Carnegie Mellon
Tepper
SCHOOL OF BUSINESS

DISSERTATION

Submitted in partial fulfillment of the requirements
for the degree of

DOCTOR OF PHILOSOPHY
INDUSTRIAL ADMINISTRATION
(FINANCIAL ECONOMICS)

Titled

"ESSAYS ON FINANCIAL INTERMEDIATION AND ECONOMIC LINKAGES"

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DOCTORAL DISSERTATION

ESSAYS ON FINANCIAL INTERMEDIATION AND ECONOMIC LINKAGES

by

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Submitted to the Tepper School of Business
in partial fulfillment of the requirements for the degree of
DOCTOR OF PHILOSOPHY
at
CARNEGIE MELLON UNIVERSITY

April, 2016

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To my family

Abstract

My dissertation studies the impact of economic linkages among market participants on equilibrium outcomes such as asset prices and returns as well as investors' welfare.

The first essay—titled "Inter-firm Relationships and Asset Prices"—studies the asset pricing properties that stem from the propagation of shocks within a network economy and the extent to which such a propagation mechanism quantitatively explains asset market phenomena. I show that changes in the propagation of shocks within a network economy are important to understanding variations in asset prices and returns, both in the aggregate and in the cross section. A calibrated model that matches features of customer-supplier networks in the U.S. as well as dynamic features of macroeconomic variables generates a persistent component in expected consumption growth and stochastic consumption volatility similar to the Long-Run Risks Model of Bansal and Yaron (2004). In the cross section, firms that are more central in the network command higher risk premium than firms that are less central. In the time series, firm-level return volatilities exhibit a high degree of comovement.

Implicit economic linkages among market participants also arise due to the existence of frictions in financial markets. The second essay—titled "Basket Securities in Segmented Markets"—studies the design and welfare implications of basket securities issued in markets with limited investor participation. Profit-maximizing intermediaries exploit investors' inability to trade freely across different markets, so they choose which market to specialize in. I show that when there is only one intermediary, the equilibrium may not be constrained efficient. Increasing competition among intermediaries increases the variety of baskets issued, but does not always improve investors' welfare. Although competition increases the variety of baskets issued, many of these baskets are redundant, in the sense that coordination among intermediaries could improve investors' risk-sharing opportunities. The equilibrium basket structure depends on institutional features of a market such as depth and gains from trade.

The third essay—titled “Imperfect Information Transmission from Banks to Investors: Real Implications” and joint with Nicolás Figueroa (Universidad Católica de Chile) and Oksana Leukhina (University of Washington)—proposes a general equilibrium model that features characteristics of securitization markets and study the interaction of information transmission in secondary loan markets and screening effort at loan origination. We show that increasing collateral values and asset complexity helps to explain the following pre-2008 crisis observations: (1) lax screening standards, (2) intensified ratings shopping, (3) rating inflation, and (4) the decline in the differential between yields on assets with low and high ratings. Contrary to conventional wisdom, we find that regulatory policies, such as mandatory rating and mandatory rating disclosure, may exacerbate credit misallocation.

Acknowledgements

For your unconditional support, I thank you Jeanne. I am also grateful to Burton Hollifield and Bryan Routledge for their guidance, continuous encouragement, and helpful discussions. I thank R. Ravi, Duane Seppi and Chester Spatt for their fruitful comments on my work. For their friendship and help, I thank Steven Baker, Andrés Bellofato, Francisco Cisternas, Benjamin Holcblat, Artem Neklyudov, Emilio Osambela, María Patricia Recalde, Maxime Roy, Alexander Schiller, and David Schreindorfer. Finally, I thank Lawrence Rapp for his help with all the administrative issues during the Ph.D. program.

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Chapter 1

Inter-firm Relationships and Asset Prices

Inter-firm relationships, such as strategic alliances, joint ventures, R&D partnerships, and customer-supplier relationships, are prevalent in modern economies. A growing body of empirical work highlights the importance of these relationships in the case of firms' distress and shows that they may serve as propagation mechanisms of negative shocks to individual firms.[1] For instance, consider South Africa's platinum miners' strike in 2014, which affected the world's top platinum producers, Anglo American Platinum, Impala Platinum, and Lonmin. First, platinum production decreased. Because platinum is used in many industrial applications such as oil cracking, some manufacturing firms may have faced higher production costs, as they needed to restructure their production given the lack of platinum. This, in turn, may have increased costs for some wholesale firms which, in turn, may have decreased some retailers' profits. Namely, a negative shock to a firm (or group of them) may spread to others via inter-firm relationships, and in doing so, potentially alter aggregate economic growth and volatility as well as asset prices and risk premia.

In this paper, I study the asset pricing properties that stem from the propagation of shocks within a fixed network economy and the extent to which such a propagation quantitatively explains asset market phenomena. To do so, I develop a dynamic, network-based equilibrium model in which the propagation of shocks determine, in large part, firms' cash-flow growth rates. To get a sense of the quantitative impact in asset prices of such a propagation mechanism, I calibrate the model to match features of customer-supplier networks in the U.S. as well as dynamic features of macroeconomic variables. To the best of my knowledge, this study is among the first to explore the extent to which the propagation of shocks within a fixed network economy quantitatively explains asset market phenomena.

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[1] See Hertzel et al., 2008, Boone and Ivanov, 2012, and Barrot and Sauvagnat, 2014.

The main finding of this paper is that changes in the propagation of shocks within a fixed network economy are important to understand variations in asset prices and returns, both in the aggregate and in the cross section. In the aggregate, a calibrated model generates a persistent component in expected consumption growth and stochastic consumption volatility similar to those in Bansal and Yaron, 2004. As in Bansal and Yaron, 2004, these two features, together with Epstein-Zin-Weil preferences, help explain characteristics of aggregate asset market data such as the equity premium and the low risk-free rate. The calibrated model also helps in understanding the cross section of expected returns, because it provides a mapping between firms' quantities of risk and firms' location in the network. For instance, firms that are more central in the network command a higher risk premium than firms that are less central. On average, firms in the highest quintile of centrality yield an annual excess return of 1% over those firms in the lowest quintile. This prediction is aligned with empirical results documented by Ahern, 2013 in the network of intersectoral trade. In the time series, firm-level return volatilities exhibit a high degree of comovement—which is consistent with evidence documented by Herskovic et al., 2014 and Duarte et al., 2014.

The main features of the model are as follows. The economy is composed of n firms whose cash-flow growth rates vary stochastically over time. In an otherwise standard dynamic endowment economy, firms' cash-flow growth rates are related via a network of inter-firm relationships, such as a supply chain, which is exogenous and fixed.[2] Each relationship generates benefits that increase a firm's cash-flow growth rate. However, relationships also increase a firm's exposure to negative shocks that affect other firms. In other words, the more relationships a firm is engaged in, the more benefits a firm receives and the higher its exposure to negative shocks that affect other firms in the network. To be more concrete, each firm faces a negative shock to its cash-flow growth rate, independently of others, with probability q—which is time-invariant and equal across firms—at very beginning of each period. Then, these negative shocks spread from one firm to another via inter-firm relationships in a probabilistic manner. In particular, a negative shock to firm i at period t propagates to firm j at t if there exists a sequence of relationships that connects firms i and j in which each relationship in the sequence transmits shocks at period t. For simplicity, each relationship potentially transmits shocks, independently of all other relationships, with probability p_t at period t. The value of p_t captures the relative importance of relationship-specific investments made by the average firm in a network economy. The higher the value of p_t, the more important relationships are on average, and the higher the likelihood that shocks propagate through the economy at period t. To allow changes in the propagation of shocks within the network, the propensity of inter-firm relationships to transmit shocks, p_t, is allowed to vary over time. As a consequence, the volatility of aggregate cash-flows and the correlation among firms' cash-flows are time-varying. Temporal changes in p_t capture changes in production technologies and complementarities among firms' activities. The pricing is done by a representative agent with Epstein-Zin-Weil preferences to embed the time-varying cash-flow correlation structure—which is endogeneously generated by the network—in a standard asset pricing model.

---

[2] The network of inter-firm relationships is assumed to be fixed for two reasons: (a) tractability, and (b) to capture the long-term nature of some customer-supplier relationships that allow connected firms to circumvent difficulties in contracting due to unforeseen contingencies, asymmetries of information, and specificity on firms' investments, e.g. Williamson, 1979; Williamson, 1983.

The above framework has two important properties. First, cash-flow growth rates are independent across firms in the absence of relationships. Second, if only one sequence of relationships connects two firms, the longer the sequence, the smaller the correlation between their cash-flow growth rates. Namely, the more distant two firms are in the network economy, the less related their cash-flows.

The distribution of consumption growth is shaped by two characteristics within the model: (a) the topology of the network of relationships and (b) the propensity of relationships to transmit shocks. Because the network is fixed, the calibrated model is able to generate a persistent component in expected consumption growth and stochastic consumption volatility as long as the propensity of relationships to transmit shocks, p_t, exhibits persistent time variation. The persistent time variation in p_t in the calibrated model is motivated by the high persistence exhibited by macroeconomic variables that proxy for the level of input specificity faced by the average firm within the U.S. economy. As Barrot and Sauvagnat, 2014 show, input specificity is an important driver of the propagation of shocks within customer-supplier networks. Suppliers of specific inputs are more difficult to replace in case of distress, and, thus, shocks may propagate more easily from one firm to another.[3]

In the cross section, shocks to central firms have a higher likelihood of affecting more firms than do shocks to less central firms. As a consequence, central firms are procyclical, whereas less central firms serve as a hedge against aggregate risk and command lower risk premium. Changes in the propensity of relationships to transmit shocks drive fluctuations in growth opportunities and uncertainty across firms. These fluctuations translate into changes in stock prices and returns, which produces a factor structure in returns and returns volatilities at the firm level.

This paper contributes to several strands of the literature. First, it develops a new theoretical framework that relates to a growing body of work focused on understanding the effects of economic linkages in asset pricing properties. Buraschi and Porchia, 2012 show that firms more central in a market-based network have lower price dividend ratios and higher expected returns. Using the network of intersectoral trade, Ahern, 2013 shows that firms in more central industries have greater exposure to systematic risk. Unlike these papers, my study uses relationships at the firm level to explore the asset pricing properties that stem from the propagation of shocks within a network between firms' cash-flow and the extent to which changes in such a propagation mechanism quantitatively explain asset market phenomena. Using customer-supplier networks, Kelly, Lustig, and Nieuwerburgh, 2013 propose that the size distribution and firm volatility distribution are intimately linked. However, they do not explore the equilibrium asset pricing implications of such networks. In a contemporaneous paper, Herskovic, 2015 focuses on efficiency gains that come from changes in the input-output network and how those changes are priced in equilibrium. This paper, on the other hand, focuses on how changes in the propagation of shocks within a fixed network alter equilibrium asset prices, risk premia, and stock return volatilities across firms.

I also add to a body of work that explores how granular shocks may lead to aggregate fluctuations in the presence of linkages among different sectors of the economy, e.g. Carvalho, 2010, Gabaix, 2011b, Acemoglu et al., 2012; Acemoglu, Ozdaglar, and Tahbaz-Salehi, 2015a, Carvalho and Gabaix, 2013, among others. This literature focuses mostly on analyzing changes in aggregate economic variables due to changes in the input-output network rather than exploring the asset pricing implications of linkages among firms. This paper expands this literature by exploring the asset pricing implications of linkages at the firm level and studying how changes in the propagation of shocks within a network affect aggregate variables as well as asset pricing, both in the aggregate and in the cross-section.

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[3] To calibrate the model, I use the time series of R&D/GDP and the number of patents created in the U.S. as measures of the degree of input specificity faced by the average firm in the U.S. The ratio R&D/GDP aims to proxy for the intensity of relationship-specific investments faced by the average firm, whereas the number of patents proxies for how easily the average firm can substitute its inputs whenever a supplier is under distress.

The rest of the paper is organized as follows. Section 1.1 explains the baseline model. Section 1.2 describes aggregate output and consumption growth within the baseline model. Section 1.3 derives expressions for the market return, the risk-free rate, the price of risk, firms' stock prices, and firms' quantity of risk in large network economies. Section 1.4 uses data on customer-supplier networks in the U.S. as well as macroeconomic variables related to the propagation of shocks within these networks to calibrate the baseline model. Section 1.5 shows that changes in the propagation of shocks within large network economies are quantitatively important to understand variations in asset prices and returns, both in the aggregate and in the cross section. Section 1.6 concludes. All proofs, unless otherwise stated, appear in the Appendix.

1.1 Baseline Model

1.1.1 The Environment

Consider an economy with one perishable good and an infinite time horizon. Time is discrete and indexed by $t \in \{0, 1, 2, \cdots\}$. The economy is populated by a large number of identical infinitely-lived individuals who are aggregated into a representative infinitely-lived investor with Epstein-Zin-Weil preferences who owns all assets in the economy. In each period, the single good is produced by n infinitely-lived Lucas, 1978 trees, henceforth firms, with n being potentially large. In an otherwise standard dynamic endowment economy, firms' cash-flows are related via a network of inter-firm relationships. Interdependencies among firms' cash-flows can be conveniently described by a graph consisting of a set of nodes—which represent firms—together with lines or edges joining certain pairs of nodes—which represent inter-firm relationships. To fix notation, let $\mathcal{G}_n = (\mathcal{F}_n, \mathcal{R}_n)$ denote the network of inter-firm relationships among n firms, where $\mathcal{F}_n$ denotes the set of firms and $\mathcal{R}_n$ denotes the set of inter-firm relationships among them. Because I focus on the effect of $\mathcal{G}_n$ on asset prices rather than on strategic network formation, inter-firm relationships are exogenously determined and fixed before $t = 0$.[4]

1.1.2 The network of inter-firm relationships $\mathcal{G}_n$ and firms' cash-flows

Firms' cash-flows vary stochastically over time and depend on the network of inter-firm relationships, $\mathcal{G}_n$. The following reduced form formulation of firms' cash-flows captures a simple trade-off in a parsimonious manner. The more relationships a firm is engaged in, the more benefits a firm receives and the higher its exposure to negative shocks that propagate through the network. Let $y_{i,t+1}$ denote firm i's cash-flow at $t+1$, and $Y_t \equiv \prod_{i=1}^{n} y_{i,t}^{1/n}$ denote the aggregate output of the economy at t.[5]

ASSUMPTION 1. *I assume that $y_{i,t+1}$ follows*

$$\log\left(\frac{y_{i,t+1}}{Y_t}\right) \equiv \alpha_0 + \alpha_1 d_i - \alpha_2\sqrt{n}\widetilde{\varepsilon}_{i,t+1}\ , \quad i \in \{1,\cdots,n\} \tag{1.1}$$

where parameters α_0, α_1 and α_2 are non-negative and equal across firms. Parameter d_i represents the number of relationships of firm i in $\mathcal{G}_n$, i.e. firm i's degree in $\mathcal{G}_n$. This parameter may differ across firms. The term $\sqrt{n}$ is included as a normalization factor in equation (1.1), which helps to characterize the equilibrium distribution of aggregate consumption growth later on. Uncertainty in $y_{i,t+1}$ is introduced by a Bernoulli random variable $\widetilde{\varepsilon}_{i,t+1}$, which equals one if firm i faces a negative shock at $t+1$ and zero otherwise. Given that

$$\log\left(\frac{y_{i,t+1}}{Y_t}\right) = \log\left(\frac{y_{i,t+1}}{y_{i,t}}\right) + \log\left(\frac{y_{i,t}}{Y_t}\right) \tag{1.2}$$

parameter α_2 in equation (1.1) measures the instantaneous decrease in a firm's cash-flow growth when a firm faces a negative shock, whereas parameter α_1 captures the benefits a firm receives from each relationship it engages in. Parameter α_0 in equation (1.1) captures the parts of firms' cash-flow growth that are unrelated to benefits or costs associated to inter-firm relationships.[6]

---

[4] See Demange and Wooders, 2005, Goyal, 2007 and Jackson, 2008 for a detailed description of network formation models.

[5] The definition of Y_t implies that positive aggregate production requires positive production by each firm. To assume that $Y_t \equiv \prod_{i=1}^{n} y_{i,t}^{1/n}$ is similar to assuming that Y_t is proportional to $\sum_{i=1}^{n} y_{i,t}$ if n is sufficiently large and all $y_{i,t} \neq 0$. The argument follows from applying a first order Taylor series expansion to $\log(Y_t)$ in which aggregate output, $Y_t \equiv \sum_{i=1}^{n} y_{i,t}$. A different way of justifying that $Y_t \equiv \prod_{i=1}^{n} y_{i,t}^{1/n}$ is to consider that every firm produces a different perishable good and each good is necessary to produce other goods in the economy. In such an environment, one obtains asset pricing properties similar to the ones obtained in this paper if the representative investor has preferences over a Cobb-Douglas consumption aggregator of the form $C_t \equiv \prod_{i=1}^{n} c_{i,t}^{1/n}$, where $c_{i,t}$ represents consumption of the good produced by firm i at time t.

To complete the description of $y_{i,t+1}$, it is necessary to understand how inter-firm relationships affect the distribution of $\widetilde{\varepsilon}_{i,t+1}$ at $t+1$. Such a distribution is determined by the following random-network model. First, each firm faces a negative shock to its cash-flow growth, independently of other firms, with probability q at the very beginning of each period. A negative shock to firm i at $t+1$ propagates to firm j at $t+1$ if there exists a path of relationships in $\mathcal{G}_n$ that connects firms i and j in which each relationship in the path transmits shocks at $t+1$. A path is a sequence of inter-firm relationships that connects a sequence of firms that are each distinct from one another. Each relationship transmits shocks, independently of all other relationships, with probability $\widetilde{p}_{t+1}$ at $t+1$. Variable $\widetilde{p}_{t+1}$ may vary over time.[7] I only allow negative shocks to propagate in a probabilistic manner throughout the network to focus on the propagation of shocks in the case of firms' distress. However, equation (1.1) can be modified so that positive and negative shocks propagate over the economy. The main results continue to hold as long as the decrease in firms' cash-flows due to negative shocks is larger than the increase in firms' cash-flow due to positive shocks.

The value of $\widetilde{p}_{t+1}$ captures the importance of restrictions on alternative sources of substitutable inputs for the average firm as well as the importance of relationship-specific investments made by the average firm at $t+1$. The higher the value of $\widetilde{p}_{t+1}$, the more important relationships are on average, and the higher the likelihood that shocks are transmitted via relationships at $t+1$. For example, in the context of supply chains, Barrot and Sauvagnat, 2014 show that input's specificity, switching costs, and complementaries among firms' activities may allow negative shocks to individual firms to propagate and affect other firms in a production chain. The existence of switching costs may prevent firms from restructuring their production sufficiently fast when they need to replace a supplier who is under distress, so negative shocks tend to spread from one firm to another.

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[6] Blume et al., 2013 analyze a similar trade-off in a static environment. They focus, however, on the strategic network formation features of economies in which agents receive benefits from the set of direct links they form, but these links expose them to the risk of being affected by cascades of failures. They provide asymptotic bounds on the welfare of both optimal and stable networks and show that very small amounts of "over-linking" may impose large losses in welfare to networks' participants.

[7] This random-network model can be thought of as a variation of either a reliability network or a bond percolation model in each period. In a typical reliability network model, the edges of a given network are independently removed with some probability. Remaining edges are assumed to transmit a message. A message from node i to j is transmitted as long as there is at least one path from i to j after edges removal—see Colbourn, 1987 for more details. Similarly, in a bond percolation model, edges of a given network are removed at random with some probability. Those edges that are not removed are assumed to percolate a liquid. The question in percolation is whether or not the liquid percolates from one node to another in the network—which is similar to the problem of transmitting a message in a reliability context. For more details see Grimmett, 1989, Stauffer and Aharony, 1994 and Newman, (2010, Chapter 16.1).

To sum up, equation (1.1) captures the potential consequences of some inter-firm relationships in a simple manner. Despite the fact that firms may use relationships to increase their growth opportunities via efficiency gains, these relationships may have additional consequences because they may also increase a firm's exposure to negative shocks that affect a broader set of firms in the economy. In fact, for a given set of parameters, it follows from equation (1.1) that a firm's expected cash-flow growth rate is initially an increasing function of the number of relationships of a firm, but then it becomes a decreasing function of the number of relationships of a firm because the benefits associated with relationships are eventually overcompensated by the increase in exposure to negative shocks. Despite the fact that equation (1.1) is a reduced form formulation, this feature of firms' expected cash-flow growth can also be obtained within an equilibrium context, e.g. Goyal and Moraga-González, 2001.

Given the topology of $\mathcal{G}_n$, the joint distribution of the sequence $\{\widetilde{\varepsilon}_{i,t+1}\}_{i=1}^{n}$ at $t+1$ is determined by two parameters: q, $\widetilde{p}_{t+1}$. Further, the marginal distribution of $\widetilde{\varepsilon}_{i,t+1}$ at $t+1$, conditional on $\widetilde{p}_{t+1}$, depends on q as well as the topology of $\mathcal{G}_n$ and the location of firm i in $\mathcal{G}_n$. In other words,

$$\mathbb{P}\left(\widetilde{\varepsilon}_{i,t+1}=1\middle|\widetilde{p}_{t+1}\right) \quad = \quad f\left(q,\ \text{topology of } \mathcal{G}_n, \text{location of firm } i \text{ in } \mathcal{G}_n\right) \tag{1.3}$$

where $\mathbb{P}\left(\widetilde{\varepsilon}_{i,t+1}=0\middle|\widetilde{p}_{t+1}\right) = 1-\mathbb{P}\left(\widetilde{\varepsilon}_{i,t+1}=1\middle|\widetilde{p}_{t+1}\right)$, and $f(\cdot)$ is a mapping characterized by the random-network model described above—which may be hard to describe in closed-form for general network topologies as n increases.

Despite the fact that the above mapping is hard to characterize for large n, some of its properties are easy to describe given its formulation. First, in the absence of relationships, $\mathbb{P}\left(\widetilde{\varepsilon}_{i,t+1}=1\middle|\widetilde{p}_{t+1}\right) = \mathbb{P}\left(\widetilde{\varepsilon}_{i,t+1}=1\right) = q$, $\forall\ i$ and $t+1$, so cash-flow growth rates are independent and identically distributed across firms over time. Second, if only one path of relationships exists between two firms, the longer the path, the smaller the correlation between their cash-flows growth rates. Thus, the more distant two firms are in a network in which there is at most one path between any two firms, the less related their cash-flows are. Having this feature—which is sometimes called correlation decay, e.g. Gamarnik, 2013—helps a great deal to obtain numerical solutions when n is large.

1.1.3 Changes in shock propagation within $\mathcal{G}_n$

Given a network $\mathcal{G}_n$, the correlation structure among firms' cash-flows depends, in large part, on the propensity of relationships to transmit shocks, $\widetilde{p}_t$. Sufficiently small values of $\widetilde{p}_t$ imply that shocks tend to remain locally confined and affect only negligible fractions of the economy, whereas sufficiently large values of $\widetilde{p}_t$ imply that shocks may affect a large fraction of the economy for some network topologies and, thus, alter the distribution of the pricing kernel.

To capture temporal changes in production technologies and complementaries among firms' activities, $\widetilde{p}_t$ is allowed to vary over time and follows a two state ergodic Markov process, taking on either the value p_L or p_H, with $0 \leq p_L < p_H < 1$. The transition probability matrix of $\widetilde{p}_t$, Ω_p, is defined by

$$\begin{aligned} \mathbb{P}(\widetilde{p}_{t+1} = p_H | \widetilde{p}_t = p_H) &= \psi(1-\phi) + \phi \ , \qquad (1.4) \\ \mathbb{P}(\widetilde{p}_{t+1} = p_H | \widetilde{p}_t = p_L) &= \psi(1-\phi) \end{aligned}$$

where ψ is the unconditional probability that $\widetilde{p}_t = p_H$. Parameter ϕ, which measures the persistence in $\widetilde{p}_t$, satisfies $0 \leq \phi < 1$, so $\widetilde{p}_t$ is positively autocorrelated. If $\phi = 0$, then $\widetilde{p}_t$'s are i.i.d. over time. As ϕ tends to 1, $\widetilde{p}_t$'s become perfectly positively correlated over time.

1.2 Distribution of Consumption Growth

Two components of the model are important to understanding equilibrium asset prices: (a) the topology of the network $\mathcal{G}_n$, and (b) the propensity of relationships to transmit shocks. Before I discuss the cross-sectional asset pricing properties that stem from the propagation of shocks within $\mathcal{G}_n$, I study how changes in these two components affect the distribution of aggregate consumption growth and, thus, alter the distribution of the pricing kernel. Let $\Delta\widetilde{c}_{t+1} \equiv \log\left(\frac{\widetilde{C}_{t+1}}{C_t}\right)$ and $\widetilde{x}_{t+1} \equiv \log\left(\frac{Y_{t+1}}{Y_t}\right)$ be the log consumption and output growth at $t+1$, respectively. In equilibrium, $\Delta\widetilde{c}_{t+1} =$

$\widetilde{x}_{t+1}$. It follows from the definition of aggregate output and equation (1.1) that,

$$\begin{aligned}
\Delta\widetilde{c}_{t+1} = \widetilde{x}_{t+1} &= \log\left(\prod_{i=1}^{n}\left(\frac{y_{i,t+1}}{Y_t}\right)^{1/n}\right) \\
&= \sum_{i=1}^{n}\frac{1}{n}\log\left(\frac{y_{i,t+1}}{Y_t}\right) \\
&= \alpha_0 + \alpha_1 \underbrace{\left(\frac{1}{n}\sum_{i=1}^{n} d_i\right)} - \alpha_2\sqrt{n}\underbrace{\left(\frac{1}{n}\sum_{i=1}^{n}\widetilde{\varepsilon}_{i,t+1}\right)} \\
&= \alpha_0 + \alpha_1 \qquad \bar{d} \qquad - \alpha_2\sqrt{n} \qquad \widetilde{W}_{n,t+1}\ , \qquad (1.5)
\end{aligned}$$

where $\bar{d}$ denotes the average number of relationships per firm in the economy, whereas $\widetilde{W}_{n,t+1}$ denotes the average number of firms affected by negative shocks at $t+1$. It follows from equation (1.5) that the distribution of $\Delta\widetilde{c}_{t+1}$ is determined by the distribution of $\sqrt{n}\widetilde{W}_{n,t+1}$. Because the distribution of $\sqrt{n}\widetilde{W}_{n,t+1}$ is affected by $\widetilde{p}_{t+1}$ and the topology of $\mathcal{G}_n$, these two components also affect the distribution of $\Delta\widetilde{c}_{t+1}$.

To appreciate the importance of $\widetilde{p}_{t+1}$ and the topology of $\mathcal{G}_n$ in shaping the distribution of $\Delta\widetilde{c}_{t+1}$, consider the case in which there are no relationships. In this case, $\{\widetilde{\varepsilon}_{i,t+1}\}_{i=1}^{n}$ is a sequence of i.i.d. Bernoulli random variables, so $n\widetilde{W}_{n,t+1}$ follows a Binomial distribution. By the Lindeberg-Lévy Central Limit Theorem, $\sqrt{n}\widetilde{W}_{n,t+1}$ is normally distributed as n grows large. Provided the absence of relationships, the realization of $\widetilde{p}_{t+1}$ is irrelevant to determining the distribution of $\Delta\widetilde{c}_{t+1}$. It then follows from equation (1.5) that the unconditional mean and variance of $\Delta\widetilde{c}_{t+1}$ are $(\alpha_0 - \alpha_2 q)$ and $q(1-q)\alpha_2^2$, respectively.

In the presence of relationships, however, $\widetilde{p}_{t+1}$ and the topology of $\mathcal{G}_n$ affect the distribution of consumption growth in two important ways. First, all moments of the distribution of $\Delta\widetilde{c}_{t+1}$ at $t+1$ potentially depend on the realization of $\widetilde{p}_{t+1}$ and the topology of $\mathcal{G}_n$. Second, the sequence $\{\widetilde{\varepsilon}_{i,t+1}\}_{i=1}^{n}$ at $t+1$ is a sequence of dependent random variables, so the conditions under which a Central Limit Theorem (CLT) holds may not be satisfied for large n. In fact, relationships may generate convoluted interdependencies among firms' cash-flows, which it makes difficult to characterize the distribution of $\Delta\widetilde{c}_{t+1}$ for general network topologies and large n.

In general, there is no guarantee that $\Delta\widetilde{c}_{t+1}$ is normally distributed, despite the fact that $\Delta\widetilde{c}_{t+1}$

comes from aggregating shocks to individual firms, as in Acemoglu, Ozdaglar, and Tahbaz-Salehi, 2015a. In fact, for a large variety of network topologies, simulation shows that the distribution of $\Delta\widetilde{c}_{t+1}$ may differ from a normal distribution for large n. In particular, if $\widetilde{p}_{t+1}$ is sufficiently close to 1 and $\mathcal{G}_n$ is locally connected—i.e., there is at least one path between any two firms in an arbitrarily large neighborhood around any given firm—then a non-negligible fraction of the economy is almost surely affected by shocks to individual firms. Therefore, the distribution of $\Delta\widetilde{c}_{t+1}$ may exhibit thicker tails than a normal distribution would. Figure A.1 illustrates the previous point. Figure A.1(a) depicts an economy with $n = 5$ firms, whereas figure A.1(b) depicts the empirical probability density function of $\sqrt{n}\widetilde{W}_{n,t+1}$ for different values of $\widetilde{p}_{t+1}$. As figure A.1(b) shows, the distribution of $\sqrt{n}\widetilde{W}_{n,t+1}$ may differ from a normal distribution for large values of $\widetilde{p}_{t+1}$. In particular, as $\widetilde{p}_{t+1}$ tends to one, the distribution of $\sqrt{n}\widetilde{W}_{n,t+1}$ tends to be bimodal.

Despite the existence of relationships and the convoluted dependencies they may generate among firms' cash-flows, the topology of $\mathcal{G}_n$ and $\widetilde{p}_{t+1}$ can be restricted so that $\Delta\widetilde{c}_{t+1}$ is normally distributed as n grows large. In such a case, keeping track of temporal changes of the whole distribution of $\Delta\widetilde{c}_{t+1}$ is equivalent to keeping track of temporal changes in only averages and standard deviations. In particular, if shocks tend to remain locally confined—i.e., shocks only propagate over fractions of the economy that become negligible as n grows large—the sequence $\{\widetilde{\varepsilon}_{i,t+1}\}_{i=1}^{n}$ at $t+1$ becomes a sequence of weakly dependent random variables to which a CLT can be applied. Then, the dynamics of consumption growth can be recast as a version of Hamilton, 1989's Markov-switching model.

To fix the notation, let $\mathcal{G}_{n+1}$ denote the network $\mathcal{G}_n$, to which I add one new firm and all the relationships the new firm may have with existing firms within $\mathcal{G}_n$. The following proposition imposes sufficient conditions on: (a) the limiting topology of the sequence of networks $\{\mathcal{G}_n\}_{n=1}^{\infty}$, $\mathcal{G}_\infty \equiv \lim_{n\to\infty} \mathcal{G}_n$, and (b) the propensity of relationships to transmit shocks, $\widetilde{p}_{t+1}$, so that $\Delta\widetilde{c}_{t+1}$ is normally distributed as n grows large.

PROPOSITION 1 (Asymptotic Normality of $\Delta\widetilde{c}_{t+1}$)**.** *Given $q > 0$ and a sequence of networks of inter-firm relationships, $\{\mathcal{G}_n\}_{n\geq 1}$, with limiting topology $\mathcal{G}_\infty$, define p_c as*

$$p_c(\mathcal{G}_\infty) \quad = \quad \sup_{p\in(0,1)} \left\{ p : \lim_{n\to\infty} P_q(n) = 0 \right\} \tag{1.6}$$

where $P_q(n)$ denotes the probability that a shock to any given firm within $\mathcal{G}_n$ also affects αn firms via shock propagation, with $\alpha > 0$. If $\widetilde{p}_{t+1} < p_c(\mathcal{G}_\infty)$, then $\sqrt{n}\widetilde{W}_{n,t+1}$ and $\Delta\widetilde{c}_{t+1}$ are normally distributed at $t+1$ as n grows large.

Let $\mu_{c,t+1}$ and $\sigma_{c,t+1}$ denote the mean and volatility of $\Delta\widetilde{c}_{t+1}$, conditional on knowing $\widetilde{p}_{t+1}$ at $t+1$. Under the conditions of proposition 1, the distribution of $\Delta\widetilde{c}_{t+1}$ can be characterized in terms of the pair $(\mu_{c,t+1}, \sigma_{c,t+1})$. Because the network is fixed, the dynamics of $(\mu_{c,t+1}, \sigma_{c,t+1})$ is fully determined by the dynamics of $\widetilde{p}_{t+1}$. Thus, the economy follows a Markov process with a continuum of values for aggregate consumption and its growth rate, $\Delta\widetilde{c}_{t+1}$, but only two values for the first two moments of the distribution of $\Delta\widetilde{c}_{t+1}$, as in Kandel and Stambaugh, 1991.

The following corollaries provide a more detailed characterization of those large network economies in which $\Delta\widetilde{c}_{t+1}$ is normally distributed. In particular, they report the limiting topology of the sequence of networks $\{\mathcal{G}_n\}_{n=1}^{\infty}$, $\mathcal{G}_\infty$, and the value of the critical probability p_c in proposition 1. Corollary 1 focuses on large networks in which all firms have the same number of relationships.

COROLLARY 1 (Symmetric Networks)**.** *Given a sequence of networks of inter-firm relationships, $\{\mathcal{G}_n\}_{n\geq 1}$, with limiting topology $\mathcal{G}_\infty$,*[8]

- $p_c = 1 - 2\sin\left(\frac{\pi}{18}\right) \approx 0.65$ *if $\mathcal{G}_\infty$ is the two dimensional honeycomb lattice.*
- $p_c = \frac{1}{2}$ *if $\mathcal{G}_\infty$ is the two dimensional square lattice.*
- $p_c = 2\sin\left(\frac{\pi}{18}\right) \approx 0.34$ *if $\mathcal{G}_\infty$ is the two dimensional triangular lattice.*
- $p_c = \frac{1}{z-1}$ *if $\mathcal{G}_\infty$ is the Bethe lattice with z neighbors per each firm.*

Figure A.2 illustrates each of the network economies considered in corollary 1. Corollary 2 focuses on large networks in which the number of relationships may differ across firms.

COROLLARY 2 (Asymmetric Networks)**.** *Given a sequence of networks of inter-firm relationships, $\{\mathcal{G}_n\}_{n\geq 1}$,*

- $p_c = \frac{1}{\textit{branching number of } \mathcal{G}_\infty}$ *if $\mathcal{G}_\infty$ is a tree. The branching number of a tree is the average number of relationships per firm in a tree.*[9]

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[8] A lattice is a graph whose drawing can be embedded in $\mathbb{R}^n$. The two dimensional honeycomb lattice is a graph in 2D that resembles a honeycomb. The two dimensional square lattice is a graph that resembles the $\mathbb{Z}^2$ grid. The two dimensional triangular lattice is a graph in 2D in which each node has 6 neighbors.

[9] A tree is a network in which any two firms are connected by exactly one path. A forest is a network whose components are trees.

- $p_c = \frac{1}{e_M}$ *if* $\mathcal{G}_n$ *is sparse and locally treelike.* $\mathcal{G}_n$ *is said to be sparse if the number of relationships in* $\mathcal{G}_n$ *increases linearly with* n*, as* n *increases.* $\mathcal{G}_n$ *is said to be locally treelike if an arbitrarily large neighborhood around any given firm takes the form of a tree. Parameter* e_M *is the leading eigenvalue of the matrix*

$$M_n = \begin{pmatrix} A_n & \mathbb{I}_n - D_n \\ \mathbb{I}_n & 0 \end{pmatrix} \tag{1.7}$$

where A_n *is the adjacency matrix of* $\mathcal{G}_n$*, i.e. the* $n \times n$ *matrix in which* $A_{ij} = 1$ *if there is a relationship between firms* i *and* j *and zero otherwise.* $\mathbb{I}_n$ *is the* $n \times n$ *identity matrix, and* D_n *is the diagonal matrix that contains the number of relationships per firm along the diagonal.*

1.3 Equilibrium Asset Prices

To see what the network $\mathcal{G}_n$ and $\widetilde{p}_{t+1}$ imply for equilibrium asset prices, both in the aggregate and in the cross-section, I embed the cash-flows correlation structure that is endogenously generated by the network in a standard asset pricing model. The representative investor has Epstein-Zin-Weil recursive preferences to account for asset pricing phenomena that are challenging to address with power utility preferences. The asset pricing restrictions on the gross return of firm i, $\widetilde{R}_{i,t+1}$, are

$$\mathbb{E}_t\left(\widetilde{M}_{t+1}\widetilde{R}_{i,t+1}\right) = 1 \tag{1.8}$$

where $\widetilde{M}_{t+1} \equiv \left[\beta\left(e^{\Delta\widetilde{c}_{t+1}}\right)^{-\rho}\right]^{\frac{1-\gamma}{1-\rho}}\left[\widetilde{R}_{a,t+1}\right]^{\frac{1-\gamma}{1-\rho}-1}$ represents the pricing kernel at $t+1$ and $\widetilde{R}_{a,t+1}$ is the gross return on aggregate wealth—an asset that delivers aggregate consumption as its dividend each period. Parameter $\rho > 0$, $\rho \neq 1$, represents the inverse of the inter-temporal elasticity of substitution, $\gamma > 0$ is the coefficient of relative risk aversion for static gambles, and $\beta > 0$ measures the subjective discount factor under certainty.[10]

To solve the model, I look for equilibrium asset prices so that price-dividend ratios are stationary, as in Mehra and Prescott, 1985, Weil, 1989, and Kandel and Stambaugh, 1991, among many others. Because equilibrium values are time invariant functions of the state of the economy—which is determined by the state of the propensity of relationships to transmit shocks—the index t can be eliminated. Hereinafter, c denotes the current level of aggregate consumption, y denotes the current level of aggregate output, and s denotes the current state of the propensity of relationships to transmit shocks.

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[10] If $\gamma = \rho$, these recursive preferences collapse to the standard case of VNM time-additive expected utility. The functional form of the Euler equation when $\rho = 1$ is different from the one shown in equation (1.8). See Weil, (1989, Appendix A) for details. I use the standard terminology to describe γ and ρ. However, Garcia, Renault, and Semenov, 2006 and Hansen et al., 2007 indicate that this interpretation may not be correct if $\rho \neq \gamma$.

I first solve for the price of aggregate wealth and the risk-free rate. These expressions are then used to solve for equilibrium asset prices and expected excess returns in the cross-section. The conditions under which proposition 1 and corollaries 1 or 2 hold are not needed to be satisfied in what follows. If those conditions are satisfied, however, the conditional expectations that appear in the following propositions can be computed in closed-form. Otherwise, I use simulation to compute those conditional expectations.

The following proposition determines the current price of aggregate wealth.

PROPOSITION 2 (Price of Aggregate Wealth)**.** *Let* $P_a(c,s)$ *denote the current price of aggregate wealth.* $P_a(c,s) = w_s^a c$*, where* w_s^a *is the solution of the following non-linear system of equations,*

$$w_s^a \quad = \quad \beta \left(\sum_{s' \in \{H,L\}} \omega_{s,s'} \mathbb{E}\left(e^{(1-\gamma)\Delta\widetilde{c}_{t+1}} \middle| p_{s'} \right) \left(w_{s'}^a + 1\right)^{\frac{1-\gamma}{1-\rho}} \right)^{\frac{1-\rho}{1-\gamma}}, \qquad s = \{H, L\} \tag{1.9}$$

where $\mathbb{E}\left(\cdot \middle| p_{s'}\right)$ *denotes the conditional expectation operator if the propensity of relationships to transmit shocks during the next period is* $p_{s'}$*, and* $\omega_{s,s'}$ *represents the* (s,s') *element of* Ω_p*.*

I restrict my analysis to the set of model primitives in which the existence of a non negative solution of (1.9) is ensured.[11] The expected period gross return of aggregate wealth in the current

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[11] Provided that $e^{\Delta\widetilde{c}_t}$ is positive for all t, parameters ρ and γ need to be restricted so that the function $h(\cdot)$ defined as

$$h\left(w_i^a\right) \quad \equiv \quad \beta \left(\sum_{j \in \{H,L\}} \omega_{i,j} \mathbb{E}\left(e^{(1-\gamma)\Delta\widetilde{c}_{t+1}} \middle| p_j \right) \left(w_j^a + 1\right)^{\frac{1-\gamma}{1-\rho}} \right)^{\frac{1-\rho}{1-\gamma}}$$

is continuous. If $h(\cdot)$ is continuous, the system of equations (1.9) has a solution by Brouwer's Fixed Point Theorem. Further restrictions in the set of parameter values can be imposed such that the solution of the system of equations is unique.

state is then

$$\mathbb{E}\left(R_a|s\right) \;=\; \sum_{s'\in\{H,L\}} \omega_{s,s'} \frac{w^a_{s'}+1}{w^a_s} \mathbb{E}\left(e^{\Delta\widetilde{c}_{t+1}}\middle|p_{s'}\right), \qquad s=\{H,L\}\,. \tag{1.10}$$

It follows from equations (1.9) and (1.10) that the price and expected period return of aggregate wealth are driven by the dynamics of $\widetilde{p}_t$. In particular, temporal changes in $\widetilde{p}_t$ convey temporal changes in the distribution of aggregate consumption growth, which, in turn, manifest in the price and the expected period return of aggregate wealth. The dynamics of $\widetilde{p}_t$, parameterized by ψ and ϕ, also impact the price and the expected period return of aggregate wealth via $\omega_{s,s'}$, because these two parameters determine: (a) how frequently the economy is in a state in which relationships transmit shocks more often, and (b) how frequently changes in the propensity of relationships to transmit shocks occur.

I next consider the risk-free asset, which pays one unit of the consumption good during the next period with certainty.

PROPOSITION 3 (Risk-free Rate)**.** *Let $R_f(s)$ denote the period gross return of the risk-free asset in the current state. $R_f(s)$ solves*

$$\frac{1}{R_f(s)} \;=\; \beta^{\frac{1-\gamma}{1-\rho}} \left(\sum_{s'\in\{H,L\}} \omega_{s,s'} \mathbb{E}\left(e^{-\gamma\Delta\widetilde{c}_{t+1}}\middle|p_{s'}\right) \left(\frac{w^a_{s'}+1}{w^a_s}\right)^{\frac{\rho-\gamma}{1-\rho}} \right) \qquad s=\{H,L\} \tag{1.11}$$

where w^a_s are the solutions of the system of equations (1.9).

It follows from equation (1.11) that the equilibrium risk-free rate is also driven by the dynamics of $\widetilde{p}_t$, because changes in $\widetilde{p}_t$ drive changes in the distribution of consumption growth and prices of aggregate wealth.

Using the previous expressions, I now study what the network $\mathcal{G}_n$ and $\widetilde{p}_t$ imply for the cross-section of asset prices and risk premia. The following proposition determines the (ex-dividend) stock price of firm i and its expected period return.

