

# MODELING THE DISPERSION AND GAIN OF RF WIRELESS CHANNELS INSIDE REVERBERANT ENCLOSURES

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*I dedicate this work to my family—my parents, Paul and Carole, and my sister, Sarah—for their  
endless love and support throughout my education and life.*

# Abstract

Sensor and instrumentation networks operating inside aircraft wings, unmanned air vehicle (UAV) fuselage, small submarine craft and/or automobile engine compartments could be significantly enhanced or enabled by implementing radio communications within these spaces. A wired infrastructure can be cumbersome and expensive to install and maintain inside challenging environments such as aircraft and submarines. Furthermore, wire bundles and assemblies can be heavy, and flight vehicle environments are particularly sensitive to mass distribution. Wireless sensor and instrumentation networks would offer an alternative to their wired counterparts, and could alleviate many of the aforementioned concerns associated with wired assemblies. Furthermore, wireless sensor networks could be quickly deployed in these environments, enabling rapid-prototype instrumentation and sensor systems. The nodes of this network are envisioned to be quite small such that the environments they are measuring are not appreciably affected by the presence of the nodes themselves. Perhaps the size of a dime, these sensor nodes would nominally contain all of the sensors, processors, antennas, communications subsystems and energy storage elements necessary to perform the required instrumentation operations in these proposed environments. However, the interior enclosures of aircraft wings, UAV fuselage and small submarines are highly reverberant to propagating electromagnetic waves due to the metallic walls surrounding the space, and energy that is transmitted within the enclosure by a radio device can be expected to remain in the space for long durations in time. This lingering of energy can cause the communications channel dispersion in the enclosure to be considerable, potentially limiting and/or degrading communications. The work described in this dissertation studies the wireless RF communications channel native to these enclosures to better understand the unique characteristics of this channel, referred to as the *enclosed space radio channel*. For the purposes of this research, the enclosures considered are generally described as being 1) several meters or less per dimension, 2) enclosed by metallic boundaries on many or all sides, and 3) filled with non-uniform objects and/or obstructions. Measurements of the channel dispersion in



two representative enclosures are made between 200 MHz and 20.0 GHz, utilizing the metrics of RMS delay spread, mean excess delay, and the 50% and 90% coherence bandwidth. A model for the channel dispersion is formed based on simple descriptions of the enclosure, and it is shown that this model matches the measured data well. A model for the gain of the enclosed space channel is developed and presented, and this model also demonstrates a good match to the gain measured in the channel. The dynamic range of the enclosed space channel gain is measured and modeled, and both theory and the measurements show that the dynamic range is bounded in a high-frequency limit. Both the dispersion model and gain model utilize the enclosure quality factor as a critical parameter, and through the quality factor an important tradeoff between gain and dispersion is seen. Measurements of the enclosures used in this work show average RMS delay spreads of 100-300 ns and 50% coherence bandwidths of 1-5 MHz, indicating that the dispersion in these enclosures is less than the dispersion seen in the outdoor mobile communications channel, but greater than the dispersion seen in the indoor wireless communications channel. Measurements of the gain in these enclosures show average values between -10 and -30 dB—considerably larger than the gain typically seen in the indoor or outdoor channels. Single-carrier communications experiments using real signals of several different modulation formats are made between 3.0 and 18.0 GHz, and the results show that data rates up to 5 Mbps are readily obtainable. The orthogonal frequency division multiplexing (OFDM) multicarrier modulation scheme is evaluated in the enclosed space channel, and results suggest that data rates up to 54 Mbps are possible using the IEEE 802.11a communications standard.

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# Chapter 1

## Introduction

### 1.1 The Enclosed Space Radio Channel

Consider a wireless sensor network operating in an environment such as the inside of an aircraft wing, an UAV fuselage, a small submarine hull or an automobile engine compartment. Such wireless sensor networks could perform instrumentation functions inside the environment, with each network node embedding the appropriate sensors, power source, and communications devices as required. Instrumentation systems that both currently use wireline communications and those that would otherwise be enabled by a wireless connectivity could be enhanced by such a development. For instance, one could imagine the development of several wireless rapid-prototype instrumentation systems. In the aircraft domain, wireless instrumentation systems could provide rapidly configured operational integrity testing for flight tests. Analogously in the automotive domain, wireless instrumentation could enable rapidly deployed fault test systems to diagnose automotive problems or characterize system performance. The distribution of mass in flight vehicles is a critical consideration, and wiring harnesses can add a substantial amount of weight to aircraft. Furthermore, wire harness deployments in any environment can be expensive in terms of the financial resources required to build these wire harness assemblies and also in terms of the expense of human capital required to install such assemblies. A system of communications designed for use in these environments would nominally be required to operate without making modifications to the environment, neglecting changes associated with the introduction of the sensor/communications node itself to the space. In other words, an important requirement of such wireless systems would be

to accept the channel presented by the enclosed space as-is, making no initial modifications. Such sensor/communications nodes would nominally be quite small—perhaps the size of a dime—so as to not appreciably disturb the environment-under-test and would nominally contain all of the sensors, processors, communications subsystems, antennas and energy storage elements required for its intended application. Wireless instrumentation systems would accordingly offer an alternative to wired instrumentation deployments, while additionally providing a degree of flexibility to install instrumentation systems in a rapid fashion and in a manner that does not appreciably change the characteristics of the environment in which they are designed to instrument. An important system design requirement is that each communications node be designed to consume a very small amount of power, possibly requiring it to operate for a long period of time without renewing its energy source. Each node's energy source will also be required to supply the node's instrumentation sensors, further restricting the communications energy resource.

Referring to the previously described environments generally as *enclosed space environments*, one can identify several common characteristics; these spaces are 1) several meters or less per dimension, 2) surrounded by reflective boundaries on many or all sides, 3) are filled with non-uniform barriers and obstructions. Considering the characteristics just described, *enclosed space environments* could also be called reverberant enclosed spaces, or equivalently, reverberant cavities, on account of the reflective nature of the metallic enclosure walls. For such wireless instrumentation networks to operate feasibly and reliably in these environments, the reverberant enclosed spaces in which such systems would operate needs to be understood as a radio communications channel. Channel characterization studies are needed to provide insight to the designers of such wireless instrumentation systems so that these systems can be created to function robustly and reliably in these novel radio communications environments. As a result of the many reflection surfaces imposed by the enclosure walls and the contained objects, multiple copies of a transmitted electromagnetic wave will arrive at the the receive antenna, each arriving at a slightly different time, since each copy of the wave can encounter a different path length on its way to the receiver. The phenomena of these multiple copies of the transmitted signal is known as *multipath* in the wireless communications community, and despite the small dimensions of these spaces, replicas of a transmitted wave can continue to arrive at the receiver for surprisingly long durations of time before decaying to an insignificant level. Mutlipath propagation is also known as *dispersion* since it disperses a trans-

mitted piece of information across several delays in time and is the causal factor of *inter-symbol interference* (ISI), or the presence of a replica of a previous symbol in the current symbol's duration. The presence of inter-symbol interference resulting from multipath is not always deleterious: If a mitigation technique such as equalization is used, this spread over multiple symbol times yields an implicit form of diversity—if any one of the paths fades below the detection threshold, there are several others from which the transmitted symbol can be successfully extracted. The particularly reflective boundaries that surround the enclosed spaces additionally ensure that energy is maintained within these spaces more efficiently than in spaces with less reflective walls (e.g. an office with sheetrock or concrete walls that allow significant amounts of energy to pass through their surfaces). Relatively high energy densities result from these highly reflective boundaries, suggesting that the power gain between a transmitter and receiver will be unusually high as compared to the gain of an indoor or outdoor radio channel.

Wireless sensor networks have been the focus of many recent studies that have considered the transport capacity of an overall wireless sensor network; a representative paper in this regard is presented in [4]. Wireless sensor networks that are required to transport information from many different sensor nodes to one central location have also been considered [5]. These contributions to the field of wireless sensor networks characterize the transport capacity of networks operating on top of the physical layer designed to accommodate the outdoor or indoor wireless channel environments. Transport capacity is an important metric for network designers to estimate, since a measure of the transport capacity will determine how much traffic (and therefore what kind of traffic) the wireless network can support. However, the wireless channel associated with reverberant enclosed space environment displays many distinguishing characteristics when compared to the indoor and outdoor channel environments, as the reader will see further in this report. To better understand the performance and capacity of wireless sensor networks operating in reverberant enclosed spaces, the characteristics of the enclosed space radio channel needs to be explored. What dispersion can a radio communications designer expect to see in a reverberant enclosed space environment? How much gain can that designer plan on to properly gauge the transmit power and receiver sensitivity required to obtain an adequate link in the enclosed space channel? How does the dispersion and gain of a RF wireless communications channel relate to the physical properties of the enclosed space, such as volume, surface area and average conductivity? This work presents a detailed characterization of

the dispersion and gain of the enclosed space RF wireless communications channel towards the end of answering these questions. The discussion that follows will introduce the campaign that seeks to characterize and explain the nature of the enclosed space radio channel dispersion and gain, providing architects of wireless communications networks that would operate in these spaces key design insight. The discussion will begin by putting the proposed work into context with previous channel characterization studies followed by an illustration of the specific methods employed towards the investigation of the enclosed space radio channel.

