and Neis (2005). In Appendix B.3, I show that a full description of the over-the-counter market leads to expressions for the holding costs that increase with debt maturity, although these expressions are not available in closed-form.

2.2.4 Valuation of an Unlevered Firm

Before examining levered firms, the main focus of this study, I first consider unlevered firms. Later, this helps to identify how debt in place influences firms' investments. The management of an unlevered firm works to maximize the firm value under the risk-neutral measure

$$U\left(A_t, K_t\right) = \max_{I_s} E_t^Q \left[\int_t^\infty e^{-r(s-t)} \left((1-\tau)A_s K_s - I_s - \theta I_s^2 + \tau\delta K_s\right) ds\right]. \tag{2.9}$$

The Hamilton-Jacobi-Bellman (HJB) equation for this problem is[2]

$$rU = \max_I (I - \delta K)U_K + \mu^Q A U_A + \frac{1}{2}\sigma^2 A^2 U_{AA} + (1-\tau)AK - I - \theta I^2 + \tau\delta K \tag{2.10}$$

and the optimal investment is $I^*(A,K) = (U_K - 1)/2\theta$. In addition to production activity, the firm is assumed to have an option to liquidate capital stock for lK, where $l < 1$. Thus, when the firm

[2] U_X denotes the first-order partial derivative of U with respect to X, $\frac{\partial U}{\partial X}$, for $X = \{A, K\}$. Also U_{XX} denotes the second-order partial derivative.

faces a sufficiently low level of productivity, it finds exercising the option to liquidate preferable to continuing operations. For now, I suppose that the stopping threshold of productivity is given by $A_L(K)$. I discuss later how to determine the threshold.

Following Abel and Eberly (1994), I find that a linear form of firm value with respect to capital, $U(A,K) = q_U(A)K + J_U(A)$, satisfies the above differential equation and yields two differential equations after collecting terms in K:

$$\begin{aligned} (r+\delta)q_U - \mu^Q A q_U' - \frac{1}{2}\sigma^2 A^2 q_U'' - (1-\tau)A - \tau\delta &= 0 \\ -rJ_U + \mu^Q A J_U' + \frac{1}{2}\sigma^2 A^2 J_U'' + \frac{(q_U-1)^2}{4\theta} &= 0 \end{aligned} \tag{2.11}$$

where[3] the optimal investment is $(q_U-1)/2\theta$. In Appendix B.4, I show that q_U and J_U are

$$\begin{aligned} q_U(A) =& \frac{(1-\tau)A}{r+\delta-\mu^Q} + \frac{\tau\delta}{r+\delta} + \left(l - \frac{(1-\tau)A_L}{r+\delta-\mu^Q} - \frac{\tau\delta}{r+\delta}\right)\left(\frac{A}{A_L}\right)^{\chi_1} \\ J_U(A) =& \phi_1\left(l - \frac{(1-\tau)A_L}{r+\delta-\mu^Q} - \frac{\tau\delta}{r+\delta}\right)^2\left[\left(\frac{A}{A_L}\right)^{2\chi_1} - \left(\frac{A}{A_L}\right)^{\chi_2}\right] + \phi_2\left(l - \frac{(1-\tau)A_L}{r+\delta-\mu^Q} - \frac{\tau\delta}{r+\delta}\right)\left[\frac{A^{\chi_1+1}}{A_L^{\chi_1}} - \frac{A^{\chi_2}}{A_L^{\chi_2-1}}\right] \\ &+\phi_3\left(l - \frac{(1-\tau)A_L}{r+\delta-\mu^Q} - \frac{\tau\delta}{r+\delta}\right)\left[\left(\frac{A}{A_L}\right)^{\chi_1} - \left(\frac{A}{A_L}\right)^{\chi_2}\right] + \phi_4\left[A^2 - \frac{A^{\chi_2}}{A_L^{\chi_2-2}}\right] + \phi_5\left[A - \frac{A^{\chi_2}}{A_L^{\chi_2-1}}\right] \\ &+\frac{1}{4\theta r}\left(\frac{\tau\delta}{r+\delta} - 1\right)^2\left[1 - \left(\frac{A}{A_L}\right)^{\chi_2}\right] \end{aligned} \tag{2.12}$$

if conditions for the value of growth options to be positive, $r > 2\mu^Q + \sigma^2$ and $r > \mu^Q$, are satisfied. The parameters $[\chi_i]_{i=1}^2$ and $[\phi_i]_{i=1}^5$ are given in Appendix B.4.

The expression of marginal value of capital, q_U, is economically intuitive: the first two terms represent the expected present value of cash flows that a unit of capital stock will deliver through operating profits and tax shields from now on. The last term is the present value of the option to liquidate capital. Upon liquidation, the firm receives the liquidated value net of the opportunity cost from stopping operations, $l-(1-\tau)/A_L(r+\delta-\mu_j^Q)-\tau\delta/(r+\delta)$, for a unit capital. $(A/A_L)^{\chi_1}$ is the value of the Arrow-Debreu security that pays when the firm liquidates. The optimal investment, $(q_U-1)/2\theta$, is consistent with the insight of the Q-theory. In particular, the amount of investment is the level at which the marginal value of installing a unit capital is equal to the marginal cost.

Within the firm value, $q_U(A)K$ represents assets in place, that is, the present value of cash flows

[3] q_U and J_U are functions of a single variable, A. q_U' denotes the first-order derivative with respect to A , and q_U'' denotes the second-order derivative.

that current capital stock K will deliver in the future. The remaining term $J_U(A)$ captures the value attributable to growth opportunities. When the productivity level is sufficiently high, $J_U(A)$ increases with A by the order of two. The quadratic dependence of $J_U(A)$ on A stems from the capital installment that is a linear function of A and the operating profit that is a product of the capital stock and the productivity level.

Finally, I state the liquidation decision of the unlevered firm. Liquidation is chosen by the firm to maximize the firm value. Thus, the liquidation threshold of productivity at capital stock K, $A_L(K)$, is determined by the smooth-pasting condition

$$U_A(A_L(K), K) = 0. \tag{2.13}$$

An analytic expression for $A_L(K)$ is provided in Appendix B.4.1.

2.2.5 Valuation of a Levered Firm

Now I examine levered firms; first, I study a levered firm's investment decisions given the firm's debt structure (P, m, C) and then examine the optimal financing decision. The management of the levered firm works in the interest of shareholders and decides at each instant whether to stop operation and if not, how much to invest. The value of equity, denoted by $S(A_t, K_t)$, is given by

$$S(A_t, K_t) = \max_{I_s} E_t^Q \left[\int_t^T e^{-r(s-t)} \left((1-\tau) A_s K_s - I_s - \theta I_s^2 - \frac{P}{m} + \frac{D(A_s, K_s)}{m} - (1-\tau)C + \tau \delta K \right) ds \right] \tag{2.14}$$

where the stopping time T is defined as $T = \inf_t \{A_t \leq A_D(K_t)\}$, and $A_D(K)$ is the stopping threshold[4] at capital stock K. Determining the optimal stopping threshold will be discussed later. For now, the threshold can be considered as exogenously given. Note that the cash flow to equity consists of EBIT, interest payment to debtholders, tax shields, and cash flow from rolling over debt.

The HJB equation for the equity value is

$$rS = \max_I (I - \delta K) S_k + \mu^Q A S_A + \frac{1}{2}\sigma^2 A^2 S_{AA} + (1-\tau)AK - I - \theta I^2 - \frac{P}{m} + \frac{D}{m} - (1-\tau)C + \tau\delta K \tag{2.15}$$

[4]Depending on the debt amount, the firm's liquidations is accompanied by defaulting on debt or not.

with boundary condition

$$S(A_D(K), K) = \max(0, lK - P). \tag{2.16}$$

The above condition represents residual claims for shareholders at the stopping time. The residual claims are contingent on the amount of existing debt in relation to the liquidated value of capital. If the liquidated value is large enough to repay debtholders, shareholders pay the face value of debt from the liquidation and receive the residual. Otherwise, shareholders default on debt and receive zero. Given the equity value, the optimal investment is $(S_K - 1)/2\theta$.

The value of debt with maturity m satisfies the following equation

$$(r + h(m))D = (I - \delta K)D_K + \mu^Q A D_A + \frac{1}{2}\sigma^2 A^2 D_{AA} + \frac{P}{m} - \frac{D}{m} + C. \tag{2.17}$$

The left-hand side is the required return by debtholders, which is the sum of the riskless return and the liquidity spread $h(m)$, which was discussed in Section 2.2.3. The first three terms on the right-hand side are capital gains on debt, which depend on the firm's state and investment. Note that investment decisions by the management also impact the debt value. Therefore, in equilibrium, the management should take into consideration the effect of investment on debt as well, while making investment decisions. The next two terms in the equation (2.17) are cash inflow to debtholders from the firm's refinancing and the cash inflow is the opposite of the rollover cash flow to equity. The last term is the interest payment to debtholders.

The debt value must also satisfy the following boundary conditions:

$$\begin{aligned} \lim_{A \to \infty} D(A, K) &= \frac{P + mC}{1 + m(r + h(m))} \\ D(A_D, K) &= \min(lK, P). \end{aligned} \tag{2.18}$$

The first condition states that if the firm faces an extremely high productivity shock, debt becomes riskless[5]. On the other hand, if the productivity is low enough to hit the stopping threshold $A_D(K)$, the manager opts to stop operation, leaving debtholders the liquidated value of capital or the par value of debt as stated in the second condition.

[5]From equation (2.17), the value of riskless debt wth (P, m, C) can be shown to be $\frac{P+mC}{1+m(r+h(m))}$

As a result, the values of equity and debt are expressed as a system of two differential equations and boundary conditions. Analogous to the solution in the unlevered benchmark, I guess that in equilibrium, the equity and debt values are linear functions of capital and verify that the linear forms satisfy the system of equations in Appendix B.5. The next proposition gives the values of equity and debt and investment policy.

Proposition 1 *Given the stopping threshold A_D, the debt value is*

$$D(A,K) = q_D(A)K + J_D(A) \tag{2.19}$$

where

$$q_D(A) = \begin{cases} l\left(\frac{A}{A_D}\right)^{\gamma_1} & \text{if } lK < P \\ 0 & \text{if } lK \geq P. \end{cases}$$

The equity value is

$$S(A,K) = q_S(A)K + J_S(A) \tag{2.20}$$

where

$$q_S(A) = \begin{cases} \frac{(1-\tau)A}{r+\delta-\mu^Q} + \frac{\tau\delta}{r+\delta} + \left(\frac{l}{mh+1} - \frac{(1-\tau)A_D}{r+\delta-\mu^Q} - \frac{\tau\delta}{r+\delta}\right)\left(\frac{A}{A_D}\right)^{\gamma_2} - \frac{l}{mh+1}\left(\frac{A}{A_D}\right)^{\gamma_1} & \text{if } lK < P \\ \frac{(1-\tau)A}{r+\delta-\mu^Q} + \frac{\tau\delta}{r+\delta} + \left(l - \frac{(1-\tau)A_D}{r+\delta-\mu^Q} - \frac{\tau\delta}{r+\delta}\right)\left(\frac{A}{A_D}\right)^{\gamma_2} & \text{if } lK \geq P. \end{cases}$$

The investment policy is

$$I(A,K) = \frac{q_S(A) - 1}{2\theta}. \tag{2.21}$$

The analytic expressions of $J_D(A)$, and $J_S(A)$ are provided in Appendix B.5.

The analytic expressions have economic interpretations. The marginal value of capital to debtholders, $q_D(A)$, represents the value that accrues to debtholders from installing a unit of capital. Then shareholders obtain a reduced value from capital investment by the marginal value to debt, leading to the underinvestment. Specifically, when the liquidated value of the current capital stock is smaller than the par value of debt, $lK < P$, the payment to debtholders at default changes according to the capital stock. Hence, additional capital installment creates value for debtholders. When $lK \geq P$, the capital installment does not influence the debt value. Thus, I expect that debt in place does not distort the incentive for shareholders to invest in such a case. If $lK < P$, the unit capital impacts the debt value through two channels: first, it increases the claim of debtholders at default by l and second, it alters default probability through increase in the market value of debt and following change in rollover cash flows.

The $q_s(A)$ is the marginal value of capital to shareholders, and the value also depends on the difference between the amount of existing debt and the liquidated value of capital. If $lK \geq P$ an additional capital stock does not contribute any value to debtholders. The marginal value to equity is identical to that of the unlevered firm; the marginal value consists of the expected present value of cash flows that a unit capital will deliver through operating profits and tax shields and the value of liquidation option that a unit capital provides.

In contrast, if $lK < P$, the terminal value to debtholders upon default is determined by capital stock, so a part of value from investment accrues to debtholders. The last term in $q_S(A)$, $l/(mh+1)(A/A_D)^{\gamma_1}$, captures the wealth transfer from shareholders to debtholders, after illiquidity discounts are considered. This transfer is one of the two channels through which capital structure impacts investment, by lowering the marginal value of capital to equity. Due to the lowered marginal value, the levered firm invests less than the unlevered firm that is otherwise the same, unless tax shields from interest payment are large enough. The other channel is default likelihood. The liability to pay debtholders on default leaves less for shareholders, so shareholders' optimal decision leads to stopping earlier than in the unlevered case. The earlier stopping further decreases the marginal value of capital, thereby causing shareholders to invest less.

$J_D(A)$ stands for a fraction of the debt value that is independent of capital stock. This consists of the value of riskless debt, $(P+mC)/(mr+mh+1)$, and adjustment terms deriving from shareholders' options of liquidation and capital expansion. In the equity value, $J_S(A)$ includes the present value of growth options of the unlevered benchmark, tax shields, and adjustment terms arising from liquidation option and capital expansion.

Finally, I now characterize the firm's stopping decision. The stopping is endogenously chosen by the management to maximize the equity value subject to the limited liability of equity. Thus, at capital stock K, the stopping threshold of productivity level, $A_D(K)$, is determined by the smooth-pasting condition

$$S_A\left(A_D(K), K\right) = 0. \tag{2.22}$$

Note that the stopping threshold changes depending on capital stock. The analytic expression for the stopping threshold is presented in Appendix B.5.

2.2.6 Optimal Financing of the Levered Firm

So far I have discussed the valuation of debt and equity and the investment policy for a given debt structure (P, m, C). I now characterize the optimal capital structure.

At date 0, the firm starts operation at the initial state of productivity and capital stock, and it chooses the debt structure to maximize the present value of the firm, which is the sum of the values of equity and debt. Thus, the firm's objective is

$$\max_{P,m,C} S(A_0, K_0; P, m, C) + D(A_0, K_0; P, m, C). \tag{2.23}$$

In addition, I assume that debt is issued at par initially, leading to the following constraint

$$D(A_0, K_0; P, m, C) = P. \tag{2.24}$$

With the solutions for debt and equity, I numerically solve for the optimal capital structure.

2.3 Calibration and Quantitative Analysis

In this section, the quantitative implications of the model are discussed. First, I describe the data and the calibration of the model. With the calibrated model, I examine the impact of debt maturity on firms' investment in environments both with and without illiquidity discounts in the bond market. I then estimate agency costs for the average firm.

2.3.1 Data

To determine the stochastic discount factor, I use data on U.S. consumption and the CRSP value-weighted returns in stock market. I use a quarterly series of consumption expenditures per capita for non-durable goods and services from 1947 Q1 to 2014 Q1. Monthly returns on the aggregate stock index in the time period are used. I use the consumption stream as a proxy for the index and determine the preference parameters so that return on the asset matches the empirical average of

Table 2.1: Summary Statistics

This table presents descriptive statistics of firm-level variables. The statistics are calculated from annual variables.

Summary Statistics						
Description	Symbol					
		Mean	Std.Dev.	25%	50%	75%
Investment rate	$\frac{I}{K}$	0.1096	0.1578	0.0500	0.0800	0.1314
Productivity	A	0.4464	0.9463	0.1566	0.2808	0.4928
Tobin's Q	Q	1.9911	2.2212	1.1182	1.4687	2.1501
Capital stock	$\log(K)$	4.7766	2.5369	2.9044	4.8503	6.5868
Asset	V	5.5187	2.4069	3.7160	5.5060	7.2278
Book leverage	$\frac{D}{V}$	0.2726	0.2373	0.0873	0.2053	0.3991
Debt maturity	m	5.0601	2.9187	2.5332	4.8227	7.4455

index returns. A detailed discussion on the consumption stream is provided in section B.2.

I use data from COMPUSTAT for corporate investment and financing. The initial sample consists of firm-quarter observations from 1987 to 2013 and includes only manufacturing firms (SIC 2000-3999). The variables of interests are measured in standard ways in the literature. Investment is capital expenditures (CAPXY) net of sales of property, plant, and equipment (SPPIVY). The investment-capital ratio in quarter t is the investment in quarter t normalized by gross capital stock (PPEGTQ) in quarter $t-1$. Due to limited availability, annual observations are used for debt amount and its maturity. The total book value of existing debt is measured by the sum of debt in current liabilities (DLCC) and long-term debt (DLTT). The pseudo-market leverage is then the ratio of the total debt to the sum of the debt and the market value of equity, which is the product of share price (PRCC) and number of shares outstanding (CSHO). Based on the assumed linear production technology, the productivity level is earnings before interest, taxes, depreciation and amortization (EBITDA) divided by the capital stock. Following Gala and Gomes (2012), I measure Tobin's Q by the market value of assets (the book value of assets plus the market value of common stock minus the book value of common stock) divided by the book value of assets.

From the initial sample, I exclude firm-quarter observations where the capital stock, book value of assets (ATQ), and sales (SALEQ) are either negative or zero. I also exclude observations with no debt in place or extreme year-to-year changes. Extreme changes are defined as having changes in leverage or investment-capital ratio in the lowest or highest 1%. After the trimming, I finally restrict firms to have more than 50 quarterly observations to be in the panel. This procedure yields 44,226 firm-quarter observations.

For debt maturity, Stohs and Mauer (1996) report that manufacturing firms use debt maturity of 3.38 years on average. They examine each of the debt obligations of the firms from Moody's Industrial Manuals, and calculate the value-weighted average of maturity. I use this estimate of debt maturity to calibrate the model of the average firm. For maturity choices of a cross section of firms, I rely on COMPUSTAT. It provides the debt amount of each firm by maturity categories: debt due in less than 1 year (DLC), and in years two to five (DD2-DD5). Debt due in more than 5 years is computed as the difference between long-term debt (DLTT) and the total debt due in years from two to five. I assume that the average maturity of the categories are 0.5 year, 1.5 years, 2.5 years, 3.5 years, 4.5 years, and 10 years. Then I compute a book-value-weighted average of maturity for a firm each year. These firm-by-firm maturity estimates will be used to empirically examine maturity choices in the cross section of firms. Table 2.1 provides the summary statistics for the variables used in the paper.

2.3.2 Calibration

The calibration is summarized in Table 2.2. First, I estimate the parameters governing consumption growth (μ_c, σ_y) via maximum likelihood. The value of β is chosen so that the discount rate for certain cash flow in one year is 0.998. Given parameters for consumption and time preference, the coefficient of relative risk aversion is chosen to match the average return on the risky asset. For other basic parameters, I employ the values used in previous studies. The corporate tax rate is $\tau = 20\%$. The capital depreciation rate is assumed to be $\delta = 10.24\%$, and the recovery value is $l = 0.9$, following Bolton, Chen, and Wang (2011).

I calibrate the remaining parameters, with the objective of reproducing the empirical moments of firms' investment and capital structure. For a given set of six parameters including productivity growth rate μ^P and volatility σ, correlation between productivity and consumption shocks ρ, investment-adjustment costs θ, and holding cost parameters κ and η, I simulate a panel of 1,000 firms that operate for 100 years. All firms start at an identical state with capital stock of $K_0 = 1$ and productivity level of $A_0 = 0.3$. This implies that the firms are also identical in the debt policy that is optimally chosen at date 0. Over time, an economy-wide consumption shock is realized as

Table 2.2: Calibration

This table reports calibrated parameters in the model. Panel A shows the parameters in the stochastic discount factor. Among them, the parameters governing consumption growth are estimated via maximum likelihood. The coefficient of relative risk aversion γ is set to 3.7818 to match the empirical average 7.41% of aggregate stock returns, while the rate of time preference β is set to 0.0020. Panel B shows calibrated parameters characterizing firm's investment and financing under the stochastic discount factor. For other basic parameters, I employ parameter values from existing literature, which include corporate tax rate, $\tau = 0.2$, depreciation rate $\delta = 0.1024$, and liquidation value $l = 0.9$.

Panel A. Estimation of Aggregate Consumption and Preference Parameters

Parameter	Symbol	Value
Mean of consumption growth	g	0.0193
Volatility of consumption growth	σ_y	0.0101
Coefficient of relative risk aversion	γ	3.7818

Panel B. Calibration of Corporate Investment and Financing Parameters

Parameter	Symbol	Value
Mean of productivity growth	μ_p	0.005
Volatility of productivity growth	σ	0.13
Correlation between productivity and consumption growth	ρ	0.3
Coefficient of investment-adjustment cost	θ	1.8
Parameters in illiquidity discounts	κ	0.003
	η	0.17

well as idiosyncratic productivity shocks to firms. Hence, firms evolve differently from each other, since they accumulate capital stock differently depending on their own realization of productivity. In cases where a firm faces a productivity level lower than its stopping threshold, the firm will choose to liquidate. For such cases, I allow a new firm to replace the liquidating firm. The new firm starts operation from the initial state (A_0, K_0). Note that here the idiosyncratic productivity shocks are the only source of differing evolution across firms. In section 2.4, on the contrary, I also consider firms' inherent heterogeneity of operating characteristics as well as different realizations of shock.

With a simulated panel, I compute cross-firm averages of seven moments: mean, autocorrelation

Table 2.3: Moments of Corporate Policies

This table compares empirical moments of firms' investment and financing with simulated moments from the calibrated model. All moments are annualized. Autocorrelation and standard deviation of investment-capital ratio are averages across firm-level estimates.

Moment	Data	Model
Investment-capital ratio		
Average	0.1096	0.1004
Standard deviation	0.0478	0.0914
Autocorrelation	0.1811	0.4810
Average leverage ratio	0.2726	0.2763
Average debt maturity (years)	3.38	3.76
1-year default rate (%)	1.13	1.67
Average liquidity spread	0.0047	0.0026

and standard deviation of investment-capital ratio within a firm, the average of market-leverage ratio, debt maturity, the 1-year default rate, and the average liquidity spreads of debt. I generate 500 simulated panels of the same length, and finally compute averages of the moments across simulations. The simulated moments are compared to the empirical counterparts. The sources of empirical counterparts are as follows. The statistics for investment-capital ratio and leverage ratio are computed from the panel data from COMPUSTAT, and the average of debt maturity is from Stohs and Mauer (1996). The cumulative default rate for 1 year is the estimate for all rated firms by Moody's, and the average liquidity spread is from Longstaff et al. (2005). Given the observed moments, I search for the set of parameter values whose simulated moments most closely resemble those of the empirical counterparts. I report the simulated and empirical moments in Table2.3.