PROPOSITION 4 (Firms' Stock Prices and Expected Period Returns)**.** *Let $P_i(y,s)$ denote the current (ex-dividend) stock price of an asset that delivers firms i's cash-flows as its dividend each period. For large n, $P_i(y,s)=v_i(s)y$, where $v_i(s)$ is the solution of the following linear system of* *equations*

$$\begin{aligned} v_i(s) &= \beta^{\frac{1-\gamma}{1-\rho}} e^{\bar{x}+\frac{\sigma_{\widetilde{x}}^2}{2}} \left(\sum_{s'\in\{H,L\}} \omega_{s,s'} \left(\frac{w^a_{s'}+1}{w^a_s}\right)^{\frac{\rho-\gamma}{1-\rho}} \mathbb{E}\left(e^{(\tau-\gamma)\Delta\widetilde{c}_{t+1}} \middle| p_{s'}\right) v_i(s') \right) \\ &+ \beta^{\frac{1-\gamma}{1-\rho}} e^{\alpha_0+\alpha_1 d_i} \left(\sum_{s'\in\{H,L\}} \omega_{s,s'} \left(\frac{w^a_{s'}+1}{w^a_s}\right)^{\frac{\rho-\gamma}{1-\rho}} \mathbb{E}\left(e^{-\gamma\Delta\widetilde{c}_{t+1}} \middle| p_{s'}\right) \left[1-\pi_i(p_{s'})\right] \right) \end{aligned} \tag{1.12}$$

where $\pi_i(p_{s'}) \equiv \mathbb{E}\left(\widetilde{\varepsilon}_{i,t+1} \middle| \widetilde{p}_{t+1} = p_{s'}\right)$ *and* $s = \{H, L\}$. *Moreover, the expected one period gross return of firm* i *is given by*

$$\mathbb{E}\left(\widetilde{R}_{i,t+1} \middle| s\right) = \frac{1}{v_i(s)} \left(\sum_{s'\in\{H,L\}} \omega_{s,s'} \left\{ v_i(s')\mathbb{E}\left(e^{\widetilde{x}_{t+1}} \middle| p_{s'}\right) + e^{\alpha_0+\alpha_1 d_i}\left(1-\pi_i(p_{s'})\right) \right\} \right). \tag{1.13}$$

To appreciate the importance of the location of a firm in $\mathcal{G}_n$ in asset prices and returns, suppose $\mathcal{G}_n$ is symmetric. Then, $d_i = \bar{d}$ and $\pi_i = \bar{\pi} \geq q$ for all i. It then follows from the second term in the right hand side of (1.12) that all firms have the same price in a given period. As equation (1.12) shows, differences in prices across firms arise solely from differences in the location of firms in $\mathcal{G}_n$. Differences in prices across firms are driven not only by the number of relationships of a firm, captured by d_i, but also by the set of firms to which a firm is connected, captured by π_i. The same applies for the cross-section of expected excess returns. Differences in expected excess returns across firms arise solely from differences across the location of firms in $\mathcal{G}_n$. To understand the cross-section of firms' risk premia, equation (1.8) can be rewritten as a beta pricing model,

$$\mathbb{E}\left(\widetilde{R}_{i,t+1} \middle| s\right) - R_f(s) = \underbrace{\left(\frac{\mathrm{Cov}\left(\widetilde{R}_{i,t+1}, \widetilde{M}_{t+1} \middle| s\right)}{\mathrm{Var}\left(\widetilde{M}_{t+1} \middle| s\right)}\right)}_{\beta_{i,\widetilde{M}}(s)} \underbrace{\left(\frac{-\mathrm{Var}\left(\widetilde{M}_{t+1} \middle| s\right)}{\mathbb{E}\left(\widetilde{M}_{t+1} \middle| s\right)}\right)}_{\lambda_{\widetilde{M}}(s)} \tag{1.14}$$

where $\beta_{i,\widetilde{M}}(s)$ and $\lambda_{\widetilde{M}}(s)$ denote the quantity of risk in firm i and the price of risk in state s, respectively. The following proposition determines $\lambda_{\widetilde{M}}(s)$.

PROPOSITION 5 (Conditional Price of Risk: $\lambda_{\widetilde{M}}(s)$). *The conditional price of risk in state* s*,*

$\lambda_{\widetilde{M}}(s)$, *equals*

$$\lambda_{\widetilde{M}}(s) = \frac{1}{R_f(s)} - R_f(s)\left(\beta^{2\left(\frac{1-\gamma}{1-\rho}\right)} \sum_{s' \in \{H,L\}} \omega_{s,s'} \left(\frac{w^a_{s'}+1}{w^a_s}\right)^{2\left(\frac{\rho-\gamma}{1-\rho}\right)} \mathbb{E}\left(e^{-2\gamma\Delta\widetilde{c}_{t+1}} \middle| p_{s'}\right)\right) \tag{1.15}$$

where $R_f(s)$ denotes the period gross return of the risk-free asset in state s.

As equation (1.15) shows, the price of risk is time-varying, because the propensity of relationships to transmit shocks varies over time. Changes in $\widetilde{p}_t$ introduce changes in the distribution of aggregate consumption growth, in the price of aggregate wealth, and in the risk-free rate, which, in turn, manifest in changes of the price of risk. To compute firms' quantities of risk, one can rearrange equation (1.14), which yields

$$\beta_{i,\widetilde{M}}(s) = \frac{\mathbb{E}\left(\widetilde{R}_{i,t+1} \middle| s\right) - R_f(s)}{\lambda_{\widetilde{M}}(s)} \tag{1.16}$$

so that firms' conditional quantities of risk can be computed from using equations (1.11), (1.13), and (1.15). As a consequence, firm i's quantity of risk is driven by (a) firm i's location in $\mathcal{G}_n$, which alters $\mathbb{E}\left(\widetilde{R}_{i,t+1} \middle| s\right)$, and (b) the dynamics of $\widetilde{p}_t$ and topology of $\mathcal{G}_n$, which alter $R_f(s)$, $\lambda_{\widetilde{M}}(s)$ and $\mathbb{E}\left(\widetilde{R}_{i,t+1} \middle| s\right)$.[12]

1.4 Calibration

So far, the model illustrates how the propagation of shocks within a network economy alters equilibrium asset prices. I now calibrate the model to get a sense of the extent to which such a propagation mechanism quantitatively explains asset market phenomena. Section 1.4.1 describes the data and the strategy I use to calibrate the network $\mathcal{G}_n$. Section 1.4.2 describes the selection of the rest of parameters in the model.

1.4.1 Description of Data and Customer-Supplier Networks

I use annual data on customer-supplier relationships among U.S. firms to pin down the topology of $\mathcal{G}_n$. Statement of Financial Accounting Standards (SFAS) No.131 requires firms to report the existence of customers who represent more than 10% of their annual sales. This information is available on COMPUSTAT files. However, these files tend to list only abbreviations of customers' names. I then resort to the Cohen and Frazzini, 2008 dataset on customer-supplier relationships—a subset of the COMPUSTAT database—in which firms' principal customers are uniquely identified.[13] Their dataset consists of 6,425 different firms, considers common stocks, and represents 26,781 unique annual customer-supplier relationships from 1980 to 2005. Customer-supplier relationships last about 3 years on average, and the distribution of firms' size resembles the size distribution of the CRSP universe over the sample period. The size distribution of firms' principal customers, however, is tilted toward large companies. The average customer size is above the 90th size percentile of CRSP firms.

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[12] In an unreported proposition I also compute firms' quantities of risk as a function of the primitives of the model.

To proxy for those relationships that relate firms' cash-flow growth rates, I consider customer-supplier relationships in which a customer represents at least 20% of a firm's annual sales. My results, however, do not qualitatively change if I decrease that threshold from 20% to 10%. Using this data, I construct undirected and non-weighted customer-supplier networks at the annual frequency over the sample period where two firms are connected in a given year if one firm represents at least 20% of another firm's sales during that year. Figures A.3(a) and A.3(b) depict the customer-supplier networks in 1980 and 1986 respectively, in which nodes represent firms and the size of each node is proportional to the number of customer-supplier relationships a firm takes part in. Table A.2 illustrates some of the characteristics of the time series of customer-supplier networks. The average number of firms per network is 388, whereas, on average, there are 281 customer-supplier relationships per network. As in many economic and social networks, the number of relationships varies dramatically across firms.

To select the benchmark topology for $\mathcal{G}_n$, I generate a large network with $n = 400$ firms so that such a network simultaneously matches some of the characteristics of the time series of customer-supplier networks reported in Table A.2. In particular, the selected topology for $\mathcal{G}_n$ matches the average size of each of the five largest components and the average empirical degree distribution of customer-supplier networks. I restrict the topology of $\mathcal{G}_n$ to be one with no cycles so that firms' probabilities of facing negative shocks in each state of the economy—$\{\pi_i(p_s)\}_{i=1}^{n}$ with

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[13] Data available at: http://www.econ.yale.edu/∼ af227/

$p_s \in \{p_L, p_H\}$ in equations (1.12) and (1.13)—are easy to compute.[14] Such restriction seems to be innocuous, because cycles are not frequent in the customer-supplier dataset. Figure A.4(a) depicts the topology of the benchmark economy, whereas figure A.4(b) depicts its degree distribution.

Selecting the network topology using this data has one important caveat. Because many firms in the economy, as well as their relationships, are not included in this dataset, one may be able to construct, in the most favorable case, a network that closely resembles only a small fraction of the aggregate economy. This is because firms need to be sufficiently large to be publicly traded and to represent at least 20% of the annual sales of a publicly traded company. To partially ensure that the topology selected in the benchmark economy provides a fair representation of the network that underlies the aggregate U.S. economy, I compare the benchmark network with networks that are uncovered using BEA input-output tables. As table A.6 shows, the network in the benchmark economy does a good job at representing some features of the U.S. input-output network, and in doing so, potentially provides a reasonable representation of the aggregate U.S. economy.[15]

1.4.2 Selecting the rest of parameter values

Given the network topology uncovered in section 1.4.1, I calibrate the rest of the parameters in the model at the monthly frequency to be consistent with the empirical literature. Table A.3 reports the key parameter values in the calibrated model.

For the sake of illustration, these parameters can be separated into four groups. Parameters in the first group define the preferences of the representative investor, which I select in line with Bansal and Yaron, 2004 so that $\beta = 0.997$, $\gamma = 10$ and $\rho = 0.65$ (IES ≈ 1.5).

Parameters in the second group define the dynamics of firms' cash-flows. I use annual data on earnings per share from COMPUSTAT to proxy for firms' cash-flows. I restrict my focus to firms mentioned in the customer-supplier dataset, because the value of parameters $\{d_i\}_{i=1}^n$ in equation (1.1)—which correspond to the number of relationships of firms—is available only for those firms. To estimate parameters α_0, α_1 and α_2 in equation (1.1), I run cross-sectional OLS regressions at the annual frequency and then compute their equivalents at the monthly frequency. To run such cross-sectional regressions, I need to determine whether firm i faces a negative shock in a given year. To do so, I explore the temporal variation of firms' cash-flows and run time series regressions at the firm level, correcting for the existence of linear time trends.[16] By doing so, I identify the years in which each firm faces a negative shock. This allows me to compute annual estimates for α_0, α_1 and α_2 from 1980 to 2004, which are depicted in figure A.5.[17] I then set parameters α_0, α_1 and α_2 to be equal to the time series average estimates. Thus, $\alpha_0 = 0.3$, $\alpha_1 = 0.1$ and $\alpha_2 = 0.07$.[18]

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[14] A cycle consists of a sequence of firms starting and ending at the same firm, with each two consecutive firms in the sequence directly connected to each other in the network.

[15] It is an empirical issue whether a network uncovered using BEA input-output tables provides a sensible representation of the network structure that underlies the U.S. economy—I leave this for future research. Another way to uncover the underlying network using the framework in this paper is to use probabilistic graphical models, which are commonly used to represent statistical relationships in large and complex systems, since my baseline model predicts certain behavior of returns covariances across stocks. For instance, one may calibrate the network using a graphical lasso estimator (GLASSO) to match observed returns covariances. In doing so, one estimates an undirected and temporally invariant network by estimating a sparse inverse covariance matrix using a lasso (L1) penalty as in Friedman, Hastie, and Tibshirani, 2008. The basic estimation strategy assumes that observations have a multivariate Gaussian distribution with mean μ and covariance matrix Σ. If the ij^{th} component of Σ^{-1} is zero, then variables i and j are conditionally independent, given the rest of the variables, which is graphically represented as the lack of an edge between variables i and j in $\mathcal{G}_n$. The normality assumption can be relaxed as in Liu et al., 2012.

Parameters in the third group define the process followed by the propagation of shock within a network economy. The calibration of these parameters has only limited guidance from prior studies. There are five parameters in this group: the coefficient that measures how frequently firms face negative idiosyncratic shocks, q; the values that $\widetilde{p}_t$ may take in each period, p_L and p_H; the coefficient that measures how frequently relationships exhibit high propensity to transmit shocks, ψ; and the coefficient that measures the persistence of the stochastic process followed by $\widetilde{p}_t$, ϕ. I choose the benchmark values in this group by either using available studies or matching important moments in data.[19]

To select parameter ϕ, I explore the time variation of macroeconomic variables that proxy for the degree of input specificity in the U.S., motivated by evidence in Barrot and Sauvagnat, 2014. Barrot and Sauvagnat, 2014 posit that input specificity is a key driver of the propagation of shocks

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[16] Namely, I run the following time series regression at the firm level,

$$\log\left(\frac{y_{i,t}}{Y_{t-1}}\right) = \beta_0 + \beta_1 * t + \epsilon_t. \tag{1.17}$$

I consider that firm i faces a negative shock at year t if $\log\left(\frac{y_{i,t}}{Y_{t-1}}\right)$ is below the value predicted by regression (1.17) for more than one standard deviation of the residuals computed from (1.17).

[17] Estimates of α_0, α_1, and α_2 are statistically significant for most of the years within the sample. In particular, out of 25 years in the sample, α_0, α_1, and α_2 are statistically significant at the 95%, 12, 8, and 20 years, respectively.

[18] To determine the benchmark values of α_0, α_1, and α_2 at the monthly frequency, I assume that $y_{i,\text{year}} = 12 \times y_{i,\text{month}}$, with $i \in \{1, \cdots, n\}$. Provided that data on firms' cash-flows is at the annual frequency, this assumption facilitates the computation of parameters α_0, α_1, and α_2 at the monthly frequency because $Y_{\text{year}} = 12 \times Y_{\text{month}}$ so that $\log\left(\frac{y_{i,\text{year}+1}}{Y_{\text{year}}}\right) = \log\left(\frac{y_{i,\text{month}+1}}{Y_{\text{month}}}\right)$.

[19] A similar strategy is used in Zhang, 2005 to pin down parameters to which there is only limited guidance from prior studies.

within supply chains. Their idea is simple. The more specific the inputs a firm uses, the more difficult it is to restructure its production if it needs to replace a supplier who is under distress, and, thus, the more likely it is that such a firm is affected by shocks to its suppliers. It is, then, natural to think that the higher the degree of input specificity faced by the average firm in the economy, the higher the likelihood that negative shocks spread from one firm to another within a customer-supplier network. To proxy for the degree of input specificity faced by the average firm in the network economy, I use the ratio of non federally funded R&D/GDP and the number of patents created in the U.S. These two measures aim to proxy for (i) the relative importance of relationship-specific investments made by the average firm, and (ii) how easily the average firm can substitute suppliers who are under distress.[20] Figure A.6 depicts the time series for R&D/GDP from 1953 to 2002 in the U.S. as well as the number of patents created in the U.S. from 1963 to 2009. I then set $\phi = 0.925$ so that the time series followed by the propensity of relationships to transmit shocks is as persistent as the time series of either R&D/GDP or the number of patents created in the U.S.[21] Finally, I select the rest of the parameters in this group by matching important moments in data. In particular, parameters q, p_L, p_H, and ψ are chosen so that the first two moments of the time-aggregated annual growth rates of consumption and dividends generated by the calibrated model are similar to those of observed annual data. I then set $q = 0.2$, $p_L = 0.38$, $p_H = 0.45$, and $\psi = 0.5$.

Parameters in the fourth group define the difference between aggregate output and consumption growth. Within the baseline model, output growth equals consumption growth at equilibrium. To provide a more realistic description of dividends and improve the fit of the calibrated model to data, I augment the baseline model so that consumption and dividends are two different processes within the benchmark economy. Similar to many others, including Cecchetti, Lam, and Mark, 1993, Abel,

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[20] Barrot and Sauvagnat, 2014 construct three measures of suppliers' specificity in their study. The first measure uses information that classifies inputs as differentiated or homogeneous, depending on whether they are sold on an organized exchange or not. The second measure uses suppliers' R&D expenses to capture the importance of relationship-specific investments, whereas the third measure uses the number of patents issued by suppliers to capture restrictions on alternative sources of substitutable inputs.

[21] To pin down the persistence of these time series, I fit autoregressive processes to the time series of non-federal R&D/GDP and the time series of the number of patents created in the U.S. by selecting the complexity of the model using the Akaike information criterion. The fitted AR models are both highly persistent. In particular, the fitted AR model for the number of patents in the U.S has a persistence parameter equal to 0.93, whereas the fitted AR model for R&D/GDP has a persistence parameter equal to 0.85. Because input's specificity may vary across industries, I also analyze (in unreported results) the time series of the average R&D/GDP and find that the time series of the average R&D/GDP is as persistent as the time series of R&D/GDP.

1999, Campbell, 1999; Campbell, 2003, and Bansal and Yaron, 2004, I assume that dividend and consumption growth jointly satisfy,

$$(\widetilde{x}_{t+1} - \bar{x}) = \tau\,(\Delta\widetilde{c}_{t+1} - \bar{c}) + \sigma_x \widetilde{\xi}_{t+1}\ , \tag{1.18}$$

where $\bar{x}$ and $\bar{c}$ are constant and represent the unconditional means of log output and consumption growth, respectively. Parameter $\tau > 0$ and $\widetilde{\xi}_{t+1}$ is i.i.d. normal with mean zero and unit variance. Thus, the representative investor is implicitly assumed to have access to labor income in the augmented model. For simplicity, $\widetilde{\xi}_{t+1}$ is independent of both $\Delta\widetilde{c}_{t+1}$ and variables $\{\widetilde{\varepsilon}_{i,t+1}\}_{i=1}^{n}$. As in Abel, 1999, parameter τ represents the leverage ratio on equity. If $\bar{x} = \bar{c} = \sigma_x = 0$, then aggregate consumption and dividend growth are specified as in Abel, 1999. If $\bar{x} = \bar{c} = \sigma_x = 0$ and $\tau = 1$, then the market portfolio is a claim to total wealth and the baseline model is recovered. I set $\bar{c} = 0.019/12$ and $\bar{x} = 0.038/12$ so that the unconditional means of consumption and dividend growth generated by the benchmark economy are similar to the ones found in data. I follow Bansal and Yaron, 2004 and set $\tau = 3$. I set $\sigma_x = 0.0262$ so that the volatility of dividends generated by the benchmark economy is similar to the one found in data.

Despite the fact that aggregate output and consumption are two different processes within the augmented model, both of these processes are still determined, in large part, by the propagation of shocks within the network. In particular, the distribution of $\widetilde{x}_{t+1}$ is fully determined by the propagation of shocks as equation (1.5) shows, whereas the distribution of $\Delta\widetilde{c}_{t+1}$ is also determined by the propagation of shocks as equation (1.18) states.

1.5 Implications of the Calibrated Model

This section studies the asset market implications of the calibrated model. It shows that changes in the propagation of shocks, within networks of inter-firm relationships that resemble customer-supplier networks, are quantitatively important to understanding variations in asset prices and returns, both in the aggregate and in the cross section.

1.5.1 Asset Market Phenomena, Network Economies, and Long-Run Risks

Table A.4 exhibits moments generated under the benchmark parameterization. Table A.4 suggests that the model does a reasonable job at matching important asset pricing moments as well as moments of consumption and dividend growth. The benchmark parameterization delivers an average annual log consumption growth of 1.8%, an annual volatility of log consumption growth of 4.7%, an average annual log dividend growth of 3.8%, and an annual volatility of log dividend growth of 14.9%, all values similar to those found in data. It also delivers an average market return of 12%, an annual volatility of the market return of 18.92%, an average risk-free rate of 2.16%, an annual volatility of the risk-free rate of 0.7%, an annual equity premium of 10%, and a Sharpe ratio of 0.52. With the exception of the volatility of the risk-free rate and Sharpe ratio, all values are aligned with those found in data.

Besides matching the above moments, the calibrated model generates a persistent component in expected consumption growth and stochastic consumption volatility similar to those assumed by the Long-Run Risks Model (LRR) of Bansal and Yaron, 2004. As Bansal and Yaron, 2004 and Bansal, Kiku, and Yaron, 2012 show, these two features, together with Epstein-Zin-Weil preferences, help to quantitatively explain an array of important asset market phenomena.[22] Table A.5 reports means and volatilities based on 300 simulated economies over 620 monthly observations of several similarity measures between time series generated with either the calibrated model or the LRR model. As table A.5 suggests, both models generate similar time series for expected consumption growth and stochastic consumption volatility.

It is important to appreciate that the persistent component in expected consumption growth and stochastic consumption volatility are endogenously generated within my model rather than exogenously imposed, as in many asset pricing models. The calibrated model generates these two features because the propensity of relationships to transmit shocks follows a persistent process, that is consistent with data, and inter-firm relationships are long-term. Despite the fact that these two features are endogenously generated, I do not claim that my model provides a complete micro-foundation of long-run risks. The reason is that inter-firm relationships are exogenous and fixed within my model. Nonetheless, this model provides a novel link between equilibrium asset prices and the propagation of firm level shocks within networks that resemble customer-supplier networks, that is consistent with the existence of long-run risks. The model suggests that changes in technologies and complementaries among firms activities within network economies are quantitatively relevant to understanding variations in asset prices and returns. This is particularly important in modern economies provided the high degree of interconnectedness among firms. In doing so, the model provides a new perspective on the potential sources of long-run risks. The framework presented in this paper is also able to nest long-run risk models under suitable assumptions as Appendix C demonstrates.

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[22] Since Bansal and Yaron, 2004, several authors have used the long-run risk framework to explain an array of market phenomena. For instance, Kiku, 2006 provides an explanation of the value premium within the long-run risks framework. Drechsler and Yaron, 2011 show that a calibrated long-run risks model generates a variance premium with time variation and return predictability that is consistent with data. Bansal and Shaliastovich, 2013 develop a long-run risks model that accounts for bond return predictability and violations of uncovered interest parity in currency markets.

1.5.2 Firms' Centrality and the Cross-Section of Risk Premia

Besides helping to explain aggregate asset market phenomena, the model helps to understand the cross-section of expected returns because it provides a mapping between firms' quantities of priced risk and firms' importance in the network. To measure the importance of a firm in the inter-firm relationships network, I define the centrality of firm i at time t as the expected number of firms that can be affected by a shock to firm i at time t. This measure captures the relative importance of firm i in transmitting shocks over the economy. Shocks to firm i may alter aggregate output and consumption growth to the extent to which they propagate over a non-negligible fraction of the economy and, thus, alter firm i's risk premium. If the economy contains no cycles, the centrality of firm i at period t, $\chi_{i,t}$, equals to

$$\chi_{i,t} = \sum_{d=1}^{L_i} n_i^j \widetilde{p}_t^j \tag{1.19}$$

where n_i^j denotes the number of firms that are at a distance j from firm i in $\mathcal{G}_n$; and $\widetilde{p}_t$ denotes the realization of $\widetilde{p}_t$ at time t. Firms i and k are said to be at a distance j if the shortest path between i and k has length j. L_i denotes the largest distance between any given firm within $\mathcal{G}_n$ and firm

i.[23]

Figure A.7(a) shows firms' conditional risk premia as a function of firms' centrality. It follows from figure A.7(a) that firms that are more central in the network command higher risk premium than firms that are less central. Figure A.7(b) shows firms' conditional quantity of risk, $\beta_{i,\widetilde{M}}$, as a function of firms' centrality. It follows from figure A.7(b) that firms that are more central have higher quantity of risk than firms that are less central. Shocks to central firms have higher likelihood of affecting more firms on average than do shocks to less central firms. As a consequence, central firms tend to be procyclical, whereas less central firms serve as a hedge against aggregate risk and, thus, command lower risk premium. On average, firms in the highest quintile of centrality yield an annual excess return of 1% over those firms in the lowest quintile—which is aligned with empirical results documented by Ahern, 2013 within the network of intersectoral trade.[24]

1.5.3 Factor Structure on Firm-Level Return Volatility

The calibrated model also generates a high degree of common time variation in return volatilities at the firm level, which is aligned with recent empirical evidence, e.g. Herskovic et al., 2014, Duarte et al., 2014. To facilitate comparison with evidence documented by Herskovic et al., 2014, figure A.8 illustrates annual total return volatility at the firm level averaged within start-of-year size quintiles. As figure A.8 shows, firms of all size exhibit similar time series volatility patterns. On average, the first principal component of the cross-section of annual return volatility accounts for 99% of the variance. Within the model, the existence of this factor structure is not surprising, because fluctuations in the propensity of relationships to transmit shocks drive changes in growth

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[23] If the network contains no cycles, the probability that k firms that are at a distance j from firm i are also affected by shocks to firm i at period t, $\mathbb{P}_i^j(k)$, is given by

$$\mathbb{P}_i^j(k) \quad = \quad \binom{n_i^j}{k} \left(\widetilde{p}_t^j\right)^k \left(1-\widetilde{p}_t^j\right)^{n_i^j-k}$$

The expected number of firms that are at a distance j from firm i and are also affected by a shock to firm i at period t is $n_i^j \widetilde{p}_t^j$. As a consequence, the expected number of firms that can be affected by shocks to firm i is given by (1.19). If there is no path between firm i and other firms within $\mathcal{G}_n$, define $L_i = \infty$.

[24] The 1% excess return comes from 200 simulated economies over 1100 monthly observations. I disregard the first 100 observations in each simulation to eliminate the potential bias coming from the initial condition. At the beginning of each year, I sort firms into five quintiles based on centrality and form five equally weighted portfolios, which I keep over the next twelve months. The 1% excess return corresponds to the average annual return of a strategy that goes long in the portfolio with those firms with the highest centrality and short in the portfolio with those firms with the lowest centrality. Despite that Ahern, 2013 uses a different network to compute his results, Table A.6 shows that the network topologies used by Ahern, 2013 are similar to the network topology used in this paper.

opportunities and uncertainty across firms, which translate into changes in prices and returns at the firm level. Provided that returns respond to a common factor—given by the propensity $\widetilde{p}_t$—firm level return volatilities inherit a factor structure.[25]

1.6 Concluding Remarks

This paper suggests that the propagation of firm level shocks within network economies are quantitatively important to understanding asset prices and returns, both in the aggregate and in the cross-section. Changes in either the network that underlies the aggregate economy or the propensity of relationships to transmit shocks within a fixed network may alter aggregate variables, such as output and consumption, which, in turn, alter equilibrium asset prices and returns.

I show that a calibrated model that matches features of customer-supplier networks in the U.S. as well as features of macroeconomic variables that aim to proxy for the propagation of firm level shocks within these networks, generates a persistent component in expected consumption growth and stochastic consumption volatility similar to that of the Long-Run Risk Model of Bansal and Yaron, 2004. As Bansal and Yaron, 2004 and Bansal, Kiku, and Yaron, 2012 show, these two features, together with Epstein-Zin-Weil preferences, help to explain characteristics of aggregate asset market data such as the equity premium and low risk-free rate. The model also helps in understanding the cross-section of expected returns, as it provides a mapping between firms' quantities of priced risk and firms' importance in the network. In the cross section, firms that are more central in the network command higher risk premium than firms that are less central. Shocks to central firms have higher likelihood of affecting more firms on average than do shocks to less central firms. As a consequence, central firms tend to be procyclical, whereas less central firms serve as a hedge against aggregate risk and, thus, command lower risk premium. In the time series, firm-level return volatilities exhibit a high degree of comovement. These two features are consistent with recent empirical evidence.

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[25] Recent empirical evidence also suggests the existence of common time variation in firm level idiosyncratic volatilities, e.g. Herskovic et al., 2014, Duarte et al., 2014. In unreported results, I explore the extent to which firm level idiosyncratic volatilities exhibit a factor structure within the calibrated model. After removing the market as a common factor of return volatilities, the high degree of common time variation in firm level return volatilities tends to disappear. On average, the first principal component of the cross-section of annual idiosyncratic volatility accounts only for 3% of the variance (see figure A.8(b)).

Chapter 2

Basket Securities in Segmented Markets

Over the past four decades, there has been a substantial increase of financial innovation and investor demand—both institutional and retail—for securities that pool different assets and whose value is determined as an aggregate of the values of those assets. The large variety of asset-backed securities, such as collateralized debt obligations and mortgage-backed securities, as well as index funds and exchange-traded funds, shows the prevalence of basket securities in modern financial markets.

In this paper, I explore how basket securities develop in an incomplete market setting where profit-maximizing intermediaries are involved in financial innovation. Specifically, I study the design and welfare implications of basket securities issued in markets with limited investor participation. The questions I address are: Which baskets are optimal for profit-maximizing intermediaries, what are the welfare implications associated with introducing such baskets, and how does competition among multiple intermediaries affect equilibrium outcomes? My analysis provides a link between the institutional features of a market, such as depth and gains from trade, and the types of basket securities that emerge in equilibrium.

In perfect capital markets the bundling activity is irrelevant. In reality, however, there are many reasons that investors do not replicate basket securities by themselves. In the literature, asymmetric information, transaction costs, and market incompleteness have been cited as possible explanations for the existence of basket securities, e.g. Subrahmanyan, 1991, Gorton and Pennacchi, 1993, Allen and Santomero, 1997, and DeMarzo, 2005a.[1] However, the cost of information and transaction costs have continuously decreased during the last thirty years, while the demand for basket securities has grown almost exponentially. In this paper, then, I focus on market incompleteness, in the sense that investors have limited access to capital markets as in Rahi and Zigrand, 2009; Rahi and Zigrand, 2010.

The assumption of limited investor participation also better captures features of today's most active basket security markets. For example, in some ABSs markets—such as CDOs and MBSs—many of the characteristics of the underlying assets are public information. More important, the selection of the underlying assets and the posterior tranching of these baskets are chosen by profit-maximizing intermediaries. However, it is not easy for a retail investor to become an intermediary, because it requires excellent distribution channels and setup costs that are typically large. ETFs are another important example of limited investor participation, because limited arbitrage trading is at the core of ETFs creation.[2] Only "authorized participants,"—typically large brokers or investment banks—are effectively able to arbitrage price differentials between ETFs and their underlying basket. If an ETF is trading at a premium compared to its underlying basket, only authorized participants can create ETF shares and deliver the underlying basket, whereas other investors cannot participate in the creation-redemption process and need instead to rely on short-long strategies.

The main features of the model are as follows. I consider a one-period economy in which trading occurs at beginning of the period and payoffs are realized at end of the period. There is one consumption good, two market segments, and initially two assets—which (random) payoffs are in units of the consumption good. A continuum of measure one of risk-averse investors is associated with each segment. Each segment is endowed with only one asset. If trading across segments is free, investors share risk perfectly and the equilibrium allocation is Pareto optimal. If markets are segmented, however, this is not necessarily true, because marginal valuations are typically not equalized at equilibrium. To capture market segmentation, trading across segments is not allowed.

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[1] If investors are asymmetrically informed, a basket security may reduce uninformed investors' trading losses because the adverse selection costs associated with baskets are typically lower than those associated with individual securities. In terms of transactions costs, basket securities are desirable because high transaction costs make it expensive for individual investors to replicate diversified portfolios on their own.

[2] In the U.S. major ETFs are more traded than any other security. ETFs' sponsors during the last five years have continually increased the variety of investment objectives and the number of funds offered. For more details about the growth of ETFs see Deville, 2008, Ferri, 2009 and Gastineau, 2010.

However, there is a risk-neutral intermediary with the ability to trade with both segments and who offers shares of one new security—the basket—in exchange for shares of the two initial assets. A basket consists of a linear combination of the two initial assets. The combination of assets is chosen by the profit-maximizing intermediary.

The first question I address is whether a monopoly intermediary would simply issue a basket to complete both market segments. The answer is not necessarily. If one asset has a small expected payoff, the issuer may not find it worthwhile to serve all investor types but instead choose to tailor her basket to one investor type. When designing a basket, the issuer seeks to both increase trading volume and increase the basket payoff—incentive which comes from the intermediary's "skin in the game." Provided that the issuer cares only about her intermediation profits, her incentives may not be aligned with those of investors. Thus, the equilibrium is not always constrained efficient.[3]

Because competition among intermediaries may improve investors' welfare, I then ask what happens when intermediaries' barriers to entry are lowered. In that case, different basket securities may coexist in equilibrium. Many of them, however, are redundant, in the sense that coordination among intermediaries may improve investors' risk-sharing opportunities. However, the non-cooperative nature of intermediaries' competition prevents coordination.

This paper relates to two strands of the literature: one on optimal security design, and the other on the creation of basket securities. Excellent surveys of security design in an incomplete market framework are Allen and Gale, 1994 and Duffie and Rahi, 1995. So far, the main focus in the literature has been on innovations introduced by agents who do not trade the securities they create. In practice, however, agents involved in financial innovation are often profit-seeking institutions that actively make markets and trade their securities across markets, e.g. Allen and Santomero, 1997. Duffie and Jackson, 1989 and Ross, 1989 are among the first studies to consider this profit-maximizing feature. In particular, Duffie and Jackson, 1989 study the incentives of exchanges that lead them to offer one contract rather than another. Ross, 1989 studies investment banks' incentives to bundle securities to lower searching costs. Further, Rahi and Zigrand, 2009; Rahi and Zigrand, 2010 study a general equilibrium model similar to mine. In Rahi and Zigrand, 2009, investors have limited access to capital markets, and strategic issuers make profits by exploiting mispricings across markets.

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[3] Elul, 1995 shows that in almost every incomplete market economy with more than one consumption good and with sufficiently many uninsured states of nature, one can introduce a set of assets that might make all agents worse-off.

The second related strand of literature is on the creation of basket securities. This literature has focused mainly on either asymmetric information or transaction costs as the cause of the creation of basket securities. For example, Gorton and Pennacchi, 1990; Gorton and Pennacchi, 1993 argue that baskets decrease uninformed investors' trading losses. Baskets decrease uninformed investors' "lemons" problem, in the sense that baskets split individual securities cash flows and eliminate the private information informed investors may have about individual securities. Along the same lines, Subrahmanyan, 1991 shows that strategic liquidity traders may prefer baskets rather than individual securities. DeMarzo, 2005a, on the other hand, considers the problem of an intermediary who may have superior information about the value of her assets and provides conditions under which intermediaries sell pools of assets, some of which are purchased by other informed intermediaries who then further pool and tranche them. Pooling and tranching allows intermediaries to leverage their capital more efficiently, enhancing the returns on their private information.

The rest of the paper is organized as follows. Section 2.1 describes the baseline model. Section 2.2 analyzes the constrained efficient allocation as a benchmark. Section 2.3 solves for the market-mediated equilibrium and characterizes its properties. Section 2.4 analyzes the effect of competition among intermediaries. Finally, section 2.5 concludes. The derivations of formulas, unless otherwise stated, appear in the Appendix.

2.1 Baseline Model

2.1.1 The Environment

Consider a one-period economy with one consumption good. Two assets, indexed by $i = \{1, 2\}$, are traded at the beginning of the period, pay at the end of the period—in units of the consumption good—and are in unit net supply. There are two market segments populated by a continuum of investors with CARA utility with absolute risk aversion $\gamma \geq 0$. Prior to trading, investors in segment one—hereafter *investors one*—are endowed with all asset 1, whereas investors in segment two—hereafter *investors two*—are endowed with all asset 2. Let $\widetilde{x}_i$ denote the (random) payoff of asset i.

ASSUMPTION 2. *Assets' payoff follow*

$$\begin{bmatrix} \widetilde{x}_1 \\ \\ \widetilde{x}_2 \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \mu_1 \\ \\ \mu_2 \end{bmatrix}, \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix} \right) \tag{2.1}$$

where μ_i and σ_i^2 denote the mean and variance of asset i's payoff. Parameter $\rho \in (-1, 1)$ denotes the correlation between $\widetilde{x}_1$ and $\widetilde{x}_2$.

To capture market segmentation, trading across market segments is not allowed. Besides investors, there is one profit-maximizing intermediary who can trade with investors in both segments. In exchange for shares of asset i, the intermediary offers shares of a new security to both segments. The new security consists of a basket that contains a fraction $\alpha_i \in (0, 1)$ of asset i.[4] If $\widetilde{b}$ denotes the (random) payoff of the basket, then

$$\widetilde{b} = \sum_{i=1}^{2} \alpha_i \widetilde{x}_i \tag{2.2}$$

The intermediary charges an exogenous intermediation fee $\theta \geq 0$ per each share of the basket she sells. Parameter θ represents the effective segmentation investors encounter when investing across different markets. If θ tends to zero, investors trade at almost no cost across markets and the equilibrium allocation tends to be Pareto optimal. However, as θ departs away from zero, trading across segments gets costly, but potentially more profitable for the intermediary, and investors' marginal valuations may not be equalized at equilibrium.

2.1.2 Agents

Investors

Let α_i be the fraction of asset i that *investors* i trade in exchange for shares of the basket and let $b_i \in [0, 1]$ denote the fraction of the basket that *investors* i buy. The optimal portfolio of

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[4] An intermediary may need both first-class distribution channels and time to market a basket to see whether there is enough demand. As a consequence, issuing several baskets at the same time may prove too costly for the intermediary, and thus, the intermediary may offer at most one basket.

investors i, (α_i^*, b_i^*), solves

$$\begin{aligned} \max_{(\alpha_i, b_i)} \quad & \mathbb{E}\left[-e^{-\gamma \widetilde{c}_i}\right] \\ st. \quad & \widetilde{c}_i = (1-\alpha_i)\widetilde{x}_i + b_i \left(\left[\sum_{j=1}^{2} \alpha_j \widetilde{x}_j \right] - \theta \right) \\ & 1 \geq b_i \geq 0 \ , \ 1 \geq \alpha_i \geq 0 \end{aligned} \tag{2.3}$$

Investors are not allowed to short-sell the basket. Otherwise, they can complete their segments at no cost and the intermediary's activity is redundant.

The Intermediary

Before trading, a risk-neutral intermediary decides the basket structure—defined by fractions (α_1^d, α_2^d)—to maximize her profits. If $\widetilde{\pi}$ denotes the (random) profits of the intermediary, it is assumed that

$$\widetilde{\pi} \;=\; \beta \left(\sum_{i=1}^{2} \alpha_i^d \widetilde{x}_i \right) + \theta \left(b_1 + b_2 \right) \tag{2.4}$$

where $\beta \geq 0$ measures the intermediary's "skin in the game" and $\theta \geq 0$ represents the basket transaction fee. Thus, the intermediary cares about the expected payoff of the basket as well as its trading volume. The basket selected by the intermediary solves

$$\begin{aligned} \max_{(\alpha_1^d, \alpha_2^d)} \quad & \mathbb{E}\left[\widetilde{\pi}\right] = \beta \left(\sum_{i=1}^{2} \alpha_i^d \mu_i \right) + \theta \left(b_1 + b_2 \right) \\ st. \quad & \alpha_i^{ub} \geq \alpha_i^d \geq \alpha_i^{lb} \ , \ i = \{1, 2\} \\ & \mathrm{E}\left[\widetilde{\pi}^*\right] \geq 0 \end{aligned} \tag{2.5}$$

where α_i^{ub} and α_i^{lb} denote the upper and lower bounds of α_i^d such that $b_i\left(\alpha_i, \alpha_j\right)$ is well-defined—i.e. $0 \leq b_i \leq 1$. The term $\widetilde{\pi}^*$ is the profit of the intermediary evaluated at the basket that maximizes her expected profits. Thus, the last restriction represents the intermediary's participation constraint.

2.1.3 Equilibrium

In equilibrium, agents maximize their expected utility at the end of period subject to their respective trading constraints.

DEFINITION 1. *An* ***equilibrium*** *is an array of fractions,* $\{\alpha_1, b_1, \alpha_2, b_2, \alpha_1^d, \alpha_1^d\}$*, such that:*

(E1) Investor's maximization: Investors i's portfolio (α_i, b_i) *solves (2.3).*

(E2) Intermediary's maximization: The basket (α_1^d, α_2^d) *solves (2.5).*

(E3) Market clearing: $\alpha_i = \alpha_i^d$ *and* $(1-\alpha_i) + \alpha_i \left(\sum_{j=1}^{2} b_j\right) = 1$*,* $i = \{1, 2\}$*.*

2.2 Constrained Efficient Allocation

This section explores the constrained efficient allocation as a benchmark. Consider a benevolent planner who needs to allocate resources among investors and the intermediary. The planner's problem can be restated as

$$\begin{aligned} \max_{(\alpha_1, b_1, \alpha_2, b_2)} \quad & \mathbb{E}\left[-e^{-\gamma \widetilde{c}_1}\right] \\ st. \quad & \mathbb{E}\left[-e^{-\gamma \widetilde{c}_2}\right] = u_0 \\ & \mathbb{E}\left[\widetilde{\pi}\right] = \pi_0 \end{aligned} \tag{2.6}$$

where $\widetilde{c}_i = (1-\alpha_i)\widetilde{x}_i + b_i \left(\left[\sum_{j=1}^{2} \alpha_j \widetilde{x}_j\right] - \theta\right)$ and $(1-\alpha_i) + \alpha_i \left(\sum_{j=1}^{2} b_j\right) = 1$, $i = \{1, 2\}$. The last restriction in problem (2.6) implies

$$\alpha_2 = \left(\frac{\pi_0 - \theta}{\beta \mu_2}\right) - \left(\frac{\mu_1}{\mu_2}\right) \alpha_1 \tag{2.7}$$

whereas $(1-\alpha_i) + \alpha_i \left(\sum_{j=1}^{2} b_j\right) = 1$ implies $b_2 = 1 - b_1$. Provided that payoffs are normally distributed and investors have CARA utility, solving problem (2.6) is equivalent to solving

$$\begin{aligned} \max_{(\alpha_1, b_1)} \quad & \mathbb{E}\left[\widetilde{c}_1\right] - \frac{\gamma}{2} \mathrm{Var}\left[\widetilde{c}_1\right] \\ st. \quad & \mathbb{E}\left[\widetilde{c}_2\right] - \frac{\gamma}{2} \mathrm{Var}\left[\widetilde{c}_2\right] = u_0^* \end{aligned} \tag{2.8}$$

Solving problem (2.8) yields

$$(1-\alpha_1+\alpha_1 b_1) \quad = \quad \frac{\mu_1}{\gamma\sigma_1^2\left(1-b_1+\rho b_1\frac{\sigma_2}{\sigma_1}\frac{\mu_1}{\mu_2}\right)}-\frac{\sigma_2}{\sigma_1}\frac{\left(\rho\sigma_1(1-b_1)+b_1\sigma_2\frac{\mu_1}{\mu_2}\right)}{\left(\sigma_1(1-b_1)+\rho b_1\sigma_2\frac{\mu_1}{\mu_2}\right)}\alpha_2 b_1 \tag{2.9}$$

which relates the fraction of asset 1 held by *investors one*, $(1-\alpha_1+\alpha_1 b_1)$, to the fraction of asset 2 held by *investors one* in the constrained efficient allocation.