## 1.2 Previously Studied Channels

Whenever radio communications are proposed in a novel environment, studies can help understand the effects of that environment on the signals transmitted within it. One of the first channels to be extensively studied is the outdoor urban propagation channel. An example of an early work studying outdoor channel multipath propagation is given by Turin, et. al. in [6], in which the author represents the channel response with a sequence of delta functions with complex weights. Cox performed some of the first empirical studies of the 910 MHz outdoor channel using a spread-spectrum sliding correlator to sound the channel [7, 8, 9, 10]. Bello provides much of the mathematical basis that can be used to analyze any time-variant channel as a filter [11]. Okamura [12] and Lee [13] have provided several of the important models used to characterize the urban propagation environment. Cox considers the large overall problem of portable radio communications and treats the many different aspects of the problem, including the channel propagation model, in [14].

Similar channel investigations and characterizations in the indoor channel have also enabled the development of indoor communications using cellular telephones and the wireless internet protocols, such as 802.11a/b/g [15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26]. Hashemi has provided an excellent overview of indoor channel studies and phenomena in the literature [27]. This paper describes efforts to characterize such parameters as the multipath channel response inter-arrival statistics, individual path amplitude and phase statistics, variations of the channel response over time and the dependence of the response on the frequency band of interest. A similar indoor channel study includes a characterization of the indoor factory and open-plan buildings, distinct from the more common indoor studies of partitioned office buildings [28, 29, 30]. The ultra-wideband indoor

channel has also recently been studied, considering how signals with bandwidths much larger than the typical band-limited signals will propagate indoors [31, 32, 33, 34, 35].

Propagation characterizations in mine and road tunnels has also been considered at length in the literature [36, 37, 38, 39, 40, 41, 42, 43, 44, 45]. Much like the enclosed space environment, mine and road tunnels present a bounded geometry within which electromagnetic waves may propagate, and the bounded nature of the space has the effect of changing the propagation characteristics as compared to an outdoor or indoor propagation environment. This comparison between the enclosed tunnel propagation characteristics and the indoor and outdoor propagation characteristics is important for two reasons: 1) In cases where mobile communications devices designed for operation in the indoor or outdoor channel environments are to be used in tunnel environments (such as mobile phone users in road or train tunnels) and 2) in the case where commercial-off-the-shelf (COTS) devices designed for indoor or outdoor channel propagation would be used in tunnel environments. In both scenarios, devices that were originally designed for one propagation environment would be employed in an alternative environment. In such applications, the alternative channel environment must be investigated to understand its propagation characteristics and to know whether or not modifications to the radio devices being employed are required to obtain reliable communications.

The heating, ventilation and air conditioning (HVAC) duct radio channel has been the focus of recent studies [46, 47, 48, 49, 50, 51, 52]. In this body of work, much has been done to evaluate the communications channel found when in-building HVAC ducts are used as waveguides of electromagnetic energy to distribute RF signals throughout a building. Numerous similarities can be identified between the HVAC and enclosed space radio channels. For instance, both channel environments can be described generally as enclosed spaces in the sense that energy contained in the enclosure has accountably few methods by which it can leave the space (i.e. dissipation in the enclosure walls, radiation from apertures and seams, absorption from internal objects). Both channel environments are initially required to be taken as-is, as their placement and configuration is for a primary purpose other than communications and modifications could deleteriously affect its designed performance. Both HVAC communications networks and enclosed space channel environments (as will be shown) demonstrate an increased level of dispersion as compared to the indoor channel environments, and this increase in dispersion must be considered when using devices designed for operation in the less dispersive indoor propagation environments or when modifying a

COTS device.

Statistical modeling of empirical campaigns is one of the common methods employed in the above cited works containing empirical measurements. Using this method, a given physical channel is excited using real signals, and the received signals are used to form an estimate of the channel impulse response. Excitations have been made using two general methods: 1) Time domain excitations and 2) frequency domain excitations. In the first method, a waveform is transmitted into the channel and the received signal is used to estimate the channel impulse response,  $h(t)$ . A commonly used method for time-domain signal excitation employs an RF impulse generator to excite the channel, and an RF power detector connected to a high speed scope to read the channel response; this general method was used in [6, 7, 35] and many of the papers referenced in [27]. The frequency domain method is typically implemented using a vector or scalar network analyzer to obtain a frequency sweep of the channel transfer function. This requires that the channel be quasi-stationary over the sweep time chosen for the instrument, but provides an instant measurement of the channel transfer function from the measured scattering matrix ( $S_{21}$ ), from which the channel impulse response can be calculated using the inverse Fourier transform. The work characterizing the HVAC communications channel referenced above uses this frequency-domain technique to estimate the channel impulse response.

Statistical characterizations that are typically performed on the channel measurements include first and second order statistical metrics based on the channel power delay profile (PDP),  $p(t) = \langle |h(t - t_0)|^2 \rangle$ , where  $t_0$  is the time offset to the first arriving propagation path (i.e. the line-of-sight path, if it is not obstructed) and  $\langle \cdot \rangle$  represent the ensemble average over many local positions of the transmitter and receiver. Two statistics that are often used are the mean excess delay,  $\bar{\tau}$  and the RMS delay spread,  $\sigma_\tau$ , which are defined as follows [53]:

$$\sigma_\tau = \sqrt{\overline{\tau^2} - \bar{\tau}^2} \quad (1.1)$$

$$\bar{\tau} = \frac{\int_0^\infty (t - t_0) p(t - t_0) dt}{\int_0^\infty p(t - t_0) dt} \quad (1.2)$$

$$\text{where } \overline{\tau^2} = \frac{\int_0^\infty (t - t_0)^2 p(t - t_0) dt}{\int_0^\infty p(t - t_0) dt}. \quad (1.3)$$

Both measurements give an indication of how long the multiple propagating copies of the transmit-

ted signal remain in the channel above a given energy level. As the RMS delay spread and the mean excess delay increase, the spread across time of the multiply received replicas increases, forcing the symbol time of a transmitted signal to increase, if inter-symbol interference is to be avoided. The metrics of the RMS delay spread and the mean excess delay require no specific shape of a channel power delay profile; they are metrics that can be applied to an arbitrary channel regardless of the environment in which it is measured.

A corresponding metric in the frequency domain is the coherence bandwidth, obtained by first evaluating the autocorrelation of the channel transfer function,  $H(\omega)$  [53]:

$$\Phi(\Delta\omega) = \frac{\int_{-\infty}^{\infty} H(\omega)H^*(\omega - \Delta\omega)d\omega}{\int_{-\infty}^{\infty} H(\omega)H^*(\omega)d\omega}. \quad (1.4)$$

The value of  $\Delta\omega$  above at which  $\Phi(\Delta\omega)$  falls below a value  $x$  for the first time is referred to as the  $x * 100\%$  coherence bandwidth. Analogous to the time-domain metrics, the coherence bandwidth is a rough estimate of how wide a transmitted spectrum may be without experiencing distortion in the channel. In a similar fashion to the RMS delay spread and the mean excess delay, the metric of coherence bandwidth can be applied to any channel, regardless of that channel's specific characteristics or shape.

Even though the above mentioned metrics can be applied to any arbitrary channel, there are no a priori guarantees that each channel's communication potential and performance for an applied modulation will be the same for the same values of RMS delay spread, mean excess delay or coherence bandwidth. After the statistics of a channel have been measured and/or characterized, the communications performance of that same channel can be evaluated using the given modulation scheme(s) considered for use in that channel. Such an evaluation could be done empirically or numerically, using a communications simulation. Empirical techniques were used to evaluate the HVAC communications channel using multicarrier transmission techniques [54]; the experiment described uses a digital signal generator to excite the HVAC channel and a vector signal analyzer to analyze the received signals and provide an estimate of the channel's communications potential. An example of the evaluation of a channel using numerical techniques can be found in [55]. In this work, the author assumes multiple different representative channel impulse responses for the outdoor urban propagation channel, and simulates several different signals transmitted through the different chan-



nels. Analytical techniques have also been used to characterize the performance of several different types of modulation schemes in multipath channels [56, 57, 58]. When measurements, simulations or analysis are assembled and different transmit signaling types are compared across the different types of explored propagation channels, one gains an insight as to how one modulation may out- or under- perform another in the same channel or how a given modulation may perform differently in different channels of similar metric (i.e. in different channels with the same RMS delay spread or mean excess delay).