2.3.3 Comparative Statics

In Table 2.4, I present comparative statics with respect to the model parameters. In this section, I highlight some of the observed relations between variables and parameters.

First, consider a variation in the mean growth rate of productivity. A higher growth in productivity implies that a unit of current capital stock will generate larger cash flows in the future, thereby increasing the marginal value of capital. Hence, the firm invests more. Expecting larger

Table 2.4: Comparative Statics

This table shows comparative statics on six model parameters. The direction of change in variable is reported for increase in one parameter, while other parameters remain at calibrated values.

	Sign of change in variable for an increase in:					
Variable	μ_p	σ	ρ	θ	κ	η
Investment-capital ratio						
Average	+	−	−	−	+/−	+/−
Standard deviation	+/−	+	−	+/−	−	+/−
Autocorrelation	+	−	+/−	−	−	−
Average leverage ratio	+/−	−	+/−	+	+/−	+/−
Average debt maturity (years)	+	−	−	−	−	−
1-year default rate (%)	−	+	+/−	+	+/−	+
Average liquidity spread	+	−	−	−	−	−

cash flows in future, the firm can safely increase its debt issuance to take advantage of tax benefits. The firm's incentive to issue more debt is confirmed by an increase in the optimal face value of debt, although I do not report these findings in the table. While raising the face value of debt, the firm adjusts debt maturity to manage default risk: the firm chooses to use longer maturity to alleviate the default risk induced by rolling over debt. Contrary to the monotonic increase in the face value of debt, the time-series average of leverage ratio does not monotonically respond, because there is another channel through which the growth rate influences the leverage ratio. Specifically, the firm with higher productivity growth experiences faster growth in firm value, which mechanically decreases leverage ratio over time. Depending on which effect dominates, an increase in the growth either decreases or increases the leverage ratio.

Next, consider the effects of a higher correlation between consumption and productivity shocks. The more correlated the two shocks are, the more systematic risk the firm is exposed to. Thus, the firm's future cash flows are discounted more, which the model captures as a decrease in the mean growth of productivity in the risk-neutral measure. Therefore, an increase in the correlation has effects similar to a decrease in the mean growth in the physical measure.

Volatility in productivity growth exerts intuitive effects on investment policy. As productivity level determines investment, higher volatility increases the standard deviation of investment. Moreover, as the random fluctuation takes a larger part in productivity dynamics, the autocorrelation of investment decreases. As for choice of capital structure, the firm responds to a raise in volatility by issuing less debt to lower the default risk caused by volatile cash flows. Less debt issuance comes with shortening debt maturity. Managing the default risk through reducing debt issuance, the firm can safely finance with shorter-term debt to avoid illiquidity discounts of long maturity. Going back to the effect on investment, higher volatility interestingly decreases average investment, because the firm with higher volatility still faces larger default risk, even after adjustments in debt structure, as suggested in the higher default rate in the table. Anticipating a higher probability of default and subsequent loss of returns on investment, the firm invests less.

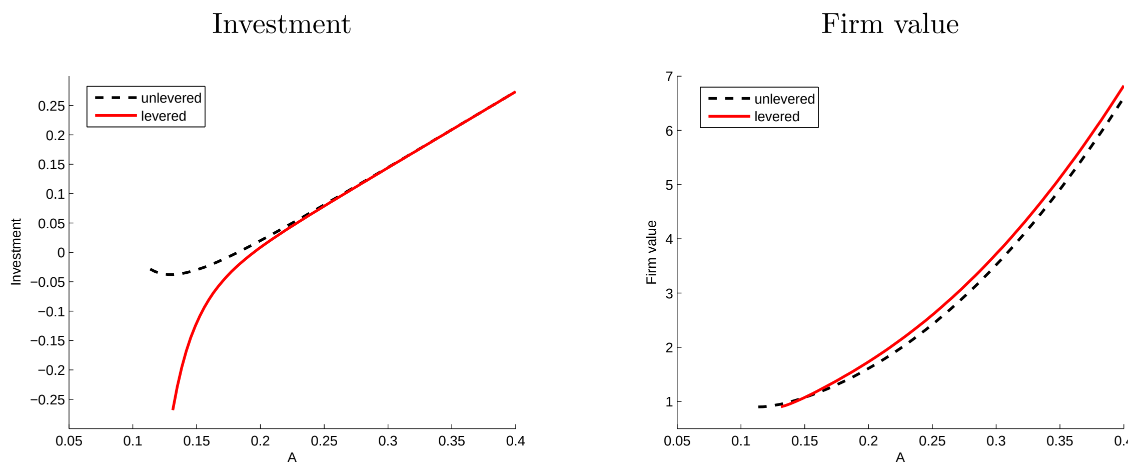
An increase in coefficient of investment-adjustment cost leads to a higher marginal cost of investment, thus lowering investment at all times. In turn, the decrease in the expected investment in the future leads to a decrease in the present value of growth opportunities, so, finally, the total firm value also goes down. In response to the firm value decrease, the firm optimally issues debt with lower face value, financing with shorter maturity to avoid illiquidity discounts. However, issuing less face value of debt does not simply translate into a decrease in the leverage ratio. In the calibrated model, the equity value falls to a greater extent than the optimal debt amount does, so the leverage ratio eventually rises.

Finally, consider an increase in the two parameters for illiquidity in the bond market. With the increase, the firm would suffer larger illiquidity discounts for a unit increase in debt maturity, so it chooses to finance with shorter-term debt, as expected. On the other hand, its negative association with liquidity spreads seems counter-intuitive. Actually, the negative association is a result of the firm's adjusting debt policy, as discussed above, in response to the change in illiquidity. By financing with shorter-term debt that is less subject to the illiquidity cost, the firm incurs lower liquidity spreads even in the more illiquid market.

2.3.4 Investment and Debt Maturity without Illiquidity Discounts

Now I examine the firm's investment and financing decisions in the calibrated model. First, I study the firm's policy in the absence of the liquidity costs. Thus, I assume for now that there is no search friction in the corporate bond market. Before discussing the impact of debt maturity choice, I illustrate how existing debt discourages investment, by comparing the unlevered firm with the

Figure 2.1: Investment and Firm Values of Unlevered and Levered Firms



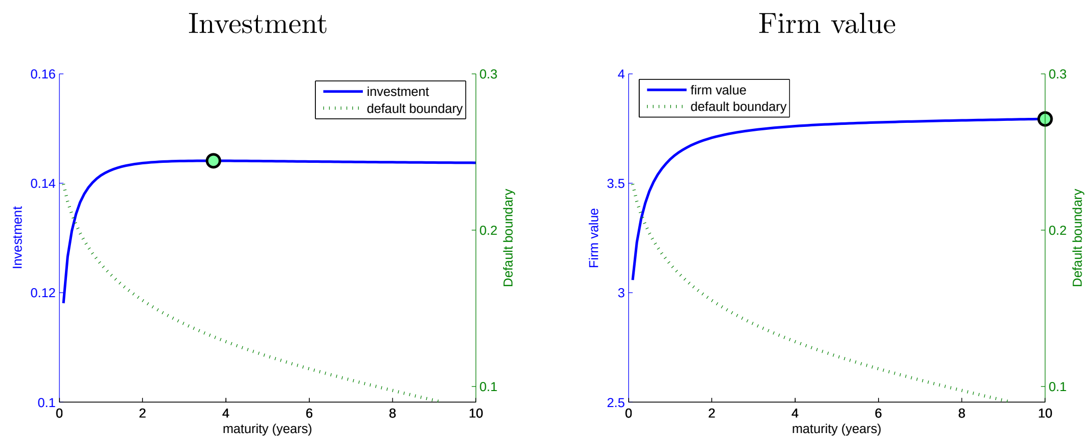
Investment and firm values are compared between an unlevered and a levered firm, when the levered firm uses a debt of $P = 1.37$,$m = 3.78$ and $C = 0.1072$. The capital stock of both firms is $K = 1$. The left panel depicts investment across productivity levels and the right panel shows the total firm value across productivity levels.

levered firm. Figure 2.1 compares in the left panel investment choices between the unlevered firm and the levered firm with a debt of $P = 1.37$ and $m = 3.78$. Each firm's investments at different levels of productivity are depicted when the two firms have a unit capital stock.

Compared to the unlevered firm, the levered firm invests far less when it faces low productivity levels near the stopping threshold. This illustrates the reduced incentive to invest due to existing debt. The levered firm invests less because a unit of capital stock has less value to shareholders, as equation (B.52) states, because of the wealth transfer to debtholders. As the firm approaches the stopping threshold, the wealth transfer takes greater effect, thus severely reducing investment incentives for shareholders. On the other hand, as productivity improves, the stopping becomes less likely, so the the difference in investment between the unlevered firm and the levered firm decreases. The right panel plots the total firm value, which is the equity value plus the debt value. Even though the two firms have the same capital stock and the same level of productivity, the levered value is lower than the unlevered value at low levels of productivity due to suboptimal investment driven by debt overhang and earlier liquidation.

Next, I vary debt maturity and examine how maturity decisions impact investment and firm value. In Figure 2.2, the left panel presents the optimal investment and the stopping thresholds

Figure 2.2: Investment and Firm Values at Different Maturities without Illiquidity Discounts



Investment, stopping thresholds and firm values are compared at different choices of maturity for a firm financing with liquid debt. In all cases, the capital stock is $K = 1$, productivity level is $A = 0.3$, and the leverage ratio is 0.37. The left panel depicts investment and default threshold, A_D, and the right panel shows the total firm value against the maturity.

at fixed capital stock $K = 1$, productivity $A = 0.3$, across different maturities. In order to control for leverage effects, I search for the face value of debt and coupon rate at each maturity so that the leverage is 37% at all maturities. I find that the firm liquidates earlier and invests less when financing with extremely short-term debt. As debt maturity increases, the firm invests more because of reduced likelihood of liquidation. At the same time, however, longer-term debt requires larger coupons and leaves less to shareholders, thus reducing the incentive for shareholders to invest. As a result, the firm invests most with debt maturity of 3.6 years. Quantitatively, the investment distortion by extremely long maturity is far smaller than the distortion by extremely short maturity, as the near-flat plot in region of long maturities shows.

The right panel shows that the firm value increases with maturity, confirming the disadvantage of short maturity in incentivizing investments. The maximum firm value is attained by choosing the longest maturity available; surprisingly, this relationship is opposite to Myers (1977) argument that shorter-term debt should be preferred to encourage investment.

Why does an extremely short-term debt discourage the firm from investing? This can be explained by the liquidation likelihood that a maturity choice induces: the continuously-refinancing firm is more likely to liquidate when financing with shorter-term debt, after controlling for the leverage. The rise in the liquidation likelihood is due to the impact on cash flow to equity of rolling over the short-term debt. When the firm's productivity deteriorates, the debt rollover incurs cash outflows from the firm, as the market value of debt is evaluated for less compared to the face value. If the firm is using short-term debt and needs to refinance more per unit time, the firm needs to pay out more to roll over debt, thus further reducing cash flows available to the firm in an already adverse state. In turn, the reduction in cash flow raises the liquidation likelihood, so the firm invests less.

Hence, in this environment without liquidity spreads in debt, the optimal choice of debt maturity is the longest maturity available in the market. However the corner choice of maturity contradicts the observed practice of capital structure, on average 3.38 years. Motivated by this conflict, I next incorporate liquidity spreads in the model.

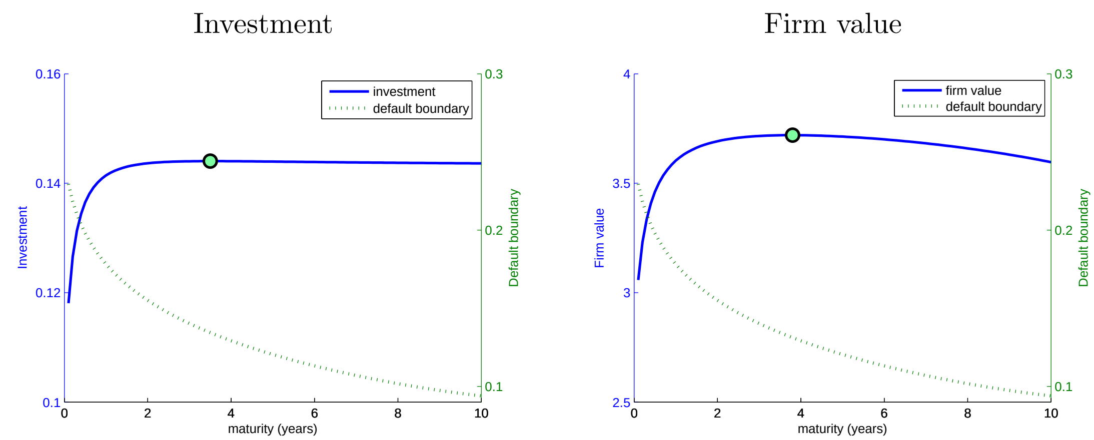
2.3.5 Investment and Debt Maturity with Illiquidity Discounts

Next, I consider the fully-specified environment where the liquidity component is included in valuing debt. In this environment with an illiquid bond market, it takes time for a liquidity-constrained bond investor to search for a counterparty to trade with. This triggers holding costs for the investor. To capture the illiquidity costs, I use the reduced-form representation of liquidity spreads discussed in section 2.2.3.

In Figure 2.3, the left panel plots investment and stopping thresholds at different choices of maturity. Firms differ only in their choice of debt maturity. They are otherwise identical with capital stock $K = 1$, productivity level $A = 0.3$, and the leverage ratio of 37%. The dependence of investment on debt maturity is similar to the previous case in a liquid bond market: a firm invests less when financing with an extremely short-term debt. As shown in the panel, the firm with the short-term debt faces a high likelihood of liquidation, due to larger cash outflows while rolling over debt in states of distress.

Surprisingly, the presence of liquidity spreads does not much alter investment choices and stopping thresholds. Since debt is issued at par, the increase in required return by debtholders to compensate illiquidity costs appears as higher coupon rates. Quantitatively, the ratio of coupon to face value C/P at $m = 3.8$ is chosen to be 0.0783 in an illiquid market, while the ratio is 0.0756 in a liquid market. At the same time, however, the firm chooses to issue lower face value of debt

Figure 2.3: Investment and Firm Values at Different Maturities with Illiquidity Discounts



Investments, stopping thresholds and firm values are compared at different choices of maturity for a firm financing with illiquid debt. For all cases, the capital stock is $K = 1$, productivity level is $A = 0.3$, and the leverage ratio is 0.37. The left panel depicts investment and default threshold, A_D, and the right panel shows the total firm value against the maturity.

$P = 1.377$ in an illiquid market, compared to 1.391 in a liquid market, because of liquidity costs of debt issuance. Given that an increase in either face value or coupon represents a larger liability to debtholders, the above debt policy in an illiquid market acts in the opposite direction in influencing the firm's liquidation. Thus, the addition of illiquidity costs has a limited effect on investment, keeping the qualitative relationship between investment and maturity unchanged. For this reason, investment is maximized near at maturity of 3.6 years.

Nevertheless, the firm finds an interior optimal maturity while maximizing the total firm value, as shown in the right panel of Figure 2.3. The optimality is a result of the trade-off among opposing forces: the rollover-induced liquidation risk, investment incentives, and illiquidity discounts for a given maturity. At very short maturities, frequent rollovers make liquidation more probable, so the firm invests less and has a low value. As the maturity increases, the liquidation likelihood falls, but the financing costs due to liquidity spreads exert a greater effect. In the case of financing with very long maturity, the firm faces large costs due to illiquidity, while the benefit of increased maturity in encouraging investment is limited, as shown by the near-flat plot of investment at long maturities. All in all, the balance among the opposing forces leads to an interior optimal choice, $m = 3.78$, which maximizes the firm value.

2.3.6 Agency Costs for the Average Firm

In this section, I use the calibrated model and estimate the agency cost for the average firm. Following Leland (1998) and Moyen (2007), the agency cost is measured as a loss in the firm value when the firm makes the second-best investment decisions that maximize the equity value, as opposed to first-best decisions that maximize the total firm value. In the previous sections, the second-best firm is studied. In the first-best case, the debt contract is different from that in the second-best case, in that shareholders can commit to subsequent investment decisions after the debt issuance. Consequently, the firm is managed for the combined interests of both shareholders and debtholders. Let $V(A_t, K_t)$ denote the total value of the first-best firm, given a debt structure (P, m, C). Then the value is

$$V(A_t, K_t) = \max_{I_s} E_t^Q \left[\int_t^T e^{-r(s-t)} \big((1-\tau)A_s K_s - I_s - \theta I_s^2 + \tau\delta K_s + \tau C - h(m)D(A_s, K_s) \big)\, ds \right] \quad (2.25)$$

where T is the stopping time $T = \inf_t \{A_t \leq A_D(K_t)\}$. Note that the rollover cash flows are dropped out here, because they are zero-net when I consider the combined value of equity and debt. The stopping time is determined to maximize the total firm value, so that the smooth-pasting condition is as follows:

$$V_A(A_D(K), K) = 0. \quad (2.26)$$

The solution methods for the first-best firm's value and investment policy are similar to the second-best case, so I do not specify here for brevity. Note that in equation (2.25), tax shields from coupon payment and liquidity spreads are included, as in the second-best case. Hence the first-best and second-best differ only in the presence of the agency problem. In these settings, any value difference between the two is attributable to the agency cost.

In Figure 2.4, I summarize the estimates of the agency cost in the two environments with and without illiquidity discounts, and at two different choices of leverage. In the top panel, the firm is assumed to have the same leverage ratio, 36.93%, at all maturities. By controlling for the leverage at different maturities, the impact of debt maturity on the agency costs is isolated from the leverage effect. To this end, face value and coupon rate are chosen at each maturity so that the market leverage of the second-best firm is the same at all maturities. The first-best firm is assumed to employ the same debt structure as the second-best, and the agency cost is defined as

$$\text{Agency costs}=\frac{\text{Firm value with FB policy} - \text{Firm value with SB policy}}{\text{Firm value with SB policy}}\times 100\ (\%). \qquad (2.27)$$

The top left panel shows the agency cost of the firm facing no liquidity costs, while the top right panel shows the cost when liquidity costs are present.

An important observation is that, depending on debt maturity, the agency cost varies significantly from near 0% to above 15%, in both cases of liquid and illiquid bond markets. Given that the leverage effect is controlled, the variation indicates that maturity choice matters in quantifying the agency costs. Another finding is that the agency costs decline with debt maturity. This relation is consistent with the result in the previous section that short-term debt leads to severe underinvestment.

This setting of forcing the firm to use the same leverage helps to confirm the importance of considering maturity, but it does not allow the firm to choose the optimal leverage. Because the leverage choice is also a significant determinant of the agency cost, the forced choice gives rise to a possible bias in the measurement.

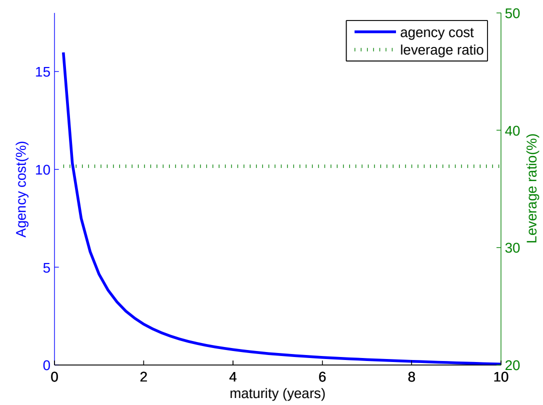
To rule out the bias, I allow the second-best firm to choose the optimal leverage at each maturity, in the bottom panel in Figure 2.4. Again, the first-best employs the same debt structure as the second-best. I find that the firm facing a liquid bond market (bottom-left panel) incurs the agency cost that increases with maturity, contrary to the above case of constant leverages. The opposite result comes from the flexibility in choosing leverage. If the firm needs to finance with short-term debt, it optimally uses lower leverage, knowing that the short maturity magnifies the volatility of rollover cash flows and liquidation is more likely to occur. On the other hand, for the firm financing with long-term debt, the adverse rollover effect is limited, so it can safely issue larger debt amount to capitalize on tax shields. With more debt in place, long maturity imposes a large agency cost on the second-best firm, which amounts to 1.09% of the total firm value at a maturity of 10 years.

I found in section 2.3.4, however, that the environment without illiquidity cannot explain the empirical choice of maturity. Given that I confirmed that debt maturity should not be ignored in quantifying agency cost, the fair measurement of the cost must be performed in an environment that

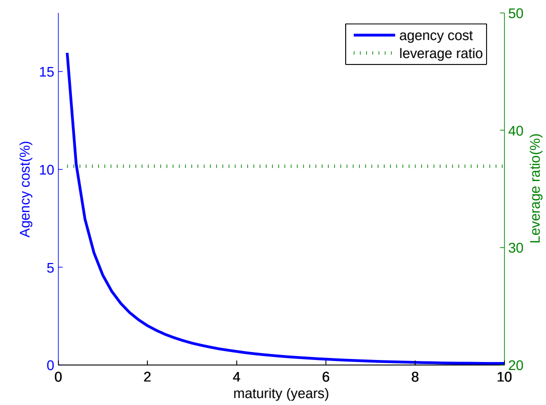
Figure 2.4: Agency Costs in Two Environments

(a) Constant leverage at all maturities

Without illiquidity discounts

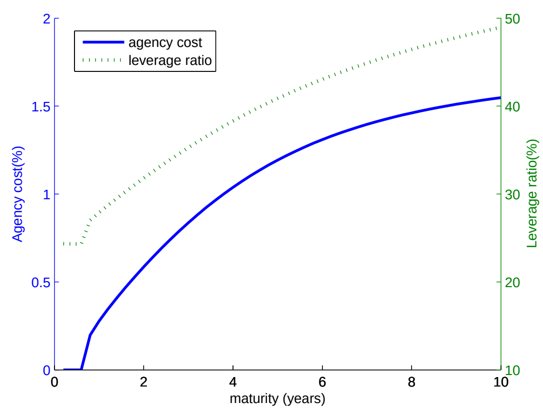


With illiquidity discounts

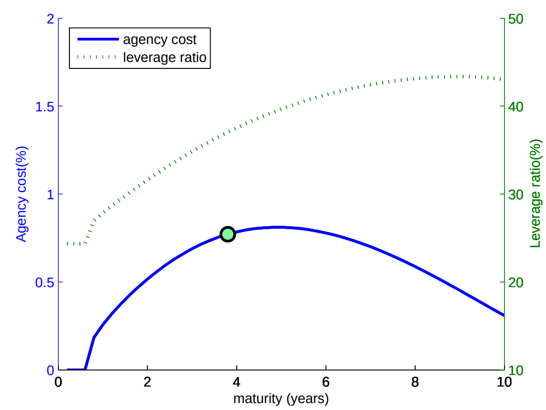


(b) Optimal leverage at each maturity

Without illiquidity discounts



With illiquidity discounts



Agency costs and leverage at different choices of maturity are plotted. In the top panel, leverage ratios at all maturities are set to be the same at 36.93% for the second-best, and the first-best uses the same debt structure as the second-best. The left panel shows results for the firm facing a liquid bond market, while the right panel shows results for the firm facing an illiquid market. In the bottom panel, optimal leverage for the second-best is used at each maturity. Agency costs are computed as a ratio of the firm value difference between the first-best and the second-best to the second-best firm value. In all cases, the capital stock is $K = 1$, and productivity level is $A = 0.3$.

is consistent with the empirical maturity choice as well as leverage. This motivates the environment with the illiquid bond market (bottom-right panel). Here I find that the optimal leverage ratio is slightly lower at all maturities than that of the firm without illiquidity discounts. This finding is intuitive: debt financing costs more due to the addition of liquidity spreads. Consistent with the intuition, the wedge in leverage ratio between the two environments increases with maturity, because longer-term debt is exposed to even larger illiquidity discounts. Regarding the agency cost, the firm in the illiquid bond market suffers lower agency costs than in a liquid bond market, as a result of its adjustment to lower leverage. Finally, at the endogenous choice of maturity of 3.78 years, which is compatible with the empirical choice, the agency costs are 0.77% of the total firm value. This indicates that the agency cost has been overestimated in previous studies that ignore the maturity choice.