2.3 Trading Equilibrium

This section studies the allocation that arises from the equilibrium of a market-mediated exchange—henceforth trading equilibrium. I then compare the trading equilibrium allocation and the constrained efficient allocation to understand the extent to which the market provides the right instruments for investors' risk-sharing.

2.3.1 Investors i's optimal portfolio

Provided that assets' payoff are normally distributed and investors have CARA utility, maximizing investor i's expected utility is equivalent to maximizing the investor i's certain equivalent $\mathbb{E}\left[\widetilde{c}_i\right]-\frac{\gamma}{2}\mathrm{Var}\left[\widetilde{c}_i\right]$. As a consequence, the first order conditions of *investors* i are given by:

$$\mu_i-\gamma\left([1-\alpha_i+\alpha_i b_i]\sigma_i^2+\alpha_j b_i\rho\sigma_i\sigma_j\right) \quad = \quad 0 \tag{2.10}$$

$$\alpha_i\mu_i+\alpha_j\mu_j-\theta-\gamma\left([1-\alpha_i+\alpha_i b_i][\alpha_i\sigma_i^2+\rho\sigma_i\sigma_j\alpha_j]+b_i\alpha_j\sigma_j[\rho\alpha_i\sigma_i+\alpha_j\sigma_j]\right) \quad = \quad 0 \tag{2.11}$$

with $j \neq i$. Equation (2.10) implies that the fraction of asset i held by *investors* i, $(1-\alpha_i+\alpha_i b_i)$, equals

$$[1-\alpha_i+\alpha_i b_i] \quad = \quad \frac{\mu_i}{\gamma\sigma_i^2}-\alpha_j b_i\left(\rho\frac{\sigma_j}{\sigma_i}\right) \tag{2.12}$$

Using equation (2.12) in equation (2.11) yields that the fraction of asset j held by *investors* i, $\alpha_j b_i$, equals

$$\alpha_j b_i = \frac{\left(\mu_j - \rho\mu_i\frac{\sigma_j}{\sigma_i}\right)}{\gamma\sigma_j^2(1-\rho^2)} - \frac{\theta}{\gamma\alpha_j\sigma_j^2(1-\rho^2)} \text{ , with } |\rho| \neq 1 \tag{2.13}$$

Using equation (2.13) in equation (2.12) yields

$$1 - \alpha_i + \alpha_i b_i \quad = \quad \frac{\mu_i}{\gamma\sigma_i^2} - \rho\frac{\sigma_j}{\sigma_i}\left\{\frac{\left(\mu_j - \rho\mu_i\frac{\sigma_j}{\sigma_i}\right)}{\gamma\sigma_j^2(1-\rho^2)} - \frac{\theta}{\gamma\alpha_j\sigma_j^2(1-\rho^2)}\right\} \text{ , with } |\rho| \neq 1 \tag{2.14}$$

If $\theta \to 0$, it follows directly from equation (2.14) that

$$(1 - \alpha_i + \alpha_i b_i) \quad \to \quad \frac{\mu_i}{\gamma\sigma_i^2} - \rho\frac{\sigma_j}{\sigma_i}\left\{\frac{\left(\mu_j - \rho\mu_i\frac{\sigma_j}{\sigma_i}\right)}{\gamma\sigma_j^2(1-\rho^2)}\right\} = \frac{1}{\gamma(1-\rho^2)}\left\{\frac{\mu_i}{\sigma_i^2} - \rho\frac{\mu_j}{\sigma_i\sigma_j}\right\} \tag{2.15}$$

and thus, the fraction of asset i held by *investors* i is increasing in μ_i, σ_j and decreasing in γ and μ_j. If $\rho\frac{\mu_j}{\sigma_j} < 2\frac{\mu_i}{\sigma_i}$, then $\frac{\partial(1-\alpha_i+\alpha_i b_i)}{\partial\sigma_i} \leq 0$ so the fraction of asset i held by *investors* i is decreasing in σ_i. On the other hand, if $\rho\frac{\mu_j}{\sigma_j} > 2\frac{\mu_i}{\sigma_i}$, the fraction of asset i held by *investors* i is increasing in σ_i. If $2\rho\mu_i > \frac{\mu_j}{\sigma_j}(1+\rho^2)$, then $\frac{\partial(1-\alpha_i+\alpha_i b_i)}{\partial\rho} \geq 0$ so the fraction of asset i held by *investors* i is increasing in ρ. On the other hand, if $2\rho\mu_1 < \frac{\mu_j}{\sigma_j}(1+\rho^2)$, then the fraction of asset i held by *investors* i is decreasing in ρ.

If $\theta \to 0$, it follows directly from equation (2.13) that

$$\alpha_j b_i \quad \to \quad \frac{\left(\mu_j - \rho\mu_i\frac{\sigma_j}{\sigma_i}\right)}{\gamma\sigma_j^2(1-\rho^2)} \tag{2.16}$$

and thus, then the fraction of asset j held by *investors* i is increasing in μ_j, σ_i and decreasing in γ and μ_i. Moreover, if $2\frac{\mu_j}{\sigma_j} > \rho\frac{\mu_i}{\sigma_i}$, then the fraction of asset j held by *investors* i is decreasing in σ_j. On the other hand, if $2\frac{\mu_j}{\sigma_j} < \rho\frac{\mu_i}{\sigma_i}$, then the fraction of asset j held by *investors* i is increasing in σ_j. If $\rho < \frac{1}{2}$, then the fraction of asset j held by *investors* i is decreasing in ρ. If $\rho > \frac{1}{2}$ and $\frac{\mu_i}{\sigma_i}\sigma_j^2(2\rho-1) > 2\mu_j$, then the fraction of asset j held by *investors* i is increasing in ρ.

ASSUMPTION 3. *The primitives of the model satisfy*

$$\begin{aligned}
\rho\mu_i\frac{\sigma_j}{\sigma_i} &< \mu_j \ , \ \textit{and} \\
\gamma\sigma_j^2(1-\rho^2)+\theta &\leq \mu_j - \rho\mu_i\frac{\sigma_j}{\sigma_i} \ , \ \textit{and} \\
4\theta\gamma\sigma_j^2(1-\rho^2) &\leq \left(\mu_j - \rho\mu_i\frac{\sigma_j}{\sigma_i}\right)^2
\end{aligned}$$

with $j \neq i$ *and* $i = \{1,2\}$.

REMARK 1. *It follows from equation (2.13) that* b_i *is well-defined if and only if* $\alpha_i \in \left[\alpha_i^{lb}, \alpha_i^{ub}\right]$, *with*

$$\begin{aligned}
\alpha_i^{lb} &= \frac{\theta}{\mu_j - \rho\mu_i\frac{\sigma_j}{\sigma_i}} \\
\alpha_i^{ub} &= \frac{\left(\mu_j - \rho\mu_i\frac{\sigma_j}{\sigma_i}\right) - \sqrt{\left(\mu_j - \rho\mu_i\frac{\sigma_j}{\sigma_i}\right)^2 - 4\theta\gamma\sigma_j^2(1-\rho^2)}}{2\gamma\sigma_j^2(1-\rho^2)}
\end{aligned}$$

Assumption 3 ensures that α_i^{ub} *is a real number smaller than* 1 *and* $\alpha_i^{lb} \geq 0$.

2.3.2 Equilibrium

In equilibrium $\alpha_i = \alpha_i^d$ and $\sum_{i=1}^2 b_i = 1$. As a consequence, the fractions (α_1^d, α_2^d) selected by the intermediary at equilibrium solve

$$\begin{aligned}
\max_{(\alpha_1^d,\alpha_2^d)} \quad & \mathbb{E}\left[\widetilde{\pi}\right] = \beta\left(\sum_{i=1}^2 \alpha_i\mu_i\right) + \theta \\
st. \quad & \alpha_i^{ub} \geq \alpha_i^d \geq \alpha_i^{lb} \ , \ i = \{1,2\} \\
& \mathrm{E}\left[\widetilde{\pi}^*\right] \geq 0
\end{aligned} \tag{2.17}$$

Therefore, if $\beta > 0$, then $(\alpha_1^{ub}, \alpha_2^{ub})$ maximizes the expected profits of the intermediary. Moreover, $(\alpha_1^{ub}, \alpha_2^{ub})$ corresponds to the basket that emerges in the trading equilibrium. If $\beta = 0$, then all baskets $(\alpha_1^d, \alpha_2^d) \in \left[\alpha_1^{lb}, \alpha_1^{ub}\right] \times \left[\alpha_2^{lb}, \alpha_2^{ub}\right]$ are equilibria because they yield the same expected profit for the intermediary.

If $\beta > 0$, $\mu_1 \neq \mu_2$, and $\sigma_1 \neq \sigma_2$, then $\alpha_1 \neq \alpha_2$ so that the intermediary tailors her basket to one segment to maximize her profits. If there are not clear gains from tailoring the basket—for instance, $\mu_i = \bar{\mu}$ and $\sigma_i = \bar{\sigma}$—the intermediary offers a basket with a fraction $\bar{\alpha}$ of each asset, with

$$\bar{\alpha} \quad = \quad \frac{\bar{\mu} - \sqrt{\bar{\mu}^2(1-\rho) - 4\theta\gamma\bar{\sigma}^2(1+\rho)}}{2\gamma\bar{\sigma}^2(1+\rho)} \tag{2.18}$$

In this case, both investor types trade the same fraction of their assets in exchange for the basket, and thus, the basket replicates the market portfolio as every investor holds the same portfolio after trading.

If $\beta = 0$, the intermediary does not have "skin in the game" and all baskets $(\alpha_1^d, \alpha_2^d) \in \left[\alpha_1^{lb}, \alpha_1^{ub}\right] \times \left[\alpha_2^{lb}, \alpha_2^{ub}\right]$ yield the same expected profit for the intermediary. As a consequence, the basket that replicates the market portfolio is one of the infinitely many equilibria. From this discussion follows

PROPOSITION 6. *If $\beta > 0$, then the basket does not necessarily replicate the market portfolio as the intermediary tailors her basket to maximize her profits. If $\beta = 0$, then the basket that replicates the market portfolio is one potential equilibrium.*

2.3.3 Efficiency of trading allocations

Provided that investors are not allowed to trade across segments, markets are incomplete. If markets are incomplete, there is no reason to expect that the trading equilibrium is constrained efficient. To assess the efficiency of trading allocations, I compare these allocations to the constrained efficient allocations and explore the conditions under which such allocations are equal so that markets provide the right instruments for investors' risk-sharing.

To perform such a comparison it is sufficient to compare the allocations of one investor type because of the symmetry of the problem. Consider $i = 1$. It follows from comparing the allocations in equations (2.9) and (2.12) that the trading allocation equals the constrained efficient allocation if and only if

$$\frac{\sigma_2}{\sigma_1}\frac{\mu_1}{\mu_2} \to 1 \quad \text{and} \quad \rho \to 1$$

In this case, the relationship between the fraction of asset 1 held by *investors one* and the fraction of asset 2 held by *investors one* is given by

$$(1-\alpha_1+\alpha_1 b_1) \quad \rightarrow \quad \frac{\mu_1}{\gamma\sigma_1^2}-\frac{\sigma_2}{\sigma_1}\alpha_2 b_1 \tag{2.19}$$

As a consequence, the allocations attained as the outcome of a market-mediated equilibrium tend not to be constrained efficient. Therefore, there are many cases when the price mechanism is perfectible. It then follows

PROPOSITION 7. *Baskets are not always constrained efficient. However, as investors initial endowments become similar—i.e.* $\frac{\sigma_2}{\sigma_1}\frac{\mu_1}{\mu_2} \rightarrow 1$ *and* $\rho \rightarrow 1$*—the basket allows investors to achieve constrained efficient allocations.*

2.4 Competition Among Intermediaries

Because the price mechanism is perfectible, introducing competition among intermediaries may increase investors' welfare. This section studies the impact of competition on: (a) the composition of the basket, and (b) investors' welfare.

For simplicity, consider an economy with two intermediaries in which each intermediary issues at most one basket. Before trading, intermediaries face a strategic environment that can be framed as a two-stage non-cooperative game. In the first stage, intermediaries choose whether or not to enter each market segment. At the end of the first stage, intermediaries observe who entered each segment. In the second stage, intermediaries select the composition of their baskets to maximize their profits. Immediately after, investors observe the composition of the baskets and choose whether or not to trade shares of their assets in exchange for shares of the baskets available in each segment.

Baskets and intermediaries are indexed by $k=\{1,2\}$. Let α_{ik} denote the fraction of asset i in basket k, with $\sum_{k=1}^{2}\alpha_{ik}=\alpha_i$. Let b_{ik} denote the fraction of basket k bought by *investors* i, with $\sum_{i=1}^{2}\sum_{k=1}^{2} b_{ik}=1$. For simplicity, consider that both intermediaries charge the same intermediation fee $\theta \geq 0$. The problem faced by intermediary one is given by[5]

$$\begin{aligned}
\max_{(\alpha_{11},\alpha_{21})} \quad & \mathbb{E}\left[\widetilde{\pi}_1\right] = \beta\left(\sum_{i=1}^{2}\alpha_{i1}\mu_i\right) + \theta\left(\sum_{i=1} b_{i1}\right) \\
st. \quad & \alpha_{i1}^{ub} \geq \alpha_{i1} \geq \alpha_{i1}^{lb}\ ,\ i = \{1,2\} \\
& \mathrm{E}\left[\widetilde{\pi}_1^*\right] \geq 0
\end{aligned} \tag{2.20}$$

where $\widetilde{\pi}_1^*$ is intermediary one's profit evaluated at the basket that maximizes her expected profits, whereas α_{i1}^{ub} and α_{i1}^{lb} denote the upper and lower bounds of α_{i1} so that b_{i1} are well-defined, $i = \{1,2\}$. Provided that $\sum_{i=1}^{2}\sum_{k=1}^{2} b_{ik} = 1$ at equilibrium, solving problem 2.20 is equivalent to solving

$$\begin{aligned}
\max_{(\alpha_{11},\alpha_{21})} \quad & \mathbb{E}\left[\widetilde{\pi}_1\right] = \theta + \beta\left(\sum_{i=1}^{2}\alpha_{i1}\mu_i\right) - \theta\left(\sum_{i=1} b_{i2}\right) \\
st. \quad & \alpha_{i1}^{ub} \geq \alpha_{i1} \geq \alpha_{i1}^{lb}\ ,\ i = \{1,2\} \\
& \mathrm{E}\left[\widetilde{\pi}_1^*\right] \geq 0
\end{aligned} \tag{2.21}$$

Assume the primitives of the model are such that problem 2.21 has an interior solution. Then the basket that maximizes the expected profits of intermediary one, $(\alpha_{11}^*, \alpha_{21}^*)$, satisfies the first order conditions

$$\beta\mu_1 = \theta\frac{\partial}{\partial\alpha_{11}}\left(\sum_{i=1} b_{i2}\right)\Bigg|_{\alpha_{11}=\alpha_{11}^*} \tag{2.22}$$

$$\beta\mu_2 = \theta\frac{\partial}{\partial\alpha_{21}}\left(\sum_{i=1} b_{i2}\right)\Bigg|_{\alpha_{21}=\alpha_{21}^*} \tag{2.23}$$

To compute the derivatives on the right hand side of equations (2.22) and (2.23), it is necessary to characterize the demand of both investor types for basket 2. To characterize those demands, it becomes handy to analyze the first order conditions of *investors* i's maximization problem, given by the following equations:

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[5] Provided the symmetry, the problem faced by intermediary two is analogous.

$$1-\alpha_i+\left[\sum_{k=1}^{2}\alpha_{ik}b_{ik}\right] = \frac{\mu_i}{\gamma\sigma_i^2}-\left(\rho\frac{\sigma_j}{\sigma_i}\right)\left[\sum_{k=1}^{2}\alpha_{jk}b_{ik}\right] \ , \text{ with } j\neq i \tag{2.24}$$

$$\left[\sum_{k=1}^{2}\alpha_{jk}b_{ik}\right] = \frac{\mu_j-\rho\mu_i\frac{\sigma_j}{\sigma_i}}{\gamma\sigma_j^2(1-\rho^2)}-\frac{\theta}{\gamma\left(\sum_{k=1}^{2}\alpha_{jk}\right)\sigma_j^2(1-\rho^2)} \tag{2.25}$$

Note that equation (2.25) implies that

$$\alpha_{21}b_{11}+\alpha_{22}b_{12} = \frac{\mu_2-\rho\mu_1\frac{\sigma_2}{\sigma_1}}{\gamma\sigma_2^2(1-\rho^2)}-\frac{\theta}{\gamma\left(\sum_{k=1}^{2}\alpha_{2k}\right)\sigma_2^2(1-\rho^2)} \ , \text{ and} \tag{2.26}$$

$$\alpha_{11}b_{21}+\alpha_{12}b_{22} = \frac{\mu_1-\rho\mu_2\frac{\sigma_1}{\sigma_2}}{\gamma\sigma_1^2(1-\rho^2)}-\frac{\theta}{\gamma\left(\sum_{k=1}^{2}\alpha_{1k}\right)\sigma_1^2(1-\rho^2)} \tag{2.27}$$

Differentiating equation (2.26) with respect to α_{11} and α_{21} implies

$$\alpha_{11} \ : \ \alpha_{21}\frac{\partial b_{11}}{\partial\alpha_{11}}+\alpha_{22}\frac{\partial b_{12}}{\partial\alpha_{11}}=0 \tag{2.28}$$

$$\alpha_{21} \ : \ b_{11}+\alpha_{21}\frac{\partial b_{11}}{\partial\alpha_{21}}+\alpha_{22}\frac{\partial b_{12}}{\partial\alpha_{21}}=\frac{\theta}{\gamma\left(\sum_{k=1}^{2}\alpha_{2k}\right)^2\sigma_2^2(1-\rho^2)} \tag{2.29}$$

whereas differentiating equation (2.27) with respect to α_{11} and α_{21} implies

$$\alpha_{11} \ : \ b_{21}+\alpha_{11}\frac{\partial b_{21}}{\partial\alpha_{11}}+\alpha_{12}\frac{\partial b_{22}}{\partial\alpha_{11}}=\frac{\theta}{\gamma\left(\sum_{k=1}^{2}\alpha_{1k}\right)^2\sigma_1^2(1-\rho^2)} \tag{2.30}$$

$$\alpha_{21} \ : \ \alpha_{11}\frac{\partial b_{21}}{\partial\alpha_{21}}+\alpha_{12}\frac{\partial b_{22}}{\partial\alpha_{21}}=0 \tag{2.31}$$

Using equations (2.28) and (2.30) into equation (2.22) yields

$$\frac{\alpha_{11}}{\alpha_{12}} = \frac{1}{\frac{\partial b_{21}}{\partial\alpha_{11}}}\left\{\frac{\theta}{\gamma\alpha_{12}\alpha_1^2\sigma_1^2(1-\rho^2)}-\frac{\alpha_{21}}{\alpha_{22}}\frac{\partial b_{11}}{\partial\alpha_{11}}-\frac{b_{21}}{\alpha_{12}}-\beta\frac{\mu_1}{\theta}\right\} \tag{2.32}$$

Similarly, using equations (2.29) and (2.31) into equation (2.23) yields

$$\frac{\alpha_{21}}{\alpha_{22}} = \frac{1}{\frac{\partial b_{11}}{\partial\alpha_{21}}}\left\{\frac{\theta}{\gamma\alpha_{22}\alpha_2^2\sigma_2^2(1-\rho^2)}-\frac{\alpha_{11}}{\alpha_{12}}\frac{\partial b_{21}}{\partial\alpha_{21}}-\frac{b_{11}}{\alpha_{22}}-\beta\frac{\mu_2}{\theta}\right\} \tag{2.33}$$

Solving the system of equations (2.32) and (2.33) yields

$$\alpha^*_{11} = \phi_1 \times \alpha^*_{12} \tag{2.34}$$

$$\alpha^*_{21} = \phi_2 \times \alpha^*_{22} \tag{2.35}$$

where ϕ_1 and ϕ_2 are two positive constants.[6]

If $\phi_1 = \phi_2$, then the two baskets are equivalent in their spanning role. In other words, both intermediaries issue the same basket. On the other hand, if $\phi_1 \neq \phi_2$, then intermediaries issue different baskets. In that case, however, the increased variety of baskets issued does not always improve investors' welfare. The coexistence of several baskets may be redundant, in the sense that cooperation among issuers may increase investors' welfare. To see this, suppose that intermediaries cooperate and perfectly split the market demand so that each basket serves one investor type. These baskets potentially allow investors to achieve constrained efficient allocations. It follows from inspection, however, that such a situation may not be sustained at equilibrium. To see this, suppose intermediary i tailors her basket such that *investors* i strictly prefers basket i over basket j, with $j \neq i$ and $i = \{1, 2\}$. If trading between intermediaries is not allowed, then basket j cannot be composed of asset i, since only *investors* i are endowed with asset i. Because investors demand baskets only for risk-sharing purposes, no investor type will demand such customized baskets. On the other hand, if trading between intermediaries is allowed, investors may not buy baskets shares if the sum of the intermediation fees is sufficiently high. Therefore, investors may benefit from cooperation between intermediaries. However, the non-cooperative character of competition among intermediaries prevents cooperation, as $\phi_1 \neq \phi_2$ for most primitives of the model. Thus, issuers tend to introduce redundant baskets which do not necessarily increase investors' risk-sharing opportunities. It then follows

PROPOSITION 8. *Under duopoly competition, intermediaries tend to introduce different baskets that do not necessarily increase investors' risk-sharing opportunities.*

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[6] See appendix for the formal definition of both constants as a function of the primitives of the model.

2.5 Concluding Remarks

I present a general equilibrium model of basket securities in segmented markets and explore the design and welfare implications of the introduction of these securities for different investor types. If there is one intermediary, I find that the market-mediated equilibrium may not be constrained efficient, because the intermediary not only seeks to maximize trading volume but also seeks to increase the payoff of the basket—incentive that comes from her “skin in the game.” I then analyze how competition among intermediaries affects the basket structure and investors’ welfare. I show that competition can generate the coexistence of several baskets that do not necessarily improve investors’ risk-sharing opportunities.

Chapter 3

Imperfect Information Transmission from Banks to Investors: Real Implications

with Nicolás Figueroa and Oksana Leukhina

The five year economic expansion leading up to the 2008 financial crisis witnessed an unprecedented growth of securitization markets. Several empirical papers—e.g., Keys et al., 2010b, Purnanandam, 2011, Bord and Santos, 2011, Keys, Seru, and Vig, 2012—document that the spectacular rise of securitization directly contributed to relaxed screening standards, which lends support to economists' public opinion regarding the adverse consequences of securitization on the originator's incentives to screen their borrowers, e.g., Stiglitz, 2007, Blinder, 2007.[1] Despite the evidence described above, our theoretical understanding of the real implications of markets for loan-backed assets remains limited, as noted by Gorton and Metrick, 2011. To fill this gap, we propose a general equilibrium model with borrowers, banks, and investors to study the real implications of the information asymmetry between banks, whose screening choices impact economic activity, and investors in secondary markets, who provide funds and bear the risks.

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[1] Purnanandam, 2011 uncovers the fact that banks with greater involvement in secondary markets originated excessively poor-quality mortgages. Keys et al., 2010b and Keys, Seru, and Vig, 2012 explore loan variation to borrowers with credit scores around 620—a threshold commonly used in securitization. They find that loans with credit scores of 620+ default at the rate 10-25% higher than loans with credit scores of 620-. Mian and Sufi, 2009 provide evidence from zip-code level data on subprime lending. Using data from the U.S. Shared National Credit Program, Bord and Santos, 2011 find that loans sold to CLOs at the time of issue are more likely to default.

The main features of the model are as follows. Borrowers, heterogenous in their credit worthiness, rely on bank financing to develop their projects. Borrowers' credit worthiness is unobserved to investors. Banks alone have the technology to screen and identify repaying borrowers at loan origination. Banks are heterogenous in their screening costs, which are unobserved to investors. To raise funds, banks sell a fraction of their loans to investors in secondary markets. Because investors do not possess information about borrowers' credit worthiness, banks may use a costly imperfect rating technology to transmit information about the quality of their loans to investors in a credible manner. Profit-maximizing banks choose whether or not to screen, rate, and disclose their ratings in secondary markets. Finally, banks and investors trade in competitive secondary loan markets, where loan prices are determined, which in turn, determine banks' screening and rating incentives.

Our main findings are as follows. First, it is the price differential between loans with different credit ratings that disciplines banks' screening at loan origination. Therefore, understanding what determines this price differential is crucial for understanding credit allocation in the economy. If the rating technology was perfect, i.e., the true loan type would be revealed with certainty, the first best outcome would be achieved. However, since the rating technology is imperfect, the price differential between loans with different ratings is not sufficiently large to induce an efficient level of screening. Interestingly, we find that a credit rating's informativeness decreases as more holders of low quality loans use the rating technology. This in turn reduces both the price differential between assets with different ratings and banks' screening at loan origination.

Second, we find that: (1) an increase in banks' collateral values, (2) an increase in the fraction of repaying borrowers, (3) a decrease in banks' skin in the game, and (4) a decrease in credit ratings' precision all unambiguously reduce banks' screening efforts at loan origination. A decrease in credit ratings' precision deserves further comment. This exercise aims to reflect an increase in the complexity of assets offered in secondary markets.[2] The direct implication is that, since rating mistakes are more likely to happen, banks with low quality loans tend to shop for ratings more intensely. Such strategic behavior further reduces credit ratings' informativeness, which in turn reduces the price differential on loans with different credit ratings, and thus relaxes banks' screening at loan origination.

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[2] Such increase in asset complexity may be implemented by banks through securitization.

Our model provides new insights on the following pre-2008 financial crisis' observations: (1) lax lending standards, (2) more intense use of ratings (shopping for ratings), (3) the rise in default probabilities conditional on investment grade ratings (inflation rating), and (4) a drop in the differential between yields on assets with high and low ratings. The first observation seems to be common to all credit expansions and exacerbated by the rise of secondary markets in the pre-2008 crisis period, e.g., Asea and Blomberg, 1998, Berger and Udell, 2004, Lown and Morgan, 2006 and Rajan, 1994. The second observation refers to the idea that issuers may expend resources to ensure that their assets receive high ratings. Thus, banks may expend resources to acquire information on how to best structure their securities, e.g., Brunnermeier, 2009, or solicit ratings from several agencies and disclose only the best rate. Bongaerts, Cremers, and Goetzmann, 2012, Griffin, Nickerson, and Tang, 2013 and Benmelech and Dlugosz, 2009b find evidence consistent with the rating shopping idea. Benmelech and Dlugosz, 2009a, Benmelech and Dlugosz, 2009b and Griffin and Tang, 2012 provide empirical evidence consistent with the third observation. Several theoretical models rationalize inflation rating via rating shopping, e.g. Faure-Grimaud, Peyrache, and Quesada, 2009, Skreta and Veldkamp, 2009, Farhi, Josh, and Tirole, 2013. What is novel in our model is the presence of a feedback effect on screening effort at the loan origination stage. Finally, figures C.1 and C.2 provide evidence consistent with the fourth observation.[3]

We also analyze two policies of interest: (1) mandatory rating and (2) mandatory rating disclosure. Interestingly, both policies turn out to be counterproductive. With mandatory rating, banks with low quality assets increase their rating activity, which decreases the informativeness of a good rating, which in turn, reduces banks' screening efforts at loan origination and exacerbates credit misallocation. With mandatory rating disclosure, banks that sell low quality assets are encouraged to rate their assets because the payoff of assets with high ratings is much larger than the payoff of assets with low ratings. Therefore, the rating activity intensifies, once again reducing the informativeness of a good rating, which reduces both banks' screening efforts at loan origination and banks' incentives to rate their assets. This further aggravates the level of credit misallocation in the economy.

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[3] In addition, Benmelech and Dlugosz, 2009a find that most of such CLOs were initially rated as AAA using data on 744 cash-flow CLOs issued from 2000 to 2007. In their sample as a whole, 71% of issuance is rated AAA; 5% is AA, 6% is A, 5% is BBB, 2% is BB, 0.1% is B, and 11% is NR(unrated). Along the same lines, Griffin and Tang, 2012 document that most CDOs notes issued prior to mid-2007 were AAA rated. Using data from one of the three major credit rating agencies, they access to information about 916 CDOs issued between Jan 1997 and Dec 2007 with total note face value of 612.8 billion. From the 916 CDOs, they obtain data about 5,466 rated tranches. Among all rated issuances, 84.1% are AAA, 14.5% are non-AAA investment grade (6.0% AA, 4.6% A, and 4.0% BBB), and 1.4% are below investment grade. The average CDO has 75.5% rated AAA (super-senior tranches are counted as AAA rated).

We contribute to the literature on information in secondary loan markets through the analysis of how the strategic behavior of loan originators alters information transmission in secondary markets. In our model, there is a natural rate of information garbling, given by the imperfect nature of the rating technology. However, this natural rate is augmented by banks who hold low quality loans and rate them, strategically hiding low credit ratings, and therefore decreasing credit ratings' informativeness. Skreta and Veldkamp, 2009 model the idea of shopping for ratings, and study the interaction between ratings and asset prices. However, they assume that investors are naive, in the sense that investors do not consider the possibility that loan originators hide information.

As in Holmstrom and Tirole, 1997, we assume that banks have access to a special technology, which allows them to screen their borrowers. As such, banks are not simply channels through which savings are allocated to borrowers, but firms that use their technologies to maximize their own profits. As with any firm, their use of screening and the rating technology are dictated by prices, which determine banks' screening activity and, hence, both credit allocation and aggregate productivity for the whole economy.[4]

The rest of the paper is organized as follows. Section 3.1 describes the baseline model. Section 3.2 characterizes the equilibrium, its existence and uniqueness, compares the equilibrium outcome in a decentralized economy to the constrained efficient allocation, and presents comparative statistics results and their empirical relevance. Section 3.3 studies mandatory rating and mandatory rating disclosure policies. Finally, section 3.4 concludes. All proofs, unless otherwise stated, appear in the Appendix.

3.1 Baseline Model

Consider an economy populated by risk-neutral borrowers, banks, and investors. Borrowers rely on bank financing to develop their projects, and their credit worthiness is costly observed. Banks alone have the technology to screen and identify repaying borrowers. To raise funds, banks sell a fraction $0 < (1-\rho) \leq 1$ of their loans to investors in competitive secondary markets.[5] Because investors in secondary markets do not possess information about the credit worthiness of borrowers that underlie loans, banks may use a rating technology—which reveals the true type of a borrower that underlies a loan with probability r—to transmit information about the quality of their loans in a credible manner. The importance of studying the macroeconomic implications of this type of informational asymmetry between banks and investors is discussed in Gorton, 2009. Our goal is to examine its implications for the allocation of loanable funds, i.e., the composition of financed borrowers.

---

[4] In Holmstrom and Tirole, 1997 banks can monitor projects they finance.

The model period can be subdivided into three stages occurring in the following order.

1. **Screening of Borrowers.** Banks choose whether or not to engage in costly screening of borrowers when originating loans, taking prices in secondary markets as given. Upon origination, banks learn a borrower's credit worthiness. This stage determines the composition of borrowers.

2. **Rating of Assets.** Banks choose whether or not to engage in costly rating of their loans, taking prices in secondary markets as given. This stage determines how much information is produced to mitigate the information asymmetry between banks and investors in secondary markets.

3. **Trade in Secondary Markets.** Banks and investors trade in competitive secondary markets, in which loan prices are determined.

3.1.1 Borrowers

There is a continuum of measure 1 of potential borrowers of unobserved type, each of whom seeks financing in the amount of 1 unit of funds to develop their projects. Potential borrowers are of unobservable type $\theta \in \{G, B\}$, represented in proportions μ_0 and $(1-\mu_0)$, respectively. Let W_θ denote the repayment of a borrower of type θ on a loan. We assume that only borrowers of type G fully repay their loans. In other words,

ASSUMPTION 4. $W_G > 1 > W_B$

---

[5] We take the presence of secondary loan markets as given and we do not attempt to explain their emergence, e.g Parlour and Plantin, 2008.

3.1.2 Banks

There is a continuum of measure 1 of profit-maximizing banks, heterogeneous in their screening cost $k \sim F[0,1]$, which is unobserved to investors. F is continuous and represents the cumulative distribution function of banks' screening costs. Each bank faces its own pool of potential borrowers of type $\theta \in \{G, B\}$, represented in proportions μ_0 and $1-\mu_0$, respectively. Banks have the option of using the screening technology at cost k, which guarantees financing of a borrower of type G. Otherwise, banks make loans at random. Lending to a borrower of type θ may also be interpreted as standing in for extending a large basket of loans that generates W_θ as total repayment.

Once the borrower is financed, a bank learns its type with certainty. However, information about the type of borrower that underlies a loan is not available to investors in secondary markets.[6] To convey that information in a credible manner, banks decide whether or not to employ a rating technology at fixed cost c, which reveals the true loan type with probability $\frac{1}{2} < r < 1$. Because banks holding a loan with an underlying borrower of type G are more likely to obtain a good rating, ratings are valuable signals in secondary markets, and thus, loans with a good rating sell at a premium.

ASSUMPTION 5. $c < (1-\rho)(W_G - W_B)$

Paying c to rate a loan may be interpreted as engaging in a costly process that results, with some positive probability, in the enhancement of the perceived value of a bank's loan in secondary markets. In practice, the process of getting all rating agencies to assign a good rating—e.g., AAA rating—to a large share of a loan basket is costly because it involves hiring consultants to obtain information regarding the rating process of each agency as well as decomposing the loan basket into tranches in a way that maximizes positive outcomes. Our assumption $r > \frac{1}{2}$ captures the idea that banks with better loans are more likely to succeed in this process. We also assume that bad ratings are available for free to all banks, which rules out the signalling value of bad ratings and ensures that only good ratings are revealed in equilibrium.[7] Because investors do not observe the screening cost of originating banks, prices in secondary markets are conditioned only on the presence of a good rating.

---

[6] In theory, the information asymmetry between banks and investors may be resolved if originating banks retain the most risky junior tranche of their loan basket, thereby sending a credible signal to asset buyers, e.g., DeMarzo, 2005b. In practice, however, retaining a junior tranche does not appear to accomplish this purpose, as it can be combined with shorting of a senior tranche. However, the senior tranches are the ones typically retained, e.g., Beltran, Cordell, and Thomas, 2013. Rating agencies, on the other hand, are used extensively to signal asset values.

[7] Later on, we relax this assumption when studying mandatory rating disclosure in section 3.3.

Banks' Rating and Screening Decision

In what follows, we analyze banks' rating and screening decisions. Let P_{GR} and P_{NR} denote the price on a loan with a good rating and no rating—or a hidden bad rating—in secondary markets, respectively. Taking these prices as given, banks choose their screening and rating strategies to maximize their profits.

Rating Strategy:

Let f_θ denote the probability that a bank with a loan with an underlying borrower of type θ—henceforth loan of type θ—uses the rating technology. Consider a bank with a loan of type B. If the bank rates its loan, it receives a good rating with probability $(1-r)$, reveals it and sells a fraction $(1-\rho)$ of the loan at $(1-\rho)P_{GR}$. On the other hand, the bank receives a bad rating with probability r, hides it and sells a fraction $(1-\rho)$ of the loan at $(1-\rho)P_{NR}$. A bank that holds a loan of type B chooses to rate it, if the expected gain of selling a fraction $(1-\rho)$ of the loan exceeds the cost of using the rating technology. If the expected gain falls short of the associated costs, on the other hand, a fraction $(1-\rho)$ of the loan is sold with no rating attached to it, while the mixed strategy is possible otherwise. In other words,

$$\begin{array}{rcll} f_B = 1 & & (1-\rho)\left(\left[(1-r)\,P_{GR} + rP_{NR}\right] - P_{NR}\right) & > c, \\ f_B = 0 & \text{if} & \text{...} & < c, \\ f_B \in (0,1) & & \text{...} & = c. \end{array} \tag{3.1}$$

We restrict attention to the range of parameter values that ensure that banks with loans of type G always rate their loans at equilibrium, i.e., $f_G = 1$:

$$(1-\rho)\left(rP_{GR} + (1-r)P_{NR} - P_{NR}\right) > c \tag{3.2}$$

Screening Strategy

Taking prices P_{GR} and P_{NR} as given, banks choose whether or not to screen their potential borrowers. Banks are heterogenous in their screening cost k, with $k \sim F[0,1]$. Let R_θ denote the expected payoff of a bank that finances a borrower of type θ. Because a bank with a loan of type G always rates its loan, the expected payoff from financing a borrower of type G is given in equation (3.3). With probability r, the bank obtains a type G rating, reveals it, and sells a fraction $(1-\rho)$ of the loan for $(1-\rho)P_{GR}$. With probability $(1-r)$, the bank obtains a type B rating, hides it, and sells a fraction $(1-\rho)$ of the loan for $(1-\rho)P_{NR}$. Because the bank holds a fraction ρ of the loan, the bank also obtains ρW_G. The expenses are the rating cost associated with the fraction of the loan that is sold in secondary markets $(1-\rho)c$ and the loan amount 1. The situation for banks that finance type B borrowers is similar, and the expected payoff from financing a borrower of type B is given in equation (3.4). With probability $(1-r)\,f_B$, the bank obtains a type G rating, reveals it, and sells a fraction $(1-\rho)$ of the loan for $(1-\rho)P_{GR}$. With probability $[1-(1-r)f_B]$, the bank sells a fraction $(1-\rho)$ of the loan for $(1-\rho)P_{NR}$. The latter case includes both, the case of the unlucky rating draw and the case of forgoing the use of the rating technology. Because the bank holds a fraction ρ of the loan, the bank also obtains ρW_B. The expenditures are the expected rating costs $(1-\rho)f_B c$ and the loan amount 1. Therefore,

$$\begin{aligned} R_G &= (1-\rho)\left[rP_{GR}+(1-r)P_{NR}-c\right]+\rho W_G-1, && (3.3)\\ R_B &= (1-\rho)\left[(1-r)f_B P_{GR}+\left[1-(1-r)f_B\right]P_{NR}-f_B c\right]+\rho W_B-1. && (3.4) \end{aligned}$$

A bank that decides to screen its potential borrowers finances a borrower of type G with certainty. If that bank faces a screening cost of k, its ex-ante expected payoff is R_G-k. On the other hand, a bank that does not screen its borrowers lends at random, and, thus, it lends to a borrower of type G with probability μ_0. The ex-ante payoff for this bank is then $\mu_0 R_G+(1-\mu_0)R_B$. Because banks are profit-maximizing and risk-neutral, a bank that faces a screening cost of k chooses to screen whenever

$$R_G-k \geq \mu_0 R_G+(1-\mu_0)R_B \tag{3.5}$$

and lends at random otherwise. As a consequence, the bank that is indifferent between these two choices—i.e. the marginal screener—is the bank with a screening cost

$$
\begin{aligned}
\bar{k} &= (1-\mu_0)(R_G - R_B) \\
&= (1-\mu_0)\left[(1-\rho)\left\{[r-(1-r)f_B][P_{GR}-P_{NR}] - c(1-f_B)\right\} + \rho(W_G - W_B)\right]
\end{aligned} \tag{3.6}
$$

Therefore, banks that face small screening costs, e.g., $k < \bar{k}$, screen and finance only borrowers of type G. Banks that face large screening costs do not screen, and, thus, they finance borrowers of type G with probability μ_0. Because banks' screening costs are described by F, the measure of borrowers of type G financed in equilibrium—which we denote by $\mu(P_{GR,}P_{NR})$ to emphasize its dependence on prices—is given by

$$
\mu\left(P_{GR}, P_{NR}\right) = F(\bar{k}) + (1 - F(\bar{k}))\mu_0. \tag{3.7}
$$

Provided that W_θ is related to the productivity of a project developed by a borrower of type θ, the equilibrium object $\mu\left(P_{GR}, P_{NR}\right)$ may be interpreted as the average productivity of a sector that relies on bank financing. As equation (3.7) states, banks directly affect aggregate productivity through the credit allocation margin because they perform the important service of screening borrowers—in the spirit of Boyd and Prescott, 1986 and Holmstrom and Tirole, 1997.

3.1.3 Investors

A large number of risk-neutral investors buy loans in competitive secondary markets. Because investors have neither information regarding the type of borrowers that underlie loans nor banks' screening costs, investors' beliefs regarding loans types are conditioned only on ratings.

Investors' Beliefs and Loan Prices in Secondary Markets

Let $\Pr_{G|GR}$ denote the probability that a loan with a good rating is a loan of type G. Let $\Pr_{G|NR}$ denote the probability that a loan with no rating is a loan of type G. Considering loans correctly and incorrectly rated, $\Pr_{G|GR}$ is given by the fraction of loans of type G among loans that received a good rating. Similarly, $\Pr_{G|NR}$ is given by the fraction of loans of type G among loans with no rating. In other words,

$$
\begin{aligned}
Pr_{G|GR} &= \frac{\mu r f_G}{\mu r f_G + (1-\mu) f_B (1-r)}, && (3.8)\\
Pr_{G|NR} &= \frac{\mu[(1-f_G) + (1-r) f_G]}{\mu[(1-f_G) + (1-r) f_G] + (1-\mu)[(1-f_B) + f_B r]}. && (3.9)
\end{aligned}
$$

where μ denotes the fraction of borrowers of type G financed at loan origination. Because markets are competitive, investors make zero profits, which implies that prices on loans on secondary markets reflect their expected payoffs. In other words,

$$
\begin{aligned}
P_{GR} &= W_G Pr_{G|GR} + W_B[1 - Pr_{G|GR}] = Pr_{G|GR} \Delta W + W_B, && (3.10)\\
P_{NR} &= W_G Pr_{G|NR} + W_B[1 - Pr_{G|NR}] = Pr_{G|NR} \Delta W + W_B, && (3.11)
\end{aligned}
$$

where $\Delta W = W_G - W_B$. Investors are then willing to pay a premium for loans with a good rating. The size of that premium depends on the beliefs that a good rating induces about the quality of the borrower that underlies a loan.[8]

3.1.4 Equilibrium

Thus far, we have described banks' optimal rating and screening strategies for given prices P_{GR} and P_{NR}. We have also discussed how P_{GR} and P_{NR} are related to investors' beliefs regarding loans types. To complete our definition of equilibrium, we also require that investors' beliefs are consistent with equilibrium outcomes.