### 1.3 Research Approach

The strategies applied to and the results understood from previous radio channel studies provide important devices and insights with which the enclosed space channel can be investigated and characterized. However, the results found with respect to each of the previously studied channels are unique to each of those channels, just as the characteristics of the enclosed space radio channel will likewise be unique. The dispersion and gain of the enclosed space channel needs to be understood in a manner that would allow architects of a wireless sensor networks and instrumentation systems to design reliable and robust communications systems to accommodate the challenges presented by the channel propagation environment. Knowledge of channel characteristics could be used to either design custom physical-layer communications devices or to modify COTS devices so that reliable and robust communications can be obtained inside of reverberant enclosed spaces. Thus, *the principle goal of the research presented in this dissertation is to devise a generally applicable channel model based on physical principles to predict the channel dispersion and gain, given an arbitrary enclosed space environment.*

To achieve a channel model that is as pervasively applicable as possible, several general approaches are required. One such approach is to form a channel model that considers only an environment's general geometric descriptions. Because the specific geometry of any particular enclosed space is unique, and because the environment cannot be initially modified to better suit the installed communications network, the enclosed space channel model would be most flexible if it were to describe the dispersion and gain as a function of broad geometric parameters. Accordingly, the characterizations presented herein will utilize such broad parameters as the volume enclosed by the

space, the surface area of that space (including the surface area associated with any internal objects) and the conductivity of the enclosure and object surfaces, but will not take as an input parameter the specific geometric arrangement of the space itself. Another important general approach is the characterization of dispersion and gain over many frequency bands, allowing for an arbitrary operation with respect to frequency. The growing number of wireless communications products in several different unlicensed bands allow enclosed space communications designers the opportunity to choose between several different operation frequency bands, and each band will present distinct dispersion and gain characteristics. Furthermore, many enclosed space environments are electromagnetically isolated from the external environment, and operation in arbitrary frequency bands is therefore feasible. It is in this manner that the operation frequency provides an additional degree of freedom in the enclosed space communications system design that is not typically available in previously considered radio channels (due to the specific government regulations under which such devices are allowed to operate in a specific nation or region), and this work will study the enclosed space as a function of frequency so that designers to better utilize this additional degree of freedom.

When considering many of the target environments for enclosed space communications, i.e. unmanned aircraft fuselage and wings, large aircraft wings, small submarine craft hulls, etc., several geometric similarities can be identified. The fuselage and hulls of these vehicles can all be described as roughly cylindrical in nature. Aircraft wings have a flattened cylindrical character that tapers along its length. To emulate these environments, two experimental platforms—one cylindrical and one conical in shape—will be used to provide a representative environment in which to make enclosed space channel measurements. These platforms, shown in Figs. 1.1 and 1.2 and described in detail in Chapter 3 and Appendix C, are made of aluminum and are segmented to allow internal objects to be placed within each. Though the model presented in this work is anticipated to describe the general character of dispersion and gain of any outer enclosure shape, this work will specifically consider just the cylindrical and conical enclosures. However, it is important to note that the spirit of broad geometric descriptions mentioned previously will not be violated, as the model presented for dispersion and gain will not depend on an enclosure's specific geometric configuration.

The content of this dissertation is organized as follows. Chapter 2 will produce the relevant background material for the cavity resonator and reverberation chamber electromagnetic environments. Reverberation chambers are test facilities that are used to study the electromagnetic suscep-



Figure 1.1: Cylindrical enclosed-space experimental platform shown with a wooden meter stick for perspective.



Figure 1.2: Conical enclosed-space experimental platform shown with a wooden meter stick for perspective.

tibility and/or radiation characteristics of a device-under-test. They can be large enough to house large aircraft or small enough to fit nicely on a lab bench for small electronics testing. Most reverberation chambers are rectangular with walls made of metal (often aluminum)—thus forming an electromagnetic resonant cavity—and the electromagnetic boundary conditions inside the chamber are varied by means of a metallic stirrer so that every point located on the object under test is exposed to an uniform average field strength. The electromagnetic environments of reverberation chambers and reverberant enclosed spaces are indeed very similar, and much can be learned about the enclosed space radio channel by first understanding the theory of reverberation chambers. Chapter 3 will describe the experimental setup and procedures used to study the enclosed space radio channel. Just as the quality factor of a reverberation chamber is a critical characteristic from which much of its performance can be surmised, it will be clear to the reader that the quality factor of the enclosed space radio channel is central to modeling the dispersion and gain of the channel, and measurements of the quality factor in the enclosed space platforms will be presented in Chapter 4. A model for and measurements of the enclosed space channel dispersion will then be presented and discussed in Chapter 5. Chapter 6 will present the measurements and model for the average power gain of the enclosed space radio channel, as well as present measurements and a model for the dynamic range of the power gain seen in the channel. As motivated previously, the performance of each type of communications signaling in every radio channel is unique, and Chapter 7 will benchmark the dispersion and gain results by performing several sets of communications experiments using real signals transmitted through the enclosed space channel. The main results of the work and the future work envisioned by its investigators will be presented in Chapter 8. Several appendices also accompany this work. Appendix A presents the complete eigenfrequency and eigenfunction solutions for the classical geometries of cavity resonators: Rectangular, cylindrical and spherical. Appendix B presents the eigenfrequency and eigenfunction solutions for the truncated conical cavity resonator, which includes solutions for the resonant frequencies found via both the eigenvalue solutions and the much more straightforward WKB method. The presentation of the complete solutions for the truncated conical cavity resonator with the additional WKB solution has not previously been presented in the literature, to the knowledge of the investigators. Finally, Appendix C contains the design documents used to manufacture the enclosed space platforms shown in Figs. 1.1 and 1.2.

## Chapter 2

# Background

While it appears that communications within reverberant enclosed spaces has not been previously considered at length, the electromagnetic environment of reverberant enclosures is very well studied. Cavity resonators are electromagnetic devices that store energy in electric and magnetic fields inside of the enclosure and have been used for many purposes, a large number of which are associated with filter applications. Reverberation chambers are electromagnetic environments that are used in one instance to evaluate devices- or systems-under-test by exposing the test object to a large electromagnetic field in the chamber. Both the cavity resonator and reverberation chamber environments have several associated studies that expose investigators to a very thorough rendering of the electromagnetic properties of reverberant enclosures, including the statistical distribution of eigenfrequencies and eigenfunctions in a reverberant space, the density of eigenstates within that space and the quality factor associated with that space. Several important results originally derived for the environment of the cavity resonator and the reverberation chamber are presented below, providing the reader with the tools that are necessary for the study of enclosed reverberant spaces as wireless communications channels.

### 2.1 Cavity Resonators

Consider an enclosed space environment of arbitrary geometry with metallic walls that does not contain any internal objects, excited and sensed using antennas contained within the enclosure. Such a space could also be called a cavity resonator and would exhibit properties such as fundamental

(i.e. dominant) and higher order resonances; that is, the cavity will only allow certain frequencies of energy to be maintained in the enclosure. Energy at frequencies between resonant modes or below the fundamental mode will be evanescent within the cavity. If the space were to be of a separable geometry (i.e. rectangular prism, cylindrical prism, sphere, etc.), the resonance frequencies and field distributions could be obtained deterministically in a straightforward manner. Appendix A contains the field solutions and resonance frequency relations for the rectangular, cylindrical, spherical and conical cavity resonators. If the constitutive parameters of the enclosure surface are known (i.e. conductivity, permeability, etc.), quality factors for each mode can also be found deterministically by integrating the energy density and dissipated power density over the enclosure volume and surface area, respectively.

Cavity resonators can also be described statistically in terms of resonant frequency spacing, mode density and field amplitude. Such statistical descriptions are useful when deterministic descriptions of a cavity are impractical or unnecessary, or when the geometry is complex and non-separable, i.e. not a standard geometry (e.g. cylinder, sphere) nor a close variant (e.g. annulus, cone, hemisphere). Knowledge of the field amplitude distributions, mode density and frequency spacing distributions is essential to the understanding of the electromagnetic environment of reverberant enclosed spaces and the related wireless communications channel. In the following sections, the statistical properties of cavity eigenfrequency spacing, mode density and field amplitudes are presented, starting with a overview of cavity mode density.