2.4 Structural Estimation

In order to characterize agency costs at an economy-level, I need to study a cross-section of firms, because the agency cost for the average firm may not represent the economy-wide average of the costs. With this in mind, I turn to the model estimation of firm-specific parameters in a cross-section of firms, which govern each firm's investment and financing decisions. With the estimated firm-specific parameters, I present a distribution of agency costs for manufacturing firms in the next section.

2.4.1 Estimation Strategy

I assume that firms in the economy differ from each other in productivity growth profiles and investment-adjustment costs. I estimate these firm-specific parameters using maximum likelihood, where the estimation exploits the model predictions for firms' choices for capital structure and investment.

A firm j is characterized by the set of parameters $\Theta_j = \left(\mu_j^p, \sigma_j, \rho_j, \theta_j\right)$, which are productivity growth rate and volatility, correlation between the productivity and consumption shocks, and investment adjustment costs, respectively. For other model parameters including the stochastic discount factor, depreciation rate δ, corporate tax rate τ, liquidated value of capital stock l, and illiquidity parameters in the bond market (κ, η), I assume that they are common to all firms in the economy and maintain the calibrated values.

The estimation is performed in two stages. In the first stage, a subset of parameters, $\Theta_{j,1} = \left(\mu_j^P, \sigma_j, \rho_j\right)$ is estimated with the time series of realized productivity. The second stage uses the estimated subset of parameters and estimates the remaining parameter θ_j, with the observed investment and financing decisions. In particular, the assumption that productivity growth evolves according to geometric Brownian motion leads to the log-normal conditional distribution of productivity. Suppose that the firm j has n_j-long time series of observed productivity, $[A_{j,t}]_{t=1}^{n_j}$, and that the time series of consumption shock for the corresponding period is $[\Delta W_t^y]_{t=1}^{n_j}$ [6]. Then, the log-likelihood function in the first stage is

$$\ln\mathcal{L}_1\left(\Theta_{j,1}; [A_{j,t}]_{t=1}^{n_j}, [\Delta W_t^y]_{t=1}^{n_j}\right) = \sum_{t=2}^{n_j} \ln f_1\left(A_{j,t}|A_{j,t-1}; \Theta_{j,1}, [\Delta W_t^y]_{t=2}^{n_j}\right) \tag{2.28}$$

where f_1 is the probability density function of the log-normal distribution. The parameter estimate $\widehat{\Theta}_{j,1}$ is the parameter set that maximizes the likelihood.

In the second stage, I use the estimated subset, $\widehat{\Theta}_{j,1}$, and estimate the remaining parameter θ_j. The likelihood function $\mathcal{L}_2$ of the parameter set θ_j is based on the joint probability of observing debt structure d_j and the time series of investment-capital ratio $[i_{j,t}]_{t=1}^{n_j}$. The debt structure d_j is a set of two observables, leverage ratio z_j and debt maturity m_j. The investment at time t, $i_{j,t}$, is the amount of investment divided by the previous capital stock, $I_{j,t}/K_{j,t-1}$. Note that in the estimation, I include only one-time choice of debt structure, while a time series is used for investment choices. This is to make the estimation consistent with the model feature that the debt structure is initially chosen and kept the same afterwards, while the firm continuously adjusts investment after the debt issuance. The initial capital stock at time 0 is normalized to 1. The firm chooses the debt structure at time 0, which is optimal at the state of (A_0, K_0), given the firm-specific parameters.

Given the parameter set θ_j, the probability in the physical measure of observing the debt struc-

[6] I obtain the time series of consumption shock $[\Delta W_t^y]_{t=1}^{n_j}$ via the estimation of parameters in consumption dynamics.

ture $d_j = (z_j, m_j)$ and the investment $[i_{j,t}]_{t=1}^{n_j}$ is given by the product of conditional probabilities

$$f_2\left(d_j, [i_{j,t}]_{t=1}^{n_j}; \theta_j, \widehat{\Theta}_{j,1}, [\Delta W_t^y]_{t=1}^{n_j}\right) = f_2\left(i_{j,1}, d_j; \theta_j, \widehat{\Theta}_{j,1}, [\Delta W_t^y]_{t=1}^{n_j}\right) \times \prod_{t=2}^{n_j} f_2\left(i_{j,t} | [i_{j,k}]_{k=1}^{t-1}, d_j; \theta_j, \widehat{\Theta}_{j,1}, [\Delta W_t^y]_{t=1}^{n_j}\right). \quad (2.29)$$

Note that $i_{j,t}$ does not satisfy the Markov property, because investment at time t depends on current capital stock at t, which has been accumulated through the entire history of past investments. Thus, the conditional probability at time t depends on not only the most recent past realization at $t-1$ but also further prior realizations starting at time 1. The log-likelihood is then

$$\ln\mathcal{L}_2\left(\theta_j; d_j, [i_{j,t}]_{t=1}^{n_j}, \widehat{\Theta}_{j,1}, [\Delta W_t^y]_{t=1}^{n_j}\right) = \ln f_2\left(i_{j,1}, d_j; \theta_j, \widehat{\Theta}_{j,1}, [\Delta W_t^y]_{t=1}^{n_j}\right) + \sum_{t=2}^{n_j} \ln f_2\left(i_{j,t} | [i_{j,k}]_{k=1}^{t-1}, d_j; \theta_j, \widehat{\Theta}_{j,1}, [\Delta W_t^y]_{t=1}^{n_j}\right). \quad (2.30)$$

The model enables us to derive analytic expressions for the probability densities, which are presented in Appendix B.6. Here I provide an overview of calculating the probability. Once the parameter set is determined, there is one to one correspondence between investment at time t and realized productivity from the model structure. Additionally, I know that the conditional distribution of productivity shock is a log-normal distribution, from the assumption of geometric-Brownian motion. Consequently, I can convert the distribution of productivity into the distribution of investment-capital ratio, through the transformation of random variables.

With the log likelihood, I define the maximum likelihood estimator for the parameters of firm j:

$$\widehat{\theta}_j = \underset{\theta_j}{\operatorname{argmax}} \ln\mathcal{L}_2\left(\theta_j; d_j, [i_{j,t}]_{t=1}^{n_j}, \widehat{\Theta}_{j,1}, [\Delta W_t^y]_{t=1}^{n_j}\right). \quad (2.31)$$

For reliability of estimation, I include in the sample only firms with at least 50 quarterly observations, resulting in 592 firms. The maximum likelihood procedure is repeated for each firm j, and I obtain the firm-specific parameter estimates $\widehat{\Theta_j}$.

To investigate reliability of the parameter estimates, I compute standard errors for each of estimated parameters for each firm. Because the data on each firm in the estimation is small so asymptotic distributions of estimators cannot be applied, I use bootstrapping to compute standard errors. In particular, I generate 30 random samples for each firm. A random sample s for firm j consists of 60 quarterly consumption shocks at economy-level, $\left[\Delta W_t^{y,(s)}\right]_{t=1}^{60}$, productivity shocks to the firm, $\left[A_{j,t}^{(s)}\right]_{t=1}^{60}$, and resulting investment decisions, $\left[i_{j,t}^{(s)}\right]_{t=1}^{60}$. One sample generates a set of parameter estimates, so I obtain 30 different sets of parameters in total. The bootstrap estimate of standard error is the sample standard deviation of these parameter estimates.

Note that the estimation involves multiple steps and that in each step, the parameter estimated in the previous steps are regarded as constant. Hence, standard error of the parameter estimated in a following step should depend on variability of the estimate of previous parameters. To reflect the dependency, I estimate in a sequential way as follows:

1. With consumption shocks $\left[\Delta W_t^{y,(s)}\right]_{t=1}^{60}$, I estimate the parameters governing consumption growth, $\left[\widehat{g}^{(s)}, \widehat{\sigma_y}^{(s)}\right]$. The coefficient of risk aversion $\widehat{\gamma}^{(s)}$ is found to match the expected return on the S&P 500.

2. Given time-series of both consumption growth and firm j's productivity shocks and the parameter estimate $\left[\widehat{g}^{(s)}, \widehat{\sigma_y}^{(s)}, \widehat{\gamma}^{(s)}\right]$ from step 1, I estimate the parameters governing productivity growth, $\widehat{\Theta}_{j,1}^{(s)} \left(= \left[\widehat{\mu_j^P}^{(s)}, \widehat{\sigma_j}^{(s)}, \widehat{\rho_j}^{(s)}\right]\right)$.

3. Given time-series of firm j's investment and debt structure and the parameters of $\left[\widehat{g}^{(s)}, \widehat{\sigma_y}^{(s)}, \widehat{\gamma}^{(s)}\right]$ and $\widehat{\Theta}_{j,1}^{(s)}$ from step 1 and 2, I estimate the remaining parameter $\widehat{\theta_j}^{(s)}$.

The standard error of parameter estimate $\widehat{\chi}$ is

$$\text{S.E.}\left(\widehat{\chi}\right) = \frac{1}{S-1} \sum_{s=1}^{S} \left(\widehat{\chi}^{(s)} - \overline{\widehat{\chi}^{(s)}}\right) \tag{2.32}$$

where S is the number of random samples and $\overline{\widehat{\chi}^{(s)}} = 1/S \sum_{s=1}^{S} \widehat{\chi}^{(s)}$ is the average of the parameter estimates.

2.4.2 Estimation Results

Figure 2.5 presents the results from the firm-level estimation for 592 manufacturing firms. With respect to each of the four parameters, the firms display considerable differences from each other. As

Table 2.5: Statistics for Firm-Specific Parameter Estimates

The table presents summary statistics for firm-specific parameters of 592 firms estimated by the maximum likelihood. The parameters are growth rate μ^P and volatility σ of geometric Brownian processes of productivity, correlation ρ between productivity and consumption growth, and coefficient of investment-adjustment cost θ. Panel A reports the cross-sectional moments for the parameters, and Panel B reports correlations among parameters.

A. Summary Statistics

	Mean	Std.Dev.	25th	50th	75th
μ^P	0.0080	0.0058	0.0026	0.0067	0.0136
σ	0.1644	0.0683	0.1047	0.1818	0.2109
ρ	0.2239	0.5677	-0.2775	0.3412	0.7193
θ	0.2899	0.3668	0.0777	0.1751	0.3767

B. Correlation of Parameters

	μ^P	σ	ρ	θ
μ^P	1			
σ	0.6623	1		
ρ	0.0275	0.2059	1	
θ	0.4168	0.3207	0.0357	1

these parameters characterize each firm's operation and hence determine investment and financing policies, the dispersion implies the possibility of heterogeneous agency costs among the firms. Thus, in order to quantify the cost at the economy-level, I could not rely on the estimate for the average firm. Instead, the estimated parameter set of each firm can be used to measure the agency cost for the individual firm. Collecting such firm-by-firm costs, I can attain a precise picture of the agency costs in the economy.

In Table 2.5, I show summary statistics and correlations for the estimated parameters. Panel A confirms that firms show significant cross-sectional variations in all of the four primitive parameters. Panel B shows correlations among the estimated parameters. Some pairs of parameters show fairly large correlation; for example, the correlation between productivity growth rate and volatility is as large as 0.66 in absolute magnitude. The reported nontrivial correlation underscores the importance of joint estimation of the parameters in studying firm decisions. Otherwise, I am likely to reach a biased conclusions about the relations between firms' underlying parameters and observables.

Table 2.6: Statistics for Standard Errors of Firm-level Parameter Estimates

The table presents summary statistics for standard errors of firm-specific parameter estimates of 592 manufacturing firms. For each firm, standard errors of four parameter estimates are obtained via bootstrapping method. From the parameter estimate, 30 simulates are generated and the parameter set is estimated for each simulation. The resulting sample standard deviation of each parameter estimates represent the standard error.

	Mean	Std.Dev.	25%	50%	75%
SE(μ^P)	0.0198	0.0094	0.0143	0.0182	0.0231
SE(σ)	0.0134	0.0058	0.0088	0.0134	0.0179
SE(ρ)	0.0807	0.0422	0.0455	0.0796	0.1156
SE(θ)	0.7803	2.7698	0.0179	0.0794	0.4044

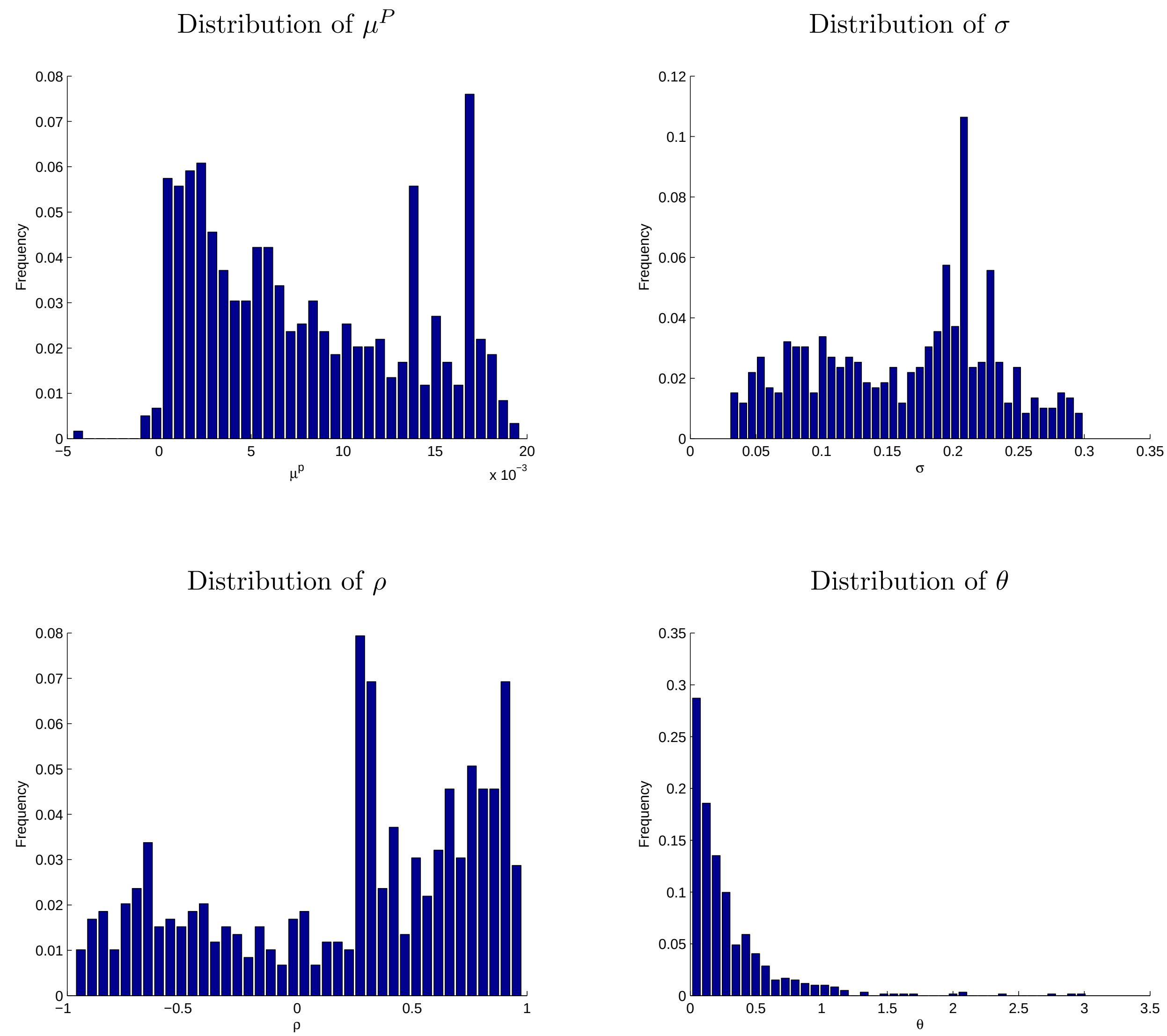
I present distributions of standard errors of parameter estimates for each of the firms in Table 2.6. As the estimated parameters vary across firms, the standard errors also show cross-sectional variations. Considering the magnitudes of parameter estimates, volatility in productivity growth σ and correlation between consumption and productivity growths ρ are fairly precisely estimated. The average standard error of investment adjustment costs θ seems large compared to the parameter estimates, but the quantile values indicate that a highly positive skewness drives the large average and that more than half of estimates are relatively precisely estimated. Looking at standard errors of productivity growth μ^P, the relative magnitude to parameter estimates are quite large. In future research, I need to consider how to improve the precision of the growth estimates.

2.5 Economy-wide Agency Costs

2.5.1 Cross-section of Agency Costs

I present the cross-sectional distribution of the agency costs in Figure 2.6. These costs are computed from firm-level estimates in section 2.4.2. Recall that the agency cost is defined as loss in firm value to the equity-value-maximizing firm, the second-best, compared to the total-value-maximizing firm, the first-best. I assume that the first-best and the second-best employ the same debt structure, which is optimal to the second-best. I then measure the value difference coming from investment and default decisions chosen by the two firms at the debt structure. The value loss is presented as

Figure 2.5: Cross-Sectional Distributions for the Firm-Specific Parameter Estimates

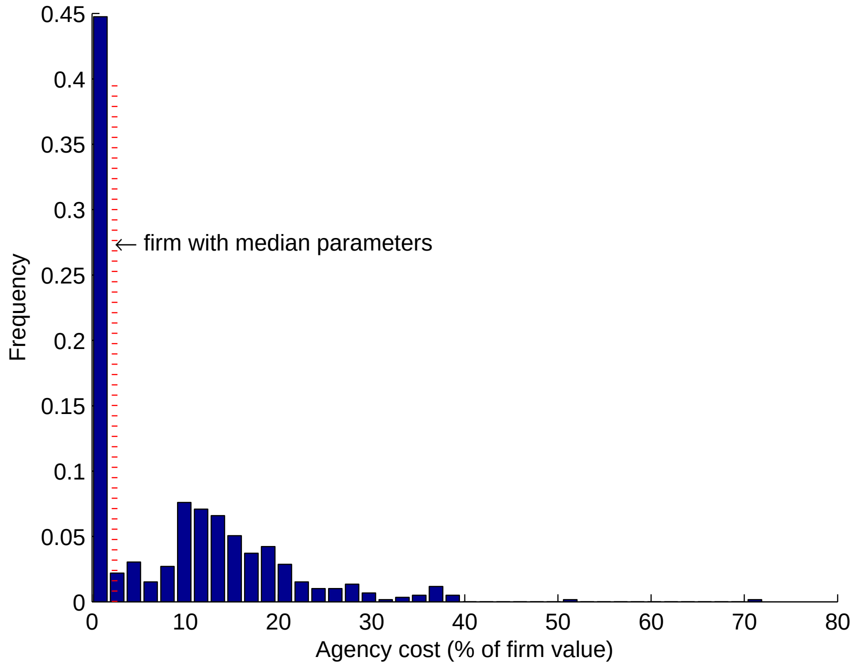


This figure shows the cross-sectional distributions of firm-specific parameter estimates for 592 manufacturing firms. The parameters are mean μ and volatility σ of productivity growth, correlation ρ between productivity and consumption growth, and coefficient of investment-adjustment cost θ.

percentages of the second-best firm values.

As I expect from the heterogeneity in firm-level parameter estimates, the agency costs also display an economically significant variation across firms: the minimum cost is as small as almost 0% and the maximum cost is as large as 72.03%. However, 58% of firms' agency costs are within 10% of firm values, and the distribution shows a noticeable positive skewness.

Figure 2.6: Cross-Sectional Distribution of Agency Costs

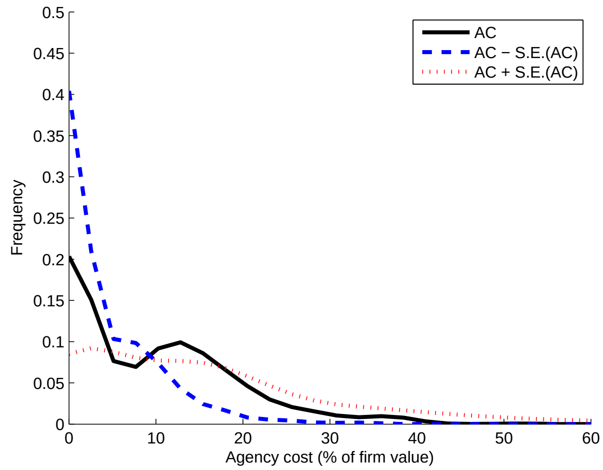


This figure shows the cross-sectional distribution of agency costs for 592 manufacturing firms.

Table 2.7 reports summary statistics for the agency costs. Panel A reports statistics when each firm chooses its own optimal debt structure. The value-weighted average of the agency costs is 7.28%, where the market values of asset are used as weights. The standard deviation of 9.87% confirms heterogeneous agency costs among firms. Both value-weighted and equally-weighted averages are significantly larger than the cost of 0.77% for the average firm. This sizable difference comes from firm heterogeneity combined with nonlinear dependence of agency costs on the firm attributes. In particular, there are two sources of the heterogeneity. Firstly, firms differ from each other in operating characteristics, as the distribution of parameter estimates in section 2.4.2 reveal. Secondly, the differences in operations causes a heterogeneity in optimal debt policy. This makes the distribution of the agency costs even more dispersed.

In order to disentangle the compounding effects, I perform a counterfactual exercise in Panel B, where all firms are forced to use the identical debt structure with a leverage ratio of 0.24 and debt maturity of 5.45 years. Given the same debt structure, the resulting cross-sectional dispersion in the

Figure 2.7: Confidence Interval of the Agency Costs



This figure shows the cross-sectional distributions of a-standard-error-larger agency costs for each firm and the distribution of a-standard-error-smaller agency costs for each firm. For display purposes, each cross-sectional distribution is estimated by standard kernel density estimation process.

costs is attributable to heterogeneity in only operating characteristics. As expected, the resulting distribution shows less dispersion with a standard deviation of 6.34%, lower than 9.87% in Panel A, confirming the compounding effect of debt structure choices. However, the equally-weighted mean is still as high as 6.12%. This indicates that the heterogeneity in operating characteristics is the major factor making the economy-wide average cost substantially higher than the cost for the average firm.