DEFINITION 2. *An equilibrium is given by a screening cost cutoff $\bar{k}^*$—which defines the marginal screener—, rating strategies $f_G^* = 1$ and f_B^*, a measure of borrowers of type G financed at loan origination, μ^*, investors' beliefs $\left\{Pr^*_{G|GR}, Pr^*_{G|NR}\right\}$, and prices P^*_{GR} and P^*_{NR} satisfying the following conditions:*

1. *Given prices P^*_{GR} and P^*_{NR}, banks with screening costs $k \leq \bar{k}^*$ find it optimal to screen their borrowers and to rate their loans according to f^*_θ. Namely, the screening condition (3.5) holds*

---

[8] This premium could be interpreted as Duffie's (2009) *lemon premium*, where an offer by a bank to sell a loan is associated with a drop in price, since it is assumed that the bank has private information. There is also the *moral hazard premium*, where a sale is associated with a drop in price because the bank has fewer incentives to control the credit risk of the loan, which we do not consider in this paper.

with "$\geq$" for these banks while rating conditions (3.1) *and* (3.2) *are satisfied.*

2. *Given prices P^*_{GR} and P^*_{NR}, banks with screening costs $k > \bar{k}^*$ find it optimal to lend at random and to rate their loans according to f^*_θ. Namely, the screening condition (3.5) holds with " $<$ " for these banks while rating conditions (3.1) and (3.2) are satisfied.*

3. *The fraction of borrowers of type G financed at loan origination, μ^*, is determined by banks' optimal screening strategies, as summarized by $\bar{k}^*$:*

$$\mu^* \quad = \quad F(\bar{k}^*) + (1 - F(\bar{k}^*))\mu_0. \tag{3.12}$$

4. *Given investors' beliefs $\left\{Pr^*_{G|GR}, Pr^*_{G|NR}\right\}$, prices P^*_{GR} and P^*_{NR} reflect expected payoffs as described in equations (3.10) and (3.11).*

5. *Investors' beliefs $\left\{Pr^*_{G|GR}, Pr^*_{G|NR}\right\}$ are consistent with the equilibrium outcomes so that equations (3.8) and (3.9) hold (whenever possible).*

3.2 Equilibrium Characterization

We solve for the equilibrium quantities and prices as follows. We first study banks' rating strategy in section 3.2.1. Given a measure of borrowers of type G financed at loan origination, μ, we derive the optimal rating strategy f^*_B as a function of screening costs, c, banks' skin in the game, ρ, borrowers' payoff differential ΔW, and the precision of the rating technology, r, in Lemma 1. Then, we study the behavior of f^*_B as a function μ^*—the equilibrium measure of borrowers of type G financed at loan origination—in Lemma 2 to understand how credit allocation impacts banks' rating activity. Once the equilibrium relationship $f^*_B(\mu^*)$ is derived, section 3.2.2 characterizes the cutoff $\bar{k}^*(\mu^*)$, which defines the set of banks that screen their borrowers. In section 3.2.3, μ^* is found as a fixed point of equation (3.12). Its existence and uniqueness are derived in Proposition 9. Section 3.2.4 compares the credit allocation in the decentralized economy to the constrained efficient allocation. Finally, section 3.2.5 derives the partial effects of changes in the primitives of the model on credit allocation at equilibrium.

3.2.1 Rating Strategy

We now characterize the rating strategy, f_B, for a fixed μ. It is helpful in understanding how the rating accuracy, r, and the cost of screening relative to the payoff differential, defined as

$$\tilde{c} \equiv \frac{c}{(1-\rho)\Delta W}, \tag{3.13}$$

affect the rating decision. Note that it is the cost of screening relative to the payoff differential that matters here, because the rating decision depends on the rating cost relative to the price gain implied by the positive rating outcome, $(1-\rho)(P_{GR}-P_{NR})$, which, in turn, reflects the payoff differential, $(1-\rho)\Delta W$.

LEMMA 1 (Rating Strategies f_B and f_G as a function of r, ρ, c, and ΔW)**.** *For a given measure of borrowers of type G financed at loan origination, μ, the rating strategy f_B can be summarized as follows:*

$$f_B = \begin{cases} 1 & \tilde{c} < \frac{\mu(1-\mu)(1-r)(2r-1)}{[r-\mu(2r-1)][\mu(2r-1)+(1-r)]} \\ f_B^{mix} \in (0,1) \quad \text{if } \frac{\mu(1-\mu)(1-r)(2r-1)}{[r-\mu(2r-1)][\mu(2r-1)+(1-r)]} < & \tilde{c} < \frac{(1-r)(1-\mu)}{1-r\mu} \\ 0 \quad \frac{(1-r)(1-\mu)}{1-r\mu} < & \tilde{c}, \end{cases}$$

where $f_B^{mix} = \frac{\tilde{c}(1-2\mu r)+\mu(1-r)-\sqrt{(\tilde{c}+\mu)^2-\mu^2 r(2-r)-(6-4r)\tilde{c}r\mu}}{2\tilde{c}(1-r)(1-\mu)}$.[9] *Moreover,*

$$f_G = 1 \quad \text{if } \tilde{c} < \frac{r\,(1-\mu)}{1-r\mu}.$$

Figure C.2 helps to illustrate lemma 1. For $\mu = \frac{1}{2}$, it depicts the optimal rating strategy, in the space of parameters r and $\tilde{c}$. Generally speaking, small values of $\tilde{c}$ induce holders of loans of type B to rate their loan because the rating technology is imperfect, and, thus, holders of loans of type B may obtain a good rating. Increasing r increases the precision of the rating technology, thereby reducing the likelihood of incorrectly rated loans. In general, increasing r reduces the incentives to rate loans of type B, except for the lower left hand corner of figure C.2 where the lower curve is upward sloping due to the dominant effect of $(1-\rho)(P_{GR}-P_{NR})$.

---

[9] In light of (3.1) , f_B^{mix} is found from $(1-\rho)(1-r)\Delta W \Delta \Pr = c$, after substituting investors' beliefs from (3.10) and (3.11).

We now study how credit allocation impacts the rating activity. Because we focus on the parameter space that ensures that $f_G = 1$ in equilibrium, the range of admissible μ is between 0 and

$$\bar{\mu} \equiv \frac{r - \tilde{c}}{r(1 - \tilde{c})},$$

which is found by solving for μ from (1) rewritten with equality. Lemma 2 gives a general characterization of the optimal rating strategy $f_B(\mu) : [0, \bar{\mu}] \rightarrow [0, 1]$ as function of μ. We show that if $\mu = 0$, then $f_B = 0$. f_B is increasing for small values of μ and may or may not reach 1 before it becomes a decreasing function of μ. For sufficiently large values of μ, $f_B = 0$ and stays at zero as μ increases further.

LEMMA 2 (Rating strategy f_B as a function of μ)**.** *If* $\tilde{c} \leq (1 - r)(2r - 1)$, *then*

$$f_B = \begin{cases} 0 & \mu = 0, \\ f_B^{mix}(\mu) \in (0, 1) & \mu \in (0, \mu_1), \\ 1 \qquad\qquad\qquad \textit{if} & \mu \in (\mu_1, \mu_2), \\ f_B^{mix}(\mu) \in (0, 1) & \mu \in (\mu_2, \mu_3), \\ 0 & \mu \geq \mu_3, \end{cases}$$

where constants μ_1*,* μ_2*, and* μ_3 *are defined as*

$$\begin{aligned} \mu_1 &= \frac{1}{2} - \frac{1}{2}\frac{(1 - r)(2r - 1) - \tilde{c}}{(2r - 1)(\tilde{c} + 1 - 2\tilde{c}r - r)} \\ \mu_2 &= \frac{1}{2} + \frac{1}{2}\frac{(1 - r)(2r - 1) - \tilde{c}}{(2r - 1)(\tilde{c} + 1 - 2\tilde{c}r - r)} \\ \mu_3 &= \frac{1 - r - \tilde{c}}{1 - r - r\tilde{c}}. \end{aligned} \tag{3.14}$$

Moreover, f_B *is an increasing function of* μ *in the interval* $(0, \mu_1)$ *and a decreasing function of* μ *in the interval* (μ_2, μ_3)*.*

If instead $\tilde{c} > (1 - r)(2r - 1)$, *then*

$$f_B(\mu) = \begin{cases} 0 & \mu = 0, \\ f_B^{mix}(\mu) \in (0, 1) \quad \textit{if} & \mu \in (0, \mu_3), \\ 0 & \mu \in (\mu_3, \bar{\mu}), \end{cases}$$

and f_B *is an increasing function of* μ *in the interval* $(0, \mu_{\max})$ *and a decreasing function of* μ *in the interval* $(\mu_{\max}, \mu_3)$*, where* $\mu_{\max}$ *is defined as*

$$\mu_{\max} = \frac{1}{2}\frac{(1-r)^2 - \tilde{c}^2}{(1-r)^2 - \tilde{c}\,(2r-1)\,(1-r)}.$$

To understand the shape of $f_B(\mu)$, recall that banks' rating activity is disciplined by the premium paid in secondary markets on loans with a good rating, which in turn depends on the informativeness of a good rating, defined as $\Delta\,\mathrm{Pr} \equiv \mathrm{Pr}_{G|GR} - \mathrm{Pr}_{G|NR}$. If μ is small, there are very few loans of type G in secondary markets, so a good rating does little to raise investors' beliefs regarding a loan's quality. At these low levels of μ, an increase in μ has a positive effect on the informativeness of a good rating, as good ratings become more valuable signals. The opposite happens for large values of μ, implying that most loans are of type G, and so an increase in μ deteriorates the value of a good rating.

3.2.2 Rating Strategies and Credit Allocation

To understand how banks' rating activity impacts credit allocation, it is illustrative to analyze the cutoff $\bar{k}$,

$$\begin{aligned}
\bar{k} &= (1-\mu_0)\,[R_G - R_B] \\
&= (1-\mu_0)(1-\rho)\,[(P_{GR} - P_{NR})\,(r - (1-r)\,f_B) - (1-f_B)\,c] + (1-\mu_0)\rho\Delta W \\
&= (1-\mu_0)(1-\rho)\left[\Delta W\,\underbrace{\left[Pr_{G|GR} - Pr_{G|NR}\right]}_{\Delta\,\mathrm{Pr}}(r-(1-r)\,f_B) - (1-f_B)\,c\right] + (1-\mu_0)\rho\Delta W
\end{aligned} \tag{3.15}$$

which defines the marginal screener. It follows from the second line in equation (3.15) that it is the price differential between loans with a good rating and loans with no ratings, $P_{GR} - P_{NR}$, that disciplines banks' screening effort at loan origination. Because the price differential is proportional to the informativeness of a good rating, $\Delta\,\mathrm{Pr}$, if $\Delta\,\mathrm{Pr}$ is large, then a good rating implies a large gain in the perceived quality of the loan in secondary markets, and, thus, more banks screen their borrowers. On the other hand, if $\Delta\,\mathrm{Pr}$ is small, then the premium on a loan with a good rate is not sufficiently large, and, thus, fewer banks screen their borrowers.

3.2.3 Existence and Uniqueness of Equilibrium

To solve for the equilibrium measure of borrowers of type G financed at loan origination, we need to solve for equation (3.12). The equilibrium relationship $\bar{k}^*(\mu^*)$ is obtained from (3.15) by substituting for investors' beliefs from equations (3.8) and (3.9) and using the equilibrium relationship $f_B(\mu)$, summarized in Lemma 2. To prove the existence and uniqueness of the equilibrium, it suffices to show that there is a unique solution μ^* to equation (3.12). The proof is formalized in Proposition 9.

PROPOSITION 9 (Existence and uniqueness of equilibrium)**.** *Denote the right hand side of the equilibrium condition* (3.12) *by* $H : [0, \bar{\mu}] \to [\mu_0, 1]$,

$$H(\mu) \equiv F(\bar{k}(\mu)) + \left[1 - F(\bar{k}(\mu)\right] \mu_0. \tag{3.16}$$

Assume

$$F\left((1-\mu_0)\rho[c+\Delta W]\right) + \left[1 - F\left((1-\mu_0)\rho[c+\Delta W]\right)\right] \mu_0 < \bar{\mu}.$$

If $\tilde{c} \leq (1-r)(2r-1)$, *then also assume*

$$(1-\rho)\bar{f}(1-\mu_0)^2 \Delta W \frac{(2r-1)^2}{r(1-r)} \sqrt{\frac{1-r-\tilde{c}/(2r-1)}{1-r-\tilde{c}(2r-1)}} \Big/ \left(\frac{1-r}{1-r-\tilde{c}(2r-1)}\right)^2 < 1,$$

where $\bar{f} \equiv \sup_{k\in[0,1]} F'(k)$. *Then* $H'(\mu) \leq 1$ *and there exists a unique equilibrium.*

3.2.4 Comparison to the Constrained Efficient Allocation

It is instructive to compare equilibrium outcomes in the decentralized economy to the constrained efficient allocation to see whether the market may induce the optimal level of screening. To do so, we need to specify a few more details about the production structure of the economy. For simplicity, assume that c is simply a transfer from banks to the rating technology and there are no additional costs associated with borrowers' production—apart from the unit of funds extended. Suppose further that a loan to a borrower of θ results in output production in the amount of Y_θ, where $Y_G > Y_B$. Finally, assume $W_G - W_B \leq Y_G - Y_B$.[10] The economy total output is then given by:

---

[10] This relationship would hold under either debt or equity financing contracts. Modeling the bank contracts in more detail is outside the scope of this paper and irrelevant for our analysis.

$$Y = F(\bar{k})Y_G + (1 - F(\bar{k}))(\mu_0 Y_G + (1-\mu_0)Y_B) - \int_0^{\bar{k}} k dF(k) - 1. \tag{3.17}$$

The first two terms in equation (3.17) represent the total output produced by all borrowers, whereas the last two terms represent inputs involved in screening and in the production of borrowers' projects.

The constrained efficient allocation is given by the screening activity $\bar{k}$ that maximizes Y in equation (3.17)—which corresponds to the choice of a benevolent planner who owns the banking technology and optimally chooses the level of screening at loan origination. The social marginal gain of screening by any bank is $(1-\mu_0)(Y_G - Y_B)$, because borrowers of type G are financed with probability μ_0 even if a bank does not screen. The marginal cost for a given bank is k. Therefore, it is socially optimal for a bank with screening cost k to screen whenever $(1-\mu_0)(Y_G - Y_B) > k$. The most productive bank faces a screening cost $k = 0$; therefore, it is always efficient for such a bank to screen. On the other hand, the least productive bank faces a screening cost of 1. If $(1-\mu_0)(Y_G - Y_B) > 1$, then even the least productive bank should screen, and so should the rest of the banks. Otherwise, there exists a cutoff marginal screener $\bar{k}^{ef} \in (0, 1)$ satisfying $\bar{k}^{ef} = (1-\mu_0)(Y_G - Y_B)$. Formally, the socially efficient marginal screener is given by

$$\bar{k}^{ef} = \min\{(1-\mu_0)(Y_G - Y_B), 1\} \tag{3.18}$$

and the implied socially efficient measure of credit allocation is given by

$$\mu^{ef} = F(\bar{k}^{ef}) + \left[1 - F(\bar{k}^{ef})\right] \mu_0. \tag{3.19}$$

In the decentralized economy, it is never the case that all banks screen, i.e., we have $\bar{k} < 1$. If this was the case, all loans would be resold at W_G, which would imply that banks would have no incentives to screen—which is a contradiction. Moreover, even if $k^{ef} < 1$, the level of screening in the decentralized economy falls short of the socially efficient level as long as $r < 1$ as Proposition 10 shows.

PROPOSITION 10 (Comparison to the Constrained Efficient Outcomes)**.** *In equilibrium, the* *level of screening activity is less than efficient, and resources are misallocated,*

$$\bar{k}^* < \bar{k}^{ef} \text{ and } \mu^* < \mu^{ef},$$

whenever $r < 1$.

3.2.5 Comparative Statics Analysis

This section derives the partial effects of changes in the primitives of the model on the equilibrium level of resource allocation, μ^*. We first analyze the changes introduced in μ^* by a change in ΔW—which can be interpreted as changes in either the relative productivity of repaying borrowers or a change in the value of collateral recovered in the case of defaulting borrowers.

The intuition suggests that an increase in ΔW should increase the measure of banks that screen as it directly increases $R_G - R_B$. This is, indeed, true if the parameter values are such that banks with loans of type B choose a pure rating strategy, i.e., $f_B \in \{0,1\}$. In other words, for small changes in ΔW, the rating behavior of banks with loans of type B remains unaffected, so no additional effects are operating. In the case of a mixed rating strategy, however, the positive direct effect of ΔW on screening activity may be offset by an increase in the rating activity of banks with loans of type B as lemma 3 shows.

LEMMA 3 (Comparative statics of $\bar{k}^*$ and μ^* with respect to the loan payoff differential, ΔW)**.** *The following effects on screening effort* $\bar{k}^*$ *and the measure of borrowers of type G financed* μ^* *hold in equilibrium:*

a) If $f_B(\mu^*) \in (0,1)$ *and* $\left[1 + c\frac{\partial f_B^{mix}}{\partial \Delta W}\right] \leq 0$*, then* $\bar{k}^*$ *and* μ^* *are weakly decreasing functions of* ΔW*.*

b) If $f_B(\mu^*) \in (0,1)$ *and* $\left[1 + c\frac{\partial f_B^{mix}}{\partial \Delta W}\right] > 0$*, then* $\bar{k}^*$ *and* μ^* *are increasing functions of* ΔW*.*

c) If $f_B(\mu^*) \in \{0,1\}$*, then* $\bar{k}^*$ *and* μ^* *are strictly increasing functions of* ΔW*.*

Within our model, the rise in collateral values in the mortgage market leading up to the 2008 crisis can be interpreted as a decline in ΔW, as more value would be recovered from defaulting borrowers in the case of higher housing prices. As a consequence, our model provides insight into why the rise in collateral values in mortgage markets worked to weaken banks' screening incentives when originating mortgage loans, thereby contributing to a worsening pool of financed borrowers.

We now analyze the impact of changes in the precision of the rating technology, r, on screening activity and, therefore, on credit allocation. As lemma 4 shows, changes in r have an unambiguous effect on screening and credit allocation. An increase in the rating precision directly increases the payoff to screening by increasing the probability that banks with loans of type G receive a good rating and sell at a premium as well as decreasing the probability that banks with loans of type B are incorrectly rated and sell at a premium.

LEMMA 4 (Comparative statics of $\bar{k}^*$ and μ^* with respect to the rating precision, r)**.** *In equilibrium, screening effort $\bar{k}^*$ and the measure of borrowers of type G financed μ^* are strictly increasing functions of r.*

Figure C.3 numerically illustrates the effects on equilibrium quantities of decreasing the rating precision, which can be interpreted as a result of increased asset complexity—a widespread phenomenon that took place prior to the 2008 financial crisis. In all panels of figure C.3, r decreases as we move along the horizontal axis. As figure C.3 (panel b) shows, as r decreases, f_B tend to increase because banks with loans of type B rate their loans more often provided that ratings are less accurate. For a sufficiently precise rating, however, banks holding loans of type B never rate their loans, whereas for a sufficiently small rating precision, everyone engages in rating. The informativeness of a good rating, depicted in figure C.3 (panel f), decreases through the direct effect of ratings becoming more prone to error. But in the region where banks with loans of type B play a mixed rating strategy, the informativeness of a good rate decreases faster and, therefore, the premium paid on loans with a good rate declines faster. In such a region, investors' beliefs deteriorate due to both—the direct effect of less precise ratings and the indirect effect of intensified rating behavior. This is also the region where the fraction of banks screening and the measure of loans of type G financed decline most rapidly as shown in figure C.3 (panels a and b). This is not surprising, because the screening decision depends on the premium paid on loans with a good rate as well as on the probability of obtaining a good rating, both of which decline.

It is important to note that the measure of loans with a good rate may increase in equilibrium, despite the fact that the actual measure of loans of type G financed at loan origination declines as figure C.3 (panels a and e) show. Such equilibrium behavior is due to the intensified use of ratings by banks holding loans of type B. As the rating precision decreases and more banks rate their loans—hoping to obtain a good rating in order to sell at a premium—, the measure of loans with a good rate increases.

The above comparative statics analysis helps rationalize several phenomena observed prior to the 2008 financial crisis, such as: (1) lax screening standards; (2) an intensified use of ratings, or shopping for a high rating; (3) the rise in default probability on loan baskets with investment grade—securities which can be interpreted as loans with high rates within the model—as shown in figure 3 (panel d); and (4) historically low spreads between high yield and investment grade securities—spreads that can be interpreted as the price differential between loans with a good rating and loans with no rating within the model—as shown in figure C.3 (panel f). Figure C.3 (panel e) even suggests that the fraction of assets receiving an investment grade may rise despite the worsening of credit allocation.

To complete our comparative analysis, it remains to consider changes in the cost of rating, c, banks' skin in the game, ρ, and the initial distribution of borrowers' types, μ_0. Lemma 5 studies the effect of changes in c on screening and credit allocation.

LEMMA 5 (Comparative statics of $\bar{k}^*$ and μ^* with respect to the rating cost, c)**.** *The following effects on screening effort, $\bar{k}^*$, and the measure of borrowers of type G that get financed, μ^*, hold in equilibrium:*

a) If $f_B(\mu^*) = 0$, then $\bar{k}^*$ and μ^* are strictly decreasing functions of c.

b) If $f_B(\mu^*) \in (0, 1)$, then $\bar{k}^*$ and μ^* are strictly increasing functions of c.

c) If $f_B(\mu^*) = 1$, then $\bar{k}^*$ and μ^* are independent of c.

Because $f_G = 1$, as the cost of rating c increases, holding a loan of type G becomes relatively less profitable compared to holding a loan of type B if $f_B < 1$, thereby weakening the incentive to screen. Thus, if $f_B = 0$, then $\bar{k}^*$ and μ^* are strictly decreasing functions of c. If $f_B = 1$, however, the incentives to screen are unaffected because both loan types are rated with certainty, so the rating costs increase by the same amount, and, thus, $\bar{k}^*$ and μ^* are independent of c. If $f_B \in (0, 1)$, there is an extra effect associated with an increase in c. An increase in c reduces the rating incentive of holders of loans of type B, thereby increasing the informativeness of a good rating and the incentive to screen. As lemma 5 shows, this effect dominates other effects, and, thus, $\bar{k}^*$ and μ^* are strictly increasing functions of c if $f_B \in (0, 1)$.

We now analyze the impact of changes in banks' skin in the game, ρ, on screening activity and, therefore, on credit allocation. As lemma 6 shows, changes in ρ have an unambiguous effect on screening and credit allocation. An increase in banks' skin in the game increases directly the payoff to screening.

LEMMA 6 (Comparative statics of $\bar{k}^*$ and μ^* with respect to banks' *skin in the game*, ρ)**.** *In equilibrium, screening effort, $\bar{k}^*$, and the measure of borrowers of type G financed, μ^*, are strictly increasing functions of ρ.*

Finally, we present what may be the most surprising result. An improvement in the initial pool of borrowers, μ_0, may lead to a worse credit allocation, μ^*. On the one hand, an increase in μ_0 has a direct positive effect on μ^*. Ceteris paribus, it increases the measure of borrowers of type G in the economy as equation (3.12) states. On the other hand, there is an indirect effect through banks' screening behavior. The better the ex-ante distribution of borrowers, the weaker banks' screening incentives.

LEMMA 7 (Comparative statics $\bar{k}^*$ and μ^* with respect to the initial distribution of borrowers, μ_0)**.** *Let $\Delta R \equiv R_G - R_B$. The following effects on screening effort $\bar{k}^*$ and the measure of borrowers of type G that get financed, μ^*, hold in equilibrium:*

a) $\bar{k}^*$ is a strictly decreasing function of μ_0.

b) μ^* is a weakly increasing function of μ_0 if and only if

$$1 - F(\bar{k}(\mu^*)) \geq (1 - \mu_0) F_k(\bar{k}(\mu^*)) \Delta R(\mu^*). \tag{3.20}$$

Condition (3.20) captures the relative size of the two effects. The positive direct effect accounts for $1 - F(\bar{k})$, as it operates only through measure $1 - F(\bar{k})$ of banks that lend at random. The negative indirect effect equals $(1 - \mu_0) F_k(\bar{k}) \Delta R$ and accounts for the measure of banks that do not screen anymore—given the increase in μ_0—, $F_k(\bar{k})$, multiplied by the expected loss due to lax screening activity, $(1 - \mu_0) \Delta R$.

3.3 Policy Experiments

This section analyzes mandatory rating and mandatory rating disclosure as two policies that may align the decentralized equilibrium allocation with the constrained efficient one. We find that, contrary to conventional wisdom, both policies worsen credit misallocation.

3.3.1 Mandatory Rating

Consider a mandatory rating policy in which all loans sold in secondary markets must be rated. As a consequence, if a bank does not show its rating, then it means that this bank did not receive a good rating. Within our model, this is equivalent to setting $f_B = f_G = 1$. Let μ^*_{MR} denote the measure of borrowers of type G financed at loan origination under the mandatory rating regime. It then follows from the model equilibrium,

PROPOSITION 11 (Mandatory Rating)**.** *Under the mandatory rating policy, the misallocation of resources worsens, i.e.,*

$$\mu^*_{MR} \leq \mu^* \leq \mu^{ef}.$$

and the following comparative statics results hold:

$$\frac{\partial \mu^*_{MR}}{\partial r} > 0, \quad \frac{\partial \mu^*_{MR}}{\partial \rho} > 0, \quad \frac{\partial \mu^*_{MR}}{\partial c} = 0, \quad \frac{\partial \mu^*_{MR}}{\partial \Delta W} > 0.$$

and $\frac{\partial \mu^*_{MR}}{\partial \mu_0} > 0$ *if and only if inequality* (3.20) *holds.*

Thus, under *mandatory rating*, the rating activity of holders of loans of type B is intensified, thereby decreasing the informativeness of a good rating, which in turn, decreases the premium paid on loans with good ratings, discouraging screening at loan origination and further exacerbating credit misallocation.

3.3.2 Mandatory Rating Disclosure

We now consider a mandatory rating disclosure policy in which banks are free to choose whether or not to rate their loans. However, should a rating be obtained, it must be disclosed. In this case, investors can differentiate among loans with a good rating, loans with a bad rating, and unrated loans, all of which potentially trade at distinct prices that we denote by P_{GR}, P_{BR}, and P_{NR}, respectively.

A bank with a loan of type G chooses to rate it, i.e., $f_G = 1$, if and only if

$$(1-\rho)\left[rP_{GR} + (1-r)P_{BR} - P_{NR}\right] > c. \tag{3.21}$$

As in the benchmark model, we consider only the range of parameter values for which $f_G = 1$. Similarly, a bank with a loan of type B chooses to rate it, i.e., $f_B = 1$, if and only if

$$(1-\rho)\left[(1-r)P_{GR} + rP_{BR} - P_{NR}\right] > c. \tag{3.22}$$

Banks' expected payoffs from lending to a borrower of type $\theta \in \{B, G\}$ are then given by

$$\begin{aligned} R_G &= (1-\rho)\left[f_G\left[rP_{GR} + (1-r)P_{BR} - c\right] + (1-f_G)P_{NR}\right] + \rho W_G - 1, & (3.23) \\ R_B &= (1-\rho)\left[f_B\left[(1-r)P_{GR} + rP_{BR} - c\right] + (1-f_B)P_{NR}\right] + \rho W_B - 1. & (3.24) \end{aligned}$$

For a given measure of loans of type G financed at loan origination, μ, and strategies (f_G, f_B), investors' beliefs must satisfy

$$\begin{aligned} \Pr_{G|GR} &= \frac{\mu r f_G}{\mu r f_G + (1-\mu)f_B(1-r)}, & (3.25) \\ \Pr_{G|BR} &= \frac{\mu(1-r)f_G}{\mu(1-r)f_G + (1-\mu)f_B r}, & (3.26) \\ \Pr_{G|NR} &= \frac{\mu(1-f_G)}{\mu(1-f_G) + (1-\mu)(1-f_B)}. & (3.27) \end{aligned}$$

where $\Pr_{G|GR}$, $\Pr_{G|BR}$ and $\Pr_{G|NR}$ denote the probabilities that a loan with a good, bad, and no rating is a loan of type G, respectively. Competition among investors implies that prices are determined as the expected payoffs. Thus, we have

$$\begin{aligned} P_{GR} &= W_G \Pr_{G|GR} + W_B[1 - \Pr_{G|GR}] = \Delta W \Pr_{G|GR} + W_B, & (3.28) \\ P_{BR} &= W_G \Pr_{G|BR} + W_B[1 - \Pr_{G|BR}] = \Delta W \Pr_{G|BR} + W_B, & (3.29) \\ P_{NR} &= W_G \Pr_{G|NR} + W_B[1 - \Pr_{G|NR}] = \Delta W \Pr_{G|NR} + W_B. & (3.30) \end{aligned}$$

Other than the fact that three types of loans are traded in secondary markets, the equilibrium in the economy with mandatory rating disclosure is defined as in the benchmark model. In other words, taking R_G and R_B—as defined in equations (3.23) and (3.24)—banks make screening decisions. Then, the measure of borrowers of type G finance at equilibrium is found as a solution to $\mu = F(\bar{k}(\mu)) + (1 - F(\bar{k}(\mu)))\mu_0$, where $\bar{k} \equiv (1-\mu_0)(R_G - R_B)$.

Lemma 8 shows that under mandatory rating disclosure f_B is never 0. In other words, banks with loans of type B always weakly prefer to rate their loans.[11] The intuition is as follows. Assume that $f_B = 0$ and $f_G > 0$. In this case, disclosing any rating, whether it is good or bad, indicates that a loan is of type G. As a consequence, loan prices incentivize banks with loans of type B to rate their loan, which contradicts the assumption that $f_B = 0$.

LEMMA 8 (Rating strategy f_B as a function of r, ρ, c, and ΔW)**.** *For a given measure of borrowers of type G financed at loan origination, μ, the rating strategy f_B can be summarized as follows:*

$$f_B = \begin{cases} 1 & \tilde{c} < \frac{\mu r(1-r)}{[\mu r+(1-\mu)(1-r)][\mu(1-r)+(1-\mu)r]} \\ f_B^{mix} \in (0,1) \quad \text{if } \frac{\mu r(1-r)}{[\mu r+(1-\mu)(1-r)][\mu(1-r)+(1-\mu)r]} \leq & \tilde{c} < 1 \end{cases}$$

where the mixed strategy is given by $f_B^{mix} = \mu \left(\frac{r(1-r)(1+2\tilde{c})-\tilde{c}+\sqrt{\tilde{c}^2(1-2r)^2-2\tilde{c}(1-2r)^2 r(1-r)+r^2(1-r)^2}}{2\tilde{c}r(1-r)(1-\mu)} \right)$.

The main result is formalized in proposition 12. We show that, under *mandatory rating disclosure*, the incentive to screen weakens, and credit misallocation gets even worse. Intuitively, holders of loans of type B are encouraged to rate their assets, as the lack of an observable rating can no longer be passed off as an undisclosed false rating. Therefore, rating activity intensifies, thereby reducing the informational value of a good rating, decreasing the expected return to screening, and compounding the credit misallocation in the economy.

PROPOSITION 12 (Mandatory Rating Disclosure)**.** *Let μ^*_{MD} denote the proportion of borrowers of type G financed at loan origination under a mandatory rating disclosure policy. Under the mandatory rating disclosure policy, the resource misallocation worsens, i.e.,*

$$\mu^*_{MD} \leq \mu^*.$$

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[11] An equilibrium in which $f_B = f_G = 0$ is possible, but we do not consider such a case.

3.4 Concluding Remarks

We develop a general equilibrium model to study the interaction of information production in secondary loan markets and screening intensity at loan origination. The model provides insight into why screening efforts at loan origination may be less than optimal, and shows that screening efforts unambiguously decrease as a result of a rise in collateral values, an increase in the fraction of repaying borrowers, a decrease in ratings' precision, and a decrease in banks' skin in the game. The model provides new insight into several pre-2008 financial crisis empirical observations, such as: (1) lax screening standards, (2) intensified ratings shopping, (3) rating inflation, and (4) the decline in the differential between yields on assets with low and high ratings. Finally, we also investigate the role of mandatory rating and mandatory rating disclosure. We find that, at odds with conventional wisdom, both policies may exacerbate credit misallocation.

Appendices

Appendix A

Inter-firm Relationships and Asset Prices

A.1 Proofs

This section contains the proofs of propositions and corollaries in the paper. The following computations consider two assumptions:

- Firm i's output at time $t+1$, $y_{i,t+1}$, follows

  $$\log\left(\frac{y_{i,t+1}}{Y_t}\right) \quad \equiv \quad \alpha_0 + \alpha_1 d_i - \alpha_2 \sqrt{n}\widetilde{\varepsilon}_{i,t+1} \tag{A.1}$$

  where $\widetilde{\varepsilon}_{i,t+1}$ denotes a Bernoulli random variable which equals one if firm i faces a negative shock at $t+1$ and zero otherwise. For a given $\mathcal{G}_n$, parameter d_i denotes firm i's degree. Parameters α_0, α_1 and α_2 are non-negative real numbers.

- Let $\widetilde{x}_{t+1} \equiv \log\left(\frac{Y_{t+1}}{Y_t}\right)$ be the log output growth rate of the economy at time $t+1$, and let $\Delta\widetilde{c}_{t+1} \equiv \log\left(\frac{\widetilde{C}_{t+1}}{C_t}\right)$, be the log aggregate consumption growth rate. The processes for $\widetilde{x}_{t+1}$ and $\Delta\widetilde{c}_{t+1}$ satisfy

  $$\widetilde{x}_{t+1} - x^* = \tau(\Delta\widetilde{c}_{t+1} - c^*) + \sigma_x \widetilde{\xi}_{t+1} \tag{A.2}$$

  where $\bar{x}$ and $\bar{c}$ are real numbers, $\tau > 0$, $\sigma_x > 0$; and $\widetilde{\xi}_{t+1} \xrightarrow{d}$ i.i.d. $\mathcal{N}(0,1)$. Variable $\widetilde{\xi}_{t+1}$ is independent of $\Delta\widetilde{c}_{t+1}$ and $\{\widetilde{\varepsilon}_{i,t+1}\}_{i=1}^{n}$ at $t+1$.

To simplicity notation, define $\bar{x} \equiv x^* - \tau c^*$. Let s_t denote the state of $\widetilde{p}_t$ at period t. Given $\mathcal{G}_n$, s_t determines the distributions of aggregate output and consumption growth at period t. Provided that $\widetilde{p}_t$ varies over time, the distributions of aggregate output and consumption growth vary over time as well, and the dynamics of the moments of these distributions satisfy the Markov property.

Sketch of proof of Proposition 1 and Corollaries 1 and 2. Given a sequence of network topologies $\{\mathcal{G}_n\}_{n=1}^{\infty}$, with limiting topology $\mathcal{G}_\infty$, and the realization of $\widetilde{p}_t$ at time t, the goal is to find the conditions under which $\sqrt{n}\widetilde{W}_{n,t}$ is normally distributed as n grows large.

Without loss of generality, fix t so that subscript t on the sequence $\{\widetilde{\varepsilon}_{i,t}\}_{i=1}^{n}$ can be eliminated. If the sequence of Bernoulli random variables $\{\widetilde{\varepsilon}_i\}_{i=1}^{n}$ is independent, the Lindeberg-Lévy central limit theorem implies that $\sqrt{n}\widetilde{W}_n$ is normally distributed as n grows large. Consequently, if $\widetilde{p}_t = 0$ firms' cash-flows are independent and $\sqrt{n}\widetilde{W}_n$ is asymptotically normally distributed.

In the presence of inter-firm relationships, however, cash-flows of connected firms are correlated if $\widetilde{p}_t > 0$. Despite that the sequence $\{\widetilde{\varepsilon}_i\}_{i=1}^{n}$ may be dependent, $\sqrt{n}\widetilde{W}_n$ may still be asymptotically normally distributed if the dependence among variables $\{\widetilde{\varepsilon}_i\}_{i=1}^{n}$ is sufficiently weak in a sense to be defined.

To better understand the main idea behind the proof, it is illustrative to review statistical concepts such as α-mixing, stationary processes and m-dependent sequences. I do so in what follows. For the sequence $\{\widetilde{\varepsilon}_i\}_{i=1}^{n}$, let α_n be a non-negative number such that

$$\left|\mathbb{P}(A \cap B) - \mathbb{P}(A)\mathbb{P}(B)\right| \quad \leq \quad \alpha_n \tag{A.3}$$

with $A \in \sigma(\widetilde{\varepsilon}_1, \cdots, \widetilde{\varepsilon}_k)$, $B \in \sigma(\widetilde{\varepsilon}_{k+n}, \widetilde{\varepsilon}_{k+n+1}, \cdots)$, $k \geq 1$ and $n \geq 1$; where $\sigma(\cdot)$ denotes the σ-algebra defined on the power set of $\{0,1\}^n \equiv \{0,1\} \times \cdots \times \{0,1\}$. The sequence $\{\widetilde{\varepsilon}_i\}_{i=1}^{n}$ is said to be α-mixing if $\alpha_n \to 0$ as n grows large. In other words, $\widetilde{\varepsilon}_k$ and $\widetilde{\varepsilon}_{k+n}$ are approximately independent for large n. The sequence is said to be stationary if the distribution of $(\widetilde{\varepsilon}_l, \widetilde{\varepsilon}_{l+1}, \cdots, \widetilde{\varepsilon}_{l+j})$ does not depend on l. If $\{\widetilde{\varepsilon}_i\}_{i=1}^{n}$ is α-mixing and stationary, $\sqrt{n}\widetilde{W}_n$ follows a normal distribution as n grows large—see Billingsley, (1995, Theorem 27.4). A special case of the above result occurs if there exists an ordering of the sequence $\{\widetilde{\varepsilon}_i\}_{i=1}^{n}$ such that the dependence between variables $\widetilde{\varepsilon}_k$ and $\widetilde{\varepsilon}_j$ decreases as the distance between them increases in such an ordering. In particular, if there exists such an ordering and a positive $m \geq 0$ such that $(\widetilde{\varepsilon}_1, \cdots, \widetilde{\varepsilon}_k)$ and $(\widetilde{\varepsilon}_{1,k+s}, \cdots, \widetilde{\varepsilon}_{k+s+l})$ are independent whenever $s > m$, the sequence $\{\widetilde{\varepsilon}_i\}_{i=1}^{n}$ is said to be m-dependent in which case $\sqrt{n}\widetilde{W}_n$ follows a normal distribution for large n. An independent sequence is 0-dependent using this terminology.

In what follows, I apply the same idea behind a m-dependent sequence. In particular, I impose that negative shocks tend to remain locally confined as n grows large so that there always exist an index ordering $\mathcal{I}$ that makes the sequence $\{\widetilde{\varepsilon}_i\}_{i\in\mathcal{I}}$ to be m-dependent, in the sense described above.

For a given network topology, let $0 < p_c \leq 1$ be a real number such that for all $\widetilde{p}_t < p_c$, negative shocks only spread over clusters of firms of finite size. Provided that the size of such clusters becomes negligible compared to the size of the economy as n grows large, and there is an infinite number of small clusters, all independent among each other, $\sqrt{n}\widetilde{W}_n$ is normally distributed as n grows large. For instance, define m as the largest expected diameter of such clusters and the corresponding index ordering $\mathcal{I}$ such that whenever $s > m$, $(\widetilde{\varepsilon}_1, \cdots, \widetilde{\varepsilon}_k)$ and $(\widetilde{\varepsilon}_{1,k+s}, \cdots, \widetilde{\varepsilon}_{k+s+l})$ are independent in $\{\widetilde{\varepsilon}_i\}_{i\in\mathcal{I}}$.

To find the threshold p_c, it is illustrative to compute the probability that at least one negative shock spreads over $n-1$ different firms. Let P_n denote such a probability. Given how shocks spread from one firm to another, P_n

equals

$$
\begin{aligned}
P_n &= (1-(1-q)^n)\,\mathbb{P}\,[\text{there is at least one open walk connecting } n \text{ firms }] \qquad \text{(A.4)}\\
&\approx (1-e^{-nq})\mathbb{P}\,[\text{there is at least one open walk connecting } n \text{ firms }] \quad (\text{for large } n)
\end{aligned}
$$

where q is the probability that a firm faces a negative idiosyncratic shock. A walk is a sequence of relationships which connect a sequence of firms that may not be all distinct from one another. A walk is considered to be open at t if all the relationships that compose the walk transmit negative shocks at t.

I focus on the limit of P_n as n grows large. Provided that $0 < q < 1$, the first term in the right-hand side of (A.4) tends to 1 as $n \to \infty$ at an exponential rate. As a consequence, if the second term in the right-hand side of (A.4) tends to 0 as n grows large, negative idiosyncratic shocks tend to remain locally confined since, almost surely, no firm belongs to an infinite open walk. Then, to determine the conditions under which a CLT-type of result applies is related to determine the probability, as $n \to \infty$, that a given firm belongs to an infinite open walk. Given a sequence $\{\mathcal{G}_n\}_n$, with limiting distribution $\mathcal{G}_\infty$, define p_c as

$$
p_c(\mathcal{G}_\infty) = \sup_{p\in(0,1)} \left\{p : \lim_{n\to\infty} P_n = 0\right\} \qquad \text{(A.5)}
$$

I write $p_c = p_c(\mathcal{G}_\infty)$ since p_c may depend on the network topology in the limit. Therefore, if $\widetilde{p}_t < p_c$ then $\sqrt{n}\widetilde{W}_n$ follows a normal distribution as n grows large since all open walks are almost surely finite and their size distribution has a tail which tend to decrease with n sufficiently fast.[1]

To prove normality, condition $\widetilde{p}_t < p_c$ may be stronger than necessary. Imposing such a condition, however, greatly facilitates the proof since the determination of p_c has been extensively studied in percolation theory, e.g. Grimmett, 1989 and Stauffer and Aharony, 1994. In percolation, p_c is sometimes called the critical probability or critical phenomenon of the model, because it indicates the arrival of an infinite connected component as $n \to \infty$ within a particular model.

To illustrate how p_c can be determined, consider the following two simple examples:

- Imagine n firms are arranged in a straight line and each relationship may transmit shocks with probability p. The probability that the line is open is p^n, which tends to zero as $n \to \infty$, so that $p_c = 1$.
- Suppose n firms are arranged in a circle. The probability that the circle is open tends to zero as $n \to \infty$. Think about putting the endpoints of an infinitely line together. Thus, $p_c = 1$.

Taking results from bond percolation, Table A.1 reports critical probabilities for several symmetric network topologies. As Table A.1 shows, p_c varies across networks. For instance, if $\mathcal{G}_\infty$ is the two dimensional honeycomb lattice then $p_c = 1 - 2\sin\left(\frac{\pi}{18}\right) \approx 0.65$ whereas if $\mathcal{G}_\infty$ is the two dimensional square lattice then $p_c = \frac{1}{2}$.