### 2.1.1 Cavity Mode Density

Consider the dispersion relation for a rectangular cavity resonator with dimensions  $a$ ,  $b$  and  $d$  in the  $x$ ,  $y$  and  $z$  directions, respectively:

$$k^2 = k_x^2 + k_y^2 + k_z^2 \quad (2.1)$$

$$k_x = \frac{m\pi}{a}, k_y = \frac{n\pi}{b}, k_z = \frac{p\pi}{d}. \quad (2.2)$$

Since  $k = \frac{2\pi f}{v}$ , where  $v$  is the speed of light in the medium, it is easily seen that the dispersion relation describes the allowable resonances in the rectangular resonator. If  $k_x$ ,  $k_y$  and  $k_z$  were to be plotted on a three dimensional axis in  $k$ -space, the octant where all axes are positive would contain

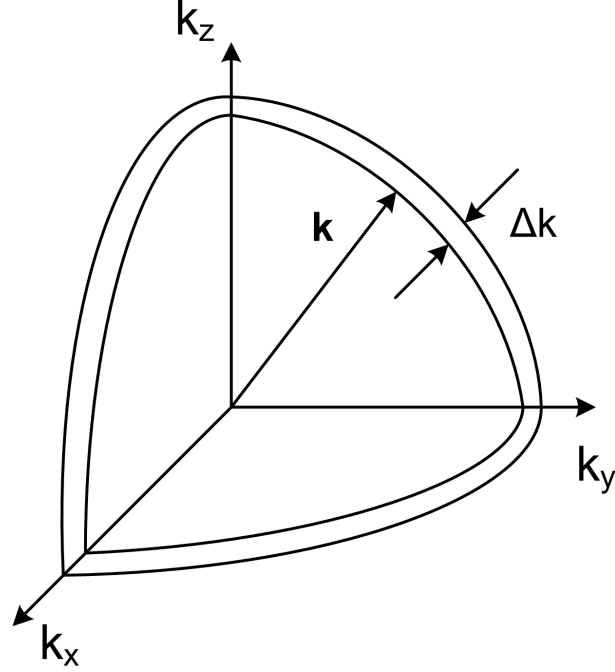


Figure 2.1: Shell of integration in the  $\hat{k}$ -space for the derivation of mode density in resonant spaces. [1]

a lattice of points corresponding to the allowable triplets of the rectangular cavity, and  $\bar{k}$  is the vector from the origin to the point corresponding to the triplet  $(m, n, p)$ . Such a plot is found in Fig. 2.1, where one can see that as  $|\bar{k}|$  increases, the number of points (modes) contained within the shell  $|\bar{k}| + \Delta k$  increases without bound, leading to the description of mode density, or the number of modes per unit  $\hat{k}$ -space. It has been shown in [1] that the smoothed mode density in  $\hat{k}$ -space for a rectangular cavity resonator is

$$\bar{D}_{rect}(k) = \frac{abd}{\pi^2} k^2 - \frac{a+b+d}{2\pi}, \quad (2.3)$$

which can be equivalently solved for frequency, giving the average number of modes per unit spectral width:

$$\bar{D}_{rect}(f) = 8\pi abd \frac{f^2}{v^3} - \frac{a+b+d}{v}. \quad (2.4)$$

These results agree with the results presented in [59], which also compares these solutions to those for the mode density in acoustical resonators. The first term in (2.4) is a well-known term called the

asymptotic Weyl expression for eigenfrequency densities:

$$\bar{D}_0(f) = \frac{8\pi V f^2}{v^3}. \quad (2.5)$$

Weyl showed in 1912 that for high frequencies ( $f \gg c/\sqrt[3]{V}$ ), (2.5) describes the smooth mode density in a resonant cavity independent of that cavity's exact geometry [60, 61, 62, 63]. The other terms in (2.4) are specific to the rectangular geometry, fine-tuning the smoothed mode density expression for the specific case of the rectangular geometry; however, note that these additional terms do not change the behavior of the mode density expression over frequency. An expression that can be used to calculate the smoothed mode density for any arbitrary geometry and  $f > c/\sqrt[3]{V}$  is derived in [64], and given below in the spherical coordinate system:

$$\bar{D}(f) = \frac{8\pi V f^2}{c^3} - \frac{2}{3\pi v} \int_{surface} \left[ \frac{1}{R_\phi(\bar{r})} + \frac{1}{R_\theta(\bar{r})} \right] ds + \frac{1}{6\pi v} \int_{edges} \frac{\pi^2 + 5\psi^2(\bar{r}) - 6\pi\psi(\bar{r})}{\psi(\bar{r})} dl. \quad (2.6)$$

In (2.6),  $R_\phi$  and  $R_\theta$  are the radius of curvature in the  $\hat{\phi}$  and  $\hat{\theta}$  directions, respectively. The internal angle of a given edge segment  $dl$  located at the point  $\bar{r}$  is given by  $\psi(\bar{r})$ . The coordinate system assumed in (2.6) has its origin located inside the cavity,  $R_{\phi,\theta} > 0$  represent concave surfaces with respect to the origin, and  $\lim_{R_{\phi,\theta} \rightarrow \infty} \int_{surface} R_{\phi,\theta}^{-1} ds = 0$ . The expression in (2.6) could be evaluated for a circular cylinder cavity, the result of which is as follows, where  $d$  is the cylinder length and  $a$  is the radius [64]:

$$\bar{D}_{cyl}(f) = \bar{D}_0 - \frac{1}{v} \left( \frac{4}{3}d + \pi a \right) = \frac{8\pi V f^2}{c^3} - \frac{1}{v} \left( \frac{4}{3}d + \pi a \right). \quad (2.7)$$

Beyond a standard geometry such as a cylindrical or rectangular cavity, experimental work has been performed in [65] in a non-uniform geometry that confirms that the mode density as predicted by (2.6) indeed matches the empirically measured (by counting resonant peaks in spectrum measurements) mode density profile very well.



### 2.1.2 Cavity Eigenfrequency Distributions

The mode density function,  $D(f)$ , is related to the mode counting function,  $N(f)$ , through the following relationship:

$$D(f) = \frac{dN(f)}{df}, \quad N(f) = \int_0^f D(\gamma) d\gamma + \text{const.} \quad (2.8)$$

The mode counting function,  $N(f)$ , is simply a cumulative step function (i.e. "staircase" function), which increases by one in value each time a new mode is encountered as frequency increases. In the case of mode degeneracies (modes that have identical resonant frequencies but orthogonal field patterns), the step function would increase by the number of degenerate modes at that frequency. These sudden changes in  $N(f)$  cause rapid changes in the mode density function  $D(f)$  that depart from the smooth behavior described by the smoothed mode density function,  $\overline{D}(f)$ ; these rapid changes, or oscillations in the mode density have been studied in [66, 67]. Separable geometries (i.e. geometries whose wave equations can be solved using the separation of variables technique) are known to experience mode degeneracies, as is shown for the several separable geometries examined in Appendix A.

While the exact position of resonant modes in the frequency spectrum of separable geometry cavity resonators can be found using the dispersion relation specific to that geometry, an alternative to a deterministic representation of the spectral mode locations is a stochastic representation of the relative positions of adjacent modes in the frequency spectrum. Let the space between two adjacent modes, normalized by the average spacing, be defined with the variable  $s$ ,

$$s = \frac{f_{n+1} - f_n}{\overline{D}^{-1}(f)}, \quad (2.9)$$

where  $\overline{D}^{-1}(f)$  is the inverse smoothed mode density (i.e. the average space between resonant modes). Now further consider that  $s$  is a random variable described by some distribution,  $p_s(s)$ . Quantum level distribution statistics give a Poisson distribution for the normalized level spacings of separable systems, and Deus, et. al. have shown in [65] that these statistics fit well to the normalized eigenfrequency spacings of separable geometry microwave electromagnetic cavities,  $s$ , as well. It was shown in the same work that the statistical distribution known to describe classically chaotic

quantum system level statistics, i.e. the Wigner distribution, also fits the statistics of the normalized eigenfrequency spacings,  $s$ , in non-separable microwave cavity geometries well [65]. Thus, the random variable  $s$  can be written as follows:

$$f_s(s) = \begin{cases} \exp(-s) & \text{for separable geometries} \\ \frac{\pi}{2} s \exp\left(-\frac{\pi}{4} s^2\right) & \text{for non-separable geometries} \end{cases}. \quad (2.10)$$

Furthermore, the Brody distribution [68] can be used to represent either the Poisson or Wigner distribution by introducing a parameter,  $\beta$ , which can cause the Brody distribution to take either the Poisson or Wigner form, or a hybrid form in between either distribution:

$$f_\beta(s) = a s^\beta \exp(-b s^{\beta+1}), \quad (2.11)$$

$$a = (\beta + 1)b, \quad b = \left[ \Gamma\left(\frac{\beta + 2}{\beta + 1}\right) \right]^{\beta+1}. \quad (2.12)$$

The parameter  $\beta$  in the Brody distribution is called the repulsion parameter; when  $\beta = 0$ , the Brody distribution becomes a Poisson distribution, and when  $\beta = 1$ , the Brody distribution is equivalent to the Wigner distribution. The repulsion parameter acquires its name from the phenomena it describes: Repulsion of modes along the frequency axis. Separable geometries exhibit numerous degeneracies, i.e. modes with zero inter-modal spacing along the frequency axis ( $s = 0$ ); the Poisson distribution accordingly shows a maximum at ( $s = 0$ ), suggesting the tendency for "mode-attraction". Conversely, the Wigner distribution—equivalent to the Rayleigh distribution with a Rayleigh parameter of  $\sqrt{2/\pi}$ —shows a maximum away from ( $s = 0$ ), meaning a separation in frequency greater than zero is much more likely than no separation at all (i.e. degenerate modes), implying that a phenomena of "mode-repulsion" exists in non-separable geometries.