Returning to the setting where each firm chooses its optimal debt structure, Figure 2.7 shows the cross-sectional distributions of the agency costs considering a variability of the measured cost for each firm. I generate each firm's distribution of agency costs via bootstrapping method; given the firm's parameter estimate, 30 simulated paths of investment and financing decisions are generated and the parameters are reestimated, leading to 30 different values of agency costs. Then I obtain one-standard-error-higher/lower values of agency costs. With the calculated firm-level confidence interval, I plot the cross-sectional distributions for two extreme cases: the first case when all firms incur a-standard-error-larger agency costs and the second case when all firms incur a-standard-deviation-smaller agency costs. Since the agency cost is a function of the parameter estimates, some of which standard deviations are not negligible, the difference in the cross-sectional distribution of

Table 2.7: Cross-Sectional Distribution for Agency Costs

The table presents summary statistics for the agency costs for 592 firms computed from the firm-level parameter estimates. The agency costs are defined as percentage loss in the firm value to the equity-value-maximizing firms compared to the total-value-maximizing firms. Panel A reports statistics when each firm uses its own optimal debt structure. Panel B reports statistics when all firm use the identical debt structure with leverage ratio of 0.24 and debt maturity of 5.45 years.

A. Firms with Optimal Debt Structures

	VW-mean	EW-mean	Std.Dev.	25th	50th	75th
Agency Costs (% of firm value)	7.28	8.40	9.86	0.17	5.40	14.32

B. Firms with Identical Debt Structures

	VW-mean	EW-mean	Std.Dev.	25th	50th	75th
Agency Costs (% of firm value)	5.38	6.12	6.35	0.37	5.29	11.41

the agency costs between the first and second cases is not negligible either; the value-weighted average in the first case is 13.97%, while the average in the second case is 3.15%. However, still the economy-wide average of the agency costs in even the second case is greater than the cost for the average firm, confirming the larger agency costs at the economy-level.

2.5.2 Characterizing the Estimated Agency Costs

In this section, I characterize the firm-level agency costs by examining how the costs are related to firm observables. This regression can be understood as approximating the structural estimation without going through the full steps, by directly relating the input observables in the estimation to the resulting agency costs. Moreover, this exercise enables us to reevaluate the connection between agency costs and firm characteristics that the empirical literature often uses as a rationale in determining optimal capital structure.

Table 2.8 reports results from the regression of the agency costs on firm-specific observables. To do the cross-sectional regression, I take the time-series averages of observables for each firm. In specifications (1) and (2), I look into how agency costs and capital structure are related. According to the result, the leverage ratio is positively associated with agency costs, while debt maturity is negatively associated; quantitatively, a standard deviation increase in leverage ratio leads to 5.16% increase in agency cost, while a standard deviation increase in debt maturity leads to 3.93% decrease in the cost.

Even though the direction of the relation between the leverage and agency cost is the same as the well-established finding that larger leverage discourages investment, this regression result reflects more than the direct channel. In fact, the leverage optimally chosen by firms reflects underlying firm characteristics, so the unobservable relation between the firm characteristics and agency costs is indirectly captured by the relation between the cost and the leverage. For example, based on Table 2.4, firms that choose a larger leverage are those with lower volatility in productivity or with a higher coefficient of investment-adjustment costs. Such characteristics lead to smaller agency costs, if capital structure is controlled. Hence, the coefficient on leverage in the regression is a result of the opposing direct and indirect channels, and this indicates that the direct channel dominates in the firm panel. The negative association between debt maturity and agency cost reflects the direct channel identified in section 2.3.5 that longer maturity distorts investment incentives less than shorter maturity. The relation also underscores the importance of considering debt maturity in calculating agency costs.

Tobin's Q has a remarkable relation with agency costs. The specification (3) shows the negative association between Tobin's Q and the agency costs. Given that Tobin's Q has been used as a proxy for growth opportunities and that firms with high Tobin's Q are expected to suffer more agency costs, the negative association seems counterintuitive. I might consider that the negative association results from those firms' conservative debt policy. However, still in the counterfactual measurement of agency costs where all firms use the identical debt structure, the negative relation remains statistically significant, even though I do not report here. In fact, the counterintuitive relation comes from the misspecified regression; when earnings-to-asset ratio is included in the specification (4), the coefficient on Tobin's Q becomes insignificant. Higher earnings-to-asset ratio is directly related to lower debt overhang, because firms with higher earning-to-asset ratio have lower default probabilities. The direct negative relation is captured by Tobin's Q in specification (3) due to the two variables' large correlation of 0.62.

I also include other variables that literature has used to characterize agency cost – investment-

Table 2.8: Regression of the Estimated Agency Costs on Firm Characteristics

The table presents regressions of the estimated firm-level agency costs on firm-specific variables: leverage, debt maturity, investment-capital ratio, Q, asset size, and earnings-asset ratio. T-statistics are presented in parentheses below parameter estimates. *, **, *** denotes significance at 10%, 5%, 1%, respectively.

Specifications	(1)	(2)	(3)	(4)	(5)
Leverage	0.2645***	0.3262***		0.2868***	0.3083***
	(11.34)	(14.97)		(11.92)	(12.60)
Debt maturity		−0.0245***		−0.0232***	−0.0152***
		(-11.40)		(-11.00)	(-6.04)
Tobin's Q			−0.0345***	0.0018	0.0039
			(-7.06)	(0.32)	(0.69)
Earnings-asset ratio				−0.3653***	−0.3187***
				(-4.70)	(-4.05)
Investment-capital ratio					$8.46e^{-5}$***
					(3.23)
Capital stock					−0.0125***
					(-5.76)
R^2	0.1795	0.3283	0.0781	0.3611	0.4064
adj-R^2	0.1782	0.3260	0.0765	0.3567	0.3998
observations	592	592	592	592	592

capital ratio and capital stock. The two characteristics are also statistically significant predictor of agency costs. Especially, the association between investment-capital ratio and agency cost is worth mentioning. When I fix a specific firm, debt overhang causes the firm to invest less, so I might expect lower investment-capital ratio to indicate larger agency cost. However, in the cross-section of firms, the financing friction is not the only factor contributing to inter-firm differences in investment-capital ratio; heterogeneity in operating characteristics also causes differences. In fact, Table 2.4 suggests that firms with large investment are those with large productivity growth and low investment-adjustment costs. If such firms become free of agency conflicts, an increase in investment would be greater than an increase for firms with other characteristics. As a result, firms that show a large investment-capital ratio are those facing huge value loss due to agency conflict.

2.6 Conclusion

Debt maturity choice affects how existing debt distorts investment. To examine the relation between endogenous debt maturity and investment in a dynamic setting, I build a structural model where a firm jointly determines investment and financing. The calibrated model predicts that financing with shorter maturity results in greater debt overhang, which opposes Myers (1977). This is because more frequent rollovers associated with shorter maturity increase cash flow variability and thus raise default risk. The calibrated model also shows that agency costs significantly vary across debt maturity choices, controlling for leverage. The average firm with leverage and debt maturity matching the empirical averages incurs an agency cost of 0.77% of firm value. The estimate indicates that the cost has been overestimated in previous studies that do not consider firms' flexibility in choosing debt maturity.

In order to quantify the debt overhang costs at an economy level, I use panel data of manufacturing firms to estimate firm-level parameters via maximum likelihood. The resulting cross-sectional distribution of agency costs is positively skewed and shows nontrivial dispersion with a standard deviation of 9.86%. The cross-sectional dispersion arises from heterogeneity in firm characteristics, and due to nonlinear dependence of agency costs, the dispersion results in a higher cross-sectional average of the costs. The economy-wide average of the agency costs is 7.28%, which is considerably higher than the cost for the average firm.

Chapter 3

Does Corporate Investment Respond to Time-Varying Risk? Empirical Evidence

3.1 Introduction

In corporate capital budgeting, the Net Present Value (NPV) rule is widely used. Along with the NPV rule, the Capital Asset Pricing Model (CAPM) has been a standard framework for firms in determining an appropriate discount rate. The standard CAPM is based on a static environment and, of course, results in a static discount rate. In recent literature in macro-asset pricing including Cochrane (2011), however, researchers point out the importance of dynamic properties of risk premium when explaining the observed features in asset returns. This innovation in understanding of asset pricing has not yet been fully incorporated in business education for corporate finance, and whether to consider the dynamic risk premium or not has a significant value implication: Chapter 1 shows that, when the underlying economy features time-varying risk, a firm's ignoring this time-variation leads to sub-optimal investment and, consequently, a 15% loss in growth options.

Given this value implication, a question rises: do firms actually ignore the time-varying risk in their investment decisions? To answer this question, I examine the empirical relation between corporate investment and time-varying risk. If firm managers follow the CAPM to the full extent, adjusting their investment rule to the time-varying risk would be impossible. In practice, however, managers tend to round up the cost of capital, and it is possible that the managers' ad hoc adjustment happens to be consistent with the theoretical discount rates that the asset pricing literature predicts. If so, we expect an inverse relationship between corporate investment and time varying risk. All other things being equal, a rise in risk premium raises the cost of capital, so the NPV of a new project decreases, thereby lowering the probability of capital investment. In this study, I investigate whether this inverse relationship holds in observed investment decisions by corporations.

To study this empirical relation, we need a measurable variable that represents firm-specific time-varying risk. I propose using option prices on individual equities to infer the expected return on equity or the risk premium implied by the financial market. Based on the design of option contracts, option prices naturally contain information helpful in determining the risk premium of equity. Specifically, these prices reflect the joint distribution of the future price of the underlying asset and the market participants' marginal rate of substitution, and the joint distribution determines the required return on equity.

Here I describe the procedure to construct the firm-specific discount rate. For each quarter, I collect the market prices of individual equity options for a firm. Then, these prices are used to compute the implied risk premium for the corresponding quarter. Specifically, I compute the state-price density by using the theoretical result of Breeden and Litzenberger (1978) that the second derivative of call options having the same maturity with respect to strike price is proportional to the state-price density. To implement the theory, I follow the approach suggested by Ait-Sahalia and Lo (2000), and estimate the state-price density nonparametrically. Next, I estimate the physical probability density of stock returns by looking at historical returns for the past 5 years. By comparing the risk neutral with the physical density, the stochastic discount factor at each future stock price is obtained, and the stochastic discount factor enters into the Euler equation to yield the expected risk premium.

The firm-specific risk premiums are estimated for large-cap manufacturing firms constituting the S&P 100. The first finding is that the cross-sectional time-series of the risk premium fluctuate countercyclically. From the factor analysis, I identify the first principal component among these time-series and find that the component is negatively correlated with consumption growth with coefficient of -0.52. This countercyclicality is consistent with the expected pattern for market-wide risk premium in theoretical research such as Campbell and Cochrane (1999) and Bansal and Yaron (2004).

Next, this study looks at how capital investments of the manufacturing companies respond to fluctuations in the estimated risk premium. The key result is that, as theory predicts, the sample of companies negatively adjust their investments to an increase in risk premium, but they respond to the lagged risk premium by 3 to 5 quarters. This suggests that even though CAPM fails to reflect the time-varying risk, firm's arbitrary adjustment somehow results in time-varying discount rates in a consistent way that option market perceives the risk. Why firms respond with delay is indecisive in this study; the delayed response may reflect the sub-optimally slow response by firm management or it is optimally chosen considering the time to build on capital stock. Furthermore, when the investment decision is examined individually firm by firm, I find 6 out of 42 companies strongly positively adapt investment decisions to the risk premium, in the opposite way of theory prediction. This finding indicates that at least some firms ignore the time-varying risk premium make capital budgeting decisions sub-optimally.

This study's estimate of risk premium can be used in practical capital budgeting. Apart from historical approaches including CAPM and other factor models, this estimate is a forward-looking measure in that it incorporates market participants' assessments of distribution of future asset returns. Additionally, the estimate is obtained nonparametrically, so it is free of the bias that parametric restrictions may cause, especially when the actual dynamics of prices differ from the parametric assumptions.

The paper is organized as follows. Section 3.2 presents the methodology to estimate firm-specific risk premium. In Section 3.3, I discuss data and empirical results. Section 3.4 concludes.

3.2 Methodology

This section presents how to estimate the firm-specific risk premium implicit in option prices. Because the option prices reflect the current expectation of future prices of underlying assets and investors' risk aversion, options can be used to determine the risk premium. Specifically, by applying the Euler equation, we know that the expected return on an asset is expressed as follows:

$$E_t\left[R_{t,T}\right] = R^f_{t,T} - R^f_{t,T}\mathrm{Cov}\left(M_{t,T}, R_{t,T}\right) \tag{3.1}$$

where $R^f_{t,T}$ is the risk-free return, $M_{t,T}$ is the stochastic discount factor, and $R_{t,T}$ is return on the asset. According to Breeden and Litzenberger (1978), option prices are informative of the state-price density, which is closely related to the stochastic discount factor. In particular, the state-price density of the asset price at time T can be obtained from a cross-section of options having the the same maturity T and different strike prices. In addition, we can use historical prices of the asset and estimate the physical probability density of the future price. By comparing these two densities for a certain state of the price, I estimate the stochastic discount factor at that state. By repeating this procedure state by state to construct the joint distribution of the stochastic discount factor and future price (or return), I compute the risk premium on a quarterly basis; for each quarter, I use price information in that quarter to provide the risk premium specific for the quarter.

3.2.1 Converting American Option Prices to European Option Prices

Computing the state-price density requires European option prices in the approach of Breeden and Litzenberger (1978). All individual equity options included in the data, however, are American options. Thus, I need to convert American option prices to European option prices. To make the conversions, I employ the implied binomial tree approach of Rubinstein (1994) and follow the procedure suggested by Tian (2011). The basic idea is to calibrate the binomial tree of underlying asset price so that the model predicts American option prices close to the observed prices, while the probability distribution at maturity is as smooth as possible. With the calibrated binomial tree, I calculate the early-exercise premium for each American option and recover the price of a corresponding European option by subtracting the premium. The detailed procedure is explained in Appendix C.1.

3.2.2 Estimating Risk-Neutral Density

The Arrow-Debreu security that pays at a certain state of future price can be replicated by combining call options with different strike prices. Accordingly, the state-price density at S_T, denoted by

$q_t(S_T)$, is proportional to the second-derivative of option prices with respect to strike price, i.e.,

$$q_t(S_T) = \exp(r^f_{t,T}\tau)\frac{\partial^2 H}{\partial X^2} \tag{3.2}$$

where H is the European option price, X is the strike price, and τ is the time to maturity. Hence, if we have an option pricing formula that is twice-differentiable with respect to strike price, the state-price density can be estimated.

Ait-Sahalia and Lo (1998) introduce the nonparametric estimation of option prices and state-price density. By relaxing a parametric assumption on the distribution of underlying asset prices, the nonparametric estimation addresses a possible bias in a parametric estimation, especially when the assumed parametric distribution differs from the actual dynamics of the prices. In particular, they show that the nonparametric estimator captures salient features in the option market including the volatility smile, which a standard parametrized model, such as log-normal distribution, fails to explain. Because every feature in the option market is informative of risk aversion and the resulting risk premium, I employ this nonparametric approach.

For practical purpose of obtaining efficient estimators with a limited number of observations, Ait-Sahalia and Lo (2000) propose semiparametric estimators where option prices are given by the extended Black-Scholes formula in which the implied volatility is a nonparametric function. Assuming that the implied volatility is a function of both the moneyness of the option and the time to maturity, the call option price is

$$H\left(S_t, X, \tau, r^f_{t,T}, \delta_{t,T}\right) = H_{BS}\left(F_{t,T}, X, \tau, r^f_{t,T}, \sigma\left(X/F_{t,T}, \tau\right)\right) \tag{3.3}$$

where $\delta_{t,T}$ is the dividend yield, $F_{t,T}$ is the forward-price of the asset, σ is implied volatility, and $H_{BS}(\cdot)$ is the Black-Scholes option price. Using observed option prices in quarter m, the implied volatility for that quarter is estimated by the kernel regression, as follows:

$$\widehat{\sigma_m}\left(X/F_{t,T}, \tau\right) = \frac{\sum_{i=1}^n k_{X/F}\left(\frac{X/F_{t,T} - X_i/F_{t_i,T_i}}{h_{X/F}}\right) k_\tau\left(\frac{\tau - \tau_i}{h_\tau}\right)\sigma_i}{\sum_{i=1}^n k_{X/F}\left(\frac{X/F_{t,T} - X_i/F_{t_i,T_i}}{h_{X/F}}\right) k_\tau\left(\frac{\tau - \tau_i}{h_\tau}\right)} \tag{3.4}$$

where σ_i is the implied volatility of observation i in quarter m, $k_{X/F}$ and k_τ are the kernel functions, and $h_{X/F}$ and h_τ are bandwidth parameters. I use the Gaussian kernel, $k(z) = 1/\sqrt{2\pi}\exp\left(-z^2/2\right)$. The bandwidth parameters are chosen to minimize the sum of squared errors of observations as suggested in Hardle (1994).

Taking the second derivative of the option price, it follows that the state-price density of S_T is

$$q_t(S_T) = e^{r^f_{t,T}\tau}\left[\frac{\partial^2 H_{BS}\left(F_{t,T}, X, \tau, r^f_{t,T}, \widehat{\sigma_m}\left(X/F_{t,T}, \tau\right)\right)}{\partial X^2}\right]_{X=S_T}. \tag{3.5}$$

3.2.3 Estimating Physical Density

The physical density is estimated from historical returns on stocks. For each quarter, I collect the time series of daily returns on a firm's stock during the past 5 years and estimate the physical density of returns over period τ. Let r_τ denote the τ-period continuously compounded return. The kernel estimator of the physical density for quarter m is

$$\widehat{g_m}(r_\tau) = \frac{1}{Nh_r}\sum_{i=1}^{N} k_r\left(\frac{r_\tau - r_{t_i,t_i+\tau}}{h_r}\right). \tag{3.6}$$

The density of returns is converted to the density of prices, as follows [1] :

$$\widehat{f}_t(S_T) = \frac{\widehat{g_m}(\log(S_T/S_t))}{S_T}. \tag{3.8}$$

3.2.4 Estimating Risk Premium

I propose an estimator of risk premium, $\widehat{R^p_t}$, which combines information from the state-price density and the physical density. First, by comparing the two densities, we obtain the stochastic discount factor, as follows:

$$\widehat{M_{t,T}}(S_T) = \frac{1}{R^f_{t,T}}\frac{\widehat{q}_t(S_T)}{\widehat{p}_t(S_T)}. \tag{3.9}$$

[1] From the density of returns, we can compute the cumulative distribution function of stock prices:

$$\Pr(S_T \le S) = \Pr(S_t e^{r_\tau} \le S) = \Pr(r_\tau \le \log(S/S_t)) = \int_{-\infty}^{\log(S/S_t)} g(r_\tau)dr_\tau. \tag{3.7}$$

The density of price is then

$$f(S) = \frac{\partial \Pr(S_T \le S)}{\partial S} = \frac{g(\log(S/S_t))}{S}.$$

Table 3.1: Summary Statistics

This table presents descriptive statistics of firm-level variables. The statistics are calculated from annualized variables.

Summary Statistics					
Variable	Mean	Std.Dev.	25%	50%	75%
INVEST_t	0.0364	0.2627	-0.0357	0.0266	0.0916
$E_t[R^p]$ (%)	5.23	8.29	-0.50	4.03	9.66
SIZE_t	10.0338	1.2526	9.5186	10.2080	10.6548
ROA_t (%)	8.18	15.49	4.41	8.39	13.55
LEV_t	0.2143	0.1353	0.1243	0.2026	0.2904

Plugging the stochastic discount factor into the Euler equation, the risk premium is

$$\widehat{E_t}\left[R^p\right] = -R^f_{t,T}\text{Cov}\left(\widehat{M_{t,T}}(S_T), \frac{S_T}{S_t}\right). \tag{3.10}$$

Appendix C.2 provides the asymptotic distribution of the risk premium.

3.3 Empirical Results

3.3.1 Data

My analysis here is focused on large-cap US manufacturing companies (SIC 2000-3999) that constitute the S&P 100 Index on March 31, 2015. Data on individual equity options are obtained from OptionMetrics for the years 1996 through 2013. I impose a condition: for a company to be in the sample, options on the company's equity should have at least 10 quarters-long history of trading during that period. As a result, 42 companies are in the final sample. To extract the state-price density from option prices, we need sets of options having the same maturity date but different strike prices. Thus, only the sets with at least 5 different strike prices are included in the sample. Consequently, each of the 42 firms has, on average, 867 observations of option price for each quarter. At the same time, I use daily stock returns from CRSP for these companies to estimate the physical density. For each quarter, I look back to daily returns during the past 5 years and obtain 1280 daily returns on average per quarter.

For the 42 companies in the sample, I obtain quarterly financial statements from COMPUSTAT.

Following Arif and Lee (2014), a firm's investment is defined as the change in the net operating assets (NOA), which are current assets (ACTQ) plus property, plant and equipment (PPENTQ) minus current liability (LCTQ). Then, the investment is scaled by average total assets (ATQ), resulting in firm i's investment at time t defined as

$$\text{INVEST}_{i,t} = \frac{\text{NOA}_{i,t} - \text{NOA}_{i,t-1}}{\frac{1}{2}\left(\text{ATQ}_{i,t-1} + \text{ATQ}_{i,t}\right)}. \tag{3.11}$$

Other firm-level variables are measured in standard ways in literature. Book leverage ratio ($\text{LEV}_{i,t}$) is the sum of debt in current liabilities (DLCQ) and long-term debt (DLTTQ) divided by total assets. Profitability of a firm is measured by return on assets ($\text{ROA}_{i,t}$), which is current net income (NIQ) divided by previous total assets (ATQ). Firm size ($\text{SIZE}_{i,t}$) is measured by the log of total assets.

In addition to the estimated risk premium, I also include other variables representing discount rates. Returns on the S%P 500 Index are used as returns on the stock market, and the risk-free returns are from 10-year treasury constant maturity rates from FRED. Table 3.1 presents descriptive statistics of these variables.