The previous analysis determines conditions under which $\sqrt{n}\widetilde{W}_n$ is normally distributed for some large symmetric networks. But what happens in other network topologies? In particular, under what conditions is $\sqrt{n}\widetilde{W}_n$ asymptotically normally distributed in large asymmetric networks? Using random walks on trees, Lyons, 1990 shows that if $\mathcal{G}_\infty$ is a tree then

---

[1] For instance, if $\mathcal{G}_n = \mathbb{L}^d$, where $\mathbb{L}^d$ represents the d-dimensional lattice, the probability that an open walk has size n is proportional to $\exp(-\zeta(p)n)$—see Grimmett, (1989, Chapters 5 and 7).

$$p_c = \frac{1}{\text{branching number of } \mathcal{G}_\infty} \tag{A.6}$$

where the branching number of a tree is the average number of branches per node in a tree.[2] A tree is a connected graph in which two given nodes are connected by exactly one path. A tree is said to be z-regular if each node has degree z. If $\mathcal{G}_\infty$ is an z-regular tree, the average number of branches per node is $z-1$ so $p_c = \frac{1}{z-1}$; which is consistent with Table A.1.[3]

One can generalize the previous result for topologies where $\mathcal{G}_\infty$ is sparse and locally treelike. G_n is said to be sparse if G_n has m edges and $m = O(n)$. Notation $m = O(n)$ indicates that m grows, at most, linearly with n so there exists a positive number c such that $\left|\frac{m}{n}\right| < c$ for all n. Namely, G_n is sparse if only a small fraction of the possible $\frac{n(n-1)}{2}$ edges are present. G_∞ is said to be locally treelike if in the limit an arbitrarily large neighborhood around any node takes the form of a tree. Using the previous idea and reformulating percolation in trees as a message passing process, Karrer, Newman, and Zdeborová, 2014 shows that if $\mathcal{G}_\infty$ is sparse and locally treelike then

$$p_c = \frac{1}{\epsilon_H} \tag{A.7}$$

where ϵ_H is the leading eigenvalue of the $2n \times 2n$ matrix

$$M = \begin{pmatrix} A & \mathbb{I} - D \\ \mathbb{I} & 0 \end{pmatrix} \tag{A.8}$$

where A is the adjacency matrix that represents $\mathcal{G}_n$, $\mathbb{I}$ is the $n \times n$ identity matrix, and D is the diagonal matrix with the number of relationships per firm along the diagonal, e.g. Karrer, Newman, and Zdeborová, 2014. Parameter e_H is always real. For a sparse network this matrix is also sparse, with only $2m + 2n$ nonzero elements, which permits rapid numerical calculation of the leading eigenvalue. For the network that characterize the benchmark economy one obtains

$$e_H = 1 \quad \rightarrow \quad p_c \approx 1 \tag{A.9}$$

$$\text{branching number} = 1.185 \quad \rightarrow \quad p_c \approx 0.85 \tag{A.10}$$

□

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[2] For a concrete definition of the branching number see Lyons, (1990, page 935).

[3] To motivate the previous result, it is informative to compute the percolation threshold in the Bethe lattice with z neighbors per every node. Start at the root and check whether there is a chance of finding an infinite open path from the root. Starting from the root, one has $(z-1)$ new edges emanating from each new node in each layer of the lattice. Each of these $(z-1)$ new edges leads to one new node, which is affected with probability p. On average, $(z-1)p$ nodes are affected at each layer of the lattice. If $(z-1)p < 1$ then the average number of affected nodes decreases in each layer by a factor of $(z-1)p$. As a consequence, if $(z-1)p < 1$ the probability of finding an infinite open path goes to zero exponentially in the path length. Thus, $p_c = \frac{1}{z-1}$ for the Bethe lattice with z neighbors for every node.

Proof of Proposition 2. I look for an equilibrium such that the price dividend ratio is stationary. I conjecture that if c is the current aggregate consumption and s the current state of $\widetilde{p}_t$, then $P_a(c,s) = w^a_s c$, in which P_a is the price of aggregate wealth and w^a_s a number that depends on state s. If $s_t = s$ and $s_{t+1} = s'$, the realized gross return at time $t+1$ of the asset that delivers aggregate consumption as its dividend each period, $\widetilde{R}_{a,t+1}$, equals

$$\widetilde{R}_{a,t+1} = \frac{\widetilde{P}_{a,t+1} + \widetilde{C}_{t+1}}{P_{a,t}} = \frac{w^a_{s'} + 1}{w^a_s}\frac{\widetilde{C}_{t+1}}{C_t} \tag{A.11}$$

Setting $\widetilde{R}_{i,t+1} = \widetilde{R}_{a,t+1}$ in equation (1.8) yields,

$$\begin{aligned} \mathbb{E}_t\left(\left[\beta\left(\frac{\widetilde{C}_{t+1}}{C_t}\right)^{-\rho}\right]^{\frac{1-\gamma}{1-\rho}}\left[\widetilde{R}_{a,t+1}\right]^{\frac{1-\gamma}{1-\rho}}\right) &= 1 \\ \Rightarrow \mathbb{E}\left(\left[\beta\left(\frac{\widetilde{C}_{t+1}}{C_t}\right)^{-\rho}\right]^{\frac{1-\gamma}{1-\rho}}\left[\frac{w^a_{s'}+1}{w^a_s}\frac{\widetilde{C}_{t+1}}{C_t}\right]^{\frac{1-\gamma}{1-\rho}}\middle| p_s\right) &= 1 \end{aligned} \tag{A.12}$$

Provided that s_t follows a Markov process, equation (A.12) can be rewritten as

$$\beta^{\frac{1-\gamma}{1-\rho}}\left(\sum_{s'=H,L}\omega_{s,s'}\mathbb{E}\left(\left(\frac{\widetilde{C}_{t+1}}{C_t}\right)^{1-\gamma}\middle| p_{s'}\right)\left(\frac{w^a_{s'}+1}{w^a_s}\right)^{\frac{1-\gamma}{1-\rho}}\right) = 1 \tag{A.13}$$

Reordering equation (A.13) yields,

$$w^a_s = \beta\left(\sum_{s'=H,L}\omega_{s,s'}\mathbb{E}\left(e^{(1-\gamma)\Delta\widetilde{c}_{t+1}}\middle| p_{s'}\right)(w^a_{s'}+1)^{\frac{1-\gamma}{1-\rho}}\right)^{\frac{1-\rho}{1-\gamma}} \qquad s = H, L \tag{A.14}$$

which completes the proof.

REMARK 2. *If $\sqrt{n}\widetilde{W}_{n,t+1}$ is normally distributed, then*

$$\mathbb{E}\left(e^{(1-\gamma)\Delta\widetilde{c}_{t+1}}\middle| s\right) = \exp\left(\frac{(1-\gamma)(\alpha_0 + \alpha_1\bar{d} - \alpha_2\mu_s - \bar{x})}{\tau} + \frac{(1-\gamma)^2}{2}\left(\frac{\alpha_2^2\sigma_s^2 - \sigma_x^2}{\tau^2}\right)\right) \qquad s = H, L \tag{A.15}$$

where

$$\begin{aligned} \mu_H &\equiv \lim_{n\to\infty}\mathbb{E}\left(\sum_{i=1}^n\frac{\widetilde{\varepsilon}_{i,t+1}}{\sqrt{n}}\middle|\widetilde{p}_{t+1} = p_H\right) \quad \text{and} \quad \sigma_H^2 \equiv \lim_{n\to\infty} Var\left(\sum_{i=1}^n\frac{\widetilde{\varepsilon}_{i,t+1}}{\sqrt{n}}\middle|\widetilde{p}_{t+1} = p_H\right) \\ \mu_L &\equiv \lim_{n\to\infty}\mathbb{E}\left(\sum_{i=1}^n\frac{\widetilde{\varepsilon}_{i,t+1}}{\sqrt{n}}\middle|\widetilde{p}_{t+1} = p_L\right) \quad \text{and} \quad \sigma_L^2 \equiv \lim_{n\to\infty} Var\left(\sum_{i=1}^n\frac{\widetilde{\varepsilon}_{i,t+1}}{\sqrt{n}}\middle|\widetilde{p}_{t+1} = p_L\right) \end{aligned}$$

and the above constants are assumed to be finite so that equation (A.15) is well-defined.

REMARK 3 (Price of Market Return)**.** *If consumption and output growth differ I compute the price of the market return as follows. I conjecture that if y is the current aggregate output and s the current state of $\widetilde{p}_t$, then $P_m(c,s) = w^m_s y$, where P_m is the price of the market portfolio and w^m_s a number that depends on state s. If $s_t = s$ and $s_{t+1} = s'$, then the realized gross return at time $t+1$ of the asset that delivers aggregate output as its dividend each period,*

$\widetilde{R}_{m,t+1}$, equals

$$\widetilde{R}_{m,t+1} \quad = \quad \frac{\widetilde{P}_{m,t+1}+Y_{t+1}}{P_{m,t}} \quad = \quad \frac{w^m_{s'}+1}{w^m_s}\frac{Y_{t+1}}{Y_t} \tag{A.16}$$

Setting $\widetilde{R}_{i,t+1}=\widetilde{R}_{m,t+1}$ in equation (1.8) yields,

$$\begin{aligned}
\mathbb{E}_t\left(\left[\beta\left(\frac{\widetilde{C}_{t+1}}{C_t}\right)^{-\rho}\right]^{\frac{1-\gamma}{1-\rho}}\left[\widetilde{R}_{a,t+1}\right]^{\frac{1-\gamma}{1-\rho}-1}\widetilde{R}_{m,t+1}\right) \quad &= \quad 1 \\
\Rightarrow \mathbb{E}\left(\left[\beta\left(\frac{\widetilde{C}_{t+1}}{C_t}\right)^{-\rho}\right]^{\frac{1-\gamma}{1-\rho}}\left[\frac{w^a_{s'}+1}{w^a_s}\left(\frac{\widetilde{C}_{t+1}}{C_t}\right)\right]^{\frac{1-\gamma}{1-\rho}-1}\left(\frac{w^m_{s'}+1}{w^m_s}\widetilde{X}_{t+1}\right)\Big| p_s\right) \quad &= \quad 1 \qquad \text{(A.17)}
\end{aligned}$$

where $\widetilde{X}_{t+1}=\frac{Y_{t+1}}{Y_t}$. Provided that s_t follows a Markov process, equation (A.17) can be rewritten as

$$\beta^{\frac{1-\gamma}{1-\rho}}\left(\sum_{s'=\{H,L\}}\omega_{s,s'}\mathbb{E}\left(\left(\frac{\widetilde{C}_{t+1}}{C_t}\right)^{-\gamma}\widetilde{X}_{t+1}\Big|p_{s'}\right)\left(\frac{w^a_{s'}+1}{w^a_s}\right)^{\frac{1-\gamma}{1-\rho}-1}\left(\frac{w^m_{s'}+1}{w^m_s}\right)\right) \quad = \quad 1 \tag{A.18}$$

Reordering equation (A.18) yields,

$$w^m_s \quad = \quad \beta^{\frac{1-\gamma}{1-\rho}}\left(\sum_{s'\in\{H,L\}}\omega_{s,s'}\mathbb{E}\left(e^{-\gamma\Delta\widetilde{c}_{t+1}+\widetilde{x}_{t+1}}\big|p_{s'}\right)\left(\frac{w^a_{s'}+1}{w^a_s}\right)^{\frac{1-\gamma}{1-\rho}-1}(w^m_{s'}+1)\right) \qquad s=\{H,L\} \tag{A.19}$$

It follows from (A.2) that $-\gamma\Delta\widetilde{c}_{t+1}+\widetilde{x}_{t+1}=\bar{x}+(\tau-\gamma)\Delta\widetilde{c}_{t+1}+\sigma_x\widetilde{\xi}_{t+1}$. Therefore, (A.19) equals to

$$w^m_s \quad = \quad \beta^{\frac{1-\gamma}{1-\rho}}e^{\bar{x}+\frac{\sigma_x^2}{2}}\left(\sum_{s'\in\{H,L\}}\omega_{s,s'}\mathbb{E}\left(e^{(\tau-\gamma)\Delta\widetilde{c}_{t+1}}\big|p_{s'}\right)\left(\frac{w^a_{s'}+1}{w^a_s}\right)^{\frac{1-\gamma}{1-\rho}-1}(w^m_{s'}+1)\right) \qquad s=\{H,L\} \tag{A.20}$$

□

Proof of Proposition 3. Setting $\widetilde{R}_{i,t+1}=R_f$ in equation (1.8) yields,

$$\mathbb{E}\left(\left[\beta\left(\frac{\widetilde{C}_{t+1}}{C_t}\right)^{-\rho}\right]^{\frac{1-\gamma}{1-\rho}}\left[\widetilde{R}_{a,t+1}\right]^{\frac{1-\gamma}{1-\rho}-1}\Big|p_s\right) \quad = \quad \frac{1}{R_f(s)}, \qquad s=\{H,L\}\,. \tag{A.21}$$

Provided that s_t follows a Markov process and $P_a(c,s)=w^a_s c$, the left hand side of equation (A.21) can be rewritten as the following sum

$$\beta^{\frac{1-\gamma}{1-\rho}}\left(\sum_{s'=H,L}\omega_{s,s'}\mathbb{E}\left(\left(\frac{\widetilde{C}_{t+1}}{C_t}\right)^{-\gamma}\Big|p_{s'}\right)\left(\frac{w^a_{s'}+1}{w^a_s}\right)^{\frac{\rho-\gamma}{1-\rho}}\right)$$

Therefore,

$$\frac{1}{R_f(s)} \quad = \quad \beta^{\frac{1-\gamma}{1-\rho}}\left(\sum_{s'=H,L}\omega_{s,s'}\mathbb{E}\left(e^{-\gamma\Delta\widetilde{c}_{t+1}}\big|p_{s'}\right)\left(\frac{w^a_{s'}+1}{w^a_s}\right)^{\frac{\rho-\gamma}{1-\rho}}\right), \qquad s=\{H,L\}$$

which completes the proof ☐

Proof of Proposition 4. Consider $s_t = s$ and $s_{t+1} = s'$. Equation (1.8) can be rewritten as,

$$P_{i,t} \quad = \quad \mathbb{E}_t\left(\widetilde{M}_{t+1}\left(\widetilde{P}_{i,t+1} + y_{i,t+1}\right)\right) \qquad i = 1, \cdots, n \tag{A.22}$$

where

$$\widetilde{M}_{t+1} \quad \equiv \quad \left[\beta\left(\frac{\widetilde{C}_{t+1}}{C_t}\right)^{-\rho}\right]^{\frac{1-\gamma}{1-\rho}} \left[\widetilde{R}_{a,t+1}\right]^{\frac{1-\gamma}{1-\rho}-1}$$

represents the pricing kernel. Dividing equation (A.22) by Y_t yields

$$\frac{P_{i,t}}{Y_t} \quad = \quad \mathbb{E}_t\left(\widetilde{M}_{t+1}\widetilde{X}_{t+1}\frac{\widetilde{P}_{i,t+1}}{Y_{t+1}}\right) + \mathbb{E}_t\left(\widetilde{M}_{t+1}\frac{y_{i,t+1}}{Y_t}\right) \qquad i = 1, \cdots, n \tag{A.23}$$

which can be rewritten as

$$v_{i,t} \quad = \quad \mathbb{E}_t\left(\widetilde{M}_{t+1}\widetilde{X}_{t+1}v_{i,t+1}\right) + \mathbb{E}_t\left(\widetilde{M}_{t+1}\frac{y_{i,t+1}}{Y_t}\right) \qquad i = 1, \cdots, n \tag{A.24}$$

with $v_{i,t} \equiv v_i(s) \equiv \frac{P_{i,t}}{Y_t}$. Provided that s_t follows a Markov process and $P_a(c,s) = w_s^a c$, the first term in the right hand side of equation (A.24) can be rewritten as

$$\mathbb{E}_t\left(\widetilde{M}_{t+1}\widetilde{X}_{t+1}v_{i,t+1}\right) \quad = \quad \beta^{\frac{1-\gamma}{1-\rho}} e^{\bar{x}+\frac{\sigma_x^2}{2}} \left(\sum_{s'=H,L} \omega_{s,s'} \left(\frac{w_{s'}^a + 1}{w_s^a}\right)^{\frac{\rho-\gamma}{1-\rho}} \mathbb{E}\left(e^{(\tau-\gamma)\Delta\tilde{c}_{t+1}} \middle| p_{s'}\right) v_i(s')\right) \tag{A.25}$$

whereas the second term in the right hand side of equation (A.24) can be rewritten as

$$\mathbb{E}_t\left(\widetilde{M}_{t+1}\frac{y_{i,t+1}}{Y_t}\right) \quad = \quad e^{\alpha_0+\alpha_1 d_i}\mathbb{E}_t\left(\widetilde{M}_{t+1}e^{-\alpha_2\sqrt{n}\tilde{\varepsilon}_{i,t+1}}\right) \tag{A.26}$$

The expectation term in the right hand side of equation (A.26) can be written as

$$\begin{aligned}
\mathbb{E}_t\left(\widetilde{M}_{t+1}e^{-\alpha_2\sqrt{n}\tilde{\varepsilon}_{i,t+1}}\right) \quad &= \quad \beta^{\frac{1-\gamma}{1-\rho}}\left(\sum_{s'=H,L}\omega_{s,s'}\mathbb{E}\left(\left(\frac{\widetilde{C}_{t+1}}{C_t}\right)^{-\gamma} e^{-\alpha_2\sqrt{n}\tilde{\varepsilon}_{i,t+1}}\middle| p_{s'}\right)\left(\frac{w_{s'}^a+1}{w_s^a}\right)^{\frac{\rho-\gamma}{1-\rho}}\right) \\
&= \quad \beta^{\frac{1-\gamma}{1-\rho}}\left(\sum_{s'=H,L}\omega_{s,s'}\mathbb{E}\left(e^{-\gamma\Delta\tilde{c}_{t+1}-\alpha_2\sqrt{n}\tilde{\varepsilon}_{i,t+1}}\middle| p_{s'}\right)\left(\frac{w_{s'}^a+1}{w_s^a}\right)^{\frac{\rho-\gamma}{1-\rho}}\right)
\end{aligned}$$

As a consequence,

$$\begin{aligned}
v_i(s) \quad &= \quad \beta^{\frac{1-\gamma}{1-\rho}} e^{\bar{x}+\frac{\sigma_x^2}{2}} \left(\sum_{s'=H,L} \omega_{s,s'} \left(\frac{w_{s'}^a + 1}{w_s^a}\right)^{\frac{\rho-\gamma}{1-\rho}} \mathbb{E}\left(e^{(\tau-\gamma)\Delta\tilde{c}_{t+1}} \middle| p_{s'}\right) v_i(s')\right) \\
&+ \quad \beta^{\frac{1-\gamma}{1-\rho}} e^{\alpha_0+\alpha_1 d_i}\left(\sum_{s'=H,L}\omega_{s,s'}\mathbb{E}\left(e^{-\gamma\Delta\tilde{c}_{t+1}-\alpha_2\sqrt{n}\tilde{\varepsilon}_{i,t+1}}\middle| p_{s'}\right)\left(\frac{w_{s'}^a+1}{w_s^a}\right)^{\frac{\rho-\gamma}{1-\rho}}\right) \qquad i = 1, \cdots, n
\end{aligned}$$

Define $\pi_i(s') \equiv \mathbb{E}\left[\widetilde{\varepsilon}_{i,t+1} | s_{t+1} = s'\right]$. It is worth noting that

$$\begin{aligned} -\gamma\Delta\widetilde{c}_{t+1} - \alpha_2\sqrt{n}\widetilde{\varepsilon}_{i,t+1} &= -\gamma \underbrace{\left(\frac{1}{\tau}\left\{\alpha_0 + \alpha_1\bar{d} - \alpha_2\left(\sum_{j\neq i}\frac{\widetilde{\varepsilon}_{j,t+1}}{\sqrt{n}}\right) - \sigma_x\widetilde{\xi}_{t+1} - \bar{x}\right\}\right)}_{\Delta\widetilde{c}_{-i,t+1}} - \alpha_2\sqrt{n}\left(1 - \frac{\gamma}{\tau n}\right)\widetilde{\varepsilon}_{i,t+1} \\ &= -\gamma \quad \Delta\widetilde{c}_{-i,t+1} \quad - \alpha_2\sqrt{n}\left(1 - \frac{\gamma}{\tau n}\right)\widetilde{\varepsilon}_{i,t+1} \end{aligned} \tag{A.27}$$

Since $\Delta\widetilde{c}_{-i,t+1}$ and $\widetilde{\varepsilon}_{i,t+1}$ are independent

$$\begin{aligned} \mathbb{E}\left(e^{-\gamma\Delta\widetilde{c}_{-i,t+1} - \alpha_2\sqrt{n}\left(1-\frac{\gamma}{\tau n}\right)\widetilde{\varepsilon}_{i,t+1}}\middle| s\right) &= \mathbb{E}\left(e^{-\gamma\Delta\widetilde{c}_{-i,t+1}}\middle| p_s\right)\left\{\pi_i(s)e^{-\alpha_2\sqrt{n}\left(1-\frac{\gamma}{\tau n}\right)} + (1-\pi_i(s))\right\} \\ &\approx \mathbb{E}\left(e^{-\gamma\Delta\widetilde{c}_{t+1}}\middle| p_s\right)(1-\pi_i(s)) \end{aligned} \tag{A.28}$$

where the last approximation is accurate for large n. If the distribution of $\Delta\widetilde{c}_{t+1}$ is known, the expectation of $-\gamma\Delta\widetilde{c}_{t+1} - \alpha_2\sqrt{n}\widetilde{\varepsilon}_{i,t+1}$ can be approximated using equation (A.28). Therefore,

$$\begin{aligned} v_i(s) &= \beta^{\frac{1-\gamma}{1-\rho}} e^{\bar{x}+\frac{\sigma_x^2}{2}}\left(\sum_{s'=H,L}\omega_{s,s'}\left(\frac{w^a_{s'}+1}{w^a_s}\right)^{\frac{\rho-\gamma}{1-\rho}}\mathbb{E}\left(e^{(\tau-\gamma)\Delta\widetilde{c}_{t+1}}\middle| p_{s'}\right)v_i(s')\right) \\ &+ \beta^{\frac{1-\gamma}{1-\rho}} e^{\alpha_0+\alpha_1 d_i}\left(\sum_{s'=H,L}\omega_{s,s'}\mathbb{E}\left(e^{-\gamma\Delta\widetilde{c}_{t+1}}\middle| p_s\right)(1-\pi_i(s))\left(\frac{w^a_{s'}+1}{w^a_s}\right)^{\frac{\rho-\gamma}{1-\rho}}\right) \qquad i = 1,\cdots,n \end{aligned}$$

which completes the proof $\square$

Proof of Proposition 5. Recall

$$\mathrm{Var}\left(\widetilde{M}_{t+1}\middle| s\right) = \mathbb{E}\left(\widetilde{M}^2_{t+1}\middle| s\right) - \mathbb{E}^2\left(\widetilde{M}_{t+1}\middle| s\right) \tag{A.29}$$

The first term in the right hand side of equation (A.29) can be rewritten as

$$\mathbb{E}\left(\widetilde{M}^2_{t+1}\middle| s\right) = \beta^{2\left(\frac{1-\gamma}{1-\rho}\right)}\left(\sum_{s'=H,L}\omega_{s,s'}\mathbb{E}\left(\left(\frac{\widetilde{C}_{t+1}}{C_t}\right)^{-2\gamma}\middle| p_{s'}\right)\left(\frac{w^a_{s'}+1}{w^a_s}\right)^{2\left(\frac{\rho-\gamma}{1-\rho}\right)}\right) \tag{A.30}$$

Provided that $\lambda_m(s) \equiv -\frac{\mathrm{Var}\left(\widetilde{M}_{t+1}\middle| s\right)}{\mathbb{E}\left(\widetilde{M}_{t+1}\middle| s\right)}$ and $\mathbb{E}\left(\widetilde{M}_{t+1}\middle| s\right) = \frac{1}{R_f(s)}$, it then follows from equation (A.29) that

$$\lambda_m(s) = \frac{1}{R_f(s)} - R_f(s)\left(\beta^{2\left(\frac{1-\gamma}{1-\rho}\right)}\sum_{s'=H,L}\omega_{s,s'}\left(\frac{w^a_{s'}+1}{w^a_s}\right)^{2\left(\frac{\rho-\gamma}{1-\rho}\right)}\mathbb{E}\left(e^{-2\gamma\Delta\widetilde{c}_{t+1}}\middle| p_{s'}\right)\right), \qquad s = \{H, L\}$$

which completes the proof $\square$

A.2 Simulation of the Model

This section describes the algorithm I use to compute firms' probabilities of facing negative shocks in each state of nature so one can compute asset prices and returns at the firm level using proposition 4. Let s_t denote the state of $\widetilde{p}_t$ at period t. To simplify the computation of probabilities $\{\pi_i(s_t)\}_{i=1}^n$, I restrict the topology of $\mathcal{G}_n$. In general topologies, computing $\{\pi_i(s_t)\}_{i=1}^n$ is hard, because the number of states that need to be considered increases exponentially with n. In economies with no cycles, however, computing $\{\pi_i(s_t)\}_{i=1}^n$ is easier. In those economies, computing $\{\pi_i(s_t)\}_{i=1}^n$ can be framed as a recursive problem as the following algorithm describes.

```
Algorithm Firms Probabilities ($G_n$, $\widetilde{p}_t, q$)
(* Description: Algorithm that computes firms' probabilities of facing negative shocks if $G_n$ is a forest *)
Input: $G_n$ (a forest), $\widetilde{p}_t$, $q$.
Output: The set of probabilities of firms facing a negative shock at time $t$, $\{\pi_i(s_t)\}_{i=1}^n$
1.  for each firm $i \in G_n$
2.          Determine the subgraph of $G_n$ wherein firm $i$ participates. Denote such a graph as $T_i$ and label firm $i$ as its root.[4]
3.          if firm $i$ has a no connections
4.                  return $\pi_i(s_t) = q$
5.            else  return Prob(i,$T_i$,$\widetilde{p}_t$,$q$)
```

where Prob(i,T_i,$\widetilde{p}_t$,q) corresponds to the following recursive program,

```
Algorithm Prob(i,$T_i$,$\widetilde{p}_t$,$q$)
(* Description: Recursive algorithm that computes firm $i$'s probability of facing a negative shock *)
Input: A node $i$ in $\mathcal{G}_n$, the tree $T_i$ wherein node $i$ is the root, $\widetilde{p}_t$ and $q$.
Output: $\pi_i(s_t)$
1.  Determine the set of children of node $i$ in $T_i$, say $\mathcal{C}_i$.[5]
2.  if $\mathcal{C}_i = \emptyset$
3.          return $\pi_i(s_t) = q$
4.     else  if every node in $\mathcal{C}_i$ has no children
5.                  return $\pi_i(s_t) = q + (1-q)\left(1 - (1-\widetilde{p}_t q)^{|\mathcal{C}_i|}\right)$
6.             else  return $\pi_i(s_t) = q + (1-q)\left(1 - \prod_{k\in\mathcal{C}_i}(1-\widetilde{p}_t \text{Prob}(k, T_{i,k}, \widetilde{p}_t, q))\right)$ [6]
```

where $\mathcal{C}_i$ denotes the cardinality of set $\mathcal{C}_i$.

---

[4] Note that such a graph is a tree provided that G_n is a forest.

[5] In a rooted tree, the parent of a node is the node connected to it on the path to the root. Every node except the root has a unique parent. A child of a node v is a node of which v is the parent.

[6] Tree $T_{i,k}$ denotes the branch of tree T_i that starts at node k.

In economies with no cycles, it is also simple to compute the first two moments of the distribution of $\sqrt{n}\widetilde{W}_{n,t+1}$ at $t+1$. Let μ_s, σ_s^2 denote the mean and variance of $\sqrt{n}\widetilde{W}_{n,t+1}$ if $s_{t+1} = s$, respectively. In other words,

$$\mu_s = \lim_{n\to\infty} \mathbb{E}\left(\sum_{i=1}^{n} \frac{\widetilde{\varepsilon}_{i,t+1}}{\sqrt{n}}\middle| s\right) = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{\pi_i(s)}{\sqrt{n}} \qquad s = L, H \tag{A.31}$$

and

$$\begin{aligned} \sigma_s^2 &= \lim_{n\to\infty} \operatorname{Var}\left(\sum_{i=1}^{n} \frac{\widetilde{\varepsilon}_{i,t+1}}{\sqrt{n}}\middle| s\right) \\ &= \lim_{n\to\infty} \left\{ \frac{1}{n}\sum_{i=1}^{n} \pi_i(s)\left(1-\pi_i(s)\right) + \frac{1}{n}\sum_{(i,j)\in\mathcal{R}_n} \operatorname{Cov}\left(\widetilde{\varepsilon}_{i,t+1}, \widetilde{\varepsilon}_{j,t+1}\middle| s_t = s\right)\right\} \qquad s = L, H \end{aligned} \tag{A.32}$$

The second equation can be simplified further. If there exists a path between firm i and j after edges are removed at time $t+1$ then $\widetilde{\varepsilon}_{i,t+1} = \widetilde{\varepsilon}_{j,t+1}$. If there is no path between firm i and j in $\mathcal{G}_n$, variables $\widetilde{\varepsilon}_{i,t+1}$ and $\widetilde{\varepsilon}_{j,t+1}$ are independent. It then follows,

$$\begin{aligned} \mathbb{E}_t\left[\widetilde{\varepsilon}_{i,t}\widetilde{\varepsilon}_{j,t}\middle| \text{there is a path between } i \text{ and } j\right] &= \operatorname{Var}_t\left[\widetilde{\varepsilon}_{i,t}\right] + \mathbb{E}_t^2\left[\widetilde{\varepsilon}_{i,t}\right] = \pi_i(s)(1-\pi_i(s)) + \pi_i^2(s) \\ \mathbb{E}_t\left[\widetilde{\varepsilon}_{i,t}\widetilde{\varepsilon}_{j,t}\middle| \text{there is no path between } i \text{ and } j\right] &= \mathbb{E}_t\left[\widetilde{\varepsilon}_{i,t}\right]\mathbb{E}_t\left[\widetilde{\varepsilon}_{j,t}\right] = \pi_i(s)\pi_j(s) \end{aligned}$$

Hence,

$$\begin{aligned} \mathbb{E}_t\left[\widetilde{\varepsilon}_{i,t}\widetilde{\varepsilon}_{j,t}\right] &= \mathbb{E}_t\left[\widetilde{\varepsilon}_{i,t}\widetilde{\varepsilon}_{j,t}\middle| \text{there is a path between } i \text{ and } j\right] \mathbb{P}\left[\text{there is a path between } i \text{ and } j \text{ at } t\right] \\ &+ \mathbb{E}_t\left[\widetilde{\varepsilon}_{i,t}\widetilde{\varepsilon}_{j,t}\middle| \text{there is no path between } i \text{ and } j\right] \mathbb{P}\left[\text{there is no path between } i \text{ and } j \text{ at } t\right] \\ &= \left(\pi_i(s)(1-\pi_i(s)) + \pi_i^2(s)\right)\mathbf{P}_{ij}(s) + \pi_i(s)\pi_j(s)\left(1-\mathbf{P}_{ij}(s)\right) \end{aligned}$$

where $\mathbf{P}_{ij}(s) \equiv \mathbb{P}\left[\text{there is a path between } i \text{ and } j \text{ if } s_t = s\right]$. Thus,

$$\begin{aligned} \operatorname{Cov}_t\left[\widetilde{\varepsilon}_{i,t}, \widetilde{\varepsilon}_{j,t}\right] &= \mathbb{E}_t\left[\widetilde{\varepsilon}_{i,t}\widetilde{\varepsilon}_{j,t}\right] - \mathbb{E}_t\left[\widetilde{\varepsilon}_{i,t}\right]\mathbb{E}_t\left[\widetilde{\varepsilon}_{j,t}\right] \\ &= \left(\pi_i(s)(1-\pi_i(s)) + \pi_i^2(s)\right)\mathbf{P}_{ij}(s) + \pi_i(s)\pi_j(s)\left(1-\mathbf{P}_{ij}(s)\right) - \pi_i(s)\pi_j(s) \\ &= \pi_i(s)\left(1-\pi_j(s)\right)\mathbf{P}_{ij}(s) \end{aligned}$$

Therefore,

$$\sigma_s^2 = \lim_{n\to\infty} \left\{ \frac{1}{n}\sum_{i=1}^{n} \pi_i(s)\left(1-\pi_i(s)\right) + \frac{1}{n}\sum_{(i,j)\in\mathcal{R}_n} \pi_i(s)\left(1-\pi_j(s)\right)\mathbf{P}_{ij}(s)\right\} \qquad s = L, H$$

To compute $\mathbf{P}_{ij}(s)$ I need to determine the set of paths that connect firms i and j on $\mathcal{G}_n$. If there is more than one path connecting firm i and j, computing $\mathbf{P}_{ij}(s)$ is difficult, because shocks can be transmitted by any of those paths which may be of different length. On the other hand, if there is only one path connecting any two given firms, say firm i and j, $\mathbf{P}_{ij}(s)$ is a function of the length of the unique path connecting firms i and j. It then becomes handy to restrict the topology of $\mathcal{G}_n$ so that it does not have cycles. The following remark describes $\mathbf{P}_{ij}(s)$ when $\mathcal{G}_n$ is a forest.

REMARK 4. *Suppose $\mathcal{G}_n$ is a forest, namely there are no cycles. Then, every component of $\mathcal{G}_n$ is a tree. Provided that any two given firms are jointed by a unique path (in case such a path exists),*

$$\boldsymbol{P}_{ij}(s) = \begin{cases} \widetilde{p}_t^{\,l_{i,j}} & \text{where } l_{i,j} \text{ is the length of the (unique) path between } i \text{ and } j \text{ in } \mathcal{G}_n \\ 0 & \text{there is no path between } i \text{ and } j \end{cases} \tag{A.33}$$

A.3 Network Economies and Long-Run Risks

This section shows how the baseline model can be recast so that it generates dynamics that are consistent with long-run risks models. In what follows, both the mean and volatility of firms' growth rate of cash-flows have a persistent component. I use approximations similar to those used by Campbell and Shiller, 1989 and Bansal and Yaron, 2004 to derive approximated solutions for equilibrium asset prices.

Recall that $P_{i,t+1}$ is the share price of firm i at $t+1$. For simplicity assume $\bar{x} = \sigma_x = 0$ and $\tau = 1$ so that the following two conditions hold at equilibrium

$$P_{a,t+1} = \sum_{i=1}^{n} P_{i,t+1} \tag{A.34}$$

$$\widetilde{c}_{t+1} = \prod_{i=1}^{n} y_{i,t+1}^{1/n} \tag{A.35}$$

Define

$$g_{i,t+1} \equiv \log\left(\frac{y_{i,t+1}}{c_t}\right) \quad , \quad g_{t+1} \equiv \log\left(\frac{\widetilde{c}_{t+1}}{c_t}\right) \tag{A.36}$$

$$z_{i,t+1} \equiv \log\left(\frac{P_{i,t+1}}{\widetilde{c}_{t+1}}\right) \quad , \quad z_{t+1} \equiv \log\left(\frac{P_{a,t+1}}{\widetilde{c}_{t+1}}\right) \tag{A.37}$$

Provided Y_{t+1} definition, it follows

$$g_{t+1} = \sum_{i=1}^{n} \frac{1}{n} g_{i,t+1} \tag{A.38}$$

Using first order Taylor approximations yields

$$z_{t+1} \approx w_0 + \sum_{i=1}^{n} w_i z_{i,t+1} \tag{A.39}$$

where $w_i \approx \mathbb{E}\left(\frac{z_{i,t}}{\sum_{j=1}^{n} z_{j,t}}\right)$, and $\sum_{i=1}^{n} w_i = 1$. The term w_0 is selected to ensure that first order approximations hold in levels as well. Define the continuous return of firm i at $t+1$ as

$$r_{i,t+1} \equiv \log\left(\frac{P_{i,t+1} + y_{i,t+1}}{P_{i,t}}\right) \tag{A.40}$$

and the continuous return on aggregate wealth at $t+1$ as:

$$r_{a,t+1} \equiv \log\left(\frac{P_{a,t+1}+\widetilde{c}_{t+1}}{P_{a,t}}\right) \tag{A.41}$$

Using first order Taylor approximations yields[7]

$$\begin{aligned} r_{i,t+1} &\approx k_i + \rho_i z_{i,t+1} - z_{i,t} + \rho_i g_{t+1} + (1-\rho_i) g_{i,t+1} && \text{(A.42)} \\ r_{a,t+1} &\approx k_m - z_t + \rho_m z_{t+1} + g_{t+1} && \text{(A.43)} \end{aligned}$$

where $\{k_i\}_{i=1}^n$ and k_m ensure that first order approximations hold in levels as well. Provided $g_{i,t+1}$ definition, $g_{i,t+1}$ can be approximated by

$$g_{i,t+1} \approx x_{i,t} + \sigma_{i,t}\eta_{i,t+1} \tag{A.44}$$

where

$$\begin{aligned} x_{i,t} &\equiv \alpha_0 + \alpha_1 d_i - \alpha_2 \mathbb{E}_t\left[\widetilde{\varepsilon}_{i,t+1}\right] && \text{(A.45)} \\ \sigma_{i,t}^2 &\equiv \alpha_2^2 \mathbb{E}_t\left[\widetilde{\varepsilon}_{i,t+1}\right]\left(1-\mathbb{E}_t\left[\widetilde{\varepsilon}_{i,t+1}\right]\right) && \text{(A.46)} \end{aligned}$$

Note that $x_{i,t}$ determines $\mathbb{E}_t[g_{i,t+1}]$ and $\sigma_{i,t}$ determines the conditional volatility of $g_{i,t+1}$, given the information at time t. Provided that $\mathcal{G}_n$ does not vary over time and $\widetilde{p}_t$ follows a two state ergodic Markov process, the processes that $x_{i,t}$ and $\sigma_{i,t}^2$ follow can be approximated by:

$$\begin{aligned} x_{i,t+1} &\approx \mu_0 + \mu_1 x_{i,t} + \mu_2 \sigma_{i,t}\zeta_{p,t+1} && \text{(A.47)} \\ \sigma_{i,t+1}^2 &\approx \nu_0 + \nu_1 \sigma_{i,t}^2 + \nu_2 \sigma_p \zeta_{p,t+1} && \text{(A.48)} \end{aligned}$$

where $0 < \mu_1 < 1$, $\mu_2 > 0$, $0 < \nu_1 < 1$ and $\nu_2 > 0$. Variable $\zeta_{p,t+1} \overset{d}{\to} \mathcal{N}(0,1)$ represents the uncertainty coming from unexpected changes in $\widetilde{p}_{t+1}$. Variables $\eta_{i,t+1} \overset{d}{\to} \mathcal{N}(0,1)$ represents the uncertainty coming from idiosyncratic productivity shocks at the firm level, with $\eta_{i,t+1} \perp\!\!\!\perp \eta_{j,t+1}$, $\forall\ j \neq i$. In the baseline model, parameter q is related to variables $\eta_{i,t+1}$ in the approximated solution. Provided how negative shocks are propagated, $\eta_{i,t+1} \perp\!\!\!\perp \zeta_{p,t+1}$, $\forall\ i$.[8]

With the above definitions and approximations at hand, I now study the asset pricing implication of inter-firm

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[7] Approximation (A.43) follows directly from Bansal and Yaron, 2004 which in turns follows from the dividend-ratio model of Campbell and Shiller, 1989. Approximation (A.42) follows from Campbell and Shiller, 1989 once noting that

$$\begin{aligned} r_{i,t+1} &\approx k_i + \log\left(\frac{y_{i,t}}{P_{i,t}}\right) - \rho_i \log\left(\frac{y_{i,t+1}}{P_{i,t+1}}\right) + \log\left(\frac{y_{i,t+1}}{y_{i,t}}\right) \\ &= k_i + \log\left(\frac{y_{i,t}}{c_{t-1}}\frac{c_t}{P_{i,t}}\frac{c_{t-1}}{c_t}\right) - \rho_i \log\left(\frac{y_{i,t+1}}{c_t}\frac{c_{t+1}}{P_{i,t+1}}\frac{c_t}{c_{t+1}}\right) + \log\left(\frac{y_{i,t+1}}{c_t}\frac{c_t}{c_{t-1}}\frac{c_{t-1}}{y_{i,t}}\right) \\ &= k_i + \rho_i z_{i,t+1} - z_{i,t} + \rho_i g_{t+1} + (1-\rho_i) g_{i,t+1} \end{aligned}$$

[8] Let $\widetilde{x}$ and $\widetilde{y}$ be two random variables. I write $\widetilde{x} \perp\!\!\!\perp \widetilde{y}$ to denote that $\widetilde{x}$ is independent of $\widetilde{y}$.

relationships. The pricing kernel equals

$$m_{t+1} \equiv \theta \ln(\delta) - \frac{\theta}{\psi} g_{t+1} + (\theta - 1) r_{a,t+1} \tag{A.49}$$

I derive firm i's price and return using the pricing kernel and the standard first order condition

$$\mathbb{E}_t \left[\exp\left(m_{t+1} + r_{i,t+1}\right)\right] = 1 \tag{A.50}$$

I first solve for the return of the market portfolio $r_{a,t+1}$ substituting $r_{i,t+1}$ by $r_{a,t+1}$. Then I solve for the risk-free rate. Finally I solve for the risk premium of firm i, $\forall\ i \in \{1, \cdots, n\}$.

Return of the Market Portfolio: Following Bansal and Yaron, 2004 I conjecture that firm i's logarithm of the price-consumption ratio follows:

$$z_{i,t} = a_0 + a_1 x_{i,t} + a_2 \sigma_{i,t}^2 \tag{A.51}$$

To solve for constants a_0, a_1 and a_2 I use equations (A.38), (A.39) and (A.43) into the Euler equation (A.50). Since $\eta_{i,t+1}$, $\zeta_{p,t+1}$ are conditionally normal, $\forall\ i \in \{1, \cdots, n\}$, $r_{a,t+1}$ and m_{t+1} are also normal. Exploiting this normality, I write down the Euler equation in terms of the state variables $\{x_{i,t}, \sigma_{i,t}\}_{i=1}^n$. As the Euler equation must hold for all values of the states variables, the terms involving $x_{i,t}$ must satisfy:

$$\frac{1}{n}\left(1 - \frac{1}{\psi}\right) - w_i a_1 + \rho_m a_1 \mu_1 w_i = 0 \tag{A.52}$$

ASSUMPTION 6. *Consider that* $\sum_{i=1}^n w_i x_{i,t} \approx \frac{1}{n}\sum_{i=1}^n x_{i,t}$.