The concept of mode repulsion can also be explained with ray tracing representations. If one were to consider a ray tracing representation of a cavity resonator (in which a launched plane wave, or ray, is traced through its allowable paths, including reflections from the cavity walls, back to the point at which it was launched), a degenerate mode is one whose path length is the same as another mode, but the specific course of the path is different. Cavities such as spheres and cubes have symmetries that support many such identical-length paths, leading to the degenerate modes, whereas a cavity with a non-regular or chaotic geometry provides few if any such paths (because of a

lack in symmetry). Degenerate modes in chaotic geometries are thus very unlikely, and suggests an inter-modal spacing distribution with a small value at zero, i.e. the Wigner (Rayleigh) distribution. In contrast, the intrinsic symmetry of many regular geometries allows for the statistical presence of such paths, leading to a distribution with a significant value at zero, i.e. the Poisson distribution.

### 2.1.3 Field Amplitude Distributions

The previous sections have discussed the statistical behavior of the eigenfrequencies in cavity resonators, as well as the mean mode density as a function of volume and frequency. The statistics of the amplitudes of the electric and magnetic fields in an overmoded cavity have also been considered in the literature. For a single mode in a cavity resonator of separable geometry, the wave equation can be solved for that geometry and the electric and magnetic field amplitudes can be calculated for that mode at a given position in the cavity. Expressions for such separable geometries can be found in Appendix A for the rectangular, cylindrical, spherical and conical cavity resonators. However, if a cavity of arbitrary (including separable) geometries were to be overmoded, characterizing the field amplitude at a given point becomes a considerably more difficult problem. Furthermore, even if the field amplitudes for all excited modes in a given cavity were found exactly (perhaps deterministically), the solution obtained becomes invalid the moment the cavity is reconfigured, even slightly. A statistical description of field amplitudes in overmoded cavities provides a balance between describing the fields and forming a model that can withstand changes in the geometric configuration of the cavity. It has been shown that power received by an antenna in an arbitrary complex overmoded cavity can be represented stochastically using the  $\Gamma$ -distribution [69]:

$$f_{P_r}(x) = \frac{x^{\alpha-1} e^{-x/\beta}}{\Gamma(\alpha) \beta^\alpha}, \quad (2.13)$$

$$\alpha(f) = \left(1 + \frac{6}{\pi N_s(f)}\right)^{-1}, \quad (2.14)$$

$$\beta(f) = \mu \left(1 + \frac{6}{\pi N_s(f)}\right), \quad (2.15)$$

where the mean and variance of the distribution (both as functions of frequency) are

$$\mu(f) = \alpha\beta = E[P_r] = \frac{\sigma_s U \nu}{3V}, \quad (2.16)$$

$$\sigma^2(f) = \alpha\beta^2 = \left(\frac{\sigma_s U \nu}{3V}\right)^2 \left(1 + \frac{6}{\pi N_s(f)}\right). \quad (2.17)$$

In the above results,  $V$  is cavity volume,  $\sigma_s$  is the free-field cross section of the receiving sensor ( $\sigma_s = P_s/S$ , where  $P_s$  is the power measured by the sensor and  $S$  is the incident power density),  $\nu$  is the speed of light in the medium inside the cavity, and  $U$  is the total energy contained in the cavity.  $N_s(f)$  represents the specific mode density, which is the average number of modes that are excited by a CW tone at a given frequency  $f$ :

$$N_s(f) = \frac{8\pi V}{Q_c(f)} \left(\frac{f}{\nu}\right)^3. \quad (2.18)$$

This value can be derived by multiplying the Weyl expression in (2.5) by the average half-power bandwidth,  $f/Q_c(f)$ , where  $Q_c(f)$  is the composite quality factor of the cavity at the frequency  $f$  (see Section 2.2.3). It is known that  $\Gamma$ -distribution with  $\alpha = 1$  takes the form of the  $\chi^2$ -distribution with 2 degrees of freedom (i.e. the exponential distribution). Notice from (2.14) and (2.18) that  $\alpha \rightarrow 1$  as the number of modes excited in the cavity gets large ( $N_s \gg 1$ ), which happens when  $f$  gets large,  $V$  gets large or  $Q_c(f)$  becomes small. Thus, in the limit of high frequency, large volume or small quality factor, the distribution of power received is given by an exponential distribution:

$$f_{P_r}(P) = \frac{3V}{\sigma_s U \nu} \exp\left\{-\frac{3V}{\sigma_s U \nu} P\right\}. \quad (2.19)$$

## 2.2 Reverberation Chambers

Much research has been done in the area of electromagnetic compatibility characterizing the performance of reverberation chambers. Reverberation chambers are typically three dimensional rectangular spaces with highly conductive metal walls that are excited with electromagnetic energy to test an enclosed device for electromagnetic interference immunity and/or generation. Reverberation chambers can be quite large, such as the NASA Langley Reverberation Chamber A shown in Fig. 2.2, as well as quite small, such as the Bluetest Reverberation chamber used to test small electronic

devices as shown in Fig. 2.3. Large chambers can be used to test the electromagnetic immunity and/or generation of very large systems, such as an entire aircraft, and small chambers—such as the one shown in Fig. 2.3—can be used for a variety of purposes, including electromagnetic immunity and compatibility testing, antenna efficiency testing [70, 71, 72], diversity antenna system testing [73, 74, 75] and emulated channel testing (simulating a Rayleigh faded environment within the chamber to test small devices such as mobile phones) [76, 77].

In their physical configurations, the reverberation chamber and the enclosed space have many traits in common. Both environments have conductive walls and non-uniform boundaries (once the objects under test are introduced to the chamber). Both applications are concerned with the properties of the electromagnetic fields excited in each respective space (i.e. field homogeneity, field strength and transient response for reverberation chambers and such radio link properties as average transmitter to receiver power loss, spatial and frequency correlations of fields, and power delay profile characteristics for enclosed spaces). The quality factor of a reverberation chamber is intuitively related to the RMS delay spread and mean excess delay of an enclosed space; higher Q values indicate less energy loss per unit time, which increases the length of a power delay profile, and consequently, the RMS delay spread and mean excess delay. Field strength for reverberation chambers is directly related to the transmitter-receiver power loss in enclosed spaces. Field correlations are also directly analogous for both environments. It seems clear that a knowledge of reverberation chamber properties could be very helpful in the study of enclosed spaces. The literature is rich with a diverse span of topics concerned with the characterization and operation of reverberation chambers. Two of the earliest published accounts of using a reverberation chamber to measure radiated microwave emissions are found in [78, 79]; a recently published overview of the properties and uses of a reverberation chamber is found in [80]. Reverberation chambers can be viewed as a form of an overmoded electromagnetic cavity with complex geometry (with internal objects-under-test in place), and an important metric for the quality of field homogeneity in a reverberation chamber is the mode density, already presented for complex cavities in Section 2.1.1. In an analogous manner, the distribution of eigenfrequencies in multimode cavities presented in Section 2.1.2 applies directly to reverberation chambers. Methods by which the mode density can be increased by altering the internal shape of the chamber using insights provided by [64] through (2.6) have also been considered [81]. The response of an antenna in the presence of the incident chamber fields when placed inside



Figure 2.2: NASA Langley Reverberation Chamber A, 2.90 m x 7.01 m x 14.33 m. Photo courtesy <http://cbunting.ecen.ceat.okstate.edu/Reverberation.htm> and NASA website, <http://www.nasa.gov>.



Figure 2.3: Bluetest reverberation chamber for small electronics testing, 0.79 m x 1.85 m x 1.06 m. Photo courtesy Bluetest website, <http://www.bluetest.se>.

the reverberation chamber has been studied assuming stochastic fields in [82, 83, 84].