3.3.2 Firm-Specific Risk Premium

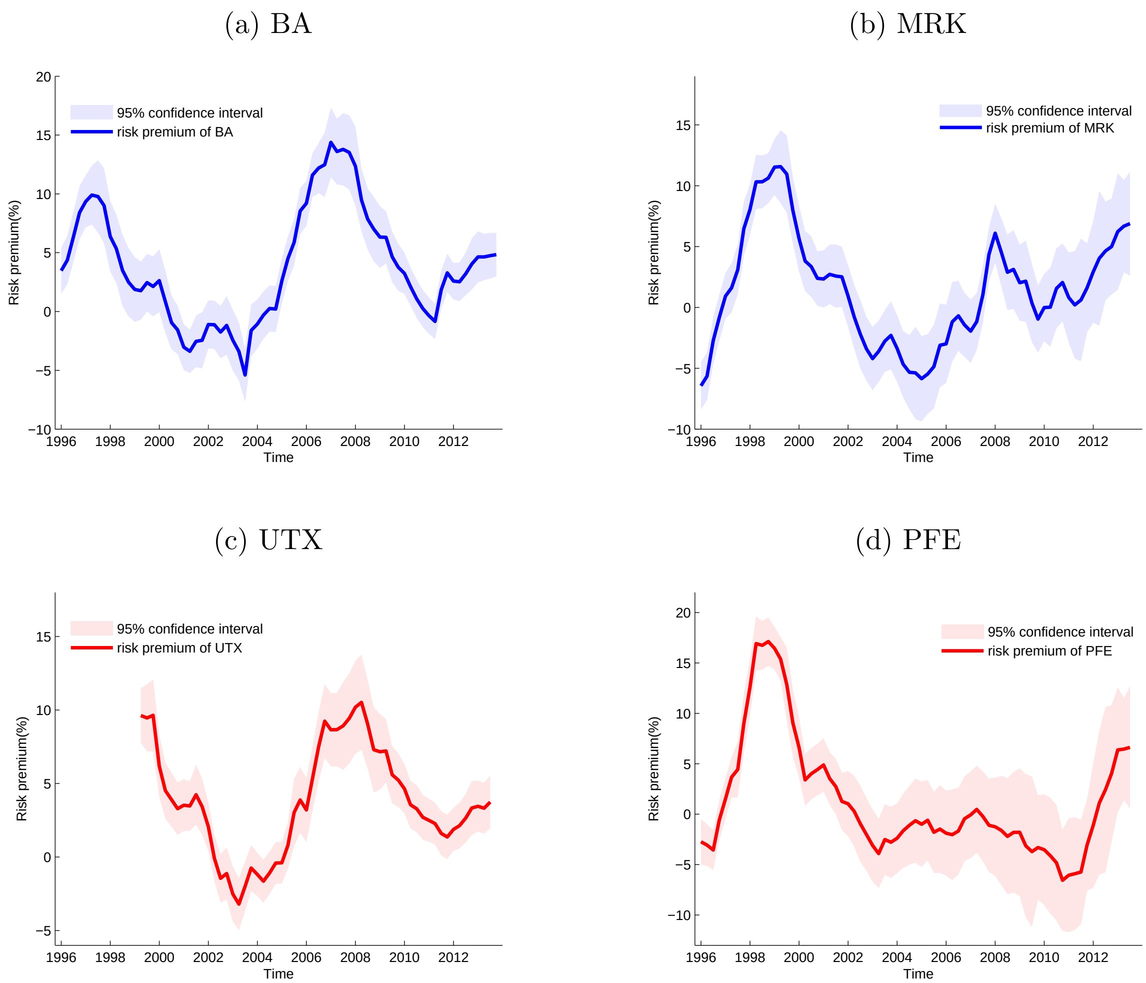
First, it is worth discussing general features of the estimated firm-specific premium before we study the relationship between the risk premium and investment. Figure 3.1 depicts time-series of the risk premium for 4 selected firms: companies in Aircraft and Parts industry, Boeing Co (BA) and United Technologies Corporation (UTX), and companies in Pharmaceutical Preparations industry, Merck & Co.,Inc. (MRK) and Pfizer Inc. (PFE).

The first observation is that all of the selected companies show significant fluctuations during 1996-2013[2]. Furthermore, the risk premiums of all are relatively high during the 2008 financial crises. When I compare two companies in the same industry, a common pattern becomes apparent, exemplified by MRK and PFE in pharmaceutical industry. The risk premiums of both companies were higher in 1998 than in 2008, but the pattern was not observed for BA in aircraft industry. The greater commonality within the industry is confirmed by correlation coefficients: the correlation

[2]The risk premium of UTX is plotted from 1999 Q2 due to data availability

Figure 3.1: Examples of Estimated Risk Premium

This figure presents time-series of the estimated risk premium for selected firms. Selected are Boeing Co (BA) and United United Technologies Corporation (UTX) in industry of Aircraft and Parts and Merck & Co.,Inc. (MRK) and Pfizer Inc. (PFE) in industry of Pharmaceutical Preparations.



(a) BA (b) MRK (c) UTX (d) PFE

coefficient between BA and UTX is 0.76, and that between MRK and PFE is 0.79. Compared to the intra-industry correlation, inter-industry correlation is low, for example, 0.25 between UTX and PFE. These findings confirm the intuition that two firms in the same industry are likely to be exposed to similar risks in business operations.

Next, with the panel data of firm-specific risk premiums, I extract a common factor for the multiple time series. Behind this exercise, the idea is that the common factor explaining the joint time-variation may capture the market-wide risk premium. From the original sample of 42 firms, I subsample 24 firms with no missing estimates of risk premium during 2001 Q1 - 2013 Q4 and perform Principal Component Analysis (PCA) on that subsample. Figure 3.2 plots the first principal component. A salient feature is that the first component generally moves countercyclically: the value of the component is relatively low during the boom of early 2000 and increases significantly during the financial crisis of 2008. Additionally, the correlation between the first component and consumption growth is -0.52. This countercyclicality is consistent with the expected pattern for market-wide risk premium in theoretical research including Campbell and Cochrane (1999) and Bansal and Yaron (2004). Moreover, the first component explains 24.8% of variances on average in firm-specific risk premium.

3.3.3 Predicting Firm-Level Investment

I examine whether variables associated with discount rates and investment opportunities forecast firm-level investments. From the NPV rule, the theory predicts that investments should respond negatively to an increase in discount rate and positively to more investment opportunities, unless the two factors are perfectly negatively correlated. Therefore, if firms correctly adjust investment decisions to time-varying risk, we should observe a negative association between investments and the firm-specific risk premium.

I run a pooled regressions with the following specification:

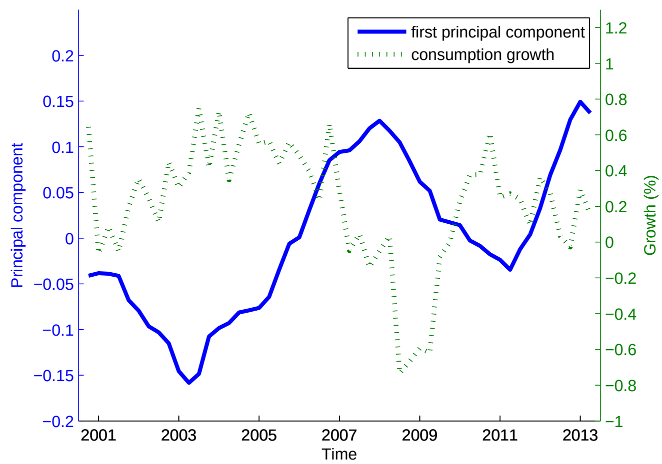
$$\text{INVEST}_{i,t} = \beta X_{i,t} + \gamma Y_{i,t-1} + \delta_i + \epsilon_{i,t+1} \tag{3.12}$$

where δ_i is a firm dummy to control for firm-specific effect, $X_{i,t}$ represents contemporaneous regressors, and $Y_{i,t-1}$ represents lagged regressors. $X_{i,t}$ includes profitability ($\text{ROA}_{i,t}$) and risk-free return (R_t^f), while $Y_{i,t-1}$ includes stock market return (R_{t-1}^m), firm size measured by the log of book value of asset (SIZE_{t-1}), the market-to-book ratio (Q_{t-1}), leverage ratio (LEV_{t-1}), and the risk premium lagged by m quarters ($E_{t-m}\left[R_i^p\right]$). Considering the possibility that firms respond to the risk premium with a delay, we regress investments on different lags of the risk premium.

Table 3.2 presents the regression results. In all specifications, both current profitability and the market-to-book ratio are significantly positively associated with investments. These findings are intuitive and conform to the results in literature including Gala and Gomes (2012). Firms invest

Figure 3.2: Consumption Growth and Common Factor in Firm-Specific Risk Premium

This figure presents the time-series of consumption growth and the common factor in firm-specific risk premium. Consumption growth is measured by real consumption per capital for nondurable goods and service from FRED. The common factor in the risk premium is the first principal component from PCA on the panel data of 24 companies' risk premium.



more in capital stock when current profits are high and firms with larger investment opportunities represented by a higher market-to-book ratio also invest more. Leverage ratio can influence investments through two channels. First, a higher leverage induces greater agency conflict of debt overhang (Myers (1977)), so it can decrease investments. Second, controlling for expected returns on equity and debt, a higher leverage implies lower cost of capital, given that expected return on equity is usually higher than that on debt. Therefore, a rise in leverage ratio may increase investments. In the regression, the positive coefficients on leverage ratio demonstrate that the second effect dominates in the sample.

The key result from Table 3.2 is relationship between investments and different discount rates across specifications. In specification (1) and (2) where the estimated risk premium lagged by up to two quarters are used, only insignificant association between investments and the risk premium is reported, even though the association is negative as theory predicts. On the contrary, specification (3) through (5) presents statistically significant and negative association between investments and the risk premium lagged by 3 to 5 quarters. These findings indicate that firms' investment decisions

Table 3.2: Predicting Firm-Level Investment

This table presents regressions of the firm-level investment on its candidate determinant. $E_{t-m}\left[R_i^p\right]$ is the estimated risk premium of firm i in quarter $t-m$, R_{t-1}^m is actual stock market return over 1 year until quarter $t-1$, and R_t^f is 10-year treasury constant maturity rate in quarter t. Size_{t-1} is the book value of total assets, Lev_{t-1} is the book value of leverage ratio, ROA_t is return on assets and, Q_{t-1} is the market-to-book ratio. T-statistics are presented in parentheses below parameter estimates. *, **, *** denotes significance at 10%, 5%, 1%, respectively.

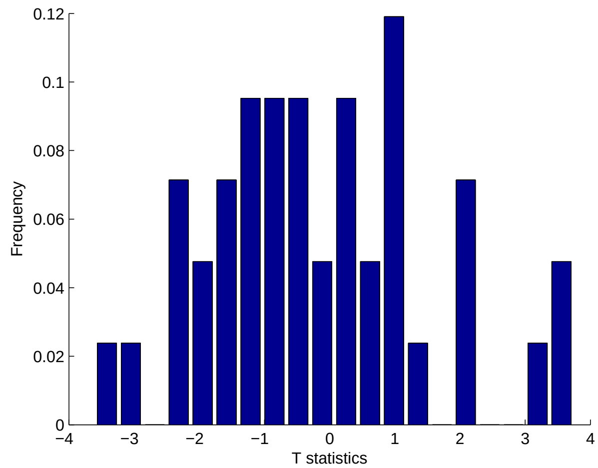
Dependent variable:	Investment-capital ratio at t						
Specifications:	(1)	(2)	(3)	(4)	(5)	(6)	(7)
$E_{t-1}\left[R_i^p\right]$	-0.057						
	(-0.81)						
$E_{t-2}\left[R_i^p\right]$		-0.099					
		(-1.44)					
$E_{t-3}\left[R_i^p\right]$			-0.137^{**}				
			(-1.97)				
$E_{t-4}\left[R_i^p\right]$				-0.081^{**}			
				(-1.98)			
$E_{t-5}\left[R_i^p\right]$					-0.088^{**}		
					(-2.16)		
$E_{t-6}\left[R_i^p\right]$						0.012	
						(0.31)	
R_{t-1}^m							-0.010
							(-1.36)
R_t^f	0.212	0.334^{**}	0.378^{**}	0.424^{***}	0.485^{***}	0.269^{*}	0.137
	(1.54)	(2.44)	(2.67)	(2.84)	(3.18)	(1.83)	(0.94)
Q_{t-1}	0.010^{***}	0.010^{***}	0.009^{***}	0.009^{***}	0.009^{***}	0.009^{***}	0.009^{***}
	(9.81)	(10.54)	(9.80)	(9.71)	(9.85)	(10.48)	(10.56)
ROA_t	0.247^{***}	0.258^{***}	0.301^{***}	0.239^{***}	0.254^{***}	0.219^{***}	0.247^{***}
	(3.95)	(4.23)	(4.67)	(3.73)	(3.84)	(3.37)	(3.95)
SIZE_{t-1}	-0.006^{**}	0.003	0.0004	-0.0003	0.001	-0.003	-0.007^{**}
	(-1.96)	(0.11)	(0.11)	(-0.07)	(0.39)	(-0.98)	(-2.21)
LEV_{t-1}	0.02^{**}	0.065^{***}	0.051^{**}	0.051^{**}	0.051^{**}	0.045^{**}	0.053^{***}
	(2.35)	(3.16)	(2.38)	(2.33)	(2.22)	(2.06)	(2.61)
adj-R^2	0.2885	0.2830	0.2734	0.2534	0.2526	0.2738	0.2738
observations	2442	2361	2312	2222	2174	2173	2442

Figure 3.3: T-Statistics of the Risk Premium in Predicting Investment

This figure presents the histogram of t-statistics of the risk premium in firm-level regression on investment for 42 manufacturing firms. The regression equation is

$$\text{INVEST}_t = \beta_0 + \beta_1 E_{t-3}\left[R^p\right] + \beta_2 R_t^f + \beta_3 Q_{t-1} + \beta_4 \text{ROA}_t + \beta_5 \text{SIZE}_{t-1} + \beta_6 \text{LEV}_{t-1} + \epsilon_t.$$

The histogram is for t-statistics of β_1 across 42 firm-level regressions.



correctly adapt to the time-varying risk premium but responds with delays. In other words, firm managers' ad hoc adjustment somehow takes into account the time-varying risk. When the risk premium is lagged by more than 5 quarters, no significant relationship between investment and the risk premium is reported.

I next look into firm-level response by performing the regression (3.12) without fixed effect, separately, firm by firm. Based on the time-series regression for each firm, we can identify firms correctly or incorrectly responding to the risk premium. Figure 3.3 shows the histogram of t-statistics for the risk premium across 42 firms. I find that at 5% significance level, 6 out of 42 firms adapt investment decisions negatively to an increase in the risk premium. Surprisingly, other 6 companies adapt positively with t-statistics higher than 1.96, in the opposite way that the theory predicts. This result indicates that at least some firms ignore the time-varying risk premium and make capital-budgeting decisions sub-optimally.

3.4 Conclusion

Capital budgeting is a core of value creation for corporations and involves numerous components in the decision-making. Given the recent advances in understanding discount rates in asset pricing literature, I empirically test whether firms take the time-varying risk into account when investing in capital. To measure the firm-specific risk premium, I use option prices on individual equities and extract the risk premium implied by the financial market. The finding is that, as theory predicts, firms negatively adjust investments to fluctuations in the risk premium, but they respond with delay of 3 to 5 quarters. Through firm-level regression, I document that there are firms that respond to the risk premium in the opposite of theory prediction. I suggest that, to avoid sub-optimal investment decisions, those firms should adopt the approach in this study to structurally determine the cost of capital.

Appendix A

A.1 The Kreps-Porteus Pricing Kernel

The pricing kernel in a representative agent model is the marginal rate of substitution between consumption at date t and consumption in state s at $t+1$. Define $\pi(s)$ as the probability of state s at $t+1$. Then the certainty equivalent is

$$\mu_t\left(U_{t+1}\right) = \left[\sum_s \pi(s) U^{\alpha}_{t+1(s)}\right]^{1/\alpha} \tag{A.1}$$

where $U_{t+1}(s)$ is continuation utility. Some derivatives of equation 1.1 and equation A.1 are :

$$\begin{aligned} \frac{\partial U_t}{\partial c_t} &= U_t^{1-\rho}(1-\beta)c_t^{\rho-1} \\ \frac{\partial U_t}{\partial \mu_t(U_{t+1})} &= U_t^{1-\rho}\beta\mu_t(U_{t+1})^{\rho-1} \\ \frac{\partial \mu_t(U_{t+1})}{\partial U_{t+1}(s)} &= \pi(s)U_{t+1}(s)^{\alpha-1}\mu_t(U_{t+1})^{1-\alpha}. \end{aligned} \tag{A.2}$$

The marginal rate of the substitution between consumption at t and consumption in state s at $t+1$ is

$$\begin{aligned} \frac{\partial U_t/\partial c_{t+1}(s)}{\partial U_t/\partial c_t} &= \frac{\left[\partial U_t/\partial \mu_t(U_{t+1})\right]\left[\partial \mu_t(U_{t+1})/\partial U_{t+1}(s)\right]\left[\partial U_{t+1}(s)/\partial c_{t+1}(s)\right]}{\partial U_t/\partial c_t} \\ &= \pi(s)\beta\left(\frac{c_{t+1}(s)}{c_t}\right)^{\rho-1}\left(\frac{U_{t+1}(s)}{\mu_t(U_{t+1})}\right)^{\alpha-\rho}. \end{aligned} \tag{A.3}$$

The rate of substitution without the probability is the pricing kernel used in the model.

A.2 Loglinear Approximation and Solution for the Scaled Utility

Note that the utility can be scaled by dividing current consumption with the use of homogeneity of both the time aggregator and the certainty equivalent function. If we define scaled utility $u_t = U_t/c_t$, then the equation can be scaled to

$$u_t = \left[(1-\beta) + \beta\mu\left(g_{t+1}u_{t+1}\right)^{\rho}\right]^{1/\rho} \tag{A.4}$$

where $g_{t+1} = c_{t+1}/c_t$ is the growth rate of consumption. A first-order approximation of $\log u_t$ around $\log u$ is

$$\begin{aligned} \log u_t &= \rho^{-1}\log\left[(1-\beta) + \beta\mu_t(g_{t+1}u_{t+1})^{\rho}\right] \\ &= \rho^{-1}\log\left[(1-\beta) + \beta e^{\log\mu_t(g_{t+1}u_{t+1})^{\rho}}\right] \\ \left[(1-\beta) + \beta e^{\log\mu_t(g_{t+1}u_{t+1})^{\rho}}\right] &\approx \kappa_0 + \kappa_1\log\mu_t(g_{t+1}u_{t+1}) \end{aligned} \tag{A.5}$$

where

$$\begin{aligned} \kappa_1 &= \frac{\beta e^{\log\mu}}{(1-\beta) + \beta e^{\rho\log\mu}} \\ \kappa_0 &= \rho^{-1}\log\left[(1-\beta) + \beta e^{\rho\log\mu}\right] - \kappa_1\log\mu. \end{aligned} \tag{A.6}$$

To get the solution for the scaled utility, guess the utility as function of x_t and v_t , in specific,

$$\log u_t = u + p_x^T x_t + p_v v_t. \tag{A.7}$$

Then, verify the function by plug the form into equation A.5 and compute for the coefficients $\{u, p_x, p_v\}$. First, compute the certainty equivalent:

$$\begin{aligned}
\mu_t(g_{t+1}u_{t+1})^\alpha &= E_t\left[(g_{t+1}u_{t+1})^\alpha\right] \\
&= E_t\left[e^{\alpha(g+e^T x_{t+1}+u+p_x^T x_{t+1}+p_v v_{t+1})}\right] \\
&= E_t\left[e^{\alpha\left(g+u+(e^T+p_x^T)(Ax_t+v_t^{1/2}Bw_{t+1})+p_v((1-\varphi)v+\varphi v_t+bw_{t+1})\right)}\right] \\
&= e^{\alpha\left(g+u+p_v(1-\varphi)v+\frac{1}{2}\alpha p_v^2 bb^T\right)+\alpha(e^T+p_x^T)Ax_t+\alpha\left(p_v\varphi+\frac{1}{2}\alpha\left(e^T+p_x^T\right)BB^T(e+p_x)\right)v_t}. \qquad \text{(A.8)}
\end{aligned}$$

Here I used $Bb^T = 0$, and $E\left[e^x\right] = e^{a+\frac{1}{2}b}$ for $x \sim N(a, b)$. Plugging this to equation A.5 leads to

$$\begin{aligned}
u + p_x^T x_t + p_v v_t &= \kappa_0 + \kappa_1\Big[g + u + p_v(1-\varphi)v + \frac{1}{2}\alpha p_v^2 bb^T + (e^T + p_x^T)Ax_t \\
&\quad + \left(p_v\varphi + \frac{1}{2}\alpha\left(e^T + p_x^T\right)BB^T\left(e + p_x\right)\right)v_t\Big] \qquad \text{(A.9)}
\end{aligned}$$

The coefficients can be solved for as follows:

$$\begin{aligned}
u &= \kappa_0 + \kappa_1\left[u + g + p_v(1-\varphi)v + \frac{\alpha}{2}p_v^2 bb^T\right] \\
p_x^T &= e^T(\kappa_1 A)(I - \kappa_1 A)^{-1} \\
p_v &= \frac{\alpha}{2}\kappa_1(1-\kappa_1\varphi)^{-1}(e+p_x)^T BB^T(e+p_x). \qquad \text{(A.10)}
\end{aligned}$$

A.3 Derivation of the Pricing Kernel

We can substitute the scaled utility into equation 1.2. The pricing kernel has the term

$$\begin{aligned}
\log(g_{t+1}u_{t+1}) - \log\mu_t(g_{t+1}u_{t+1}) &= g + e^T x_{t+1} + u + p_x^T x_{t+1} + p_v v_{t+1} \\
&\quad - \left(g + u + p_v(1-\varphi)v + \frac{1}{2}\alpha p_v^2 bb^T\right) - (e^T + p_x^T)Ax_t \\
&\quad - \left(p_v\varphi + \frac{1}{2}\alpha\left(e^T + p_x^T\right)BB^T\left(e + p_x\right)\right)v_t \\
&= v_t^{1/2}(e_t^T + p_x^T)Bw_{t+1} + p_v^T bw_{t+1} \\
&\quad -\frac{\alpha}{2}p_v^2 bb^T - \frac{\alpha}{2}\left(e^T + p_x^T\right)BB^T\left(e + p_x\right)v_t \qquad \text{(A.11)}
\end{aligned}$$

The pricing kernel follows as

$$
\begin{aligned}
\log m_{t+1} &= \log\beta + (\rho-1)\log g_{t+1} + (\alpha-\rho)\left[\log(g_{t+1}u_{t+1}) - \log\mu_t\left(g_{t+1}u_{t+1}\right)\right] \\
&= \log\beta + (\rho-1)g - (\alpha-\rho)(\alpha/2)p_v^2 bb^T \\
&\quad +(\rho-1)e^T A x_t - \left[(\alpha-\rho)(\alpha/2)(e+p_x)^T BB^T(e+p_x)\right] v_t \\
&\quad +v_t^{1/2}\left[(\rho-1)e + (\alpha-\rho)(e+p_x)\right]^T Bw_{t+1} + (\alpha-\rho)p_v w_{t+1}. \\
&\equiv \delta_0 + \delta_x^T x_t + \delta_v v_t + \lambda_x^T w_{t+1} + \lambda_v^T w_{t+1} \qquad \text{(A.12)}
\end{aligned}
$$