It is worth noting that if most firms in $\mathcal{G}_n$ have a similar number of connections, then assumption 6 is satisfied. For instance, if $\mathcal{G}_n$ is regular, i.e. all firms have the same degree, then $w_i \approx \frac{1}{n}$ for most firms in $\mathcal{G}_n$. If assumption 6 is satisfied, I then can rewrite equation (A.52) as

$$w_i \left(1 - \frac{1}{\psi}\right) - w_i a_1 + \rho_m a_1 \mu_1 w_i \approx 0 \tag{A.53}$$

as a consequence,

$$a_1 \approx \frac{\left(1 - \frac{1}{\psi}\right)}{1 - \mu_1 \rho_m} \tag{A.54}$$

ASSUMPTION 7. *Assume that for most firms in* $\mathcal{G}_n$, $\sigma_{i,t} \approx \sigma_{i,t}^2$.

Using assumption 7 and collecting all the terms that involve $\sigma_{i,t}^2$ yields

$$-w_i a_2 + \rho_m w_i a_2 \nu_1 + \frac{\theta}{2}\left(\frac{1}{n}\right)^2 \left(1 - \frac{1}{\psi}\right)^2 + \frac{\theta}{2}\rho_m^2 w_i^2 \left(a_1^2 \mu_2^2 + a_1 \mu_2 a_2 \nu_2 \sigma_p\right) \approx 0 \tag{A.55}$$

If assumption 6 is satisfied and n is sufficiently large, then $w_i \approx w_i^2$ for most firms. Then equation (A.55) can be rewritten as

$$-a_2 + \rho_m a_2 \nu_1 + \frac{\theta}{2}\left(1 - \frac{1}{\psi}\right)^2 + \frac{\theta}{2}\rho_m^2\left(a_1^2\mu_2^2 + a_1\mu_2 a_2\nu_2\sigma_p\right) \quad \approx \quad 0 \tag{A.56}$$

It then follows,

$$a_2 \quad \approx \quad \frac{\frac{\theta}{2}\left(\left(1 - \frac{1}{\psi}\right)^2 + \rho_m^2 a_1^2 \mu_2^2\right)}{1 - \nu_1\rho_m + \frac{\theta}{2}\rho_m^2 a_1\mu_2\nu_2\sigma_p} \tag{A.57}$$

Given the solution for $z_{i,t}$ the innovation to the return of aggregate wealth is given by

$$\begin{aligned} r_{a,t+1} - \mathbb{E}_t[r_{a,t+1}] &\approx \rho_m\left(a_1\mu_2\left(\sum_{i=1}^n w_i\sigma_{i,t}\right) + a_2\nu_2\sigma_p\right)\zeta_{p,t+1} + \frac{1}{n}\sum_{i=1}^n \sigma_{i,t}\eta_{i,t+1} \\ &\approx \rho_m\left(a_1\mu_2\left(\sum_{i=1}^n w_i\sigma_{i,t}\right) + a_2\nu_2\sigma_p\right)\zeta_{p,t+1} + \sum_{i=1}^n w_i\sigma_{i,t}\eta_{i,t+1} \\ &= \rho_m\Delta_{p,t}\zeta_{p,t+1} + \sum_{i=1}^n w_i\sigma_{i,t}\eta_{i,t+1} \end{aligned} \tag{A.58}$$

where $\Delta_{p,t} \equiv a_1\mu_2\left(\sum_{i=1}^n w_i\sigma_{i,t}\right) + a_2\nu_2\sigma_p$. The conditional variance of aggregate wealth is given by

$$\mathrm{Var}_t[r_{a,t+1}] \quad \approx \quad \rho_m^2\Delta_{p,t}^2 + \sum_{i=1}^n w_i^2\sigma_{i,t}^2 \tag{A.59}$$

Hereinafter, I assume that assumptions 6 and 7 are satisfied.

Pricing Kernel: Using equations (A.38) and (A.43), I rewrite the pricing kernel in terms of the state variables,

$$\begin{aligned} m_{t+1} &\equiv \theta\ln(\delta) - \frac{\theta}{\psi}g_{t+1} + (\theta-1)r_{a,t+1} \qquad \text{(A.60)} \\ &\approx \theta\ln(\delta) - \frac{\theta}{\psi}\left(\sum_{i=1}^n w_i(x_{i,t} + \sigma_{i,t}\eta_{i,t+1})\right) \\ &+ (\theta-1)\left(k_m - w_0 - \sum_{i=1}^n w_i\left(a_0 + a_1 x_{i,t} + a_2\sigma_{i,t}^2\right)\right) \\ &+ (\theta-1)\rho_m\left(w_0 + \sum_{i=1}^n w_i\left(a_0 + a_1\mu_0 + a_1\mu_2 x_{i,t} + a_1\mu_2\sigma_{i,t}\zeta_{p,t+1}\right)\right) \\ &+ (\theta-1)\rho_m\left(\sum_{i=1}^n w_i\left(a_2\nu_0 + a_2\nu_1\sigma_{i,t}^2 + a_2\nu_2\sigma_p\zeta_{p,t+1}\right)\right) \\ &+ (\theta-1)\left(\sum_{i=1}^n w_i(x_{i,t} + \sigma_{i,t}\sigma_{i,t+1})\right) \end{aligned}$$

Innovations to the pricing kernel are then given by

$$m_{t+1} - \mathbb{E}_t[m_{t+1}] \quad \approx \quad \lambda_{m,q}\left(\sum_{i=1}^n w_i\sigma_{i,t}\eta_{i,t+1}\right) + \lambda_{m,p}\Delta_{p,t}\zeta_{p,t+1} \tag{A.61}$$

where λ's represent the aggregate market prices of risk for each source of risk, namely $\{\eta_{i,t+1}\}_{i=1}^n$ and $\zeta_{p,t+1}$, which are defined as

$$\begin{aligned} \lambda_{m,q} &\equiv \theta\left(1-\frac{1}{\psi}\right)-1 \\ \lambda_{m,p} &\equiv (\theta-1)\rho_m \end{aligned}$$

It follows from equation (A.61) that the conditional variance of the pricing kernel is given by

$$\mathrm{Var}_t[m_{t+1}] \approx \lambda_{m,q}^2\left(\sum_{i=1}^{n} w_i^2\sigma_{i,t}^2\right)+\lambda_{m,p}^2\Delta_{p,t}^2 \tag{A.62}$$

Equity Premium: The risk premium of the market return (aggregate wealth) is determined by the conditional covariance between the market portfolio and the pricing kernel. It then follows

$$\mathbb{E}_t[r_{a,t+1}-r_{f,t}] = -\mathrm{Cov}_t\left(m_{t+1}-\mathbb{E}_t[m_{t+1}], r_{a,t+1}-\mathbb{E}_t[r_{a,t+1}]\right)-\frac{1}{2}\mathrm{Var}_t(r_{a,t+1}) \tag{A.63}$$

Using equations (A.58) and (A.61) into the above equation yields

$$\mathbb{E}_t[r_{a,t+1}-r_{f,t}] \approx -\left(\lambda_{m,q}+\frac{1}{2}\right)\left(\sum_{i=1}^{n} w_i^2\sigma_{i,t}^2\right)-\rho_m\left(\lambda_{m,p}+\frac{\rho_m}{2}\right)\Delta_{p,t}^2 \tag{A.64}$$

Risk-free Rate: As in Bansal and Yaron, 2004 the risk-free rate satisfies

$$r_{f,t} = -\ln(\delta)+\frac{1}{\psi}\mathbb{E}_t[g_{t+1}]+\frac{1-\theta}{\theta}\mathbb{E}_t[r_{a,t+1}-r_{f,t}]-\frac{1}{2\theta}\mathrm{Var}_t[m_{t+1}] \tag{A.65}$$

Using equations (A.62) and (A.64) into the above equation yields,

$$\begin{aligned} r_{f,t} &\approx -\ln(\delta)+\frac{1}{\psi}\left(\sum_{i=1}^{n} w_i x_{i,t}\right) \\ &- \frac{1-\theta}{\theta}\left(\left(\lambda_{m,q}+\frac{1}{2}\right)\left(\sum_{i=1}^{n} w_i^2\sigma_{i,t}^2\right)+\rho_m\left(\lambda_{m,p}+\frac{\rho_m}{2}\right)\Delta_{p,t}^2\right) \\ &- \frac{1}{2\theta}\left(\lambda_{m,q}^2\left(\sum_{i=1}^{n} w_i^2\sigma_{i,t}^2\right)+\lambda_{m,p}^2\Delta_{p,t}^2\right) \end{aligned} \tag{A.66}$$

Risk Premium in the Cross-Section: As with the market portfolio, the risk premium of firm i is determined by the conditional covariance between firm i's return and the pricing kernel. It then follows

$$\mathbb{E}_t[r_{i,t+1}-r_{f,t}] = -\mathrm{Cov}_t\left(m_{t+1}-\mathbb{E}_t[m_{t+1}], r_{i,t+1}-\mathbb{E}_t[r_{i,t+1}]\right)-\frac{1}{2}\mathrm{Var}_t(r_{i,t+1}) \tag{A.67}$$

It becomes handy to compute the innovations on firm i's return. Using equation (A.42) it can be shown

$$\begin{aligned} r_{i,t+1} - \mathbb{E}_t[r_{i,t+1}] &\approx \rho_i \left(a_1\mu_2\sigma_{i,t} + a_2\nu_2\sigma_p\right)\zeta_{p,t+1} \\ &+ \rho_i \left(\sum_{j\neq i}^{n} w_j\sigma_{j,t}\eta_{j,t+1}\right) + (1-\rho_i(1-w_i))\sigma_{i,t}\eta_{i,t+1} \\ &= \rho_i\nabla_{p,t}\zeta_{p,t+1} + \rho_i \left(\sum_{j\neq i}^{n} w_j\sigma_{j,t}\eta_{j,t+1}\right) + (1-\rho_i(1-w_i))\sigma_{i,t}\eta_{i,t+1} \end{aligned} \tag{A.68}$$

where $\nabla_{p,t} \equiv a_1\mu_2\sigma_{i,t} + a_2\nu_2\sigma_p$. It then follows from equation (A.68)

$$\mathrm{Var}_t(r_{i,t+1}) \approx \rho_i^2\nabla_{p,t}^2 + \rho_i^2\sum_{j\neq i}^{n} w_j^2\sigma_{j,t}^2 + (1-\rho_i(1-w_i))^2\sigma_{i,t}^2 \tag{A.69}$$

Using equations (A.61), (A.68) and (A.69) into (A.67) yields

$$\begin{aligned} \mathbb{E}_t[r_{i,t+1} - r_{f,t}] &\approx -\rho_i\left(\sum_{j\neq i}^{n} w_j^2\sigma_{j,t}^2\right)\left(\lambda_{m,q} + \frac{\rho_i}{2}\right) \\ &- \left(\lambda_{m,q}w_i + \frac{1}{2}(1-\rho_i(1-w_i))\right)(1-\rho_i(1-w_i))\,\sigma_{i,t}^2 \\ &- \rho_i\nabla_{p,t}\left(\lambda_{m,p}\Delta_{p,t} + \frac{\rho_i}{2}\nabla_{p,t}\right) \end{aligned} \tag{A.70}$$

Topology of $\mathcal{G}_n$ and the Cross-Section of Risk Premia: Let e_i denote a measure of centrality of firm i in $\mathcal{G}_n$. For example, e_i may represent a firm degree, closeness, betweenness or eigenvector centrality. Differentiating equation (A.70) with respect to e_i yields

$$\begin{aligned} \frac{\partial\mathbb{E}_t[r_{i,t+1} - r_{f,t}]}{\partial e_i} &\approx -2\rho_i\left(\sum_{j\neq i}^{n} w_j\sigma_{j,t}\frac{\partial\sigma_{j,t}}{\partial e_i}\right)\left(\lambda_{m,q} + \frac{\rho_i}{2}\right) \\ &- 2\left(\lambda_{m,q}w_i + \frac{1}{2}(1-\rho_i(1-w_i))\right)(1-\rho_i(1-w_i))\,\sigma_{i,t}\frac{\partial\sigma_{i,t}}{\partial e_i} \\ &- \rho_i\lambda_{m,p}\nabla_{p,t}\frac{\partial\Delta_{p,t}}{\partial e_i} - \rho_i\left(\lambda_{m,p}\Delta_{p,t} + \rho_i\nabla_{p,t}\right)\frac{\partial\nabla_{p,t}}{\partial e_i} \end{aligned} \tag{A.71}$$

where $\frac{\partial\nabla_{p,t}}{\partial e_i} = a_1\mu_2\frac{\partial\sigma_{i,t}}{\partial e_i}$ and $\frac{\partial\Delta_{p,t}}{\partial e_i} = a_1\mu_2\left(\sum_{k=1}^{n} w_k\frac{\partial\sigma_{k,t}}{\partial e_i}\right)$.

As Bansal and Yaron, 2004, consider $\gamma = 10$ and $\psi = 1.5$. Thus, $a_1 > 0$ and $\theta < 0$. As a consequence,

- $\lambda_{m,p} < 0$
- $\lambda_{m,q} < 0$
- $\lambda_{m,q} + \frac{\rho_i}{2} < 0$
- $\lambda_{m,q}w_i + \frac{1}{2}(1-\rho_i(1-w_i)) < 0$

If either μ_2, ν_2 or σ_p are sufficiently large such that $a_2 > 0$ then

- $\nabla_{p,t} > 0$
- $\lambda_{m,p}\Delta_{p,t} + \rho_i\nabla_{p,t} < 0$

Consider further that $\sigma_{k,t}$ are weakly increasing functions of e_i, $\forall\ k \in \{1, \cdots, n\}$. If the following sum

$$-\quad 2\rho_i \left(\sum_{j\neq i}^{n} w_j \sigma_{j,t} \frac{\partial \sigma_{j,t}}{\partial e_i}\right) \left(\lambda_{m,q} + \frac{\rho_i}{2}\right) - \rho_i \lambda_{m,p} \nabla_{p,t} a_1 \mu_2 \left(\sum_{j\neq i}^{n} w_j \frac{\partial \sigma_{j,t}}{\partial e_i}\right)$$

is greater than

$$\begin{aligned} &- \quad 2\left(\lambda_{m,q} w_i + \frac{1}{2}(1-\rho_i(1-w_i))\right)(1-\rho_i(1-w_i))\,\sigma_{i,t}\frac{\partial \sigma_{i,t}}{\partial e_i} - \rho_i \lambda_{m,p} \nabla_{p,t} a_1 \mu_2 w_i \frac{\partial \sigma_{i,t}}{\partial e_i} \\ &- \quad \rho_i a_1 \mu_2 \frac{\partial \sigma_{i,t}}{\partial e_i}\left(\lambda_{m,p}\Delta_{p,t} + \rho_i \nabla_{p,t}\right) \end{aligned}$$

Then $\frac{\partial \mathbb{E}_t[r_{i,t+1} - r_{f,t}]}{\partial e_i} \geq 0$. If the above inequality holds, then firm i is more procyclical than firms with centrality scores smaller than e_i, because shocks to firm i tend to affect a higher number of firms in the economy than do shocks to firms with scores smaller than e_i. In such an environment, an increase on firm i's centrality increases the effect that firm i plays on aggregate volatility, which is measured by terms $\left(\sum_{j\neq i}^{n} w_j \sigma_{j,t} \frac{\partial \sigma_{j,t}}{\partial e_i}\right)$ and $\left(\sum_{j\neq i}^{n} w_j \frac{\partial \sigma_{j,t}}{\partial e_i}\right)$. The increase in risk tends to overcompensate the increase in firm i's growth opportunities. On the other hand, firms with small e_i tend to be less procyclical than firms with large e_i, and thus they serve as a hedge to aggregate risk.

A.4 Tables and Figures

This section contains the tables and figures mentioned in the paper and in the appendix.

Table A.1
Critical probability for different symmetric network topologies

The table reports critical probabilities for different symmetric network topologies. Besides reporting the two examples described in Appendix A, the table reproduces a subset of the values reported in Stauffer and Aharony, (1994, Table 1). The first column reports the topology of $\mathcal{G}_\infty$. The second column reports the number of neighbors of any given node in $\mathcal{G}_\infty$. The third column reports the critical probability, $p_c(\mathcal{G}_\infty)$. Despite that $\mathcal{G}_\infty$ may be highly connected, if $\widetilde{p}_t < p_c(\mathcal{G}_\infty)$ then no infinite component emerges as $n \to \infty$, and thus $\sqrt{n}\widetilde{W}_n$ is asymptotically normally distributed. For illustrative purposes, figure A.2(a) depicts a 2D Honeycomb lattice, figure A.2(b) depicts a 2D Squared lattice; figure A.2(c) depicts a 2D Triangular lattice and figure A.2(d) depitcs a Bethe lattice with $z = 3$. The Bethe lattice of degree z is defined as an infinite tree in which any node has degree z. For n finite such topologies are called Cayley Trees.

Topology of $\mathcal{G}_\infty$	**Number of neighbors**	$p_c(\mathcal{G}_\infty)$
Infinite Line (1D lattice)	2	1
Infinite Circle	2	1
2D Honeycomb lattice	3	$1 - 2\sin\left(\frac{\pi}{18}\right)$
2D Squared lattice	4	$\frac{1}{2}$
2D Triangular lattice	6	$2\sin\left(\frac{\pi}{18}\right)$
Bethe lattice	z	$\frac{1}{z-1}$

Table A.2
Characteristics of Customer-Supplier Networks

The table reports characteristics of customer-supplier networks generated at the annual frequency using the Cohen and Frazzini, 2008 dataset from 1980 to 2004. Two firms are connected in the network of year t if one of them represents at least 20% of the other firm's sales during year t. The number of components (clusters) in each network is computed via two consecutive depth-first searches. Provided that degree distributions exhibit fat tails, one can approximate them via power law distributions at least in the upper tail. Namely, the probability of a given degree d in the network of year t, $\mathbb{P}^t(d)$, can be expressed as $\mathbb{P}^t(d) = a_t d^{-\xi_t}$, where $a_t > 0$ and $\xi_t > 1$ are parameters to be estimated. The last row shows the average and standard deviation of the MLE estimators for ξ_t, over the sample period.

Characteristic	**Mean**	**Standard Deviation**
Number of firms per customer-supplier network	388	178
Number of relationships per customer-supplier network	281	154
Number of components per network	122	47
Size of the largest component	30	13
Size of the second largest component	24	11
Size of the third largest component	12	8
Size of the fourth largest component	9	7
Size of the fifth largest component	6	4
Exponent of fitted power law to the degree distribution	3.06	0.28

Table A.3
Benchmark Parameterization

The table reports the list of parameter values in the benchmark parametrization. I set $\bar{c} = 0.019/12$ and $\bar{x} = 0.038/12$ so that the unconditional means of consumption and dividend growth generated by the benchmark economy are similar to the ones found in data. I follow Bansal and Yaron, 2004 and I set $\tau = 3$. I set $\sigma_x = 0.0262$ to match the volatility of dividends. I divide the rest of parameter values into three groups. Parameters in the first group define the preferences of the representative investor: β represents the time discount factor; γ represents the coefficient of relative risk aversion for static gambles; and ρ represents the inverse of the inter-temporal elasticity of substitution. Parameters in the second group describe firms' cash-flows: α_0 measures the part of firms' cash-flows unrelated to inter-firm relationships; α_1 measures the marginal benefit a firm receives from each relationship; and α_2 measures the decrease in a firm's cash-flow if a firm faces a negative shock. Given a network topology, parameters in the third group define the stochastic process that determines the propagation of shocks within the network economy: p_L and p_H are the values that the propensity of relationships to transmit negative shocks; q measures how frequently firms face negative idiosyncratic shocks; ψ measures how frequently relationships exhibit high propensity to transmit negative shocks; and ϕ measures the persistence of the stochastic process followed by $\widetilde{p}_t$.

Preferences			**Firms' Cash-flows**			**Propagation of shocks**				
β	γ	ρ	α_0	α_1	α_2	p_L	p_H	q	ψ	ϕ
0.997	10	0.65	0.3	0.1	0.07	0.38	0.45	0.2	0.5	0.925

Table A.4
Moments under the Benchmark Parameterization

The table reports the first two moments of consumption and dividend growth as well as a set of key asset pricing moments. Column **Data** reports moments found in data. Column **Model** reports moments generated under the benchmark parametrization described in Table A.3. Column **BY2004** reports moments generated under the Long-Run Risks Model of Bansal and Yaron, 2004. Data on consumption and dividends is obtained from Robert Shiller's website http://www.econ.yale.edu/ shiller/data.htm. Moments on the return on aggregate wealth, risk-free rate, equity premium and Sharpe ratio are based on data from 1928 to 2014 and obtained from Aswath Damodaran's website: http://pages.stern.nyu.edu/∼adamodar/. The annual return on aggregate wealth is approximated by the annual return of the S&P 500 while the yield on three month T-bills is used to proxy for the return on the risk-free asset.

Moments	**Data**	**Model**	**BY2004**
Average annual log of consumption growth rate	1.9%	1.9%	1.8%
Annual volatility of log consumption rate	3.5%	4.7%	2.8%
Average annual log dividend growth rate	3.8%	3.8%	1.8%
Annual volatility of the log dividend growth rate	11.63%	14.9%	12.3%
Average annual market return (S&P 500)	11.53%	12%	7.2%
Annual volatility of the market return	19%	18.92%	19.42%
Average annual risk-free rate (3 month T-Bill)	3.53%	2.16%	0.86%
Annual volatility of risk-free rate	3%	0.7%	0.97%
Average annual equity risk premium	8%	10%	6.33%
Average annual Sharpe ratio	0.4	0.52	0.33

Table A.5
Similarities between the calibrated model and the LRR model

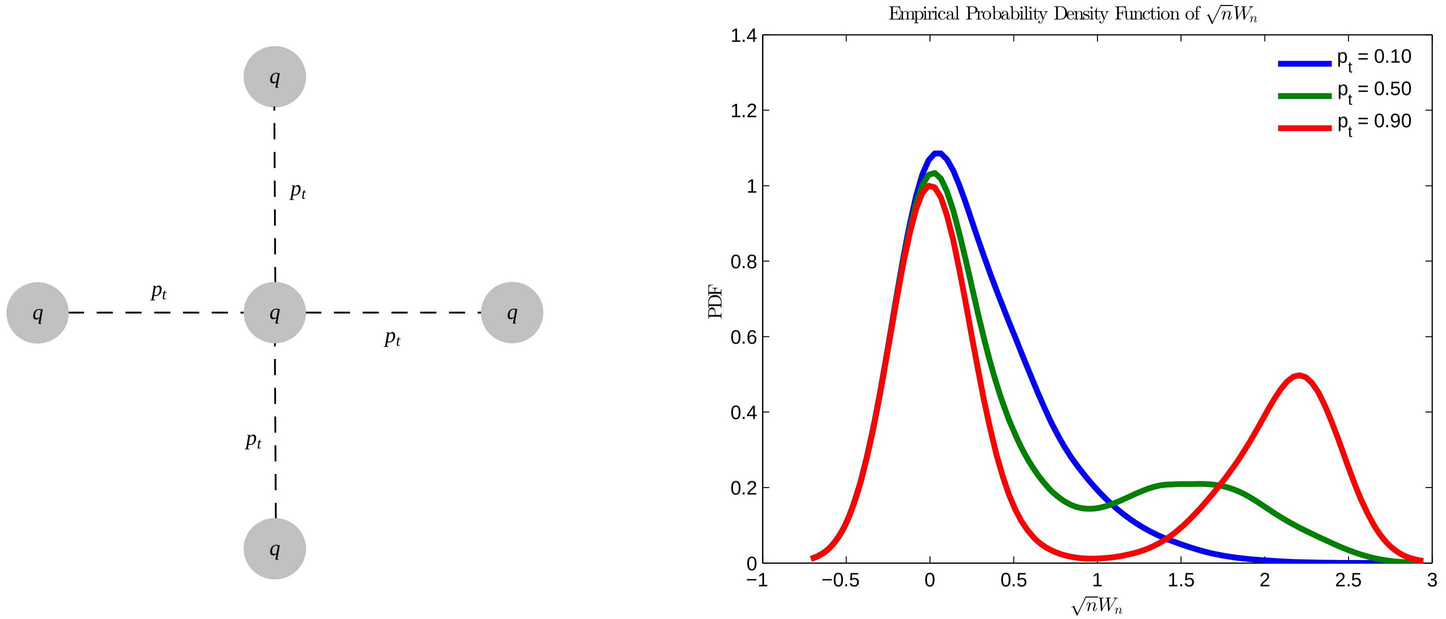
The table reports averages and standard deviations of similarity measures between time series generated with either the calibrated model or the benchmark parameterization in the LRR model of Bansal and Yaron, 2004. To compute these measures I assume that the propensity of inter-firm relationships to transmit negative shocks follows: $\widetilde{p}_{t+1} = 0.4 + 0.925(\widetilde{p}_t - 0.4) + 0.006\epsilon_{t+1}$, where ϵ_{t+1} is standard normal and i.i.d over time. Such an AR(1) process can be approximated by the 2-states Markov chain followed by $\widetilde{p}_t$ in the benchmark parameterization. To compute averages and standard deviations, I sample from the calibrated model and the LRR model to construct two finite-sample empirical distributions for each similarity measure: one for expected consumption growth, $\mathbb{E}_t\left[\Delta\widetilde{c}_{t+1}\right]$, and one for the conditional volatility of consumption growth, $\text{Vol}_t\left[\Delta\widetilde{c}_{t+1}\right]$. Reported values are based on 300 simulated samples over 620 periods. The first 500 periods in each sample are disregarded to eliminate bias coming from the initial condition. All similarity measures report scores computed as $\frac{1}{1+distance}$, where *distance* is defined according to each similarity measure. Let $\mathbf{X}_T = (X_1, \cdots, X_T)$ and $\mathbf{Y}_T = (Y_1, \cdots, Y_T)$ denote realizations from two time series, $X = \{X_t\}$ and $Y = \{Y_t\}$. The first and second similarity measures focus on the proximity between X and Y at specific points of time. The euclidean distance (ED) is defined as $\sqrt{\sum_{t=1}^{T}(X_t - Y_t)^2}$, whereas the dynamic time warping (DTW) distance is defined as $\min_r\left(\sum_{i=1}^{m}|X_{a_i} - Y_{b_i}|\right)$, where $r = ((X_{a_1}, Y_{b_1}), \cdots, (X_{a_m}, Y_{b_m}))$ is a sequence of m pairs that preserves the order of observations, i.e. $a_i < a_j$ and $b_i < b_j$ if $j > i$. DTW seeks to find a mapping such that the distance between X and Y is minimized. This way of computing distance allows two time series that are similar but locally out of phase to align in a non-linear manner. The third measure focuses on correlation-based distances. It uses the partial autocorrelation function (PACF) to define distance between time series. In particular, distance is defined as $\sqrt{(\hat{\rho}_{X_t} - \hat{\rho}_{Y_t})'\Omega(\hat{\rho}_{X_t} - \hat{\rho}_{Y_t})}$ where Ω is a matrix of weights, whereas $\hat{\rho}_{X_t}$ and $\hat{\rho}_{Y_t}$ are the estimated partial autocorrelations of X and Y, respectively. The fourth and fifth measures assume that an specific model generates both time series. The idea is to fit the specific model to each time series and then measure the dissimilarity between the fitted models. The fourth measure computes the distance between two time series as the Euclidean distance between the truncated AR operators. In this case, distance is defined as $\sqrt{\sum_{j=1}^{k}(e_{j,X_t} - e_{j,Y_t})^2}$ where $e_{X_t} = (e_{1,X_t}, \cdots, e_{k,X_t})$ and $e_{Y_t} = (e_{1,Y_t}, \cdots, e_{k,Y_t})$ denote the vectors of $AR(k)$ parameter estimators for X and Y, respectively. The fifth measure computes dissimilarity between two time series in terms of their linear predictive coding in ARIMA processes as in Kalpakis, Gada, and Puttagunta, 2001. The last measure defines distance based on nonparametric spectral estimators. Let f_{X_T} and f_{Y_T} denote the spectral densities of X_T and Y_T, respectively. In this case, the dissimilarity measure is given by a nonparametric statistic that checks the equality of the log-spectra of the two time series. It defines distance as $\sum_{k=1}^{n}\left[Z_k - \hat{\mu}(\lambda_k) - 2\log(1 + e^{Z_k - \hat{\mu}(\lambda_k)})\right] - \sum_{k=1}^{n}\left[Z_k - 2\log(1 + e^{Z_k})\right]$, where $Z_k = \log(I_{X_T}(\lambda_k)) - \log(I_{Y_T}(\lambda_k))$, and $\hat{\mu}(\lambda_k)$ is the local maximum log-likelihood estimator of $\mu(\lambda_k) = \log(f_{X_T}(\lambda_k)) - \log(f_{Y_T}(\lambda_k))$ computed with local lineal smoothers of the periodograms. All similarity measures are computed using the **R** package TSclust (see Montero and Vilar, 2014).

	$\mathbb{E}_t\left[\Delta\widetilde{c}_{t+1}\right]$		$\text{Vol}_t\left[\Delta\widetilde{c}_{t+1}\right]$	
Similarity measure	**Mean**	**Standard Deviation**	**Mean**	**Standard Deviation**
Euclidean Distance (ED)	0.958	0.012	0.974	0.008
Dynamic Time Warping	0.758	0.091	0.723	0.105
PACF	0.736	0.043	0.743	0.043
ED in AR	0.908	0.100	0.910	0.097
Linear predictive in ARIMA	0.726	0.325	0.729	0.313
Spectral distance	1.0	0.000	1.0	0.000

Table A.6
Eigenvector Centrality Summary Statistics

The table reports averages of summary statistics for log(eigenvector centrality). To compute averages in the third and fourth columns, I use customer-supplier data on years 1982, 1987, 1992, 1997 and 2002 to be consistent with the years used by Ahern, 2013. Using data reported in Ahern, (2013, Internet Appendix Table II), the second column present averages of the statistics for log(eigenvector centrality) in inter-sectoral trade networks. The third column presents averages in annual customer supplier networks in which two firms are connected if one firm represents at least 10% of the other firm's annual sales. The fourth column presents averages in annual customer supplier networks in which two firms are connected if one firm represents at least 20% of the other firm's annual sales. The fifth column reports the statistics for log(eigenvector centrality) in the network of the calibrated economy.

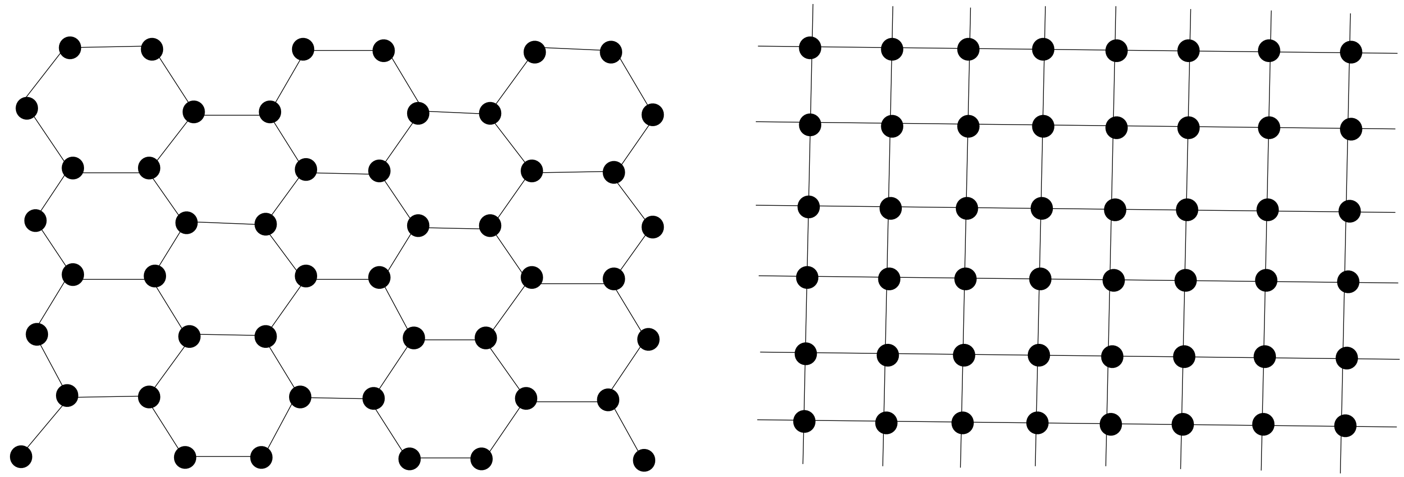
Statistic	**Inter-sectoral Networks**	**Customer Supplier Networks** (10%)	**Customer Supplier Networks** (20%)	**Calibrated Network**
Number of sectors/firms	474	750	382	400
Mean	−6.68	−6.74	−6.62	−6.09
Standard Deviation	1.48	1.07	1.31	1.71
Skewness	0.87	4.04	3.28	1.54
Kurtosis	4.45	18.50	12.38	3.70
Minimum	−10.21	−7.01	−7.01	−7.01
1st Percentile	−9.39	−7.01	−7.01	−7.01
25th Percentile	−7.71	−7.01	−7.01	−7.01
Median	−6.85	−7.01	−7.01	−6.09
75th Percentile	−5.90	−7.01	−7.01	−6.42
99th	−2.27	−1.83	−1.67	−2.30
Maximum	−0.17	−0.46	−0.34	−0.74



(a) Network Topology

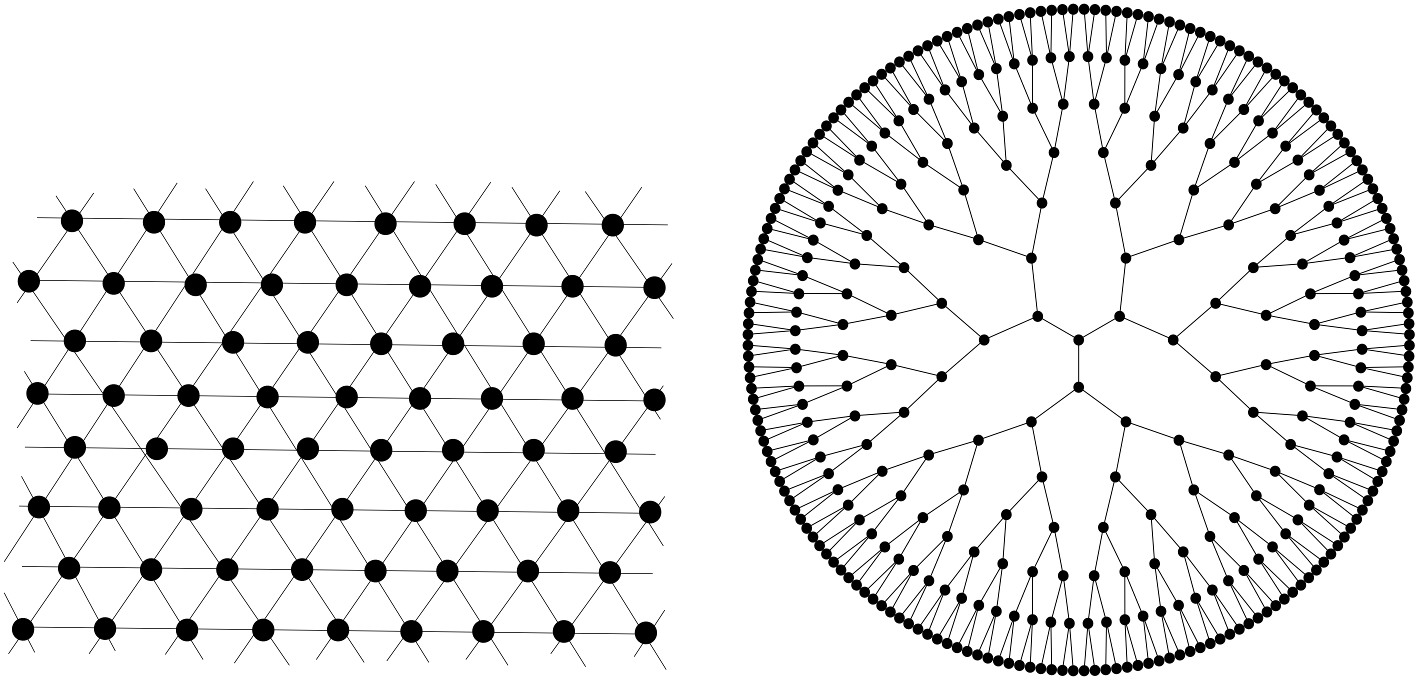
(b) Empirical density function of $\sqrt{n}\widetilde{W}_{n,t}$

Figure A.1. The figure illustrates how changes in the propensity of inter-firm relationships to transmit shocks at t, $\widetilde{p}_t$, impact the distribution of $\sqrt{n}\widetilde{W}_{n,t}$. Figure A.1(a) depicts an economy with $n = 5$ firms, whereas figure A.1(b) depicts estimates of the density function of $\sqrt{n}\widetilde{W}_{n,t}$ for different values of $\widetilde{p}_t$. These estimates are computed via normal kernel smoothing estimators using function ksdensity$(\cdot)$ in MATLAB.



(a) 2D Honeycomb lattice

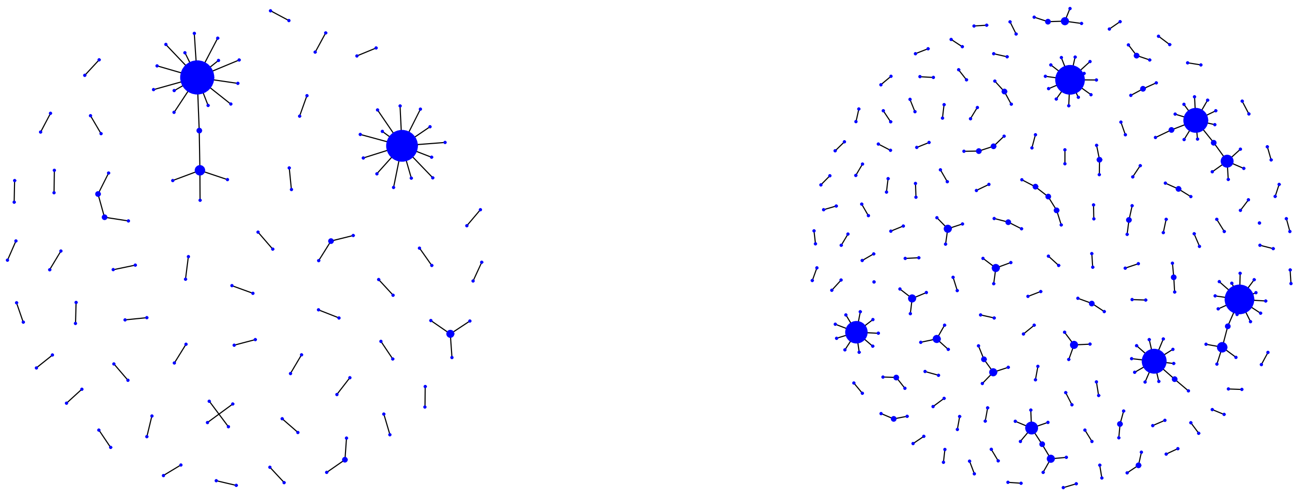
(b) 2D Square lattice



(c) 2D Triangular lattice

(d) Bethe lattice with degree 3

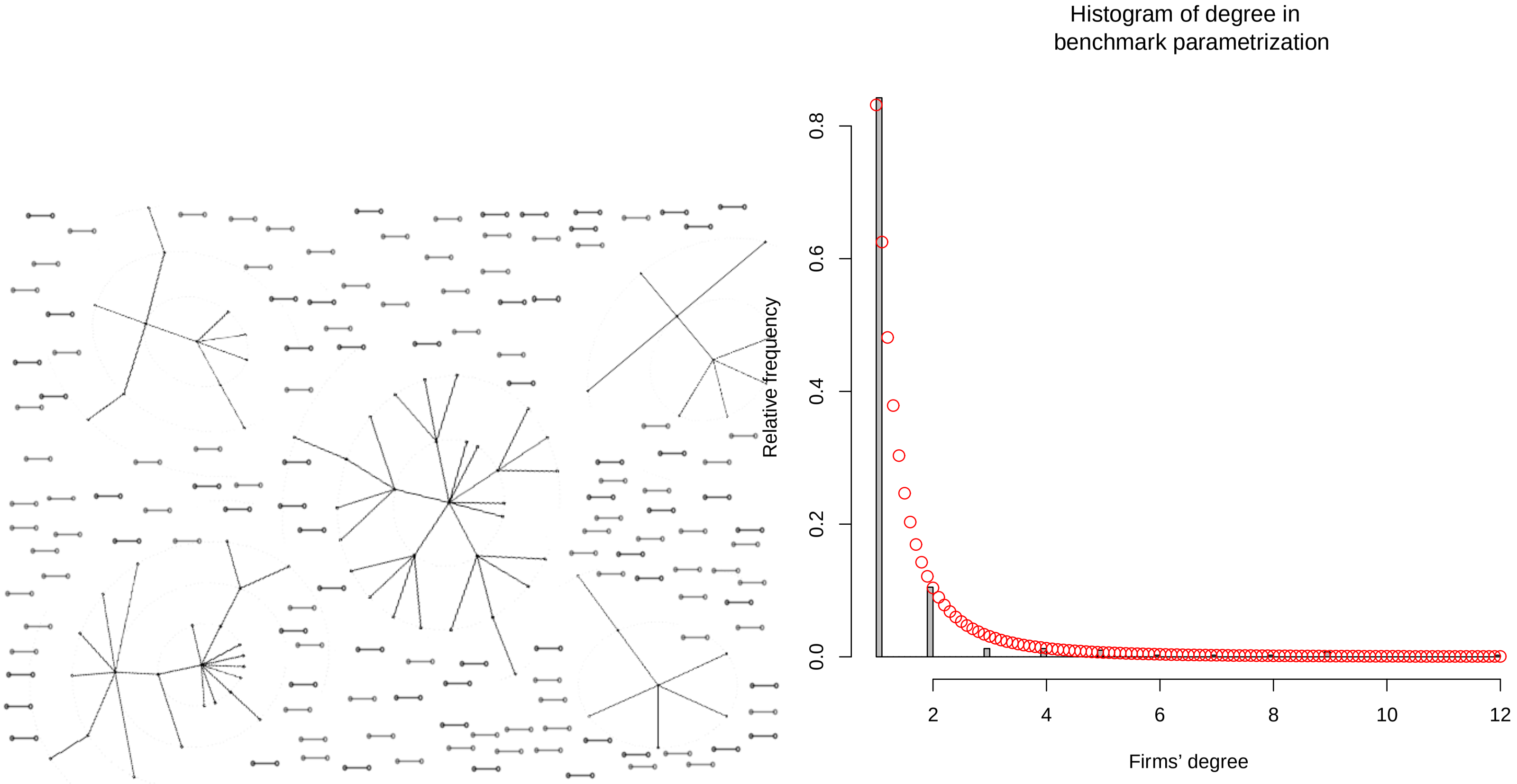
Figure A.2. The figure shows the topologies of the symmetric networks considered in corollary 1 and Table A.1.