Additionally, studies have been made on the statistics of the field components inside a reverberation chamber, as well as the spatial correlation of the fields. The power received by an antenna in a reverberation chamber, and therefore, the gain of the chamber has been considered. The composite quality factor (averaged, considering all excited modes) of a chamber—both for specific chamber geometries and independent of the chamber geometry—has been derived, allowing one to estimate the quality factor based only on a chamber volume, surface area, and the relative permeability and skin depth of the walls. These aforementioned topics will now be discussed with some detail, starting with the statistics of the fields in a reverberation chamber.

### 2.2.1 Chamber Field Statistics

One of the earliest works considering the statistics of the fields in reverberation chambers was done by Kostas and Boverie in 1991 [85]. In this study, overmoded reverberation chambers are studied in the situation where the fields are stochastically varied in the chamber by changing the position of a metal stirrer with time so that the boundary conditions—and thus the field eigenfunctions—are also varied to form an ensemble average over time. Indeed, most reverberation chambers operate using a form of mechanical stirring, although frequency stirring techniques have also been employed [86, 87]. The analysis by Kostas and Boverie proceeds by assuming that the chamber fields are a vector sum of the many modes excited in the chamber and that each in-phase and quadrature component of the electric and magnetic fields (12 components total) is a sum of many identically distributed random variables (i.e. the complex mode amplitudes), which are a function of the stirrer position over time. Since the boundary conditions imposed by the walls was not explicitly considered, the results in [85] are limited to points away from the chamber walls\*. By the Central Limit Theorem, each of these 12 components is described by the Gaussian distribution; the magnitude of any of the 6 components of the electric and magnetic fields is then described by the Rayleigh distribution ( $\chi$ -distribution with two degrees of freedom) [85],

$$f_{|E_i|}(|E_i|) = \frac{|E_i|}{\sigma^2} e^{-|E_i|^2/2\sigma^2}, \quad (2.20)$$

---

\*Hill derived the statistics of the chamber fields located at the walls of the enclosure in 2005 [88]. See the discussion near the end of this section for more details.

where  $|E_i|$  is the magnitude of any electric (or magnetic) field component, and  $\sigma^2$  is the variance of each of the six field components, which is proportional to the average energy contained in a particular field component.

Since the magnitude of the field is  $\chi$ -distributed with two degrees of freedom, it follows that the square of the field magnitude is  $\chi^2$ -distributed with two degrees of freedom, i.e. an exponential distribution [85]:

$$f_{|E|^2}(|E|^2) = \frac{1}{2\sigma^2} e^{-|E|^2/2\sigma^2}, \quad (2.21)$$

which is the same as the result found in the high frequency/high volume/low Q limit of (2.19) ( $N_s \gg 1$ ), derived in [69] for complex cavities and presented in Section 2.1.3.

Dunn has shown that each of the six components (3 orthogonal electric components and 3 orthogonal magnetic components) contain the same average energy if the fields are being considered away from the chamber walls ( $\epsilon\langle|E_i|^2\rangle/2 = \mu\langle|H_i|^2\rangle/2 = W/6$ , where  $E_i$  and  $H_i$  are any two components of the electric and magnetic fields and  $W$  is the average total energy density in the cavity); however, the fields very near to the walls do not show identical energy partitioning, due to the imposed boundary conditions [89]:

$$\frac{\mu}{2}\langle|H_{tan}|^2\rangle\Big|_{walls} = \frac{2W}{3}, \quad (2.22)$$

$$\frac{\epsilon}{2}\langle|E_{norm}|^2\rangle\Big|_{walls} = \frac{W}{3}, \quad (2.23)$$

$$\frac{\mu}{2}\langle|H_{norm}|^2\rangle\Big|_{walls} = 0, \quad (2.24)$$

$$\frac{\epsilon}{2}\langle|E_{tan}|^2\rangle\Big|_{walls} = 0. \quad (2.25)$$

where  $W$  is the average total energy density in the chamber.

An alternative to modeling the reverberation chamber as the vector sum of many excited field eigenfunctions is to model the fields as a vector sum of many different propagating plane waves, associated with the different modes in the chamber. Hill provides a derivation of the field statistics using a plane wave model in [90, 91]. Given that the chamber stirrer provides a rich ensemble of field structures, it is assumed that the angle of arrival of the ensemble of plane waves is uniformly distributed over  $4\pi$  steradians, away from the chamber walls. Under this assumption, the ensemble



of plane waves give rise to field statistics that match the results of Kostas and Boverie in [85], i.e. Rayleigh distributed in field amplitude and exponentially distributed in the field magnitude-squared. Hill derives in [90] the probability distribution functions for the real and imaginary parts of the current induced in a load connected to a receiving antenna, finding a zero-mean Gaussian distribution for each:

$$f_{I_r}(I_r) = \frac{1}{\sqrt{2\pi}\sigma_I} \exp\left[-\frac{I_r^2}{2\sigma_I^2}\right] \quad (2.26)$$

$$f_{I_i}(I_i) = \frac{1}{\sqrt{2\pi}\sigma_I} \exp\left[-\frac{I_i^2}{2\sigma_I^2}\right]. \quad (2.27)$$

Hill continues to state that the magnitude of the current induced in a load connected to the receive antenna is a  $\chi$ -distributed random variable with 2 degrees of freedom (i.e. a Rayleigh distribution) [92],

$$f_{|I|}(|I|) = \frac{|I|}{\sigma_I^2} \exp\left[-\frac{|I|^2}{2\sigma_I^2}\right], \quad (2.28)$$

and the magnitude squared of the same current is a  $\chi^2$  distributed random variable with two degrees of freedom (i.e. the exponential distribution) [92],

$$f_{|I|^2}(|I|^2) = \frac{1}{2\sigma_I^2} \exp\left[-\frac{|I|^2}{2\sigma_I^2}\right]. \quad (2.29)$$

Since the power dissipated in the receiver load is proportional to the magnitude squared of the current induced, the distribution of the power received inside a reverberant space is also exponentially distributed [90]:

$$f_{P_r}(P_r) = \frac{1}{2\sigma_I^2 R_r} \exp\left[-\frac{P_r}{2\sigma_I^2 R_r}\right], \quad (2.30)$$

where  $R_r$  is the radiation resistance of the antenna, if it is matched, or the resistance of the load, if it is not. In the above distributions, the  $\sigma_I$  parameter is defined as follows:

$$\sigma_I^2 = \frac{1}{4R_r} \frac{Uv}{V} \frac{\lambda^2}{4\pi} = E[I_r^2] = E[I_i^2], \quad (2.31)$$

where  $U$  is the total average energy in the chamber volume  $V$ ,  $R_l$  is the load resistance and  $\lambda$  is the wavelength.

Hill published results in 2005 [88] showing that the statistics of the non-zero field-components at the wall of the chamber do indeed have the same distribution as the field-components considered at points away from the walls. That is, the normal component of the electric field and the tangential components of the magnetic field at any point along the chamber wall will display a Rayleigh distribution in their magnitude statistics and a exponential distribution in their magnitude-square statistics. Hill also confirms the results that Dunn presented in [89] that show that the non-zero field components at the points along the walls contain twice the energy as the same components at points away from the walls [88]. Knowledge of these field statistics at the walls will be particularly useful in the discussion of the dynamic range of the enclosure gain, as will be presented in Chapter 6.

It should be remembered that in the above presented statistics, all investigators made the following three assumptions: 1) The reverberant enclosure is overmoded and 2) The fields are being ensemble averaged through the means of a mechanical stirring device and 3) Excitation of the reverberant space was accomplished using a CW signal. Excitation using signals other than CW tones have been considered (e.g. in [86, 87]), and this technique of chamber excitation is known as *frequency stirring* and can often be used as a substitution for mechanical stirring. In the frequency stirring method the variation of the fields in the chamber are realized by changing the excitation frequencies over time, thereby changing the field patterns in the reverberant space over time.

### 2.2.2 Spatial Field Correlations

The previous section discussed the statistics of the electric and magnetic fields at a given point inside the chamber. Spatial field correlations describe statistical dependence between two points separated by a given distance in the chamber. If the fields at two points are perfectly correlated ( $\rho = 1$ ), the fields are linearly related; if one point experiences a fade in field strength, the other will also fade. Likewise, zero correlation ( $\rho = 0$ ) indicates a statistical independence, and an intermediate value ( $0 < \rho < 1$ ) indicates some measure of statistical dependence, which increases as  $\rho \rightarrow 1$ . Such correlation functions are important for understanding the test conditions for the system-under-test in a reverberation chamber and, in the case of the enclosed space radio channel, give the required distance of separation such that two antennas experience statistically uncorrelated

channels—important for implementing diversity effectively.