A.4 Equity Returns

We define equity as the consumption stream. The return is the ratio of its value at $t+1$, measured in units of $t+1$ consumption, to the value at t, measured in units of t consumption. The value at $t+1$ is U_{t+1} expressed in c_{t+1} units:

$$
\begin{aligned}
U_{t+1}/\left(\partial U_{t+1}/\partial c_{t+1}\right) &= U_{t+1}/\left[(1-\beta)U_{t+1}^{(}1-\rho)c_{t+1}^{\rho-1}\right] \\
&= (1-\beta)^{-1}u_{t+1}^{\rho}c_{t+1} \qquad \text{(A.13)}
\end{aligned}
$$

The value at t is the certainty equivalent expressed in c_t units:

$$
\begin{aligned}
q_t^c c_t &= \frac{\partial U_t/\partial\mu_t(U_{t+1})}{\partial U_t/\partial c_t}\mu_t(U_{t+1}) = \frac{\beta\mu_t(U_{t+1})^{\rho}}{(1-\beta)c_t^{\rho}}c_t \\
&= \beta(1-\beta)^{-1}\mu_t(g_{t+1}u_{t+1})^{\rho}c_t. \qquad \text{(A.14)}
\end{aligned}
$$

The return is the ratio:

$$
\begin{aligned}
r_{t+1}^c &= \beta^{-1}\left[u_{t+1}/\mu_{t+1}(g_{t+1}u_{t+1})\right]^{\rho} g_{t+1} \\
&= \beta^{-1}\left[g_{t+1}u_{t+1}/\mu_{t+1}(g_{t+1}u_{t+1})\right]^{\rho} g_{t+1}^{1-\rho} \qquad \text{(A.15)}
\end{aligned}
$$

The log of the return is

$$\begin{aligned}\log r^c_{t+1} &= -\log\beta + (1-\rho)g - (\rho\alpha/2)p_v bb^T \\ &\quad +(1-\rho)e^T A x_t - (\rho\alpha/2)(e+p_x)^T BB^T(e+p_x)v_t \\ &\quad +v_t^{1/2}(e+\rho p_x)^T B w_{t+1} + \rho p_v b w_{t+1}. \qquad \text{(A.16)}\end{aligned}$$

The price of default-free bond which delivers 1 unit of consumption is $b^1_t = E_t[M_{t+1}]$ and the return is $r^1_{t+1} = \frac{1}{b^1_t}$. Thus the risk-free rate is

$$\log r^1_{t+1} = -(\delta_0 + \lambda_v^T\lambda_v/2) - \delta_x^T x_t - (\delta_v + \lambda_x^T\lambda_x/2)v_t. \qquad \text{(A.17)}$$

Then, the excess return of equity is

$$\begin{aligned}\log r^c_{t+1} - \log r^1_{t+1} &= (1/2)[(\alpha-\rho)^2 - \alpha^2]p_v^2 bb^T \\ &\quad + \left[\lambda_x^T\lambda_x/2 - (\alpha^2/2)(e+p_x)^T BB^T(e+p_x)\right] v_t \\ &\quad +v_t^{1/2}(e^T + \rho p_x^T)Bw_{t+1} + \rho p_v b w_{t+1} \qquad \text{(A.18)}\end{aligned}$$

A.5 Price of Elementary Assets

First, the date-t price-payout ratio of the asset that matures on the next date is

$$\begin{aligned}q^1_t &= E_t\left[m_{t+1}\frac{d_{t+1}}{d_t}\right] \\ &= E_t\left[e^{\delta_0+\delta_x^T x_t+\delta_v v_t+v_t^{1/2}\lambda_x^T w_{t+1}+\lambda_v^T w_{t+1}} e^{g+e^T(Ax_t+\beta v_t^{1/2}Bw_{t+1})-\frac{\beta^2 v_t}{2}e^T BB^T e}\right] \\ &= E_t\left[e^{\delta_0+\delta_x^T x_t+\delta_v v_t+g+e^T Ax_t-\frac{\beta^2 v_t}{2}e^T BB^T e} e^{v_t^{1/2}(\lambda_x^T+e^T B)w_{t+1}+\lambda_v^T w_{t+1}}\right] \\ &= E_t\left[e^{\delta_0+\delta_x^T x_t+\delta_v v_t+g+e^T Ax_t-\frac{\beta^2 v_t}{2}e^T BB^T e} E_{t+1}\left[e^{v_t^{1/2}(\lambda_x^T+e^T B)w_{t+1}+\lambda_v^T w_{t+1}}\right]\right] \\ &= E_t\left[e^{\delta_0+\delta_x^T x_t+\delta_v v_t+g+e^T Ax_t-\frac{\beta^2 v_t}{2}e^T BB^T e} e^{\frac{v_t}{2}(\lambda_x^T+e^T B)(\lambda_x^T+e^T B)^T+\frac{\lambda_v^T\lambda_v}{2}}\right] \\ &= e^{\delta_0+g+\frac{\lambda_v^T\lambda_v}{2}+(\delta_x^T+e^T A)x_t+\left(\delta_v+\frac{\lambda_x^T\lambda_x}{2}+\beta e^T B\lambda_x\right)v_t} \\ &\equiv e^{D_{0,1}+D_{x,1}x_t+D_{v,1}v_t} \qquad \text{(A.19)}\end{aligned}$$

Here we use the law of iterated expectation, and $Bb^T = 0$, $B\lambda_v = 0$, $b\lambda_x = 0$, and $E\left[e^x\right] = e^{a+\frac{1}{2}b}$ for $x \sim N(a, b)$.

Next, we compute the price-payout ratio of the elementary asset with maturity $s > 1$. Suppose that the price for the asset with maturity $s - 1$ is given by

$$q_t^{s-1} = e^{D_{0,s-1}+D_{x,s-1}x_t+D_{v,s-1}v_t}. \tag{A.20}$$

Then,

$$\begin{aligned}
q_t^s &= E_t\left[m_{t,t+s}\frac{d_{t+s}}{d_t}\right] \\
&= E_t\left[m_{t,t+1}\frac{d_{t+1}}{d_t}m_{t+1,t+s}\frac{d_{t+s}}{d_{t+1}}\right] \\
&= E_t\left[m_{t,t+1}\frac{d_{t+1}}{d_t}E_{t+1}\left[m_{t+1,t+s}\frac{d_{t+s}}{d_{t+1}}\right]\right] \\
&= E_t\left[m_{t,t+1}\frac{d_{t+1}}{d_t}e^{D_{0,s-1}+D_{x,s-1}x_{t+1}+D_{v,s-1}v_{t+1}}\right] \\
&= E_t\Bigg[e^{\delta_0+\delta_x^T x_t+\delta_v v_t+v_t^{1/2}\lambda_x^T w_{t+1}+\lambda_v^T w_{t+1}+g+e^T(Ax_t+\beta v_t^{1/2}Bw_{t+1})-\frac{\beta^2 v_t}{2}e^T BB^T e} \\
&\quad \times e^{D_{0,s-1}+D_{x,s-1}(Ax_t+v_t^{1/2}Bw_{t+1})+D_{v,s-1}((1-\varphi)v+\varphi v_t+bw_{t+1})}\Bigg] \\
&= e^{\delta_0+g+D_{0,s-1}+D_{v,s-1}(1-\varphi)v+\frac{(\lambda_v^T+D_{v,s-1}b)(\lambda_v^T+D_{v,s-1}b)^T}{2}+(\delta_x^T+e^T A+D_{x,s-1}A)x_t} \\
&\quad \times e^{\left(\delta_v+D_{v,s-1}\phi_v-\frac{\beta^2}{2}e^T BB^T e+\frac{(\lambda_x^T+\beta e^T B+D_{x,s-1}B)(\lambda_x^T+\beta e^T B+D_{x,s-1}B)^T}{2}\right)v_t} \\
&\equiv e^{D_{0,s}+D_{x,s}x_t+D_{v,s}v_t}
\end{aligned} \tag{A.21}$$

In this way, we determine coefficients $\{D_{0,s}, D_{x,s}, D_{v,s}\}$ for any $s > 1$ recursively from $s = 1$.

Appendix B

B.1 Derivation of Stochastic Discount Factor

Duffie and Epstein (1992) show that the standard additive expected utility function

$$E_t\left[\int_t^{\infty} e^{-\beta(s-t)} u(c_s) ds\right] \tag{B.1}$$

corresponds to the aggregator $f(c, v) = u(c) - \beta v$, where c is current consumption and v is the value function from expected future consumption. They also show that the stochastic discount factor process is given by

$$\pi_t = e^{\left(\int_0^t f_v(c_s, v_s) ds\right)} f_c(c_t, v_t). \tag{B.2}$$

For the time-separable power utility, the discount factor becomes

$$\pi_t = e^{-\beta t} c_t^{-\gamma}. \tag{B.3}$$

Applying the Ito's lemma and using the consumption process, I obtain following dynamics of the stochastic discount factor:

$$\begin{aligned} d\pi_t &= -\beta e^{-\beta t} c_t^{-\gamma} dt - \gamma e^{-\beta t} c^{-\gamma-1}(g c_t dt + \sigma_y c_t dW_t^c) + \frac{1}{2}\gamma(\gamma+1) e^{-\beta t} c^{-\gamma-2} \sigma_y^2 c_t^2 dt \\ \frac{d\pi_t}{\pi_t} &= \left(-\beta - \gamma g + \frac{1}{2}\sigma_y^2 \gamma(\gamma+1)\right) dt - \gamma \sigma_y dW_t^c \end{aligned} \tag{B.4}$$

B.2 Return on Consumption Asset

For a benchmark of risky assets, I define a consumption stream which delivers consumption at each instant. Let X_t denote the value of the asset, so

$$X_t = E_t^Q \left[\int_t^\infty e^{-rs} y_s ds \right]. \tag{B.5}$$

The value satisfies the following HJB equation:

$$rX = \left(g - \gamma\sigma_y^2\right) CX_y + \frac{1}{2}\sigma_y^2 y^2 X_{yy} + y \tag{B.6}$$

Solving the differential equation, I obtain

$$X = \frac{y}{r - g + \gamma\sigma_y^2} \tag{B.7}$$

The expected return on the asset is

$$E_t \left[\frac{dX + ydt}{X} \right] = \left(r + \gamma\sigma_y^2\right) dt \tag{B.8}$$

B.3 Illiquid bond market

I assume that investors receive liquidity shock that follows a Poisson process with the intensity of λ_1. Once receiving the shock, they prefer to sell their assets to raise funds. In the case they cannot sell their assets on arrival of the shock, they incur holding costs during the liquidity-constrained period. Following Duffie et al. (2005), the holding costs $f(\tau)$ per dollar value of debt increase with amount of time τ that they are liquidity-constrained, that is , $f'(\tau) > 0$.

Now I describe matching technology in the bond market. When a liquidity-constrained investor starts searching for a trading opportunity to sell her bond holdings, the matching event of finding another investor to trade with occurs with the intensity λ. This implies that the constrained investor faces the holding costs until bond matures or until she finds a counterparty to trade with, whichever comes first. When she invests in bond with maturity m, the average holding costs per unit time conditional on an arrival of liquidity shock are

$$\int_0^\infty \underbrace{\lambda_2 e^{-\lambda_2 t}}_{\text{prob. of matching}} \frac{1}{t} \left(\int_0^t \underbrace{\frac{1}{m} e^{-\frac{u}{m}} f(u) du}_{\text{holding cost of bond retiring at } u<t} + \underbrace{e^{-\frac{t}{m}} f(t)}_{\text{holding cost of bond outstanding at } t} \right) dt. \tag{B.9}$$

The expected holding costs $h(m)$ are then given by a product of probability of the liquidity shock λ_1 and the above average holding costs per unit time:

$$\begin{aligned} h(m) &= \lambda_1 \int_0^\infty \lambda_2 e^{-\lambda_2 t} \frac{1}{t} \left(\int_0^t \frac{1}{m} e^{-\frac{u}{m}} f(u) du + e^{-\frac{t}{m}} f(t) \right) dt \\ &= \lambda_1 \int_0^\infty \lambda_2 e^{-\lambda_2 t} \frac{1}{t} \left(\left[-e^{-\frac{u}{m}} f(u) \right]_0^t + \int_0^t e^{-\frac{u}{m}} f'(u) du + e^{-\frac{t}{m}} f(t) \right) \\ &= \lambda_1 \int_0^\infty \lambda_2 e^{-\lambda_2 t} \frac{1}{t} \left(\int_0^t e^{-\frac{u}{m}} f'(u) du \right) \end{aligned} \tag{B.10}$$

Although the expected holding costs cannot be solved algebraically with the general function of $f(\tau)$, dependence of the holding costs on maturity can be studied. Differentiating the holding costs with respect to maturity, I obtain

$$h'(m) = \lambda_1 \int_0^\infty \lambda_2 e^{-\lambda_2 t} \frac{1}{t} \left(\int_0^t \frac{u}{m^2} e^{-\frac{u}{m}} f'(u) du \right) > 0. \tag{B.11}$$

Hence, the holding costs increase with maturity.

B.4 Unlevered Firm

I begin with the differential equation of the unlevered firm's problem

$$rU = \max_I (I - \delta K) U_K + \mu^Q A U_A + \frac{1}{2} \sigma^2 A^2 U_{AA} + (1-\tau) AK - I - \theta I^2 + \tau \delta K. \tag{B.12}$$

Substituting $U(A, K) = q_U(A) K + J_U(A)$ into the above equation yields

$$r \left(K q_U + J_U \right) = \max_I (I - \delta K) q_U + \mu^Q A \left(q_U' K + J_U' \right) + \frac{1}{2} \sigma^2 A^2 \left(q_U'' K + J_U'' \right) + (1-\tau) AK - I - \theta I^2 + \tau \delta K. \tag{B.13}$$

Collecting terms in K yields

$$\left[(r+\delta)q_U - \mu^Q A q_U' - \frac{1}{2}\sigma^2 A^2 q_U'' - (1-\tau)A - \tau\delta\right] K = \max_I -rJ_U + Iq_U + \mu^Q A J_U' + \frac{1}{2}\sigma^2 A^2 J_U'' - I - \theta I^2. \tag{B.14}$$

In order for the equality to hold for every value of K, both LHS and RHS should be 0. Substituting the optimal investment $(q_U - 1)/2\theta$ for I in RHS, I have two differential equations

$$\begin{aligned} (r+\delta)q_U - \mu^Q A q_U' - \frac{1}{2}\sigma^2 A^2 q_U'' - (1-\tau)A - \tau\delta &= 0 \\ -rJ_U + \mu^Q A J_U' + \frac{1}{2}\sigma^2 A^2 J_U'' + \frac{(q_U-1)^2}{4\theta} &= 0. \end{aligned} \tag{B.15}$$

with the boundary condition that

$$q_U(A_L) = l, \text{ and } J_U(A_L) = 0 \tag{B.16}$$

from the condition that $U(A_L, K) = lK$.

Given the structure of the differential equations, I first solve for q_U and then solve for J_U, using the solution to q_U. The solution to q_U consists of the particular solution and general solution. The particular solution is

$$\frac{(1-\tau)A}{r+\delta-\mu^Q} + \frac{\tau\delta}{r+\delta}. \tag{B.17}$$

The general solution is $C_1 A^{\chi_+} + C_2 A^{\chi_-}$, where

$$\chi_\pm = \frac{-\mu^Q + \frac{1}{2}\sigma^2 \pm \sqrt{(-\mu^Q + \frac{1}{2}\sigma^2)^2 + 2\sigma^2(r+\delta)}}{\sigma^2}. \tag{B.18}$$

The coefficients C_1 and C_2 are determined by the boundary conditions. First, from the quadratic equation of $\chi_\pm$, it follows that $\chi_+ > 1$, when constraints $r > 2\mu^Q + \sigma^2$ and $r > \mu^Q$[1] are satisfied. To rule out a bubble solution to q_U, I set $C_1 = 0$. Then I have the general solution with power of χ_-, and denote the power as χ_1. The coefficient C_2 is determined from $q_U(A_D) = l$. The resulting

[1] I impose these constraints ex-post.

complete expression of $q_U(A)$ is

$$\frac{(1-\tau)A}{r+\delta-\mu^Q}+\frac{\tau\delta}{r+\delta}+\left(l-\frac{(1-\tau)A_L}{r+\delta-\mu^Q}-\frac{\tau\delta}{r+\delta}\right)\left(\frac{A}{A_L}\right)^{\chi_1}. \tag{B.19}$$

Now, I turn to solution to $J_U(A)$. It also consists of general and particular solutions. The particular solution is

$$\begin{aligned}&\phi_1\left(l-\frac{(1-\tau)A_L}{r+\delta-\mu^Q}-\frac{\tau\delta}{r+\delta}\right)^2\left(\frac{A}{A_L}\right)^{2\chi_1}+\phi_2\left(l-\frac{(1-\tau)A_L}{r+\delta-\mu^Q}-\frac{\tau\delta}{r+\delta}\right)\left(\frac{1}{A_L}\right)^{\chi_1}A^{\chi_1+1}\\&+\phi_3\left(l-\frac{(1-\tau)A_L}{r+\delta-\mu^Q}-\frac{\tau\delta}{r+\delta}\right)\left(\frac{A}{A_L}\right)^{\chi_1}+\phi_4A^2+\phi_5A+\frac{1}{4\theta r}\left(\frac{\tau\delta}{r+\delta}-1\right)^2\end{aligned} \tag{B.20}$$

where

$$\begin{aligned}\phi_1&=\frac{1}{4\theta[r-2\mu^Q\chi_1-\sigma^2\chi_1(2\chi_1-1)]}\\ \phi_2&=\frac{1-\tau}{2\theta(r+\delta-\mu^Q)[r-\mu^Q(\chi_1+1)-\frac{\sigma^2}{2}\chi_1(\chi_1+1)]}\\ \phi_3&=\frac{1}{2\theta}\left(\frac{\tau\delta}{r+\delta}-1\right)\frac{1}{r-\mu^Q\chi_1-\frac{\sigma^2}{2}\chi_1(\chi_1-1)}\\ \phi_4&=\frac{1}{4\theta}\left(\frac{1-\tau}{r+\delta-\mu^Q}\right)^2\frac{1}{r-2\mu^Q-\sigma^2}\\ \phi_5&=\frac{1}{2\theta}\left(\frac{\tau\delta}{r+\delta}-1\right)\frac{1-\tau}{(r+\delta-\mu^Q)(r-\mu^Q)}\end{aligned}$$

The general solution has the form of $C_1A^{\chi_+}+C_2A^{\chi_-}$, where

$$\chi_\pm=\frac{-\mu^Q+\frac{1}{2}\sigma^2\pm\sqrt{(-\mu^Q+\frac{1}{2}\sigma^2)^2+2\sigma^2r}}{\sigma^2}. \tag{B.21}$$

In a similar way that I solve for $q_U(A)$, the solution with power of χ_+ is dropped, and I denote χ_- as χ_2. The coefficient C_2 is determined with the boundary condition $J_U(A_L)=0$, and as a result, the complete solution to $J_U(A)$ is

$$\phi_1\left(l-\frac{(1-\tau)A_L}{r+\delta-\mu^Q}-\frac{\tau\delta}{r+\delta}\right)^2\left[\left(\frac{A}{A_L}\right)^{2\chi_1}-\left(\frac{A}{A_L}\right)^{\chi_2}\right]+\phi_2\left(l-\frac{(1-\tau)A_L}{r+\delta-\mu^Q}-\frac{\tau\delta}{r+\delta}\right)\left[\frac{A^{\chi_1+1}}{A_L^{\chi_1}}-\frac{A^{\chi_2}}{A_L^{\chi_2-1}}\right]$$
$$+\phi_3\left(l-\frac{(1-\tau)A_L}{r+\delta-\mu^Q}-\frac{\tau\delta}{r+\delta}\right)\left[\left(\frac{A}{A_L}\right)^{\chi_1}-\left(\frac{A}{A_L}\right)^{\chi_2}\right]+\phi_4\left[A^2-\frac{A^{\chi_2}}{A_L^{\chi_2-2}}\right]+\phi_5\left[A-\frac{A^{\chi_2}}{A_L^{\chi_2-1}}\right]$$
$$+\frac{1}{4\theta r}\left(\frac{\tau\delta}{r+\delta}-1\right)^2\left[1-\left(\frac{A}{A_L}\right)^{\chi_2}\right]. \tag{B.22}$$

I impose $r > 2\mu^Q + \sigma^2$ and $r > \mu^Q$ so that the present value of growth options, $J_U(A)$ is positive for all possible levels of productivity.