(a) Customer-Supplier Network in 1980

(b) Customer-Supplier Network in 1986

Figure A.3. Customer-Supplier Networks in 1980 and 1986. Nodes represent firms while lines represent customer-supplier relationships above the 20% cutoff. The size of each node is proportional to its number of relationships (degree). Figure A.3(a) consists of 131 firms and 84 inter-firm relationships. The three firms with the highest degree are: Ford, Sears and K-Mart. Figure A.3(b) consists of 316 firms and 213 inter-firm relationships. The three firms with the highest degree are: Medicare-Glaser, Ford and Sears.



(a) Topology of inter-firm relationships network

(b) Degree distribution of inter-firm relationships network

Figure A.4. Network of inter-firm relationships under benchmark parametrization. Figure A.4(a) plots the topology of the network under the benchmark parameterization. The size of the five largest components match the average size of the five largest components in customer-supplier networks. Figure A.4(b) plots the degree distribution under benchmark parametrization (shown with bars). Dots represent a power law distribution with exponent 3.

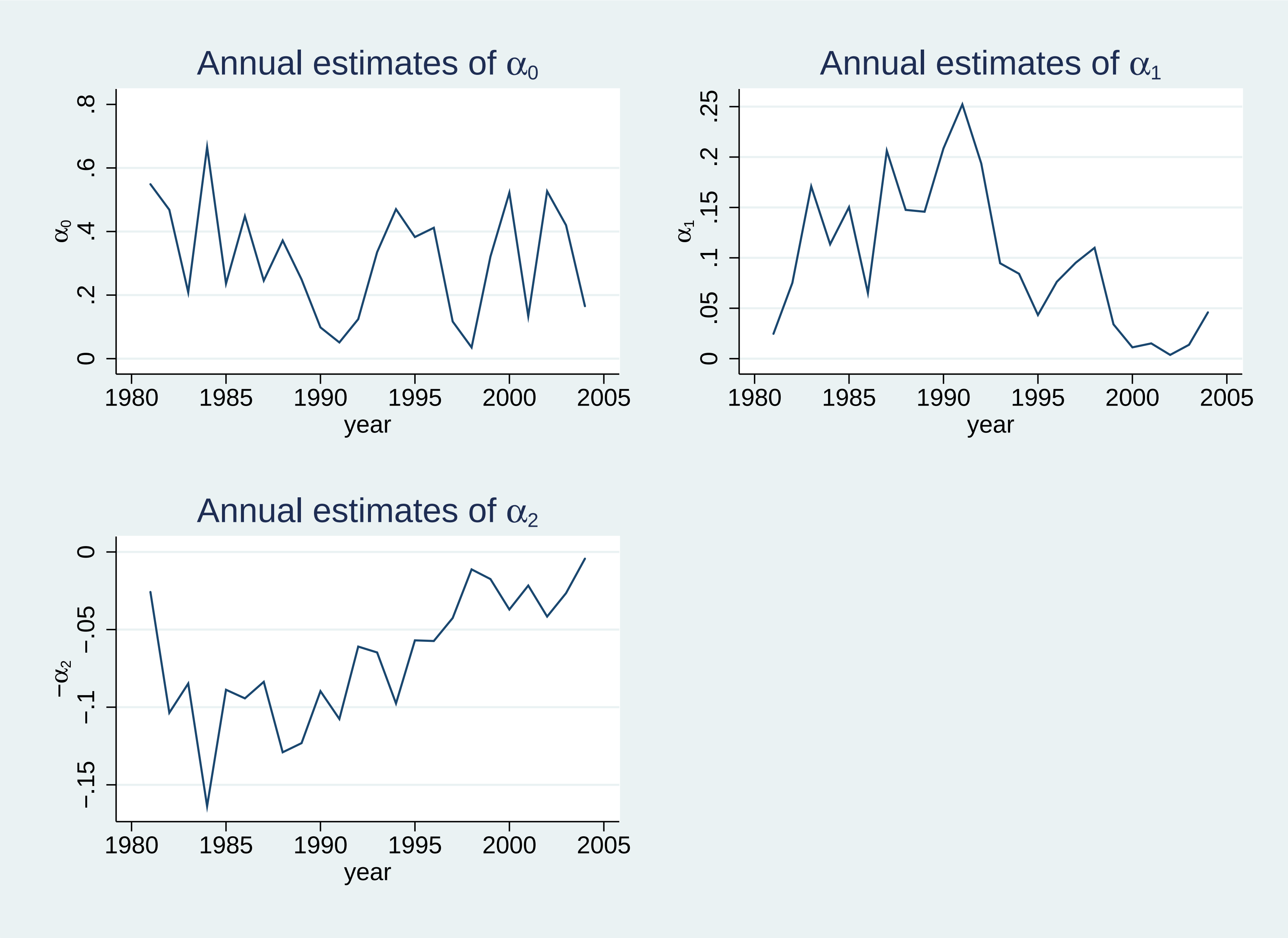
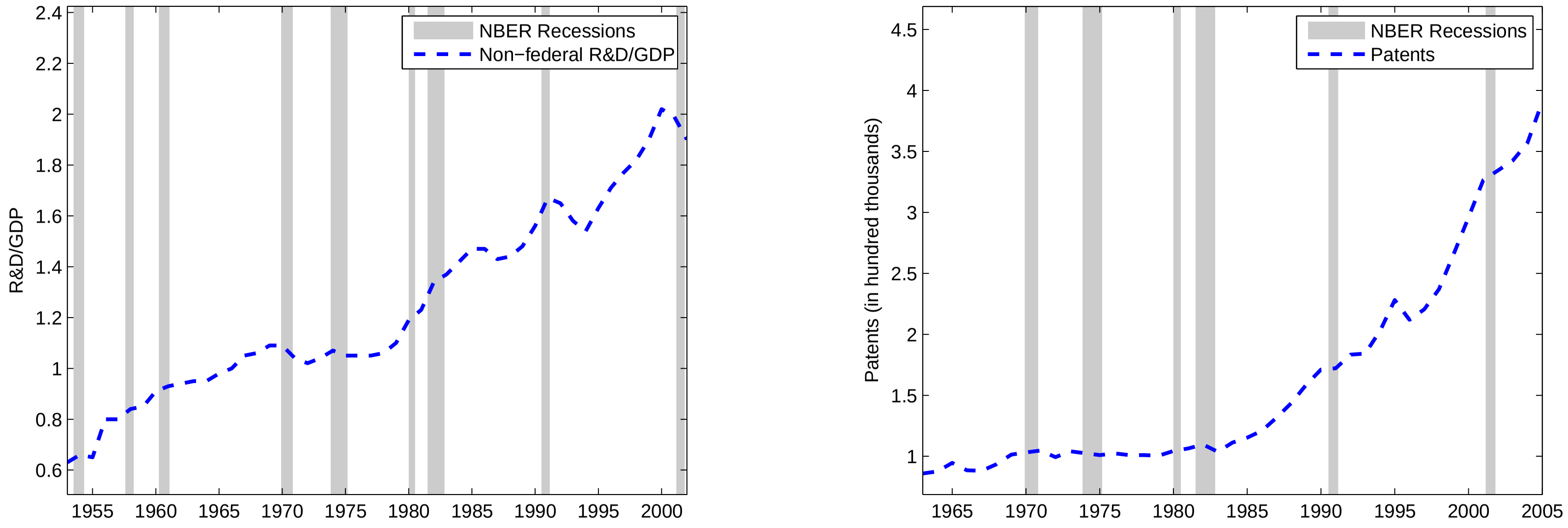


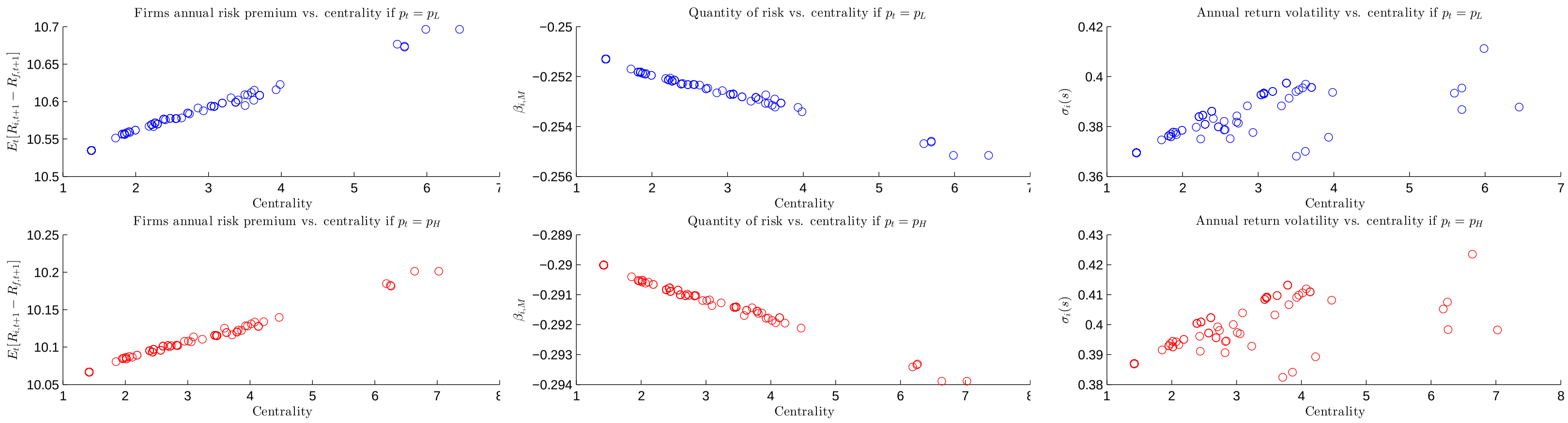
Figure A.5. The figure shows annual estimates of parameters α_0, α_1 and α_2 in equation (1.1). I estimate these parameters using data on earnings per share (EPS) from COMPUSTAT to proxy for $\{y_{i,t}\}_{i=1}^{n}$.



(a) Non federally funded R&D/GDP

(b) Number of patents created in U.S.

Figure A.6. The figure shows the time series of R&D/GDP and the number of patents created in the U.S. Figure A.6(a) depicts non-federal ratios of Research and Development (R&D) to GDP from 1953 to 2002 in the U.S. and NBER recessions. Figure A.6(b) plots the number of patents created in the U.S. from 1963 to 2009 and NBER recessions. Source: National Science Foundation, http://www.nsf.gov/statistics/.

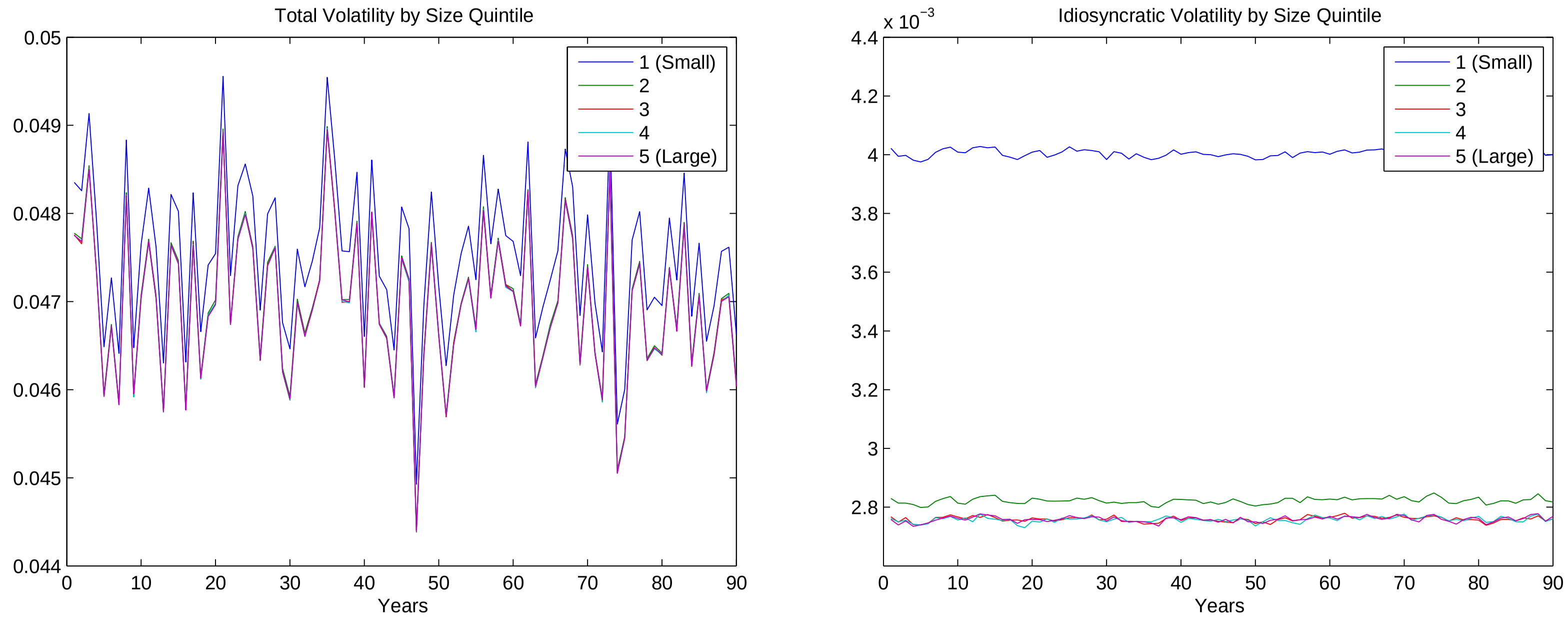


(a) Firms' risk premium

(b) Firms' quantity of risk

(c) Firms' return volatility

Figure A.7. The figure plots firms' conditional risk premium, conditional quantities of risk and conditional annualized return volatilities as a function of firms' centrality in the network. Upper panels show economies in which the propensity $\widetilde{p}_t$ attains p_L whereas bottom panels show economies in which the propensity $\widetilde{p}_t$ attains p_H. Figure A.7(a) plots firms' (conditional) annual risk premium as a function of firms centrality. Figure A.7(b) plots firms' (conditional) quantity of risk, $\beta_{i,\widetilde{M}}$, as a function of firms' centrality. Figure A.7(c) plots firms' (conditional) annual return volatility as a function of firms' centrality.



(a) Total Volatility by Size Quintile

(b) Idiosyncratic Volatility by Size Quintile

Figure A.8. Annualized firm level volatilities averaged within size quintiles. I simulate 200 panels with 400 firms over 1,500 periods and disregard the first 500 periods to eliminate biases coming from the initial condition. Within each panel, I compute firm level total volatility as the annualized standard deviation of monthly firm level realized returns. I construct firm level idiosyncratic volatility as the annualized standard deviation of residuals computed from monthly CAPM regressions of firm level excess realized returns on the excess realized return of the market portfolio. This procedure yields panels of firm-year total and idiosyncratic volatilities estimates. Then, at the beginning of each year I sort firms based on size and average annual volatilities within size quintiles. This procedure yields five time series of total and idiosyncratic volatilities per panel. Figure A.8(a) plots total volatilities per quintile whereas figure A.8(b) plots idiosyncratic volatilities per quintile averaged over the 200 panels.

Appendix B

Basket Securities in Segmented Markets

This section contains the derivations of the formulas presented in the paper. Let $\widetilde{x}_i$ denote the (random) payoff of asset i, $i = \{1, 2\}$. Let α_i denote the fraction of asset i in the basket. If $\widetilde{b}$ denotes the (random) payoff of the basket, then $\widetilde{b} = \sum_{i=1}^{2} \alpha_i \widetilde{x}_i$. Let $b_i \in [0, 1]$ denote the fraction of the basket that *investors* i buy, $i = \{1, 2\}$. If $\widetilde{\pi}$ denotes the (random) profits of the intermediary, it is assumed that

$$\widetilde{\pi} \quad = \quad \beta \left(\sum_{i=1}^{2} \alpha_i \widetilde{x}_i \right) + \theta \left(b_1 + b_2 \right) \tag{B.1}$$

where $\beta \geq 0$ measures the intermediary's "skin in the game" and $\theta > 0$ represents a basket transaction fee.

B.1 Planner's Problem

The planner's problem can be restated as

$$\begin{aligned} \max_{(\alpha_1, b_1, \alpha_2, b_2)} \quad & \mathbb{E}\left[-e^{-\gamma \widetilde{c}_1} \right] \qquad \text{(B.2)} \\ st. \quad & \mathbb{E}\left[-e^{-\gamma \widetilde{c}_2} \right] = u_0 \\ & \mathbb{E}\left[\widetilde{\pi} \right] = \pi_0 \end{aligned}$$

where $\widetilde{c}_i = (1 - \alpha_i)\widetilde{x}_i + b_i \left(\left[\sum_{j=1}^{2} \alpha_j \widetilde{x}_j \right] - \theta \right)$ and $(1 - \alpha_i) + \alpha_i \left(\sum_{j=1}^{2} b_j \right) = 1$, $i = \{1, 2\}$. Parameter γ measures investors' risk aversion. The last restriction in problem (B.2) implies

$$\alpha_2 \quad = \quad \left(\frac{\pi_0 - \theta}{\beta \mu_2} \right) - \left(\frac{\mu_1}{\mu_2} \right) \alpha_1 \tag{B.3}$$

whereas $(1-\alpha_i)+\alpha_i\left(\sum_{j=1}^{2} b_j\right)=1$ implies $b_2=1-b_1$. Provided that payoffs are normally distributed and investors have CARA utility, solving problem (B.2) is equivalent to solving

$$\begin{aligned} \max_{(\alpha_1,b_1)} \quad & \mathbb{E}\left[\widetilde{c}_1\right]-\frac{\gamma}{2}\mathrm{Var}\left[\widetilde{c}_1\right] \\ st. \quad & \mathbb{E}\left[\widetilde{c}_2\right]-\frac{\gamma}{2}\mathrm{Var}\left[\widetilde{c}_2\right]=u_0^* \end{aligned} \tag{B.4}$$

where $\alpha_2=\left(\frac{\pi_0-\theta}{\beta\mu_2}\right)-\left(\frac{\mu_1}{\mu_2}\right)\alpha_1$ and $b_2=1-b_1$. The lagrangian of problem (B.4) is given by

$$\begin{aligned} \mathcal{L} \;=\; & (1-\alpha_1+\alpha_1 b_1)\mu_1+\alpha_2 b_1\mu_2-\theta b_1-\frac{\gamma}{2}\left\{(1-\alpha_1+\alpha_1 b_1)^2\sigma_1^2+2(1-\alpha_1+\alpha_1 b_1)\alpha_2 b_1\rho\sigma_1\sigma_2+\alpha_2^2 b_1^2\sigma_2^2\right\} \\ & -\lambda\left(\alpha_1 b_2\mu_1+(1-\alpha_2+\alpha_2 b_2)\mu_2-\theta b_2-\frac{\gamma}{2}\left\{(1-\alpha_2+\alpha_2 b_2)^2\sigma_2^2+2(1-\alpha_2+\alpha_2 b_2)\alpha_1 b_2\rho\sigma_1\sigma_2+\alpha_1^2 b_2^2\sigma_1^2\right\}-u_0^*\right) \end{aligned} \tag{B.5}$$

where $\lambda\geq 0$ is the Lagrange multiplier. Define

$$\begin{aligned} y &= \alpha_2 b_1+\lambda(1-\alpha_2+\alpha_2 b_2) \\ z &= (1-\alpha_1+\alpha_1 b_1)+\lambda\alpha_1 b_2 \end{aligned}$$

The first order conditions are given by:

$$\alpha_1 \;:\; -\mu_1(1+\lambda)+\gamma\sigma_1\left(\sigma_1 b_2+\rho b_1\sigma_2\frac{\mu_1}{\mu_2}\right)z+\gamma\sigma_2\left(\rho\sigma_1 b_2+b_1\sigma_2\frac{\mu_1}{\mu_2}\right)y=0 \tag{B.6}$$

$$b_1 \;:\; (\alpha_1\mu_1+\alpha_2\mu_2-\theta)(1+\lambda)-\gamma\sigma_2\left(\alpha_1\sigma_1\rho+\alpha_2\sigma_2\right)y-\gamma\sigma_1\left(\alpha_1\sigma_1+\alpha_2\sigma_2\rho\right)z=0 \tag{B.7}$$

$$\lambda \;:\; \mathbb{E}\left[\widetilde{c}_2\right]-\frac{\gamma}{2}\mathrm{Var}\left[\widetilde{c}_2\right]=u_0^* \tag{B.8}$$

Equation (B.6) can be rewritten as

$$\begin{aligned} & -\mu_1+\gamma\sigma_1\left(\sigma_1 b_2+\rho b_1\sigma_2\frac{\mu_1}{\mu_2}\right)[1-\alpha_1+\alpha_1 b_1]+\gamma\sigma_2\left(\rho\sigma_1 b_2+b_1\sigma_2\frac{\mu_1}{\mu_2}\right)\alpha_2 b_1 \\ = \;& \lambda\left\{\mu_1-\gamma\sigma_1\left(\sigma_1 b_2+\rho b_1\sigma_2\frac{\mu_1}{\mu_2}\right)\alpha_1 b_2-\gamma\sigma_2\left(\rho\sigma_1 b_2+b_1\sigma_2\frac{\mu_1}{\mu_2}\right)[1-\alpha_2+\alpha_2 b_2]\right\} \end{aligned} \tag{B.9}$$

whereas equation (B.7) can be rewritten as

$$\begin{aligned} & \alpha_1\mu_1+\alpha_2\mu-\theta-\gamma\sigma_2\left(\alpha_1\sigma_1\rho+\alpha_2\sigma_2\right)\alpha_2 b_1-\gamma\sigma_1\left(\alpha_1\sigma_1+\alpha_2\sigma_2\rho\right)[1-\alpha_1+\alpha_1 b_1] \\ = \;& \lambda\left\{-(\alpha_1\mu_1+\alpha_2\mu_2-\theta)+\gamma\sigma_2\left(\alpha_1\sigma_1\rho+\alpha_2\sigma_2\right)[1-\alpha_2+\alpha_2 b_2]+\gamma\sigma_1\left(\alpha_1\sigma_1+\alpha_2\sigma_2\rho\right)\alpha_1 b_2\right\} \end{aligned} \tag{B.10}$$

Differentiating equation (B.8) with respect to b_2 yields

$$-(\alpha_1\mu_1+\alpha_2\mu_2-\theta)+\gamma\sigma_2\left(\alpha_1\sigma_1\rho+\alpha_2\sigma_2\right)[1-\alpha_2+\alpha_2 b_2]+\gamma\sigma_1\left(\alpha_1\sigma_1+\alpha_2\sigma_2\rho\right)\alpha_1 b_2 \;=\; 0 \tag{B.11}$$

Thus, equation (B.10) implies

$$(1-\alpha_1+\alpha_1 b_1) \;=\; \frac{(\alpha_1\mu_1+\alpha_2\mu_2-\theta)-\gamma\sigma_2(\alpha_1\sigma_1\rho+\alpha_2\sigma_2)\alpha_2 b_1}{\gamma\sigma_1(\alpha_1\sigma_1+\alpha_2\sigma_2\rho)} \tag{B.12}$$

which relates the fraction of asset one held by investors one, $(1 - \alpha_1 + \alpha_1 b_1)$, to the fraction of asset two held by investors one, $\alpha_2 b_1$. A similar relationship can be found, noting that if equation (B.8) is satisfied, then $\lambda = 0$ at the optimum. It then follows from equation (B.9),

$$(1 - \alpha_1 + \alpha_1 b_1) \quad = \quad \frac{\mu_1}{\gamma\sigma_1^2 \left(1 - b_1 + \rho b_1 \frac{\sigma_2}{\sigma_1}\frac{\mu_1}{\mu_2}\right)} - \frac{\sigma_2}{\sigma_1}\frac{\left(\rho\sigma_1(1 - b_1) + b_1\sigma_2\frac{\mu_1}{\mu_2}\right)}{\left(\sigma_1(1 - b_1) + \rho b_1\sigma_2\frac{\mu_1}{\mu_2}\right)}\alpha_2 b_1 \tag{B.13}$$

If $\frac{\sigma_2}{\sigma_1}\frac{\mu_1}{\mu_2} \to 1$ and $\rho \to 1$, it then follows from equation (B.13)

$$(1 - \alpha_1 + \alpha_1 b_1) \quad \to \quad \frac{\mu_1}{\gamma\sigma_1^2} - \frac{\sigma_2}{\sigma_1}\alpha_2 b_1 \tag{B.14}$$

B.2 Trading Equilibrium

B.2.1 Investors' Problem

The optimal portfolio of *investors one*, (α_1^*, b_1^*), solves

$$\begin{aligned} \max_{(\alpha_1, b_1)} \quad & \mathbb{E}\left[-e^{-\gamma\tilde{c}}\right] \\ st. \quad & \tilde{c} = (1 - \alpha_1)\tilde{x}_1 + b_1\left(\left[\sum_{i=1}^{2}\alpha_i\tilde{x}_i\right] - \theta\right) \\ & 1 \geq b_1 \geq 0\ ,\ 1 \geq \alpha_1 \geq 0 \end{aligned} \tag{B.15}$$

Provided that assets' payoff are normally distributed and investors have CARA utility, maximizing expected utility is equivalent to maximizing the certain equivalent $\mathbb{E}[\tilde{c}] - \frac{\gamma}{2}\mathrm{Var}[\tilde{c}]$. As a consequence, the first order conditions of *investors one* are given by:

$$\mu_1 - \gamma\left([1 - \alpha_1 + \alpha_1 b_1]\sigma_1^2 + \alpha_2 b_1 \rho\sigma_1\sigma_2\right) \quad = \quad 0 \tag{B.16}$$

$$\alpha_1\mu_1 + \alpha_2\mu_2 - \theta - \gamma\left([1 - \alpha_1 + \alpha_1 b_1][\alpha_1\sigma_1^2 + \rho\sigma_1\sigma_2\alpha_2] + b_1\alpha_2\sigma_2[\alpha_1\rho\sigma_1 + \alpha_2\sigma_2]\right) \quad = \quad 0 \tag{B.17}$$

Equation (B.16) implies that the fraction of asset one held by *investors one*, $(1 - \alpha_1 + \alpha_1 b_1)$, equals

$$1 - \alpha_1 + \alpha_1 b_1 \quad = \quad \frac{\mu_1}{\gamma\sigma_1^2} - \alpha_2 b_1 \rho\frac{\sigma_2}{\sigma_1} \tag{B.18}$$

Using equation (B.18) in equation (B.17) implies that the fraction of asset 2 held by *investors one*, $\alpha_2 b_1$, equals

$$\alpha_2 b_1 = \frac{\left(\mu_2 - \rho\mu_1\frac{\sigma_2}{\sigma_1}\right)}{\gamma\sigma_2^2(1 - \rho^2)} - \frac{\theta}{\gamma\alpha_2\sigma_2^2(1 - \rho^2)}\ ,\ \text{with } |\rho| \neq 1 \tag{B.19}$$

Using equation (B.19) in equation (B.18) yields

$$1 - \alpha_1 + \alpha_1 b_1 \quad = \quad \frac{\mu_1}{\gamma\sigma_1^2} - \rho\frac{\sigma_2}{\sigma_1}\left\{\frac{\left(\mu_2 - \rho\mu_1\frac{\sigma_2}{\sigma_1}\right)}{\gamma\sigma_2^2(1 - \rho^2)} - \frac{\theta}{\gamma\alpha_2\sigma_2^2(1 - \rho^2)}\right\}\ ,\ \text{with } |\rho| \neq 1 \tag{B.20}$$

It follows directly from equation (B.20) that if $\theta \to 0$ then

$$(1-\alpha_1+\alpha_1 b_1) \quad \to \quad \frac{\mu_1}{\gamma\sigma_1^2} - \rho\frac{\sigma_2}{\sigma_1}\left\{\frac{\left(\mu_2 - \rho\mu_1\frac{\sigma_2}{\sigma_1}\right)}{\gamma\sigma_2^2(1-\rho^2)}\right\} = \frac{1}{\gamma(1-\rho^2)}\left\{\frac{\mu_1}{\sigma_1^2} - \rho\frac{\mu_2}{\sigma_1\sigma_2}\right\} \tag{B.21}$$

Therefore, if $\theta \to 0$, then the fraction of asset one held by *investors one* is increasing in μ_1, σ_2 and decreasing in γ and μ_2. If $\rho\frac{\mu_2}{\sigma_2} < 2\frac{\mu_1}{\sigma_1}$, then $\frac{\partial(1-\alpha_1+\alpha_1 b_1)}{\partial\sigma_1} \leq 0$ so the fraction of asset one held by *investors one* is decreasing in σ_1. On the other hand, if $\rho\frac{\mu_2}{\sigma_2} > 2\frac{\mu_1}{\sigma_1}$, the fraction of asset one held by *investors one* is increasing in σ_1. If $2\rho\mu_1 > \frac{\mu_2}{\sigma_2}(1+\rho^2)$, then $\frac{\partial(1-\alpha_1+\alpha_1 b_1)}{\partial\rho} \geq 0$ so the fraction of asset one held by *investors one* is increasing in ρ. On the other hand, if $2\rho\mu_1 < \frac{\mu_2}{\sigma_2}(1+\rho^2)$, then the fraction of asset one held by *investors one* is decreasing in ρ.

It follows directly from equation (B.19) that if $\theta \to 0$, then

$$\alpha_2 b_1 \quad \to \quad \frac{\left(\mu_2 - \rho\mu_1\frac{\sigma_2}{\sigma_1}\right)}{\gamma\sigma_2^2(1-\rho^2)} \tag{B.22}$$

Therefore, if $\theta \to 0$, then the fraction of asset two held by *investors one* is increasing in μ_2, σ_1 and decreasing in γ and μ_1. Moreover, if $2\frac{\mu_2}{\sigma_2} > \rho\frac{\mu_1}{\sigma_1}$, then the fraction of asset two held by *investors one* is decreasing in σ_2. On the other hand, if $2\frac{\mu_2}{\sigma_2} < \rho\frac{\mu_1}{\sigma_1}$, then the fraction of asset two held by *investors one* is increasing in σ_2. If $\rho < \frac{1}{2}$, then the fraction of asset two held by *investors one* is decreasing in ρ. If $\rho > \frac{1}{2}$ and $\frac{\mu_1}{\sigma_1}\sigma_2^2(2\rho-1) > 2\mu_2$, then the fraction of asset two held by *investors one* is increasing in ρ.

Because the problem of *investors two* is symmetric, the fraction of asset two held by *investors two*, $(1-\alpha_2+\alpha_2 b_2)$, equals

$$1-\alpha_2+\alpha_1 b_2 \quad = \quad \frac{\mu_2}{\gamma\sigma_2^2} - \alpha_1 b_2 \rho\frac{\sigma_1}{\sigma_2} \tag{B.23}$$

whereas the fraction of asset one held by *investors two*, $\alpha_1 b_2$, equals

$$\alpha_1 b_2 \quad = \quad \frac{\left(\mu_1 - \rho\mu_2\frac{\sigma_1}{\sigma_2}\right)}{\gamma\sigma_1^2(1-\rho^2)} - \frac{\theta}{\gamma\alpha_1\sigma_1^2(1-\rho^2)} \tag{B.24}$$

B.2.2 Intermediary's Problem

The basket—defined by fractions (α_1, α_2)—selected by the intermediary solves

$$\begin{aligned} \max_{(\alpha_1,\alpha_2)} \quad & \mathbb{E}\left[\widetilde{\pi}\right] = \beta\left(\sum_{i=1}^{2}\alpha_i\mu_i\right) + \theta\left(b_1+b_2\right) \\ st. \quad & \alpha_i^{ub} \geq \alpha_i \geq \alpha_i^{lb}\ ,\ i = \{1,2\} \\ & \mathrm{E}\left[\widetilde{\pi}^*\right] \geq 0 \end{aligned} \tag{B.25}$$

where $\widetilde{\pi}^*$ is the intermediary's profit evaluated at the basket that maximizes the intermediary's expected profits. Thus, the last restriction represents the intermediary's participation constraint.

Equations (B.19) and (B.24) imply that the intermediary's expected profits are given by:

$$\mathbb{E}\left[\widetilde{\pi}\right] \quad = \quad \beta\left(\sum_{i=1}^{2}\alpha_i\mu_i\right) + \theta\left(\underbrace{\frac{\alpha_2\left(\mu_2 - \rho\mu_1\frac{\sigma_2}{\sigma_1}\right) - \theta}{\gamma\alpha_2^2\sigma_2^2(1-\rho^2)}}_{b_1} + \underbrace{\frac{\alpha_1\left(\mu_1 - \rho\mu_2\frac{\sigma_1}{\sigma_2}\right) - \theta}{\gamma\alpha_1^2\sigma_1^2(1-\rho^2)}}_{b_2}\right) \tag{B.26}$$

Because $b_1 + b_2 = 1$ at equilibrium the basket that maximizes intermediary's expected profits is given by $(\alpha_1^{ub}, \alpha_2^{ub})$.

B.3 Competition among intermediaries

Baskets and intermediaries are indexed by $k = \{1, 2\}$. Let α_{ik} denote the fraction of asset i in basket k, with $\sum_{k=1}^{2}\alpha_{ik} = \alpha_i$. Let b_{ik} denote the fraction of basket k bought by *investors* i, with $\sum_{i=1}^{2}\sum_{k=1}^{2} b_{ik} = 1$. The first order conditions of *investors* i's maximization problem are given by

$$1 - \alpha_i + \left[\sum_{k=1}^{2}\alpha_{ik}b_{ik}\right] \quad = \quad \frac{\mu_i}{\gamma\sigma_i^2} - \left(\rho\frac{\sigma_j}{\sigma_i}\right)\left[\sum_{k=1}^{2}\alpha_{jk}b_{ik}\right] \text{ , with } j \neq i \tag{B.27}$$

$$\left[\sum_{k=1}^{2}\alpha_{jk}b_{ik}\right] \quad = \quad \frac{\mu_j - \rho\mu_i\frac{\sigma_j}{\sigma_i}}{\gamma\sigma_j^2(1-\rho^2)} - \frac{\theta}{\gamma\left(\sum_{k=1}^{2}\alpha_{jk}\right)\sigma_j^2(1-\rho^2)} \tag{B.28}$$

For simplicity consider the problem faced by intermediary one—the problem faced by intermediary two is analogous. The basket—defined by fractions $(\alpha_{11}, \alpha_{21})$—selected by intermediary one solves

$$\begin{aligned} \max_{(\alpha_{11},\alpha_{21})} \quad & \mathbb{E}\left[\widetilde{\pi}\right] = \beta\left(\sum_{i=1}^{2}\alpha_{i1}\mu_i\right) + \theta\left(\sum_{i=1} b_{i1}\right) \\ st. \quad & \alpha_{i1}^{ub} \geq \alpha_{i1} \geq \alpha_{i1}^{lb} \text{ , } i = \{1,2\} \\ & \mathrm{E}\left[\widetilde{\pi}^*\right] \geq 0 \end{aligned} \tag{B.29}$$

where $\widetilde{\pi}^*$ is the intermediary's profit evaluated at the basket that maximizes her expected profits, whereas α_{i1}^{ub} and α_{i1}^{lb} denote the upper and lower bounds of α_{i1} so that b_{i1} are well-defined. Provided that $\sum_{i=1}^{2}\sum_{k=1}^{2} b_{ik} = 1$ at equilibrium, solving problem B.29 is equivalent to solving

$$\begin{aligned} \max_{(\alpha_{11},\alpha_{21})} \quad & \mathbb{E}\left[\widetilde{\pi}\right] = \theta + \beta\left(\sum_{i=1}^{2}\alpha_{i1}\mu_i\right) - \theta\left(\sum_{i=1} b_{i2}\right) \\ st. \quad & \alpha_{i1}^{ub} \geq \alpha_{i1} \geq \alpha_{i1}^{lb} \text{ , } i = \{1,2\} \\ & \mathrm{E}\left[\widetilde{\pi}^*\right] \geq 0 \end{aligned} \tag{B.30}$$

Assume the primitives of the model are such that problem B.30 has an interior solution. The basket that maximizes the expected profits of intermediary one, $(\alpha_{11}^*, \alpha_{21}^*)$, satisfies the first order conditions

$$\beta\mu_1 = \theta \frac{\partial}{\partial \alpha_{11}} \left(\sum_{i=1} b_{i2} \right) \Big|_{\alpha_{11}=\alpha_{11}^*} \tag{B.31}$$

$$\beta\mu_2 = \theta \frac{\partial}{\partial \alpha_{21}} \left(\sum_{i=1} b_{i2} \right) \Big|_{\alpha_{21}=\alpha_{21}^*} \tag{B.32}$$

Note that equation (B.28) implies that

$$\alpha_{21} b_{11} + \alpha_{22} b_{12} = \frac{\mu_2 - \rho\mu_1 \frac{\sigma_2}{\sigma_1}}{\gamma\sigma_2^2(1-\rho^2)} - \frac{\theta}{\gamma\left(\sum_{k=1}^2 \alpha_{2k}\right)\sigma_2^2(1-\rho^2)} \text{ , and} \tag{B.33}$$

$$\alpha_{11} b_{21} + \alpha_{12} b_{22} = \frac{\mu_1 - \rho\mu_2 \frac{\sigma_1}{\sigma_2}}{\gamma\sigma_1^2(1-\rho^2)} - \frac{\theta}{\gamma\left(\sum_{k=1}^2 \alpha_{1k}\right)\sigma_1^2(1-\rho^2)} \tag{B.34}$$

Differentiating equation (B.33) with respect to α_{11} and α_{21} implies

$$\alpha_{11} \quad : \quad \alpha_{21}\frac{\partial b_{11}}{\partial \alpha_{11}} + \alpha_{22}\frac{\partial b_{12}}{\partial \alpha_{11}} = 0 \tag{B.35}$$

$$\alpha_{21} \quad : \quad b_{11} + \alpha_{21}\frac{\partial b_{11}}{\partial \alpha_{21}} + \alpha_{22}\frac{\partial b_{12}}{\partial \alpha_{21}} = \frac{\theta}{\gamma\left(\sum_{k=1}^2 \alpha_{2k}\right)^2 \sigma_2^2(1-\rho^2)} \tag{B.36}$$

whereas differentiating equation (B.34) with respect to α_{11} and α_{21} implies

$$\alpha_{11} \quad : \quad b_{21} + \alpha_{11}\frac{\partial b_{21}}{\partial \alpha_{11}} + \alpha_{12}\frac{\partial b_{22}}{\partial \alpha_{11}} = \frac{\theta}{\gamma\left(\sum_{k=1}^2 \alpha_{1k}\right)^2 \sigma_1^2(1-\rho^2)} \tag{B.37}$$

$$\alpha_{21} \quad : \quad \alpha_{11}\frac{\partial b_{21}}{\partial \alpha_{21}} + \alpha_{12}\frac{\partial b_{22}}{\partial \alpha_{21}} = 0 \tag{B.38}$$

Using equations (B.35) and (B.37) into equation (B.31) yields

$$\frac{\alpha_{11}}{\alpha_{12}} = \frac{1}{\frac{\partial b_{21}}{\partial \alpha_{11}}} \left\{ \frac{\theta}{\gamma\alpha_{12}\alpha_1^2\sigma_1^2(1-\rho^2)} - \frac{\alpha_{21}}{\alpha_{22}}\frac{\partial b_{11}}{\partial \alpha_{11}} - \frac{b_{21}}{\alpha_{12}} - \beta\frac{\mu_1}{\theta} \right\} \tag{B.39}$$

Similarly, using equations (B.36) and (B.38) into equation (B.32) yields

$$\frac{\alpha_{21}}{\alpha_{22}} = \frac{1}{\frac{\partial b_{11}}{\partial \alpha_{21}}} \left\{ \frac{\theta}{\gamma\alpha_{22}\alpha_2^2\sigma_2^2(1-\rho^2)} - \frac{\alpha_{11}}{\alpha_{12}}\frac{\partial b_{21}}{\partial \alpha_{21}} - \frac{b_{11}}{\alpha_{22}} - \beta\frac{\mu_2}{\theta} \right\} \tag{B.40}$$

Using equations (B.39) and (B.40) yields

$$\frac{\alpha_{11}}{\alpha_{12}} = \frac{1}{1 - \left(\frac{\frac{\partial b_{11}}{\partial \alpha_{11}} \frac{\partial b_{21}}{\partial \alpha_{21}}}{\frac{\partial b_{11}}{\partial \alpha_{21}} \frac{\partial b_{21}}{\partial \alpha_{11}}} \right)} \left\{ \frac{1}{\frac{\partial b_{21}}{\partial \alpha_{11}}} \left[\frac{\theta}{\gamma\alpha_{12}\alpha_1^2\sigma_1^2(1-\rho^2)} - \beta\frac{\mu_1}{\theta} - \frac{b_{21}}{\alpha_{12}} - \frac{\frac{\partial b_{11}}{\partial \alpha_{11}}}{\frac{\partial b_{11}}{\partial \alpha_{21}}} \left(\frac{\theta}{\gamma\alpha_{22}\alpha_2^2\sigma_2^2(1-\rho^2)} - \beta\frac{\mu_2}{\theta} - \frac{b_{11}}{\alpha_{22}} \right) \right] \right\}$$

As a consequence,

$$\alpha_{11}^* = \phi_1 \times \alpha_{12}^* \tag{B.41}$$

$$\alpha_{21}^* = \phi_2 \times \alpha_{22}^* \tag{B.42}$$

where

$$\begin{aligned}
\phi_1 &= \frac{1}{1-\left(\frac{\frac{\partial b_{11}}{\partial \alpha_{11}}\frac{\partial b_{21}}{\partial \alpha_{21}}}{\frac{\partial b_{11}}{\partial \alpha_{21}}\frac{\partial b_{21}}{\partial \alpha_{11}}}\right)}\left\{\frac{1}{\frac{\partial b_{21}}{\partial \alpha_{11}}}\left[\frac{\theta}{\gamma\alpha_{12}\alpha_1^2\sigma_1^2(1-\rho^2)}-\beta\frac{\mu_1}{\theta}-\frac{b_{21}}{\alpha_{12}}-\frac{\frac{\partial b_{11}}{\partial \alpha_{11}}}{\frac{\partial b_{11}}{\partial \alpha_{21}}}\left(\frac{\theta}{\gamma\alpha_{22}\alpha_2^2\sigma_2^2(1-\rho^2)}-\beta\frac{\mu_2}{\theta}-\frac{b_{11}}{\alpha_{22}}\right)\right]\right\} \text{ , and} \\
\phi_2 &= \frac{1}{\frac{\partial b_{11}}{\partial \alpha_{21}}}\left\{\frac{\theta}{\gamma\alpha_{22}\alpha_2^2\sigma_2^2(1-\rho^2)}-\phi_1\frac{\partial b_{21}}{\partial \alpha_{21}}-\frac{b_{11}}{\alpha_{22}}-\beta\frac{\mu_2}{\theta}\right\}
\end{aligned}$$

Appendix C

Imperfect Information Transmission from Banks to Investors: Real Implications

C.1 Proofs

C.1.1 Proof of Lemma 1

Rating strategy f_B:

Case 1 $f_B = 1$: It follows from (3.1), that this strategy is optimal whenever $(1-\rho)(1-r)\left(P_{GR}-P_{NR}\right) > c$. Substituting for prices from the zero profit conditions (3.10) and (3.11), we have

$$(1-\rho)(1-r)\Delta W \Delta \Pr > c,$$

where $\Delta \Pr \equiv Pr_{G|GR} - Pr_{G|NR}$. Using equations (3.10) and (3.11) and $f_B = 1$ in the above inequality yields:

$$\begin{aligned} (1-\rho)(1-r)\Delta W\left[\frac{\mu r}{\mu r+(1-\mu)(1-r)}-\frac{\mu(1-r)}{\mu(1-r)+r(1-\mu)}\right] &> c, \text{ or} \\ (1-r)\frac{\mu(1-\mu)(2r-1)}{(r-\mu(2r-1))(\mu(2r-1)+(1-r))} &> \frac{c}{(1-\rho)\Delta W}. \end{aligned} \tag{C.1}$$

Case 2 $f_B = 0$: Following the same steps as in Case 1, we find this strategy is optimal whenever

$$(1-\rho)(1-r)\Delta W \Delta \Pr < c.$$

Substituting for beliefs from the consistency conditions and for $f_B = 0$, we obtain

$$\begin{aligned} (1-\rho)(1-r)\Delta W\left[1-\frac{\mu(1-r)}{\mu(1-r)+(1-\mu)}\right] &< c, \text{ or} \\ (1-r)\left[\frac{1-\mu}{1-r\mu}\right] &< \frac{c}{(1-\rho)\Delta W}. \end{aligned} \tag{C.2}$$

Case 3 $f_B \in (0,1)$**:** The mixed strategy is optimal whenever

$$(1-\rho)(1-r)\Delta W \Delta \operatorname{Pr} = c.$$

Substituting into the above equality for beliefs from (3.10) and (3.11) gives

$$(1-r)\left[\frac{\mu r}{\mu r+(1-\mu)f_B(1-r)} - \frac{\mu(1-r)}{\mu(1-r)+(1-\mu)[(1-f_B)+f_B r]}\right] = \frac{c}{(1-\rho)\Delta W}.$$

The positive solution of the above equation is given by

$$f_B^{mix} = \frac{\tilde{c}\,(1-2\mu r)+\mu\,(1-r)-\sqrt{(\tilde{c}+\mu)^2-\mu^2 r\,(2-r)-(6-4r)\,\tilde{c}r\mu}}{2\tilde{c}\,(1-r)\,(1-\mu)}. \tag{C.3}$$

The optimal strategy f_B is given by f_B^{mix} derived above, but bounded by 0 from below and 1 from above. Thus we have $f_B \in (0,1)$ whenever

$$(1-r)\frac{\mu(1-\mu)(2r-1)}{(r-\mu(2r-1))(\mu(2r-1)+(1-r))} < \frac{c}{(1-\rho)\Delta W} < (1-r)\left[\frac{1-\mu}{1-r\mu}\right].$$

The first inequality in the expression above is derived by setting $f_B^{mix} < 1$ whereas the second inequality is derived by setting $f_B^{mix} > 0$.