Hill uses the same plane wave model in a reverberation chamber as presented in the previous section to find the spatial correlation of fields as a function of a separation vector. Consider the two points in space inside the chamber,  $\bar{r}_1$  and  $\bar{r}_2$  whose separation vector is  $\bar{r} = \bar{r}_2 - \bar{r}_1 \triangleq \hat{r}|\bar{r}_2 - \bar{r}_1| = \hat{r}r$ . The longitudinal correlation function, i.e. the correlation function for the field components in the  $\hat{r}$ -direction, is given as [82, 93]

$$\rho_l(\bar{r}_1, \bar{r}_2) = \frac{\langle E_l(\bar{r}_1)E_l^*(\bar{r}_2) \rangle}{\sqrt{\langle |E_l(\bar{r}_1)|^2 \rangle \langle |E_l(\bar{r}_2)|^2 \rangle}} \quad (2.32)$$

$$= \frac{3}{(kr)^2} \left[ \frac{\sin(kr)}{kr} - \cos(kr) \right], \quad (2.33)$$

where  $k$  is the wavenumber of the fields under consideration and  $\langle \cdot \rangle$  represents the ensemble average. Likewise, the correlation function for the fields transverse to  $\hat{r}$  is given by the following expression [82, 93]:

$$\rho_t(\bar{r}_1, \bar{r}_2) = \frac{\langle E_t(\bar{r}_1)E_t^*(\bar{r}_2) \rangle}{\sqrt{\langle |E_t(\bar{r}_1)|^2 \rangle \langle |E_t(\bar{r}_2)|^2 \rangle}} \quad (2.34)$$

$$= \frac{3}{2} \left\{ \frac{\sin(kr)}{kr} - \frac{1}{(kr)^2} \left[ \frac{\sin(kr)}{kr} - \cos(kr) \right] \right\}. \quad (2.35)$$

The longitudinal (2.33) and transverse correlation (2.35) functions can be combined to find the total electric field correlation function [82, 93]:

$$\rho(\bar{r}_1, \bar{r}_2) = \frac{\langle \bar{E}(\bar{r}_1)\bar{E}^*(\bar{r}_2) \rangle}{\sqrt{\langle |\bar{E}(\bar{r}_1)|^2 \rangle \langle |\bar{E}(\bar{r}_2)|^2 \rangle}} \quad (2.36)$$

$$= \frac{1}{3} [2\rho_t(\bar{r}_1, \bar{r}_2) + \rho_l(\bar{r}_1, \bar{r}_2)] = \frac{\sin(kr)}{kr}. \quad (2.37)$$

From the above expressions, one can see that the field correlation functions in the reverberation chamber follow a  $\sin(x)/x$  behavior and are different than the free-space indoor or outdoor propagation channels, which display a monotonically decreasing behavior. Whereas in an indoor or outdoor multipath channel, a further separation in space will on average result in a decreased correlation, the  $\sin(x)/x$  correlation behavior in a reverberation chamber suggests that as  $r$  increases from zero, the above correlation functions pass through a point of zero correlation magnitude and then proceed to

an intermediate maxima. A similar study of correlations in complex cavities without considering a stirring method [94] shows the identical results for the transverse and longitudinal correlations, as well as for the total field correlation function.

The correlations of the central magnitude squared field components—which are proportional to the power induced in a receiving antenna placed in the chamber—have also been derived in [82, 93]. The correlation function for the longitudinal field component is found to be

$$\rho_{ll}(\bar{r}_1, \bar{r}_2) = \frac{\langle [ |E_l(\bar{r}_1)|^2 - \langle |E_l(\bar{r}_1)|^2 \rangle ] [ |E_l(\bar{r}_2)|^2 - \langle |E_l(\bar{r}_2)|^2 \rangle ] \rangle}{\sqrt{\langle [ |E_l(\bar{r}_1)|^2 - \langle |E_l(\bar{r}_1)|^2 \rangle ]^2 \rangle \langle [ |E_l(\bar{r}_2)|^2 - \langle |E_l(\bar{r}_2)|^2 \rangle ]^2 \rangle}} \quad (2.38)$$

$$= \rho_l^2(\bar{r}_1, \bar{r}_2) = \left\{ \frac{3}{(kr)^2} \left[ \frac{\sin(kr)}{kr} - \cos(kr) \right] \right\}^2. \quad (2.39)$$

Similarly, the correlation function for the central magnitude squared transverse field has been derived to be [82, 93]

$$\rho_{tt}(\bar{r}_1, \bar{r}_2) = \frac{\langle [ |E_t(\bar{r}_1)|^2 - \langle |E_t(\bar{r}_1)|^2 \rangle ] [ |E_t(\bar{r}_2)|^2 - \langle |E_t(\bar{r}_2)|^2 \rangle ] \rangle}{\sqrt{\langle [ |E_t(\bar{r}_1)|^2 - \langle |E_t(\bar{r}_1)|^2 \rangle ]^2 \rangle \langle [ |E_t(\bar{r}_2)|^2 - \langle |E_t(\bar{r}_2)|^2 \rangle ]^2 \rangle}} \quad (2.40)$$

$$= \rho_t^2(\bar{r}_1, \bar{r}_2) = \left\{ \frac{3}{2} \left\{ \frac{\sin(kr)}{kr} - \frac{1}{(kr)^2} \left[ \frac{\sin(kr)}{kr} - \cos(kr) \right] \right\} \right\}^2. \quad (2.41)$$

The correlation function for the central magnitude squared of the total field is found using the following expression [82, 93]:

$$\rho_{EE}(\bar{r}_1, \bar{r}_2) = \frac{\langle [ |\bar{E}(\bar{r}_1)|^2 - \langle |\bar{E}(\bar{r}_1)|^2 \rangle ] [ |\bar{E}(\bar{r}_2)|^2 - \langle |\bar{E}(\bar{r}_2)|^2 \rangle ] \rangle}{\sqrt{\langle [ |\bar{E}(\bar{r}_1)|^2 - \langle |\bar{E}(\bar{r}_1)|^2 \rangle ]^2 \rangle \langle [ |\bar{E}(\bar{r}_2)|^2 - \langle |\bar{E}(\bar{r}_2)|^2 \rangle ]^2 \rangle}} \quad (2.42)$$

$$= \frac{1}{3} [2\rho_{tt}(\bar{r}_1, \bar{r}_2) + \rho_{ll}(\bar{r}_1, \bar{r}_2)]. \quad (2.43)$$

Empirical measurements in [95, 96] showed a fairly good match to the theoretical results for the spatial correlation functions shown above; however, Hill points out in [82] that the work in [95, 96]—which used transversely oriented monopole antennas and compared the results to  $\rho^2(\bar{r}_1, \bar{r}_2)$  from (2.37)—should have compared the empirical results to  $\rho_{tt}^2(\bar{r}_1, \bar{r}_2)$  in (2.41), but those results were not available at the time of [95, 96]. Hill compares the empirical results from [95, 96] to (2.41) in [82] and finds that the fit is better than the fit observed with  $\rho^2(\bar{r}_1, \bar{r}_2)$  from (2.37).

### 2.2.3 Composite Quality Factor and Other Losses

The quality factor, or  $Q$ , is generally defined as a ratio of the energy stored per cycle to the energy lost per cycle in an energy storage element, such as a capacitor or a resonant electromagnetic cavity. It can be written as

$$Q = \frac{\omega U}{P_l}, \quad (2.44)$$

where  $\omega$  is the radian frequency of the energy stored ( $U$ ), and  $P_l$  is the power lost from the energy storage element. Qualitatively, the quality factor is a measure of how well an energy storage element "holds-on" to its contained energy over time; a large  $Q$  implies good energy containment, and a small  $Q$  implies poor energy containment, i.e. high losses are present. In a cavity resonator with an infinite number of possible modes, each mode has its own associated  $Q$  value, which reflects losses due to currents in the finite conductivity enclosure walls. In overmoded cavity resonators and reverberation chambers, a much more useful description is the composite quality factor, which is an average of all of the modal quality factors as a function of frequency. A composite quality factor allows one to represent the losses of the many modes in an overmoded resonant space (due to the finite conductivity enclosure) with a single quality factor. The first composite quality factor was derived for the rectangular cavity [1]:

$$Q_{rect} = \frac{3V}{2\mu_r S \delta_s} \left[ 1 + \frac{3\pi}{8k} \left( \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right) \right]^{-1}, \quad (2.45)$$

where  $a$ ,  $b$  and  $c$  are the dimensions of the chamber,  $k$  is the wavenumber,  $V$  is the volume of the chamber,  $S$  is the surface area of the enclosure and  $\mu_r$  and  $\delta_s$  are the relative permeability and skin depth of the enclosure, respectively. Note that in the high frequency limit, this expression reduces to

$$Q_c = \frac{3V}{2\mu_r S \delta_s} = \frac{3V}{2S} \frac{\sqrt{\pi \sigma \mu} f}{\mu_r}, \quad (2.46)$$

which is generally accepted as the average composite quality factor of an arbitrarily shaped resonant cavity. This expression has been additionally derived by averaging the losses of plane-wave

reflections at the walls over all incidence angles and polarizations [89].