B.4.1 Liquidation Decision

From the smooth pasting condition, the optimal stopping threshold $A_L(K)$ given capital stock K is where the derivative of the firm value is equal to zero:

$$q'_U(A)+J'_U(A)|_{A=A_L}=\left[\frac{(1-\tau)}{r+\delta-\mu^Q}+\left(l-\frac{(1-\tau)A_L}{r+\delta-\mu^Q}-\frac{\tau\delta}{r+\delta}\right)\left(\frac{\chi_1}{A_L}\right)\right]K \tag{B.23}$$
$$+\phi_1\left(l-\frac{(1-\tau)A_L}{r+\delta-\mu^Q}-\frac{\tau\delta}{r+\delta}\right)^2\left(\frac{2\chi_1-\chi_2}{A_L}\right)+\phi_2\left(l-\frac{(1-\tau)A_L}{r+\delta-\mu^Q}-\frac{\tau\delta}{r+\delta}\right)(\chi_1+1-\chi_2)$$
$$+\phi_3\left(l-\frac{(1-\tau)A_L}{r+\delta-\mu^Q}-\frac{\tau\delta}{r+\delta}\right)\left(\frac{\chi_1-\chi_2}{A_L}\right)+\phi_4(2-\chi_2)A_L+\phi_5(1-\chi_2)$$
$$-\frac{1}{4\theta r}\left(\frac{\tau\delta}{r+\delta}-1\right)^2\left(\frac{\chi_2}{A_L}\right)$$
$$=0. \tag{B.24}$$

Rearranging the equation in terms of A_L, I find A_L is a solution to the following quadratic equation

$$A_L\left[\phi_1\frac{(1-\tau)^2}{(r+\delta-\mu^Q)^2}(2\chi_1-\chi_2)-\phi_2\frac{1-\tau}{r+\delta-\mu^Q}(\chi_1+1-\chi_2)+\phi_4(2-\chi_2)\right] \tag{B.25}$$
$$+\left[\frac{1-\tau}{r+\delta-\mu^Q}(1-\chi_1)K+2\phi_1(2\chi_1-\chi_2)\left(\frac{\tau\delta}{r+\delta}-l\right)\frac{1-\tau}{r+\delta-\mu^Q}+\phi_2\left(l-\frac{\tau\delta}{r+\delta}\right)(\chi_1+1-\chi_2)\right]$$
$$+\left[-\phi_3(\chi_1-\chi_2)\frac{1-\tau}{r+\delta-\mu^Q}+\phi_5(1-\chi_2)\right] \tag{B.26}$$
$$+\frac{1}{A_L}\left[\left(l-\frac{\tau\delta}{r+\delta}\right)\chi_1K+\phi_1(2\chi_1-\chi_2)\left(l-\frac{\tau\delta}{r+\delta}\right)^2+\phi_3(\chi_1-\chi_2)\left(l-\frac{\tau\delta}{r+\delta}\right)-\frac{\chi_2}{4\theta r}\left(\frac{\tau\delta}{r+\delta}-1\right)^2\right]$$
$$=0. \tag{B.27}$$

B.5 Firm with Debt Contract

B.5.1 Proof of Proposition

Given the system of equations for debt and equity, I guess that the values are linear functions of capital, $S(A,K) = q_S(A)K + J_S(A)$ and $D(A,K) = q_D(A)K + J_D(A)$. Then I verify that the functional forms satisfy the equations, and simultaneously find q_S, q_D, J_S, J_D as functions of A. Substituting $S(A,K)$ and $D(A,K)$ into the equation (2.15) yields

$$r\left(Kq_S + J_S\right) = \max_I (I-\delta K)q_S + \mu^Q A\left(q_S' K + J_S'\right) + \frac{1}{2}\sigma^2 A^2\left(q_S'' K + J_S''\right) + (1-\tau)AK - I - \theta I^2 - \frac{P}{m} + \frac{q_D K + J_D}{m} - (1-\tau)C + \tau\delta K. \tag{B.28}$$

Collecting terms in K yields

$$\left[(r+\delta)q_S - \mu^Q A q_S' - \frac{1}{2}\sigma^2 A^2 q_S'' - (1-\tau)A - \frac{q_D}{m} - \tau\delta\right]K = \max_I -rJ_S + Iq_S + \mu^Q A J_S' + \frac{1}{2}\sigma^2 A^2 J_S'' - I - \theta I^2 - \frac{P}{m} + \frac{J_D}{m} - (1-\tau)C \tag{B.29}$$

and the optimal investment is $I^* = (q_S - 1)/2\theta$. In order for the equality to hold for every K, both LHS and RHS should be zero. Then,

$$\begin{aligned} (r+\delta)q_S - \mu^Q A q_S' - \frac{1}{2}\sigma^2 A^2 q_S'' - (1-\tau)A - \frac{q_D}{m} - \tau\delta &= 0 \\ -rJ_S + \mu^Q A J_S' + \frac{1}{2}\sigma^2 A^2 J_S'' + \frac{(q_S-1)^2}{4\theta} - \frac{P}{m} + \frac{J_D}{m} - (1-\tau)C &= 0 \end{aligned} \tag{B.30}$$

Plugging $D(A,K)$ into the equation (2.17) yields

$$(r+h)\left(q_D K + J_D\right) = (I - \delta K)\,q_D + \mu^Q A\left(q_D' K + J_D'\right) + \frac{1}{2}\sigma^2 A^2\left(q_D'' K + J_D''\right) + \frac{P}{m} - \frac{q_D K + J_D}{m} + C. \tag{B.31}$$

Again collecting terms in K yields

$$\left[(r+h+\delta+\frac{1}{m})q_D - \mu^Q A q_D' - \frac{1}{2}\sigma^2 A^2 q_D''\right]K = -\left(r+h+\frac{1}{m}\right)J_D + \mu^Q A J_D' + \frac{1}{2}\sigma^2 A^2 J_D'' + \frac{P}{m} + q_D\frac{q_S-1}{2\theta} + C. \tag{B.32}$$

In order for the equality to hold for every K,

$$\begin{aligned} (r+h+\delta+\frac{1}{m})q_D - \mu^Q A q_D' - \frac{1}{2}\sigma^2 A^2 q_D'' &= 0 \\ -\left(r+h+\frac{1}{m}\right)J_D + \mu^Q A J_D' + \frac{1}{2}\sigma^2 A^2 J_D'' + \frac{P}{m} + q_D\frac{q_S-1}{2\theta} + C &= 0 \end{aligned} \tag{B.33}$$

The solutions also need to satisfy the boundary conditions

$$\begin{aligned} q_S(A_D)K + J_S(A_D) &= \max(0, lK) \\ q_D(A_D)K + J_D(A_D) &= \min(lK, P) \\ \lim_{A\to\infty} q_D(A)K + J_D(A) &= \frac{P + mC}{1 + m(r+h)} \end{aligned} \tag{B.34}$$

In the following, I solve for the case where $lK < P$. The other case can be solved similarly. Given the 4 equations for 4 unknowns, first I solve for $q_D(A)$ with equation (B.33). Next, with solved $q_D(A)$, I solve for $q_S(A)$ with equation (B.30). Then, $J_D(A)$ and $J_S(A)$ are solved from equation (B.34) and (B.31), respectively.

The equation (B.33) has the general solution

$$q_D(A) = C_1 A^{\gamma_+} + C_2 A^{\gamma_-} \tag{B.35}$$

where

$$\gamma_\pm = \frac{-\mu^Q + \frac{1}{2}\sigma^2 \pm \sqrt{(-\mu^Q + \frac{1}{2}\sigma^2)^2 + 2\sigma^2(r + h + \delta + \frac{1}{m})}}{\sigma^2} \tag{B.36}$$

From the boundary condition that debt of the extremely productive firm is riskless, the debt value is independent of K when A goes to infinity. Thus, it follows that $C_1 = 0$. Let's denote γ_- by γ_1. Using the boundary condition at default, $C_2 = lA_D^{-\gamma_1}$. Thus

$$q_D(A) = l\left(\frac{A}{A_D}\right)^{\gamma_1}. \tag{B.37}$$

From equation (B.30), $q_S(A)$ consists of a general and a particular solution. The functional form of the general solution of $q_S(A)$ is similar to that of $q_D(A)$, and only negative power is chosen in order to rule out a bubble solution. [2] The negative power, say γ_2, is

$$\gamma_2 = \frac{-\mu^Q + \frac{1}{2}\sigma^2 - \sqrt{(-\mu^Q + \frac{1}{2}\sigma^2)^2 + 2\sigma^2(r + \delta)}}{\sigma^2} \tag{B.38}$$

[2] I can show that the positive power is greater than 1, if the convergence conditions, $r > 2\mu^Q + \sigma^2$ and $r > \mu^Q$ are satisfied. By applying the no bubble condition

$$\lim_{A\to\infty} \frac{S(A,K)}{A} < \infty$$

I rule out the positive power.

and the general solution is CA^{γ_2}, where C will be determined soon by the boundary condition. The particular solution of $q_S(A)$ is

$$\frac{(1-\tau)A}{r+\delta-\mu^Q} - \frac{l}{mh+1}\left(\frac{A}{A_D}\right)^{\gamma_1} + \frac{\tau\delta}{r+\delta}. \tag{B.39}$$

The boundary condition at default, $q_S(A_D) = 0$, determines C, leading to

$$q_S = \left(\frac{l}{mh+1} - \frac{(1-\tau)A_D}{r+\delta-\mu^Q} - \frac{\tau\delta}{r+\delta}\right)\left(\frac{A}{A_D}\right)^{\gamma_2} - \frac{l}{mh+1}\left(\frac{A}{A_D}\right)^{\gamma_1} + \frac{(1-\tau)A}{r+\delta-\mu^Q} + \frac{\tau\delta}{r+\delta}. \tag{B.40}$$

Now I solve for $J_D(A)$. Again, its general solution consists of power functions of A, one with positive power and the other with negative one, and the positive power is dropped from the boundary condition, $\lim_{A\to\infty} J_D \to (P+mC)/(1+mr+my)$. Denote the negative power by γ_3 that is given by

$$\gamma_3 = \frac{-\mu^Q + \frac{1}{2}\sigma^2 - \sqrt{(-\mu^Q + \frac{1}{2}\sigma^2)^2 + 2\sigma^2(r+h+\frac{1}{m})}}{\sigma^2}. \tag{B.41}$$

The particular solution of $J_D(A)$ is

$$\begin{aligned}&\frac{P+mC}{m(r+h)+1} + \alpha_1\left(\frac{l}{mh+1} - \frac{(1-\tau)A_D}{r+\delta-\mu^Q} - \frac{\tau\delta}{r+\delta}\right)\left(\frac{1}{A_D}\right)^{\gamma_1+\gamma_2} A^{\gamma_1+\gamma_2} + \alpha_2\left(\frac{1}{A_D}\right)^{2\gamma_1} A^{2\gamma_1}\\ &+ \alpha_3\left(\frac{1}{A_D}\right)^{\gamma_1} A^{\gamma_1+1} + \alpha_4\left(\frac{1}{A_D}\right)^{\gamma_1} A^{\gamma_1}.\end{aligned} \tag{B.42}$$

where

$$\begin{aligned}\alpha_1 &= \frac{l}{2\theta}\frac{1}{r+h+\frac{1}{m} - \mu^Q(\gamma_1+\gamma_2) - \frac{1}{2}\sigma^2(\gamma_1+\gamma_2)(\gamma_1+\gamma_2-1)}\\ \alpha_2 &= \frac{l^2}{2\theta}\frac{1}{-r-\frac{1}{m}-h+2\mu^Q\gamma_1+\sigma^2\gamma_1(2\gamma_1-1)}\\ \alpha_3 &= \frac{l(1-\tau)}{2\theta(r+\delta-\mu^Q)}\frac{1}{-\delta-\mu^Q-\sigma^2\gamma_1}\\ \alpha_4 &= \frac{l}{2\theta\delta}\left(1-\frac{\tau\delta}{r+\delta}\right)\end{aligned} \tag{B.43}$$

Another boundary condition that $J_D(A_D) = 0$ determines the coefficients in the general solution. Finally

$$\begin{aligned}J_D(A) &= \frac{P+mC}{m(r+h)+1}\left[1-\left(\frac{A}{A_D}\right)^{\gamma_3}\right] + \alpha_1\left(\frac{l}{mh+1} - \frac{(1-\tau)A_D}{r+\delta-\mu^Q} - \frac{\tau\delta}{r+\delta}\right)\left[\left(\frac{A}{A_D}\right)^{\gamma_1+\gamma_2} - \left(\frac{A}{A_D}\right)^{\gamma_3}\right]\\ &+\alpha_2\left[\left(\frac{A}{A_D}\right)^{2\gamma_1} - \left(\frac{A}{A_D}\right)^{\gamma_3}\right] + \alpha_3 A\left[\left(\frac{A}{A_D}\right)^{\gamma_1} - \left(\frac{A}{A_D}\right)^{\gamma_3-1}\right] + \alpha_4\left[\left(\frac{A}{A_D}\right)^{\gamma_1} - \left(\frac{A}{A_D}\right)^{\gamma_3}\right]\end{aligned} \tag{B.44}$$

Now, $J_S(A)$ is ready to be solved. The positive power of general solution is greater than 1. Again, only negative power of general solution is taken to rule out a bubble solution. The negative power, say γ_4, is

$$\gamma_4 = \frac{-\mu^Q + \frac{\sigma^2}{2} - \sqrt{(\mu^Q - \frac{\sigma^2}{2})^2 + 2r\sigma^2}}{\sigma^2}. \tag{B.45}$$

The particular solution of $J_S(A)$ is

$$\begin{aligned}
&\beta_1 \left(\frac{A}{A_D}\right)^{2\gamma_1} + \beta_2 \left(\frac{l}{mh+1} - \frac{(1-\tau)A_D}{r+\delta-\mu^Q} - \frac{\tau\delta}{r+\delta}\right)^2 \left(\frac{A}{A_D}\right)^{2\gamma_2} + \beta_3 \left(\frac{l}{mh+1} - \frac{(1-\tau)A_D}{r+\delta-\mu^Q} - \frac{\tau\delta}{r+\delta}\right) \left(\frac{A}{A_D}\right)^{\gamma_1+\gamma_2} \\
&+ \beta_4 \left(\frac{1}{A_D}\right)^{\gamma_1} A^{\gamma_1+1} + \beta_5 \left(\frac{l}{mh+1} - \frac{(1-\tau)A_D}{r+\delta-\mu^Q} - \frac{\tau\delta}{r+\delta}\right) \left(\frac{1}{A_D}\right)^{\gamma_2} A^{\gamma_2+1} + \beta_6 \left(\frac{A}{A_D}\right)^{\gamma_1} \\
&+ \beta_7 \left(\frac{l}{mh+1} - \frac{(1-\tau)A_D}{r+\delta-\mu^Q} - \frac{\tau\delta}{r+\delta}\right) \left(\frac{A}{A_D}\right)^{\gamma_2} \\
&+ \frac{1}{mh+1}\left[\alpha_1 \left(\frac{l}{mh+1} - \frac{(1-\tau)A_D}{r+\delta-\mu^Q} - \frac{\tau\delta}{r+\delta}\right) + \alpha_2 + \alpha_3 A_D + \alpha_4 + \frac{P+mC}{m(r+h)+1}\right] \left(\frac{A}{A_D}\right)^{\gamma_3} \\
&+ \beta_8 A^2 + \beta_9 A + \frac{1}{r}\left(\frac{1}{4\theta}\left(\frac{\tau\delta}{r+\delta} - 1\right)^2 - \frac{(P+mC)(r+h)}{m(r+h)+1} + \tau C\right)
\end{aligned} \tag{B.46}$$

where

$$\begin{aligned}
\beta_1 &= \frac{1}{r - 2\mu^Q\gamma_1 - \sigma^2\gamma_1(2\gamma_1 - 1)}\left(\frac{\alpha_2}{m} + \frac{l^2}{4\theta(mh+1)^2}\right) \\
\beta_2 &= \frac{1}{4\theta\left(r - 2\mu^Q\gamma_2 - \sigma^2\gamma_2(2\gamma_2 - 1)\right)} \\
\beta_3 &= \frac{1}{r - \mu^Q(\gamma_1+\gamma_2) - \frac{1}{2}\sigma^2(\gamma_1+\gamma_2)(\gamma_1+\gamma_2-1)}\left(\frac{\alpha_1}{m} - \frac{l}{2\theta(mh+1)}\right) \\
\beta_4 &= \frac{1}{r - \mu^Q(\gamma_1+1) - \frac{1}{2}\sigma^2\gamma_1(\gamma_1+1)}\left(\frac{\alpha_3}{m} - \frac{l}{2\theta(mh+1)}\frac{1-\tau}{r+\delta-\mu^Q}\right) \\
\beta_5 &= \frac{1-\tau}{2\theta(r+\delta-\mu^Q)}\frac{1}{r - \mu^Q(\gamma_2+1) - \frac{1}{2}\sigma^2\gamma_2(\gamma_2+1)} \\
\beta_6 &= \left(\frac{\alpha_4}{m} - \frac{l}{2\theta(mh+1)}\left(\frac{\tau\delta}{r+\delta} - 1\right)\right)\frac{1}{-y - \delta - \frac{1}{m}} \\
\beta_7 &= \left(-\frac{1}{2\theta\delta}\right)\left(\frac{\tau\delta}{r+\delta} - 1\right) \\
\beta_8 &= \frac{(1-\tau)^2}{4\theta(r+\delta-\mu^Q)^2\left(r - 2\mu^Q - \sigma^2\right)} \\
\beta_9 &= \frac{1-\tau}{2\theta(r+\delta-\mu^Q)(r-\mu^Q)}\left(\frac{\tau\delta}{r+\delta} - 1\right)
\end{aligned} \tag{B.47}$$

The boundary condition $J_S(A_D) = 0$ determines the coefficients in the general. Then the complete solution to $J_S(A)$ is

$$
\begin{aligned}
J_S(A) &= \beta_1 \left[\left(\frac{A}{A_D}\right)^{2\gamma_1} - \left(\frac{A}{A_D}\right)^{\gamma_4} \right] + \beta_2 \left(\frac{l}{mh+1} - \frac{(1-\tau)A_D}{r+\delta-\mu^Q} - \frac{\tau\delta}{r+\delta} \right)^2 \left[\left(\frac{A}{A_D}\right)^{2\gamma_2} - \left(\frac{A}{A_D}\right)^{\gamma_4} \right] \qquad \text{(B.48)} \\
&+ \beta_3 \left(\frac{l}{mh+1} - \frac{(1-\tau)A_D}{r+\delta-\mu^Q} - \frac{\tau\delta}{r+\delta} \right) \left[\left(\frac{A}{A_D}\right)^{\gamma_1+\gamma_2} - \left(\frac{A}{A_D}\right)^{\gamma_4} \right] + \beta_4 \left[\left(\frac{1}{A_D}\right)^{\gamma_1} A^{\gamma_1+1} - \left(\frac{1}{A_D}\right)^{\gamma_4-1} A^{\gamma_4} \right] \\
&+ \beta_5 \left(\frac{l}{mh+1} - \frac{(1-\tau)A_D}{r+\delta-\mu^Q} - \frac{\tau\delta}{r+\delta} \right) \left[\left(\frac{1}{A_D}\right)^{\gamma_2} A^{\gamma_2+1} - \left(\frac{1}{A_D}\right)^{\gamma_4-1} A^{\gamma_4} \right] + \beta_6 \left[\left(\frac{A}{A_D}\right)^{\gamma_1} - \left(\frac{A}{A_D}\right)^{\gamma_4} \right] \\
&+ \beta_7 \left(\frac{l}{mh+1} - \frac{(1-\tau)A_D}{r+\delta-\mu^Q} - \frac{\tau\delta}{r+\delta} \right) \left[\left(\frac{A}{A_D}\right)^{\gamma_2} - \left(\frac{A}{A_D}\right)^{\gamma_4} \right] \\
&+ \frac{1}{mh+1} \left[\alpha_1 \left(\frac{l}{mh+1} - \frac{(1-\tau)A_D}{r+\delta-\mu^Q} - \frac{\tau\delta}{r+\delta} \right) + \alpha_2 + \alpha_3 A_D + \alpha_4 + \frac{P+mC}{m(r+h)+1} \right] \left[\left(\frac{A}{A_D}\right)^{\gamma_3} - \left(\frac{A}{A_D}\right)^{\gamma_4} \right] \\
&+ \beta_8 \left[A^2 - \left(\frac{1}{A_D}\right)^{\gamma_4-2} A^{\gamma_4} \right] + \beta_9 \left[A - \left(\frac{1}{A_D}\right)^{\gamma_4-1} A^{\gamma_4} \right] \\
&+ \frac{1}{r} \left(\frac{1}{4\theta} \left(\frac{\tau\delta}{r+\delta} - 1 \right)^2 - \frac{(P+mC)(r+h)}{m(r+h)+1} + \tau C \right) \left[1 - \left(\frac{A}{A_D}\right)^{\gamma_4} \right]
\end{aligned}
$$

The above results can be summarized in the following proposition.

Proposition 1. Given A_D, the debt value is

$$D(A,K) = q_D(A)K + J_D(A) \qquad \text{(B.49)}$$

where

$$
q_D(A) = \begin{cases} l\left(\frac{A}{A_D}\right)^{\gamma_1} & \textit{if } lK < P \\ 0 & \textit{if } lK \geq P \end{cases} \qquad \text{(B.50)}
$$

$$
J_D(A) = \begin{cases} \frac{P+mC}{m(r+h)+1}\left[1-\left(\frac{A}{A_D}\right)^{\gamma_3}\right] + \alpha_1\left(\frac{l}{mh+1} - \frac{(1-\tau)A_D}{r+\delta-\mu^Q} - \frac{\tau\delta}{r+\delta}\right)\left[\left(\frac{A}{A_D}\right)^{\gamma_1+\gamma_2} - \left(\frac{A}{A_D}\right)^{\gamma_3}\right] & \textit{if } lK < P \\ +\alpha_2\left[\left(\frac{A}{A_D}\right)^{2\gamma_1} - \left(\frac{A}{A_D}\right)^{\gamma_3}\right] + \alpha_3 A\left[\left(\frac{A}{A_D}\right)^{\gamma_1} - \left(\frac{A}{A_D}\right)^{\gamma_3-1}\right] + \alpha_4\left[\left(\frac{A}{A_D}\right)^{\gamma_1} - \left(\frac{A}{A_D}\right)^{\gamma_3}\right] & \\ \frac{P+mC}{m(r+h)+1} + \frac{Pm(r+h)-mC}{m(r+h)+1}\left(\frac{A}{A_D}\right)^{\gamma 3} & \textit{if } lK \geq P. \end{cases}
$$

The equity value is

$$S(A,K) = q_S(A)K + J_S(A) \qquad \text{(B.51)}$$

where

$$q_S(A) = \begin{cases} \frac{(1-\tau)A}{r+\delta-\mu^Q} + \frac{\tau\delta}{r+\delta} + \left(\frac{l}{mh+1} - \frac{(1-\tau)A_D}{r+\delta-\mu^Q} - \frac{\tau\delta}{r+\delta}\right)\left(\frac{A}{A_D}\right)^{\gamma_2} - \frac{l}{mh+1}\left(\frac{A}{A_D}\right)^{\gamma_1} & \textit{if } lK < P \\ \frac{(1-\tau)A}{r+\delta-\mu^Q} + \frac{\tau\delta}{r+\delta} + \left(l - \frac{(1-\tau)A_D}{r+\delta-\mu^Q} - \frac{\tau\delta}{r+\delta}\right)\left(\frac{A}{A_D}\right)^{\gamma_2} & \textit{if } lK \geq P \end{cases} \tag{B.52}$$

$$J_S(A) = \begin{cases} \beta_1\left[\left(\frac{A}{A_D}\right)^{2\gamma_1} - \left(\frac{A}{A_D}\right)^{\gamma_4}\right] + \beta_2\left(\frac{l}{mh+1} - \frac{(1-\tau)A_D}{r+\delta-\mu^Q} - \frac{\tau\delta}{r+\delta}\right)^2\left[\left(\frac{A}{A_D}\right)^{2\gamma_2} - \left(\frac{A}{A_D}\right)^{\gamma_4}\right] & \textit{if } lK < P \\ +\beta_3\left(\frac{l}{mh+1} - \frac{(1-\tau)A_D}{r+\delta-\mu^Q} - \frac{\tau\delta}{r+\delta}\right)\left[\left(\frac{A}{A_D}\right)^{\gamma_1+\gamma_2} - \left(\frac{A}{A_D}\right)^{\gamma_4}\right] + \beta_4\left[\left(\frac{1}{A_D}\right)^{\gamma_1} A^{\gamma_1+1} - \left(\frac{1}{A_D}\right)^{\gamma_4-1} A^{\gamma_4}\right] & \\ +\beta_5\left(\frac{l}{mh+1} - \frac{(1-\tau)A_D}{r+\delta-\mu^Q} - \frac{\tau\delta}{r+\delta}\right)\left[\left(\frac{1}{A_D}\right)^{\gamma_2} A^{\gamma_2+1} - \left(\frac{1}{A_D}\right)^{\gamma_4-1} A^{\gamma_4}\right] + \beta_6\left[\left(\frac{A}{A_D}\right)^{\gamma_1} - \left(\frac{A}{A_D}\right)^{\gamma_4}\right] & \\ +\beta_7\left(\frac{l}{mh+1} - \frac{(1-\tau)A_D}{r+\delta-\mu^Q} - \frac{\tau\delta}{r+\delta}\right)\left[\left(\frac{A}{A_D}\right)^{\gamma_2} - \left(\frac{A}{A_D}\right)^{\gamma_4}\right] & \\ +\frac{1}{mh+1}\left[\alpha_1\left(\frac{l}{mh+1} - \frac{(1-\tau)A_D}{r+\delta-\mu^Q} - \frac{\tau\delta}{r+\delta}\right) + \alpha_2 + \alpha_3 A_D + \alpha_4 + \frac{P+mC}{m(r+h)+1}\right]\left[\left(\frac{A}{A_D}\right)^{\gamma_3} - \left(\frac{A}{A_D}\right)^{\gamma_4}\right] & \\ +\beta_8\left[A^2 - \left(\frac{1}{A_D}\right)^{\gamma_4-2} A^{\gamma_4}\right] + \beta_9\left[A - \left(\frac{1}{A_D}\right)^{\gamma_4-1} A^{\gamma_4}\right] & \\ +\frac{1}{r}\left(\frac{1}{4\theta}\left(\frac{\tau\delta}{r+\delta} - 1\right)^2 - \frac{(P+mC)(r+h)}{m(r+h)+1} + \tau C\right)\left[1 - \left(\frac{A}{A_D}\right)^{\gamma_4}\right] & \\ & \\ \beta_2\left(l - \frac{(1-\tau)A_D}{r+\delta-\mu^Q} - \frac{\tau\delta}{r+\delta}\right)^2\left[\left(\frac{A}{A_D}\right)^{2\gamma_2} - \left(\frac{A}{A_D}\right)^{\gamma_4}\right] & \textit{if } lK \geq P \\ +\beta_5\left(l - \frac{(1-\tau)A_D}{r+\delta-\mu^Q} - \frac{\tau\delta}{r+\delta}\right)\left[\left(\frac{1}{A_D}\right)^{\gamma_2} A^{\gamma_2+1} - \left(\frac{1}{A_D}\right)^{\gamma_4-1} A^{\gamma_4}\right] & \\ +\beta_7\left(l - \frac{(1-\tau)A_D}{r+\delta-\mu^Q} - \frac{\tau\delta}{r+\delta}\right)\left[\left(\frac{A}{A_D}\right)^{\gamma_2} - \left(\frac{A}{A_D}\right)^{\gamma_4}\right] & \\ +\frac{P(r+h)-C}{(m(r+h)+1)(-h-1/m)}\left[\left(\frac{A}{A_D}\right)^{\gamma_3} - \left(\frac{A}{A_D}\right)^{\gamma_4}\right] + \beta_8\left[A^2 - \left(\frac{1}{A_D}\right)^{\gamma_4-2} A^{\gamma_4}\right] + \beta_9\left[A - \left(\frac{1}{A_D}\right)^{\gamma_4-1} A^{\gamma_4}\right] & \\ +\frac{1}{r}\left(\frac{1}{4\theta}\left(\frac{\tau\delta}{r+\delta} - 1\right)^2 - \frac{(P+mC)(r+h)}{m(r+h)+1} + \tau C\right)\left[1 - \left(\frac{A}{A_D}\right)^{\gamma_4}\right] & \end{cases} \tag{B.53}$$

where constants $[\gamma_i]_{i=1}^4$,$[\alpha_i]_{i=1}^4$ and $[\beta_i]_{i=1}^9$ are given this section.