Rating strategy f_G**:**

To ensure that $f_G = 1$, we must impose that

$$(1-\rho)r\Delta W \Delta \operatorname{Pr} > c.$$

Because $(1-\rho)r\Delta W \Delta \operatorname{Pr} > (1-\rho)(1-r)\Delta W \Delta \operatorname{Pr} \geq c$, the above inequality holds whenever $f_B > 0$.

However, on the space of parameters that implies $f_B = 0$, we need an additional restriction. Substituting for $\Delta \operatorname{Pr}$ in the case of $f_B = 0$ and simplifying, we obtain

$$(1-\rho)r\Delta W \Delta \operatorname{Pr} = (1-\rho)r\Delta W\left[\frac{1-\mu}{1-r\mu}\right] > c.$$

C.1.2 Proof of Lemma 2

Note that $\mu_1 < \mu_{\max} < \mu_2 < \mu_3 < \bar{\mu}$. The proof follows from Lemma 1 after recognizing that μ_1 and μ_2 denote the solutions of $f_B^{mix} = 1$, whereas μ_3 is the solution to $f_B^{mix} = 0$. Whether f_B^{mix} is an increasing or decreasing function of μ follows directly from the fact that f_B^{mix} is an inverted parabola maximized at $\mu_{\max}$. If $\tilde{c} > (1-r)(2r-1)$, then f_B^{mix} never reaches 1. On the other hand, if $\tilde{c} \leq (1-r)(2r-1)$, then f_B^{mix} exceeds 1 on the interval (μ_1, μ_2), so that $f_B = 1$ within that range.

C.1.3 Proof of Proposition 9

Because $H(\cdot)$ is continuous, there exists at least one fixed point of $H(\cdot)$ whenever $H(0) > 0$ and $H(\bar{\mu}) < \bar{\mu}$. The fixed point is unique if $H'(\mu) < 1$ on the entire range of $\mu \in (0, \bar{\mu})$. The constants μ_1, μ_2, μ_3, and $\bar{\mu}$ used in the proof are defined in Lemma 2.

Existence

Recall the definition of $H : [0, \bar{\mu}] \to [\mu_0, 1]$ given in (3.16) :

$$H(\mu) = F(\bar{k}(\mu)) + \left[1 - F(\bar{k}(\mu)\right] \mu_0.$$

We substitute for the marginal screener in the expression above using

$$\bar{k}(\mu) = (1-\mu_0)\{(1-\rho)[\Delta W \Delta \Pr(\mu)[r - (1-r) f_B(\mu)] - (1 - f_B(\mu))c] + \rho \Delta W\} \tag{C.4}$$

and

$$\Delta \Pr(\mu) = \frac{\mu r}{\mu r + (1-\mu) f_B(\mu)(1-r)} - \frac{\mu(1-r)}{\mu(1-r) + (1-\mu)[(1-f_B(\mu)) + f_B(\mu) r]}$$

which come from equations (3.8) and (3.9).

We use the resulting expression to find $H(0)$. We know from Lemma 1 that $f_B(0) = 0$ and $f_G(0) = 1$. Hence, $\Delta \Pr(0) = \frac{1-\mu}{1-\mu r}$ and $\bar{k}(0) = (1-\mu_0)\left\{(1-\rho)\left[\Delta W \left(\frac{1-\mu}{1-\mu r}\right) r - c\right] + \rho \Delta W\right\} > 0$. As a consequence,

$$H(0) = F(\bar{k}(0)) + \left[1 - F(\bar{k}(0))\right] \mu_0 > F(0) + [1 - F(0)] \mu_0 > 0.$$

Our next objective is to find $H(\bar{\mu})$. By Lemma 1, $f_B(\bar{\mu}) = 0$ and $f_G(\bar{\mu}) = 1$. Hence, $\Delta \Pr(\bar{\mu}) = \frac{1-\bar{\mu}}{1-r\bar{\mu}}$ and

$$\begin{aligned}
\bar{k}(\bar{\mu}) &= (1-\mu_0)\{(1-\rho)[\Delta W \Delta \Pr(\bar{\mu})[r - (1-r) f_B(\bar{\mu})] - (1 - f_B(\bar{\mu}))c] + \rho \Delta W\} = \\
&= (1-\mu_0)\left\{(1-\rho)\left[\Delta W r \frac{1-\bar{\mu}}{1-r\bar{\mu}} - c\right] + \rho \Delta W\right\} \\
&= (1-\mu_0)(1-\rho)\left[\Delta W r \frac{1 - \frac{r-\tilde{c}}{r(1-\tilde{c})}}{1 - \frac{r-\tilde{c}}{1-\tilde{c}}} - c\right] + (1-\mu_0)\rho \Delta W \\
&= (1-\mu_0)(1-\rho)\left[\Delta W r \frac{\tilde{c}}{r} - c\right] + (1-\mu_0)\rho \Delta W = (1-\mu_0)\left\{(1-\rho)\left[\frac{c}{1-\rho} - c\right] + \rho \Delta W\right\} \\
&= (1-\mu_0)\rho[c + \Delta W] > 0.
\end{aligned}$$

By assumption,

$$H(\bar{\mu}) = F((1-\mu_0)\rho[c + \Delta W]) + [1 - F((1-\mu_0)\rho[c + \Delta W])] \mu_0 < \bar{\mu}.$$

As a consequence, $H(0) > 0$ and $H(\bar{\mu}) < \bar{\mu}$, which implies that equation $\mu = H(\mu)$ has at least one solution.

Uniqueness

Now we discuss uniqueness. Differentiating $H(\mu)$, yields

$$H'(\mu) = (1-\mu_0)F_{\bar{k}}\frac{\partial \bar{k}}{\partial \mu}.$$

Range 1. First, consider the highest admissible range of $\mu \in (\mu_3, \bar{\mu})$, where $f_B = 0$ by Lemma 1. Substituting for f_B into ($C.4$) and differentiating, we obtain $\frac{\partial \bar{k}}{\partial \mu} = \frac{-(1-\mu_0)(1-\rho)r(1-r)\Delta W}{(1-r\mu)^2}$ and, therefore,

$$H'(\mu) = -F_{\bar{k}}(1-\rho)(1-\mu_0)^2\left(\frac{r(1-r)\Delta W}{(1-r\mu)^2}\right) \leq 0.$$

Range 2. If $\tilde{c} \leq (1-r)(2r-1)$, consider $\mu \in (0, \mu_1)$ and $\mu \in (\mu_2, \mu_3)$. If instead $\tilde{c} > (1-r)(2r-1)$, consider the entire range of $\mu \in (0, \mu_3)$. By Lemma 1, we have $f_B \in (0,1)$ for these values of μ, and, therefore, $\Delta \Pr = \frac{c}{(1-\rho)(1-r)\Delta W}$. Substituting this into ($C.4$) and differentiating gives us $\frac{\partial \bar{k}}{\partial \mu} = 0$, and therefore,

$$H'(\mu) = 0.$$

Range 3. It remains to consider the range $\mu \in (\mu_1, \mu_2)$, relevant only if parameters satisfy $\tilde{c} \leq (1-r)(2r-1)$. By Lemma 1, $f_B = 1$. Substituting that into ($C.4$) and differentiating, we obtain

$$\frac{\partial \bar{k}}{\partial \mu} = (1-\rho)(1-\mu_0)\Delta W \frac{r(2r-1)^2(2\mu-1)(r-1)}{(r-\mu(2r-1))^2(\mu(2r-1)+(1-r))^2},$$

which implies

$$\begin{aligned} H'(\mu) &= (1-\rho)F_{\bar{k}}(1-\mu_0)^2\Delta W \frac{(2r-1)^2(1-2\mu)r(1-r)}{(r-\mu(2r-1))^2(\mu(2r-1)+(1-r))^2} = \\ &= (1-\rho)F_{\bar{k}}(1-\mu_0)^2\Delta W \frac{(2r-1)^2(1-2\mu)r(1-r)}{r^2(1-r)^2\left(1+\mu(1-\mu)\left(\frac{r^2+(1-r)^2}{r(1-r)}-2\right)\right)^2}. \end{aligned} \tag{C.5}$$

which is a decreasing function of μ, with a zero at $\mu = 0.5$. If instead $\mu \geq 0.5$, then $H'(\mu) \leq 0$. If $\mu < 0.5$, then $H'(\mu) > 0$, and it is maximized out at μ_1. Therefore, we can bound $H'(\mu)$ on the range of $\mu \in (\mu_1, 0.5)$ by setting $H'(\mu_1) < 1$. To do so, note that using the definition of μ_1 in (3.14), we can simplify the expression $\mu_1(1-\mu_1)$ to

$$\mu_1(1-\mu_1) = \frac{\tilde{c}r(1-r)}{(2r-1)(1-r-\tilde{c}(2r-1))}.$$

Substituting this into expression ($C.5$), we obtain

$$\begin{aligned}
H'(\mu) &< H'(\mu_1) = (1-\rho)F_{\bar{k}}(1-\mu_0)^2 \Delta W \frac{(2r-1)^2 \sqrt{\frac{(1-r)(2r-1)-\tilde{c}}{(2r-1)(\tilde{c}+1-2\tilde{c}r-r)}}}{r(1-r)\left(1+\frac{\tilde{c}r(1-r)}{(2r-1)(1-r-\tilde{c}(2r-1))}\left(\frac{r^2+(1-r)^2}{r(1-r)}-2\right)\right)^2} \\
&= (1-\rho)F_{\bar{k}}(1-\mu_0)^2 \Delta W \frac{(2r-1)^2 \sqrt{\frac{(1-r)(2r-1)-\tilde{c}}{(2r-1)(1-r-\tilde{c}(2r-1))}}}{r(1-r)\left(\frac{1-r}{(1-r-\tilde{c}(2r-1))}\right)^2} < \\
&= (1-\rho)\bar{f}(1-\mu_0)^2 \Delta W \frac{(2r-1)^2 \sqrt{\frac{(1-r)-\tilde{c}/(2r-1)}{(1-r-\tilde{c}(2r-1))}}}{r(1-r)\left(\frac{1-r}{(1-r-\tilde{c}(2r-1))}\right)^2} < 1,
\end{aligned}$$

where $\bar{f} = \sup_{k\in[0,1]} F'(k)$ and the last inequality is satisfied by assumption.

To summarize, we found that $H(\mu)$ is weakly decreasing on the entire range of $\mu \in (0, \bar{\mu})$ if $\tilde{c} > (1-r)(2r-1)$ and on the range of $\mu \in (0.5, \bar{\mu})$ if $\tilde{c} \leq (1-r)(2r-1)$. In all cases, $H'(\mu) < 1$, which ensures that equation $\mu = H(\mu)$ has exactly one solution.

C.1.4 Proof of Proposition 10

First consider the case where $(1-\mu_0)(Y_G - Y_B) \geq 1$ and so $\bar{k}^{ef} = 1$. We want to show that $\bar{k}^* < \bar{k}^{ef} = 1$. Suppose not. If $\bar{k}^* = 1$, then $\mu^* = 1$, and, thus, $P_{GR}(\mu^*) = P_{NR}(\mu^*) = W_G$. This, in turn, implies that no bank will choose to engage in costly screening, i.e., $\bar{k}^* = 0$, which is a contradiction.

Now consider the case where $(1-\mu_0)(Y_G - Y_B) < 1$ and so $\bar{k}^{ef} = (1-\mu_0)(Y_G - Y_B)$. In equilibrium, we have $\bar{k} = (1-\mu_0)(R_G - R_B)$, where

$$\begin{aligned}
R_G - R_B &= (1-\rho)\left[\Delta W \Delta Pr[r-(1-r)f_B] - (1-f_B)c\right] + \rho \Delta W \\
&\leq \Delta W\left[(1-\rho)r\Delta Pr + \rho\right] < \Delta W,
\end{aligned}$$

where the last inequality holds because $0 < \rho < 1$ and $0 < r\Delta Pr < 1$. Provided that $\Delta W \leq \Delta Y$, it then follows that $\bar{k}^{ef} > \bar{k}^*$ which completes the proof.

C.1.5 Proof of Lemma 3

From the equilibrium condition $\mu^*(\Delta W) = H(\mu^*(\Delta W), \Delta W)$, we obtain

$$\frac{\partial \mu^*}{\partial \Delta W} = \frac{H_{\Delta W}}{1 - H_\mu}.$$

By Proposition 9, we know that $1 - H_\mu > 0$. Therefore, the sign of $\frac{\partial \mu^*}{\partial \Delta W}$ is determined by the sign of $H_{\Delta W}$.

Recalling the definition of H from (3.16),

$$H(\mu, \Delta W) = F(\bar{k}(\mu, \Delta W)) + (1 - F(\bar{k}(\mu, \Delta W)))\mu_0,$$

we see that $H(\mu, \cdot)$ is increasing in ΔW if and only if $\bar{k}(\mu, \cdot)$ is increasing in ΔW.

Recalling the expression for the marginal screener from (C.4),

$$\begin{aligned}\bar{k}(\mu, \Delta W) &= (1-\mu_0)(R_G(\mu, \Delta W) - R_B(\mu, \Delta W)) \\ &= (1-\mu_0)(1-\rho)\{[r-(1-r)f_B(\mu, \Delta W)]\, \Delta W \Delta \Pr(\mu, \Delta W) - (1 - f_B(\mu, \Delta W)c)\} + (1-\mu_0)\rho\Delta W,\end{aligned}$$

note that ΔW enters it through two channels.

First, a higher ΔW implies that borrowers of type G repay relatively more compared to borrowers of type B, thereby directly increasing $R_G - R_B$ as well as the incentive to screen. Second, ΔW affects $\bar{k}$ indirectly by inducing changes in the rating intensity f_B. There are two cases to consider.

Case 1. Suppose that $f_B(\mu^*) \in (0,1)$. Then $\Delta W \Delta \Pr = \frac{c}{(1-\rho)(1-r)}$, and, therefore,

$$\bar{k}(\mu, \Delta W)) = (1-\mu_0)(1-\rho)\left\{(r-(1-r)f_B)\frac{c}{(1-\rho)(1-r)} - (1-f_b c)\right\} + (1-\mu_0)\rho\Delta W,$$

As a consequence,

$$\frac{\partial \bar{k}}{\partial \Delta W} = \rho(1-\mu_0)\left[1 + c\frac{\partial f_B}{\partial \Delta W}\right]$$

and thus, if $\left[1 + c\frac{\partial f_B}{\partial \Delta W}\right] > 0$ then $\bar{k}^*$ and μ^* are increasing in ΔW. Otherwise, $\bar{k}^*$ and μ^* are decreasing in ΔW.

Case 2. Suppose that $f_B(\mu^*) \in \{0,1\}$. Then f_B is constant in the neighborhood of μ^*, and only the positive direct effect of ΔW remains. It then follows that $\frac{\partial \bar{k}^*}{\partial \Delta W} > 0$ and $\frac{\partial \mu^*}{\partial \Delta W} > 0$.

C.1.6 Proof of Lemma 4

From the equilibrium condition $\mu^*(r) = H(\mu^*(r), r)$, we obtain

$$\frac{\partial \mu^*}{\partial r} = \frac{H_r}{1 - H_\mu}.$$

By Proposition 9, we know that $1 - H_\mu > 0$. Therefore, the sign of $\frac{\partial \mu^*}{\partial r}$ is determined by the sign of H_r.

Recalling the definition of H,

$$H(\mu, r) = F(\bar{k}(\mu, r)) + (1 - F(\bar{k}(\mu, r)))\mu_0,$$

we see that $H(\mu, \cdot)$ is increasing in r if and only if $\bar{k}(\mu, \cdot)$ is increasing in r. Recalling the expression for the marginal screener,

$$\begin{aligned}\bar{k}(\mu, r) &= (1-\mu_0)(R_G(\mu, r) - R_B(\mu, r)) \\ &= (1-\mu_0)(1-\rho)\{(r-(1-r)f_B(\mu, r))\Delta W \Delta \Pr(\mu, r) - (1-f_B)c\} + (1-\mu_0)\rho\Delta W,\end{aligned} \tag{C.6}$$

we see that r enters through two channels. An increase in the rating precision, r, directly increases the payoff to screening by increasing the probability that holders of loans of type G receive a good rating and sell loans at a premium in secondary markets. In addition, an increase in r decreases the probability that holders of loans of type B receive a good rating and sell at a premium.

There is also an indirect effect working through f_B, which influences the premium paid on a loan with a good rate. In particular, there are two cases to consider.

Case 1. Suppose that $f_B \in (0,1)$. Then $\Delta W \Delta \Pr = \frac{c}{(1-\rho)(1-r)}$. As a consequence, $\Delta \Pr$ is increasing in r, and, hence, the premium paid on loans with a good rate also increases with r. In particular, if we substitute for $\Delta \Pr$ into (C.6) we obtain $\bar{k} = (1-\rho)\frac{2r-1}{1-r}c\mu_0 + \rho\Delta W(1-\mu_0)$, which is clearly increasing in r. Thus, $\frac{\partial \bar{k}^*}{\partial r} > 0$ and $\frac{\partial \mu^*}{\partial r} > 0$.

Case 2. Suppose that $f_B \in \{0,1\}$. Then f_B is constant in the neighborhood of μ^*, and, therefore,

$$\frac{\partial(R_G - R_B)}{\partial r} = (1-\rho)\left[(1+f_B)\Delta W \Delta \Pr + (r-(1-r)f_B)\Delta W \frac{\partial \Delta \Pr}{\partial r}\right] > 0,$$

which implies that $\frac{\partial \bar{k}^*}{\partial r} > 0$ and $\frac{\partial \mu^*}{\partial r} > 0$.

C.1.7 Proof of Lemma 5

From the equilibrium condition $\mu^*(c) = H(\mu^*(c), c)$, we obtain

$$\frac{\partial \mu^*}{\partial c} = \frac{H_c}{1 - H_\mu}.$$

By Proposition 9, we know that $1 - H_\mu > 0$. Therefore, the sign of $\frac{\partial \mu^*}{\partial c}$ is determined by the sign of H_c.

Recalling the definition of H,

$$H(\mu, c) = F(\bar{k}(\mu, c)) + (1 - F(\bar{k}(\mu, c)))\mu_0,$$

we see that $H(\mu, \cdot)$ is increasing in c if and only if $\bar{k}(\mu, \cdot)$ is increasing in c.

Recalling the expression for the marginal screener,

$$\begin{aligned}\bar{k}(\mu, c) &= (1-\mu_0)(R_G(\mu, c) - R_B(\mu, c)) \\ &= (1-\mu_0)(1-\rho)\{(r-(1-r)f_B(\mu, c))\Delta W \Delta \Pr(\mu, c) - (1-f_B)c\} + (1-\mu_0)\rho\Delta W,\end{aligned}$$

we see that c enters through several channels. First, it directly increases the cost of rating a loan, and in doing so, it decreases $\bar{k}^*$. However, c also indirectly affects $\bar{k}^*$ through $f_B(\mu, c)$ and $\Delta \Pr(\mu, c)$. There are three cases to consider.

Case 1. Suppose that $f_B = 0$. Then f_B remains constant at 0 in the neighborhood of μ^*, and we have

$$\bar{k} = (1-\mu_0)(1-\rho)[r\Delta W \Delta \Pr - c] + (1-\mu_0)\rho\Delta W,$$

which is strictly decreasing in c, so the result follows.

Case 2. Suppose $f_B \in (0,1)$. Then $\Delta W \Delta \Pr = \frac{c}{(1-\rho)(1-r)}$. Substituting into the above expression, we obtain $\bar{k} = (1-\rho)\frac{2r-1}{1-r}c\mu_0 + \rho\Delta W(1-\mu_0)$, which is an increasing function of c, and, thus, the result follows.

Case 3. Suppose that $f_B = 1$. Then f_B remains constant at 1 in the neighborhood of μ^*, and we have

$$\bar{k} = (1-\mu_0)\left[(1-\rho)(2r-1)\Delta W \Delta \Pr + \rho \Delta W\right].$$

Because $\Delta \Pr$ depends on c only through f_B, which is fixed at 1, we have that $\bar{k}$ is independent of c.

C.1.8 Proof of Lemma 6

From the equilibrium condition $\mu^*(\rho) = H(\mu^*(\rho), \rho)$, we obtain

$$\frac{\partial \mu^*}{\partial \rho} = \frac{H_\rho}{1 - H_\mu}.$$

By Proposition 9, we know that $1 - H_\mu > 0$. Therefore, the sign of $\frac{\partial \mu^*}{\partial \rho}$ is determined by the sign of H_ρ.

Recalling the definition of H,

$$H(\mu, \rho) = F(\bar{k}(\mu, \rho)) + (1 - F(\bar{k}(\mu, \rho)))\mu_0,$$

we see that $H(\mu, \cdot)$ is an increasing function of ρ if and only if $\bar{k}(\mu, \cdot)$ is increasing in ρ.

Recalling the expression for the marginal screener,

$$\begin{aligned}
\bar{k}(\mu, \rho) &= (1-\mu_0)(R_G(\mu, c) - R_B(\mu, \rho)) \\
&= (1-\mu_0)(1-\rho)\{(r - (1-r)f_B(\mu, \rho))\Delta W \Delta \Pr(\mu, \rho) - (1 - f_B(\mu, \rho))c\} + (1-\mu_0)\rho \Delta W,
\end{aligned}$$

we see that ρ enters through several channels. First, it directly increases $\bar{k}^*$ because an increase in ρ increases banks' skin in the game, and, thus, banks' screening incentives increase. However, ρ also indirectly affects $\bar{k}^*$ through $f_B(\mu, \rho)$ and $\Delta \Pr(\mu, \rho)$. There are two cases to consider.

Case 1. Suppose that $f_B \in \{0, 1\}$. Then f_B remains constant at either 0 or 1 in the neighborhood of μ^*, and we have

$$\bar{k} = (1-\mu_0)(1-\rho)\{(r - (1-r)f_B(\mu, \rho))\Delta W \Delta \Pr(\mu, \rho) - (1 - f_B(\mu, \rho))c\} + (1-\mu_0)\rho \Delta W,$$

which is strictly increasing in ρ because $(r - (1-r)f_B)\Delta \Pr < 1$, so the result follows.

Case 2. Suppose $f_B \in (0, 1)$. Then $\Delta W \Delta \Pr = \frac{c}{(1-\rho)(1-r)}$. Substituting into the above expression, we obtain $\bar{k} = (1-\rho)\frac{2r-1}{1-r}c\mu_0 + \rho \Delta W(1-\mu_0)$, which is an increasing function of c, and, thus, the result follows.

C.1.9 Proof of Lemma 7

From the equilibrium condition $\mu^*(\mu_0) = H(\mu^*(\mu_0), \mu_0)$, we obtain

$$\frac{\partial \mu^*}{\partial \mu_0} = \frac{H_{\mu_0}}{1 - H_\mu}.$$

By Proposition 9, we know that $1 - H_\mu > 0$. Therefore, the sign of $\frac{\partial \mu^*}{\partial \mu_0}$ is determined by the sign of H_{μ_0}.

Recalling the definition of H,

$$H(\mu, \mu_0) = F(\bar{k}(\mu, \mu_0)) + (1 - F(\bar{k}(\mu, \mu_0)))\mu_0,$$

we obtain

$$\begin{aligned} \frac{\partial H}{\partial \mu_0} &= 1 + (1-\mu_0) F_k(\bar{k}) \frac{\partial \bar{k}}{\partial \mu_0} - F(\bar{k}) \\ &= (1 - F(\bar{k})) + (1-\mu_0) F_k(\bar{k}) \frac{\partial \bar{k}}{\partial \mu_0}. \end{aligned} \tag{C.7}$$

Employing the expression for the marginal screener, $\bar{k}(\mu, \mu_0) = (1-\mu_0)\Delta R$, we obtain $\frac{\partial \bar{k}}{\partial \mu_0} = -\Delta R + (1-\mu_0)\frac{\partial \Delta R}{\partial \mu_0}$, and, thus,

$$\frac{\partial H}{\partial \mu_0} = (1 - F(\bar{k})) + (1-\mu_0) F_k(\bar{k}) \left[-\Delta R + (1-\mu_0) \frac{\partial \Delta R}{\partial \mu_0} \right].$$

Because $\frac{\partial \Delta R}{\partial \mu_0} = 0$, it follows from the above equation that

$$\frac{\partial H}{\partial \mu_0} = (1 - F(\bar{k})) - (1-\mu_0) F_k(\bar{k}) \Delta R.$$

We then have that μ^* is a weakly increasing function of μ_0 if

$$\mu_0 \geq 1 - \left[\frac{1 - F(\bar{k})}{F_k(\bar{k})} \right] \frac{1}{\Delta R(\mu^*)},$$

and a weakly decreasing function of μ_0 otherwise.

C.1.10 Proof of Proposition 11

The first result is direct from the observation that the premium $P_{GR} - P_{NR}$ is decreasing in f_B, and, therefore, the right hand side of the equilibrium condition

$$\mu = F(\bar{k}(\mu)) + (1 - F(\bar{k}(\mu)))\mu_0$$

is smaller under the policy of mandatory rating. It then follows that the fixed point under mandatory rating must be smaller than the fixed point found in the baseline model. The comparative statics results follow immediately from Lemmas 3 to 7 proved for the benchmark model after substituting for $f_B = 1$.

C.1.11 Proof of Lemma 8

Because $r > \frac{1}{2}$, it follows from equations (3.21) and (3.22) that $f_G \geq f_B$. We first rule out the case of $f_B = 0$. Suppose there exists an equilibrium in which $f_B = 0$. Because we consider the parameter space that yields $f_G = 1$, substituting for $f_B = 0$ and $f_G = 1$ into the beliefs expressions (3.25) and (3.26), we obtain $\Pr_{G|GR} = \Pr_{G|BR} = 1$ and $\Pr_{G|NR} = 0$. It follows from equations (3.28) to (3.30) that $P_{GR} = P_{BR} = W_G$ and $P_{NR} = W_B$. Substituting for prices into (3.22), we see that it is optimal for holders of poor quality assets to rate as long as $(1-\rho)\Delta W > c$, which holds by assumption 5. This contradicts our supposition that there exists an equilibrium in which $f_B = 0$.

The remaining cases are: (a) $f_G = f_B = 1$ and (b) $f_G = 1$ and $f_B \in (0,1)$. Both cases imply $\Pr_{G|NR} = 0$ and $P_{NR} = W_B$.

Consider case (a). For both types of loans to be rated, it must be the case that $(1-\rho)(rP_{GR}+(1-r)P_{BR}-W_B) > c$ and $(1-\rho)((1-r)P_{GR}+rP_{BR}-W_B) > c$. Because $r > \frac{1}{2}$, the latter condition is sufficient to ensure that both conditions hold. Substituting for prices and beliefs and using $f_G = f_B = 1$ in $(1-\rho)((1-r)P_{GR}+rP_{BR}-W_B) > c$ yields

$$\tilde{c} < \frac{\mu r(1-r)}{[\mu r+(1-\mu)(1-r)][\mu(1-r)+(1-\mu)r]}.$$

where $\widetilde{c} \equiv \frac{c}{(1-\rho)\Delta W}$.

Consider case (b). In this case, banks with loans of type B are indifferent between rating and not rating, i.e.,

$$(1-\rho)((1-r)P_{GR}+rP_{BR}-P_{NR}) = c$$

Substituting for prices, beliefs and $f_G = 1$ into the above expression, we obtain

$$\frac{(1-r)\mu r}{\mu r+(1-\mu)f_B(1-r)} + \frac{(1-r)\mu r}{\mu(1-r)+(1-\mu)f_B r} = \tilde{c}.$$

If we solve for f_B from the equation above we get $f_B^{mix} = \mu\left(\frac{r(1-r)(1+2\widetilde{c})-\widetilde{c}+\sqrt{\widetilde{c}^2(1-2r)^2-2\widetilde{c}(1-2r)^2 r(1-r)+r^2(1-r)^2}}{2\widetilde{c}r(1-r)(1-\mu)}\right)$. Setting $f_B^{mix} > 0$ simplifies to $\widetilde{c} < 1$, whereas setting $f_B^{mix} < 1$ simplifies to $\tilde{c} > \frac{\mu r(1-r)}{[\mu r+(1-\mu)(1-r)][\mu(1-r)+(1-\mu)r]}$.

C.1.12 Proof of Proposition 12

The equilibrium proportion of borrowers of type G financed at loan origination is given by a fixed point of $H(\mu)$. Because $H(\cdot)$ is a contraction, we only need to show that $H(\cdot)$ under voluntary disclosure is larger (pointwise) than $H(\cdot)$ under mandatory disclosure to prove the result. Because $H(\mu) = F(\bar{k}(\mu))(1-\mu_0)+\mu_0$, it suffices to show that $\bar{k}$ is larger (pointwise) under voluntary disclosure than under mandatory disclosure. Because $\bar{k}$ is decreasing in f_B, it is enough to show that f_B is smaller (pointwise) under voluntary disclosure than under mandatory disclosure to prove the result.

In both cases (mandatory and voluntary disclosure) f_B is simply a truncation between the solution to the indifference condition of type B borrower and the natural limits 0 and 1. For mandatory disclosure, f_B is the solution to:

$$(1-\rho)(1-r)\left(\frac{\mu r}{\mu r+(1-\mu)f_B(1-r)} + \frac{r\mu}{\mu(1-r)+(1-\mu)f_B r}\right) = \tilde{c},$$

whereas for voluntary disclosure f_B is the solution to:

$$(1-\rho)(1-r)\left(\frac{\mu r}{\mu r+(1-\mu)f_B(1-r)} - \frac{\mu(1-r)}{\mu(1-r)+(1-\mu)(1-f_B+f_B r)}\right) = \tilde{c}.$$

It is then easy to see that f_B is larger under mandatory disclosure because the second term is positive in the first equation above, whereas the term is negative in the second equation above. Thus, the result follows.

C.2 Figures

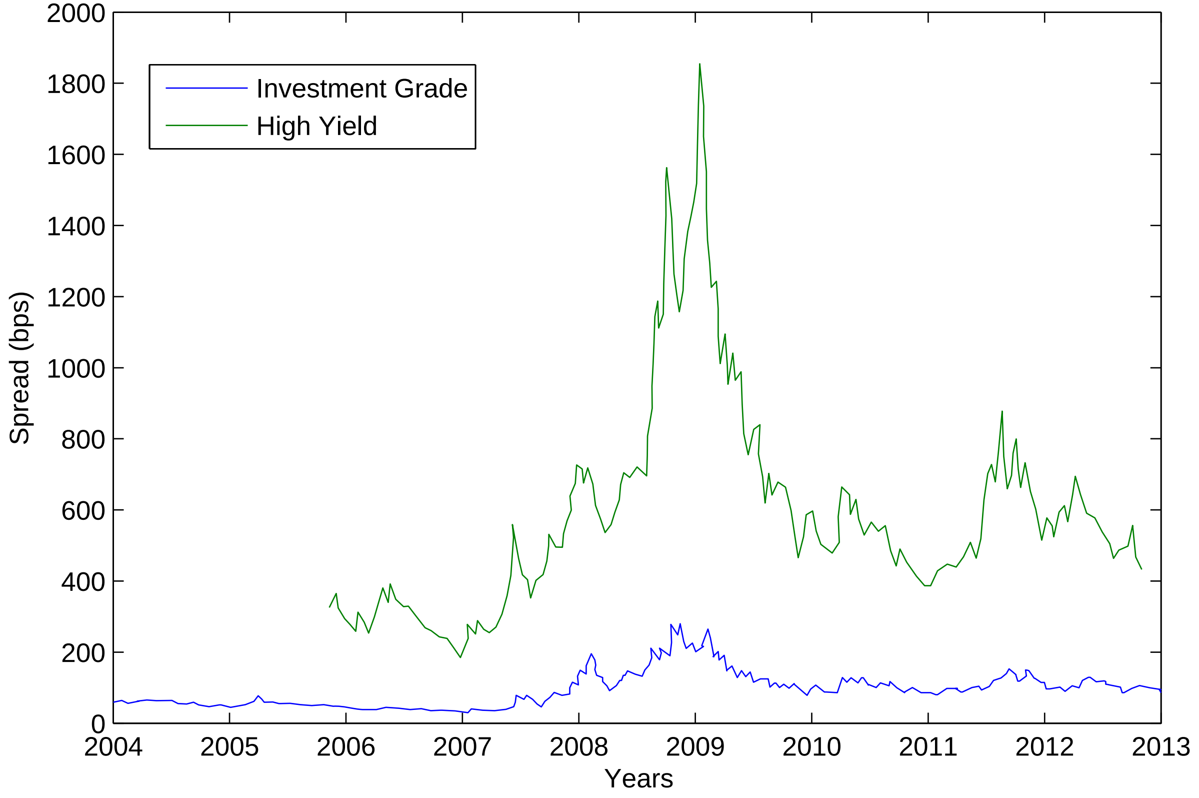


Figure C.1. The figure aims to represent the difference on prices between investment grade and high yield securities before the 2007 financial crisis. The figure shows the spread of two commonly traded credit default swap indexes which can be thought as proxies for investment grade and high yield securities prices (since investors hedge risk by trading CDS indexes). The lower curve shows the spread on the Markit CDX North America Investment Grade Index which is composed of 125 equally weighted credit default swaps on investment grade entities. The upper curve shows the spread on the Markit CDX North America High Yield Index is composed of 100 non-investment grade entities, distributed among two sub-indexes: B and BB (source: Bloomberg).

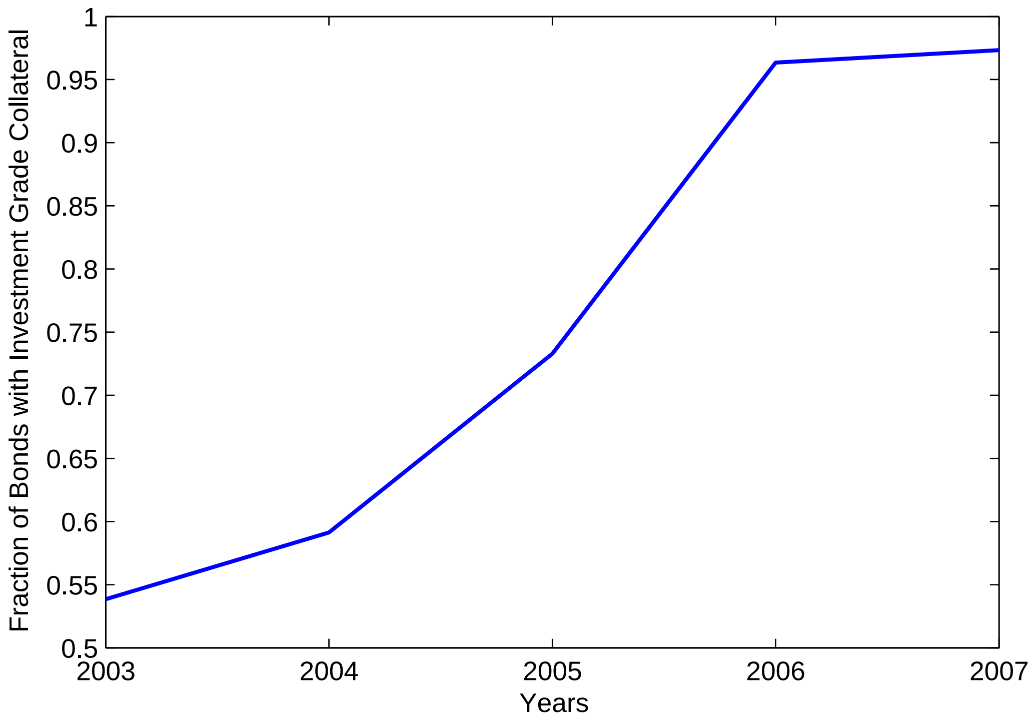


Figure C.2. The figure shows the time series of the fraction of bonds issued with investment grade collateral. For a given year we compute the above fraction only considering data on high yield and investment grade bonds. We use data on global CDOs issuance from the Securities Industry and Financial Markets Association (SIFMA) (see http://www.sifma.org/research/statistics.aspx). Investment grade bonds are defined as bonds with ratings equal to or above Baa3 from Moody's or BBB- from S&P while high yield bonds are defined as bonds with ratings below Baa3 from Moody's or BBB- from S&P.

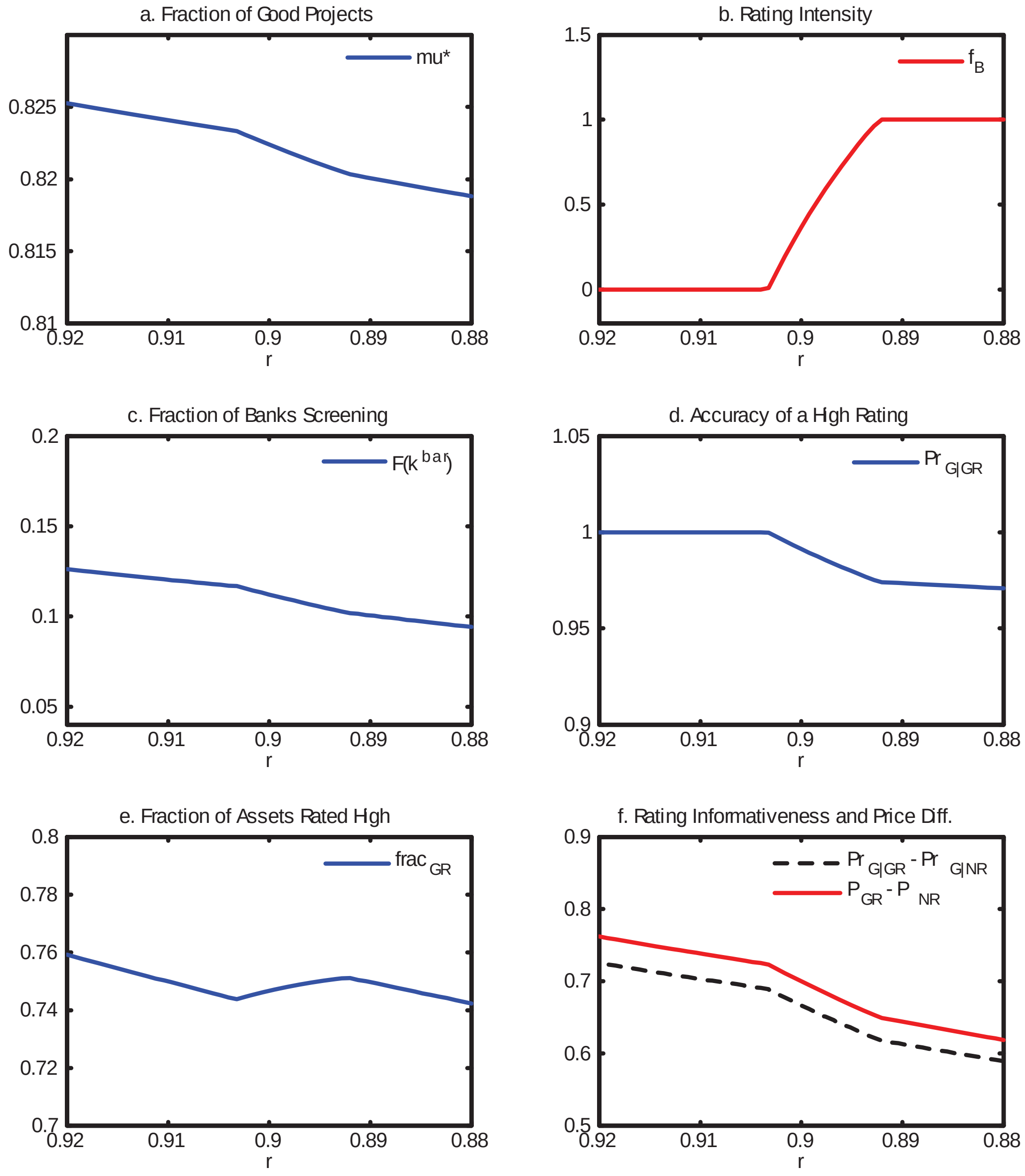


Figure C.3. The figure shows the equilibrium effects of decreasing the precision of the rating r

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