B.5.2 Stopping Decision

Here I determine the stopping threshold for the case where $lK < P$. The stopping decision for the other case can be solved similarly. Differentiating $S(A, K)$ with respect to A, and evaluating the derivative at A_D gives

$$\begin{aligned}
S'(A)|_{A=A_D} &= q'(A)K + J'(A)|_{A=A_D} \\
&= \left[\left(\frac{l}{mh+1} - \frac{(1-\tau)A_D}{r+\delta-\mu^Q} - \frac{\tau\delta}{r+\delta}\right)\frac{\gamma_2}{A_D} - \frac{l}{mh+1}\frac{\gamma_1}{A_D} + \frac{1-\tau}{r+\delta-\mu^Q}\right]K \\
&+ \beta_1\left[\frac{2\gamma_1-\gamma_4}{A_D}\right] + \beta_2\left(\frac{l}{mh+1} - \frac{(1-\tau)A_D}{r+\delta-\mu^Q} - \frac{\tau\delta}{r+\delta}\right)^2\left[\frac{2\gamma_2-\gamma_4}{A_D}\right] \\
&+ \beta_3\left(\frac{l}{mh+1} - \frac{(1-\tau)A_D}{r+\delta-\mu^Q} - \frac{\tau\delta}{r+\delta}\right)\left[\frac{\gamma_1+\gamma_2-\gamma_4}{A_D}\right] + \beta_4\left[\gamma_1+1-\gamma_4\right] \\
&+ \beta_5\left(\frac{l}{mh+1} - \frac{(1-\tau)A_D}{r+\delta-\mu^Q} - \frac{\tau\delta}{r+\delta}\right)\left[\gamma_2+1-\gamma_4\right] + \beta_6\left[\frac{\gamma_1-\gamma_4}{A_D}\right] \\
&+ \beta_7\left(\frac{l}{mh+1} - \frac{(1-\tau)A_D}{r+\delta-\mu^Q} - \frac{\tau\delta}{r+\delta}\right)\left[\frac{\gamma_2-\gamma_4}{A_D}\right] \\
&+ \frac{1}{mh+1}\left[\alpha_1\left(\frac{l}{mh+1} - \frac{(1-\tau)A_D}{r+\delta-\mu^Q} - \frac{\tau\delta}{r+\delta}\right) + \alpha_2 + \alpha_3 A_D + \alpha_4 + \frac{P+mC}{m(r+h)+1}\right]\left[\frac{\gamma_3-\gamma_4}{A_D}\right] \\
&+ \beta_8\left[(2-\gamma_4)A_D\right] + \beta_9\left[1-\gamma_4\right] + \frac{1}{r}\left(\frac{1}{4\theta}\left(\frac{\tau\delta}{r+\delta}-1\right)^2 - \frac{(P+mC)r}{m(r+h)+1} + \tau C\right)\left[-\frac{\gamma_4}{A_D}\right]
\end{aligned} \tag{B.54}$$

The stopping threshold can be obtained by solving the following quadratic equation of A_D

$$\begin{aligned}
&A_D\left[\beta_2(2\gamma_2-\gamma_4)\left(\frac{1-\tau}{r+\delta-\mu^Q}\right)^2 - \beta_5(\gamma_2+1-\gamma_4)\frac{1-\tau}{r+\delta-\mu^Q} + \beta_8(2-\gamma_4)\right] \\
&+ \left[\frac{(1-\tau)}{r+\delta-\mu^Q}(1-\gamma_2)K + 2\beta_2(2\gamma_2-\gamma_4)\frac{1-\tau}{r+\delta-\mu^Q}\left(\frac{\tau\delta}{r+\delta} - \frac{l}{mh+1}\right) - \beta_3(\gamma_1+\gamma_2-\gamma_4)\frac{1-\tau}{r+\delta-\mu^Q} + \beta_4\left[\gamma_1+1-\gamma_4\right]\right] \\
&+ \left[\beta_5(\gamma_2+1-\gamma_4)\left(\frac{l}{mh+1} - \frac{\tau\delta}{r+\delta}\right) - \beta_7(\gamma_2-\gamma_4)\frac{1-\tau}{r+\delta-\mu^Q} + \frac{\gamma_3-\gamma_4}{mh+1}\left(\alpha_3 - \alpha_1\frac{1-\tau}{r+\delta-\mu^Q}\right) + \beta_9(1-\gamma_4)\right] \\
&+ \frac{1}{A_D}\left[\frac{(\gamma_2-\gamma_1)l}{mh+1}K - \frac{\tau\delta\gamma_2}{r+\delta}K + \beta_1(2\gamma_1-\gamma_4) + \beta_2(2\gamma_2-\gamma_4)\left(\frac{l}{mh+1} - \frac{\tau\delta}{r+\delta}\right)^2 + \beta_3(\gamma_1+\gamma_2-\gamma_4)\left(\frac{l}{mh+1} - \frac{\tau\delta}{r+\delta}\right)\right] \\
&+ \frac{1}{A_D}\left[\beta_6(\gamma_1-\gamma_4) + \beta_7(\gamma_2-\gamma_4)\left(\frac{l}{mh+1} - \frac{\tau\delta}{r+\delta}\right) + \frac{\gamma_3-\gamma_4}{mh+1}\left[\alpha_1\left(\frac{l}{mh+1} - \frac{\tau\delta}{r+\delta}\right) + \alpha_2 + \alpha_4 + \frac{P+mC}{mr+mh+1}\right]\right] \\
&+ \frac{1}{A_D}\left[-\frac{\gamma_4}{r}\left(\frac{1}{4\theta}\left(\frac{\tau\delta}{r+\delta}-1\right)^2 - \frac{(P+mC)r}{mr+mh+1} + \tau C\right)\right] \\
&= 0
\end{aligned} \tag{B.55}$$

B.6 Structural Estimation

First, I derive the conditional probability density $f_2(i_{j,t}|i_{j,1:t-1}, d_j; \theta_j, \widehat{\Theta}_{j,1}, [\Delta W_t^y]_{t=1}^{n_j})$. For simplicity in notation, the firm subscript j is omitted in this section. In the model, the optimal investment is given by $(q_S(A)-1)/(2\theta)$. Thus, investment-capital ratio at time t is

$$i_t = \frac{q_S(A_t)-1}{2\theta K_t} \tag{B.56}$$

where the analytic expression of $q_S(A)$ is presented in Proposition **??**. For productivity shock A_t, I know that the conditional probability $f\left(A_t|A_{t-1};\theta,\widehat{\Theta}_1,[\Delta W_t^y]_{t=1}^n\right)$ is a log-normal distribution from the assumption of geometric Brownian motion:

$$f_1\left(A_t|A_{t-1};\theta,\widehat{\Theta}_1,[\Delta W_t^y]_{t=1}^n\right) = \frac{1}{\sqrt{2\pi(1-\rho^2)\left(\frac{\sigma^2}{4}\right)}|A_{t-1}|}\exp\left(-\frac{\left(\ln(A_t/A_{t-1})-\frac{1}{4}\left(\mu^P-\frac{1}{2}\sigma^2\right)-\rho\sigma\Delta W_y^c\right)^2}{2(1-\rho^2)\left(\frac{\sigma^2}{4}\right)}\right) \tag{B.57}$$

where the time interval between adjacent observations is one quarter, while the unit of time is one year. Note that once debt structure is given, the investment is solely determined by the state (A_t, K_t) and that A_t satisfies the Markov property. Thus, I find

$$f_2\left(i_t|i_{1:t-1},d;\theta,\widehat{\Theta}_1,[\Delta W_t^y]_{t=1}^n\right) = f_2\left(i_t|K_t,\widehat{A}_{t-1},d_j;\theta,\widehat{\Theta}_1,[W_t^y]_{t=1}^n\right) \tag{B.58}$$

where $K_t = K_0\exp\left(\sum_{k=1}^t(i_k-\delta)/4\right)$ and $\widehat{A}_{t-1}$ is the productivity implied by the investment at $t-1$ such that

$$\frac{q_S(\widehat{A}_{t-1})-1}{2\theta K_{t-1}} = i_{t-1}. \tag{B.59}$$

Given the conditional distribution of productivity, I can derive the conditional probability of investment via transformation of random variables. Specifically,

$$f_2\left(i_t|K_t,\widehat{A}_{t-1},d_j;\theta,\widehat{\Theta}_1,[\Delta W_t^y]_{t=1}^{n_j}\right) = f_1\left(\widehat{A_t}|\widehat{A}_{t-1};\theta,\widehat{\Theta}_1,[\Delta W_t^y]_{t=1}^n\right)\left|\frac{\partial i}{\partial A}\right|^{-1} \tag{B.60}$$

where $\widehat{A}_t$ is the productivity level that leads to time- t investment such that $(q_S(\widehat{A}_t)-1)/(2\theta K_t) = i_t$. The Jacobian is given by

$$\frac{di}{dA} = \begin{cases} \frac{1}{2\theta K}\left(\frac{(1-\tau)}{r+\delta-\mu^Q}+\left(\frac{l}{mh+1}-\frac{(1-\tau)A_D}{r+\delta-\mu^Q}-\frac{\tau\delta}{r+\delta}\right)\left(\frac{A}{A_D}\right)^{\gamma_2-1}\frac{\gamma_2}{A_D}-\frac{l}{mh+1}\left(\frac{A}{A_D}\right)^{\gamma_1-1}\frac{\gamma_1}{A_D}\right) & \text{if } lK < P \\ \frac{1}{2\theta K}\left(\frac{1-\tau}{r+\delta-\mu^Q}+\left(l-\frac{(1-\tau)A_D}{r+\delta-\mu^Q}-\frac{\tau\delta}{r+\delta}\right)\left(\frac{A}{A_D}\right)^{\gamma_2-1}\frac{\gamma_2}{A_D}\right) & \text{if } lK \geq P \end{cases}. \tag{B.61}$$

Next, I derive the joint probability $f_2(d,i_1;\theta,\widehat{\Theta}_1,[\Delta W_t^y]_{t=1}^n)$. I assume that the debt structure is chosen at date 0, while investment starts being observed at date 1. Using the law of total probability, the joint probability can be expressed as

$$f_2\left(d, i_1; \theta, \widehat{\Theta}_1, [\Delta W_t^y]_{t=1}^n\right) = \int f_2(d, i_1, A_0; \theta, \widehat{\Theta}_1, \Delta W_1^y) dA_0 \tag{B.62}$$
$$= \int f_2(i_1 | d, A_0; \theta, \widehat{\Theta}_1, \Delta W_1^y) f^P(d, A_0; \Theta) dA_0$$

where the conditioning on (d, A_0) is applied in the last equality. Also I exploit the property that consumption growth is i.i.d, so its growth at date $t+1$, ΔW_{t+1}^y, is independent of the growth at date t, ΔW_t^y. Hence the date $t+1$-growth is also independent of date t-investment. Since the debt structure is optimally chosen at the state A_0, the productivity A_0 can be inferred by observing the chosen debt structure. This implies the probability density $f^P(d, A_0; \Theta)$ is positive and infinite only at $\widehat{A_0}$ that leads to choosing the observed debt structure d, and zero elsewhere, while $\int f^P(d, A_0; \theta, \widehat{\Theta}_1) dA_0 = 1$. Heuristically, the probability $f_2(d, A_0; \theta, \widehat{\Theta}_1)$ acts like the Dirac delta function, so

$$f_2(d, i_1; \theta, \widehat{\Theta}_1, [\Delta W_t^y]_{t=1}^n) = f_2(i_1 | d, \widehat{A_0}; \theta, \widehat{\Theta}_1, \Delta W_1^y) \tag{B.63}$$

using the property of the delta function.

Still, how to determine $\widehat{A_0}$ remains. Since I observe two moments in debt structure consisting of leverage ratio and maturity (y, m) and want to solve for one-dimension state $\widehat{A_0}$, there is generally no exact solution to the state. Instead, I define the estimate of the implied productivity in the spirit of generalized least squares as follows:

$$\widehat{A_0} = \underset{A_0}{\operatorname{argmax}} \, (d(A_0) - d_j)' \, W \, (d(A_0) - d_j) \tag{B.64}$$

where $d(A_0)$ denotes the (2×1) vector of optimal debt structure as function of A_0, and W is the weighting matrix. I choose $W = \Sigma^{-1}$, where Σ is the covariance matrix of the two empirical moments.

Appendix C

C.1 Converting American Option Prices into European Option Prices

To extract European option prices from American option prices, I construct an implied binomial tree, following Rubinstein (1994) and Tian (2011). To start the construction, first I calibrate ending nodal probabilities of underlying asset prices, which are unconditional probabilities of reaching a particular price node at option maturity. Once the ending node probabilities are determined, I next calculate ending path probabilities, which are unconditional probabilities of following a particular path and reaching an ending price node. Assuming that path probabilities are identical among different paths as long as they start from the initial node and and reach the same ending node, the ending path probability is the nodal probability for the ending node divided by the number of paths leading to the node. With the path probabilities, I calculate the nodal probabilities of nodes a time-interval ahead of the ending nodes. After repeating the calculation toward the initial node, I construct the implied binomial tree. Details of how to calibrate the ending nodal probabilities are as follows. The basic idea is to use European option prices, given by either initial guess or the previous stage, and choose the probabilities to reproduce the option prices while achieving the maximum smoothness of probability density. In particular, I solve the following optimization problem for ending nodal probabilities $[P_j]_{j=1}^n$ for n ending nodes:

$$\min_{[P_j]_{j=1}^n} \sum_{j=2}^{n-1} (P_{j-1} - 2P_j + P_{j+1})^2 + \alpha \sum_{i=1}^{m} (V_i^{\text{model}} - V_i^{\text{market}})^2 \tag{C.1}$$

subject to conditions that $[P_j]_{j=1}^n$ are non-negative and sum to one and that the present value of underlying asset with the probabilities are equal to the current asset price. The implied binomial tree produces the model prediction of European option prices, V_i^{model}, and also the early exercise premium. I deduct the early exercise premium from the observed price of American options and obtain the pseudo-market value of European option, V_i^{market}. The pseudo-market price of European options are used in the next iteration. α is a parameter affecting the penalty for not matching option prices exactly and set to be 0.1 following TIAN. I continue the iteration until the option prices converge.

C.2 Asymptotic Distribution

In this section, I derive the asymptotic distributions of the estimated risk premium. To do so, I use the results of Ait-Sahalia and Lo (2000) who derived asymptotic distributions of estimated probability density functions both in risk-neutral and physical measure. The regularity conditions for the asymptotic distributions of nonparametric estimators to be defined are assumed to hold. For details, see Appendix A of Ait-Sahalia and Lo (2000).

C.2.1 Asymptotic Distribution of Risk-Neutral Density

First, I discuss the choice of bandwidth parameters. The bandwidths are given by

$$h_{X/F} = c_{X/F} s(X/F) n^{-1/10}, h_\tau = c_\tau s(\tau) n^{-1/6} \tag{C.2}$$

where $s(X/F)$ and $s(\tau)$ are unconditional standard deviations of the nonparametric regressors of X/F and τ, n the number of observations, $c_{X/F} = \gamma_{X/F}/\log(n)$, with $\gamma_{X/F}$ constant, and $c_\tau = \gamma_\tau/\log(n)$, with γ_τ constant. The power of n in the bandwidth are chosen to optimize the asymptotic properties following Ait-Sahalia and Lo (2000). I set $c_{X/F}$ and c_τ to minimize the sum of squared errors of the observations.

The second derivative of the implied volatility with respect to strike price has the following asymptotic distribution:

$$n^{1/2}h_{X/F}^{5/2}h_{\tau}^{1/2}\left[\frac{\partial^2\widehat{\sigma}}{\partial X^2}-\frac{\partial^2\sigma}{\partial X^2}\right]\xrightarrow{d}\mathcal{N}\left(0,\sigma^2_{d^2\sigma}\right) \tag{C.3}$$

where

$$\sigma^2_{d^2\sigma}=\frac{s^2(\tilde{Y})\left(\int_{-\infty}^{\infty}k_{X/F}^{(2)}(\omega)^2d\omega\right)\left(\int_{-\infty}^{\infty}k_{\tau}^2(\omega)d\omega\right)}{\pi(\tilde{Y})F_{t,\tau}^4} \tag{C.4}$$

where $s^2(\tilde{Y})$ is the variance of the volatility conditional on regressors $\tilde{Y}$, $\pi(\tilde{Y})$ is the marginal density of regressors. AL provide the value of kernel constants: $\int_{-\infty}^{\infty}k_{X/F}^{(2)}(\omega)^2d\omega=3/(8\sqrt{\pi})$ and $\int_{-\infty}^{\infty}k_{\tau}^2(\omega)d\omega=1/(2\sqrt{\pi})$.

Then, the risk-neutral density is asymptotically distributed as

$$n^{1/2}h_{X/F}^{5/2}h_{\tau}^{1/2}\left[\widehat{q}(S_T)-q(S_T)\right]\xrightarrow{d}\mathcal{N}\left(0,\sigma_q^2\right) \tag{C.5}$$

where

$$\sigma_q^2=\left[e^{r_f\tau}\frac{\partial H_{BS}}{\partial\sigma}\right]^2\sigma^2_{d^2\sigma}. \tag{C.6}$$

C.2.2 Asymptotic Distribution of Physical Density

The bandwidth in nonparametric regression for physical density function is

$$h_r=c_r s(r)N^{-1/5} \tag{C.7}$$

where $s(r)$ is the unconditional standard deviation of the returns, N the number of observations of the returns, $c_r=\gamma_r/\log(N)$ with γ_r constant. Again, γ_r is chosen to minimize the sum of squared errors of the observations. The density of log return is asymptotically distributed as

$$N^{1/2}h_r^{1/2}\left[\widehat{g}(r)-g(r)\right]\xrightarrow{d}\mathcal{N}\left(0,\sigma_g^2\right) \tag{C.8}$$

where $\sigma_g^2=g(r)\left(\int_{-\infty}^{\infty}k_r^2(\omega)d\omega\right)$. Then, the density of asset price is distributed as

$$N^{1/2}h_r^{1/2}\left[\widehat{f}(S_T)-f(S_T)\right]\xrightarrow{d}\mathcal{N}\left(0,\sigma_f^2\right) \tag{C.9}$$

where

$$\sigma_f^2 = \left(\int_{-\infty}^{\infty} k_r^2(\omega) d\omega \right) f(S_T)/S_T \tag{C.10}$$

where the kernel constant $\int_{-\infty}^{\infty} k_r^2(\omega) d\omega = 1/(2\sqrt{\pi})$.

C.2.3 Standard Deviation of Estimated Risk Premium

Rearranging the risk premium leads to

$$\begin{aligned}
\widehat{R_t^p} &= -R_{t,T}^f \left(E_t \left[\widehat{M_{t,T}}(S_T) \frac{S_T}{S_t} \right] - E_t \left[\widehat{M_{t,T}}(S_T) \right] E_t \left[\frac{S_T}{S_t} \right] \right). \\
&= - \int \frac{\widehat{q}_t(S_T)}{\widehat{p}_t(S_T)} \frac{S_T}{S_t} \widehat{p}_t(S_T) dS_T + \int \frac{S_T}{S_t} \widehat{p}_t(S_T) dS_T.
\end{aligned}$$

Approximating the variance of the risk premium by ignoring the correlation between risk neutral and physical density, the variance becomes

$$\frac{1}{n h_{X/F}^5 h_\tau} \int \left(\frac{S_T}{S_t} \right)^2 \sigma_q^2(S_T) dS_T + \frac{1}{N h_\tau} \int \left(\frac{S_T}{S_t} \right)^2 \sigma_f^2(S_T) dS_T. \tag{C.11}$$

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