Recall now the definition for specific mode density given in (2.18), and insert the above expression for the composite quality factor,  $Q_c$ . The following expression for the specific mode density then represents the average number of modes that exist within the average half-power bandwidth within the reverberant space, independent of specific geometry, in terms of surface area  $S$ , frequency  $f$ , material parameters  $\sigma$  &  $\mu = \mu_r \mu_0$ , and the speed of light in the medium  $v$ :

$$N_s(f) = \frac{16}{3} \frac{S}{v^3} \sqrt{\frac{\pi \mu_r}{\sigma \mu_0}} f^{\frac{5}{2}}. \quad (2.47)$$

Note that the specific mode density increases as  $f^{5/2}$  and is proportional to the enclosure surface area, but does not depend on the volume enclosed. Results presented in Section 2.1.3 showed that as the specific mode density,  $N_s$ , increases, the statistics of the field amplitude in the reverberant space become Rayleigh distributed. The specific mode density will be an important parameter to consider in understanding the dynamic range of the enclosed space radio channel, as the reader will understand in Chapter 6.

While wall losses certainly often account for the bulk of the losses in a cavity or reverberation chamber, additional causes of loss include apertures, antennas and absorption in materials other than the walls. Just as the losses due to the walls are represented using a Q factor, the losses due to each other mechanism are similarly represented using associated quality factors. The net quality factor of a cavity or chamber is found by combining the individual Q's like resistors in parallel [97]:

$$Q_{net}^{-1} = Q_{walls}^{-1} + Q_{antenna}^{-1} + Q_{aperture}^{-1} + Q_{absorb}^{-1} + \dots \quad (2.48)$$

For instance, the loss due to the antenna is found by considering the effective area of that antenna and the power density inside the cavity. The effective area of an antenna has been shown to be [98]  $A_e = m \eta_a \lambda^2 \bar{D} / 8\pi$ , which includes a polarization mismatch factor of 1/2 [99], an impedance mismatch factor  $m$  and an antenna efficiency factor  $\eta_a$ .  $\bar{D}$  is the average directive gain of the antenna used, which when averaged over  $4\pi$  steradians is equal to one, since it is assumed that the plane-wave arrival in an enclosed reverberant space is isotropic. The power density in the cavity is  $S = Uv/V$ —where  $U$  is the energy stored in the enclosure,  $V$  is the volume of the enclosure and  $v$  is

the speed of light in the medium inside the enclosure—and so the quality factor due to the antenna (located at points away from the walls) can be written as [97],

$$Q_{ant} = \frac{\omega U}{P_{ant}} = \frac{\omega U}{S A_e} = \frac{2}{\pi} \left( \frac{\omega}{v} \right)^3 \frac{V}{m \eta_a} = 16\pi^2 \left( \frac{f}{v} \right)^3 \frac{V}{m \eta_a}. \quad (2.49)$$

The impedance mismatch factor  $m$  accounts for the mismatch between the antenna and its feedline and is expressed as

$$m = 1 - \left| \frac{Z_{ant} - Z_0}{Z_{ant} + Z_0} \right|^2 \quad (2.50)$$

where  $Z_{ant}$  and  $Z_0$  are the antenna and feedline impedances, respectively. Note from (2.49) that the antenna quality factor does not explicitly depend on the directive gain pattern of the antenna, since it was assumed that the average value of the directive gain is one (isotropic antenna illumination).

### 2.2.4 Chamber Gain

When a reverberation chamber is excited and sensed with two distinct antennas, the gain seen between those two antennas can be analytically described. For the enclosed space radio channel, this gain is an important measure to understanding the amount of power that is needed at the transmitter for the receiver to successfully determine the message that was sent through the channel. The gain,  $G$ , is found by examining the ratio of the net quality factor (assumed here to be due to wall and antenna losses only, i.e.  $Q_{net}^{-1} = Q_{ant}^{-1} + Q_c^{-1}$ ) and the antenna quality factor, taken from (2.46) and (2.49) [96, 100]:

$$G = \frac{P_{ant}}{P_{in}} = \frac{\omega U / P_{net}}{\omega U / P_{ant}} = \frac{Q_{net}}{Q_{ant}} = \frac{1}{1 + Q_{ant} / Q_c} \quad (2.51)$$

$$= \left\{ 1 + \frac{2}{\pi} \left( \frac{\omega}{v} \right)^3 \frac{V}{m} \left[ \frac{3V}{2\mu_r S \delta_s} \right]^{-1} \right\}^{-1} = \left\{ 1 + \frac{4}{3} \sqrt{\frac{2\mu_r}{\mu_0 \sigma}} \frac{S}{m v^3 \pi} \omega^{5/2} \right\}^{-1}, \quad (2.52)$$

where it is assumed that the total power lost in the system is equal to the total power put into the system (power conservation). The above expression is useful for predicting the gain in a reverberation chamber, based only on the chamber's constitutive parameters, such as surface area and conductivity. Since the enclosed space radio channel is closely related to the reverberation chamber

in the electromagnetic sense, the gain of the enclosed space radio channel can be found in a similar manner, as will be presented in [Chapter 6](#).



## Chapter 3

# Experimental Methods and Platforms

### 3.1 Physical Research Platforms

Empirical measurements inside the enclosed space channel serve the dual purpose of both establishing a fundamental characteristic of the enclosed space channel and confirming the theoretical models that will be developed to describe the channel. Analogous empirical studies have served to characterize and confirm the outdoor, [7, 8, 9], indoor [27] and indoor ultra-wide band [35] channels. To support the empirical measurements needed to form a channel model for the enclosed space environment, two experimental platforms, one cylindrical in shape and one conical in shape, have been designed and are depicted in Figs. 1.1 and 1.2, respectively. These environments are not intended to be a complete set of representative environments with which to study the enclosed space channel, but rather are representative platforms whose characteristics match the critical environmental qualities of the enclosed space channel, identified previously as 1) several meters or less per dimension, 2) surrounded on many or all sides by highly reflective (metallic) boundaries, and 3) consist of non-uniform geometries containing a non-regular distribution of objects. As many of the environments envisioned for enclosed space communications are unavailable to maintain in a university laboratory—i.e. unmanned air vehicles, an aircraft wing, small submarine vehicles, etc.—these platforms provide a similar electromagnetic environment with which to perform and interpret experiments in order to gain critical insights into the channel behavior.

Table 3.1 shows the relevant specifications of each of the spaces. Each apparatus is made of 3.175 mm (0.125 inch) aluminum, whose inner surfaces have been bead blasted, and whose outer

| PARAMETER        | CYLINDER                   | CONE                                             |
|------------------|----------------------------|--------------------------------------------------|
| Length:          | 1.397 m (55 in.)           | 1.524 m (60 in.) (axial)                         |
| Diameter:        | 0.4572 m (18 in.)          | 0.1143 m (4.5 in.) min./ 0.5563m (21.9 in.) max. |
| Segments:        | 5                          | 5                                                |
| Material:        | 3.175 mm (0.125 in.) Alum. | 3.175 mm (0.125 in.) Alum.                       |
| Fund. Resonance: | 400 MHz                    | 433 MHz                                          |

Table 3.1: Specifications for enclosed space research assets.

surfaces have been power coated for durability and presentation. Each design is segmented (shown in Fig. 3.1) so that its internal configuration (position of antennas, arrangement of internal objects, etc.) can be changed efficiently. Furthermore, the segmentation of the devices allows for plates fashioned with seams or apertures to be placed between segments, forming micro-spaces coupled by the seams and/or apertures. Each segment has flanges that allow adjacent segments to be attached quickly and with mechanical integrity. Furthermore, design consideration ensured that when the seams of adjacent sections were affixed to one another, no significant gaps in the seams would exist that could allow stray energy to escape from or enter into the enclosures. As shown in Fig. 3.2, the inside edge of the seams of the platforms were carefully designed to ensure a smooth internal interface so that when the platforms are bolted together, the empty space can be accurately modeled using the cavity resonator theories (Appendix A). Using these theories the fundamental resonance of the cylindrical cavity is found to be approx. 400 MHz, and the fundamental resonance of the conical cavity is found to be approx. 433 MHz (see Appendix B). Each of these platforms were formed segment-by-segment, by cutting the appropriate shape out of 3.175 mm (0.125 in.) aluminum, followed by a precision bending process to form each cylindrical and conical section individually. The flanges were then added, as well as the end caps, and special care was taken to ensure that the aluminum welds were made on the inside of both environments and were smoothed to blend into the surrounding surfaces. The design documents that were used to create the cylinder and conical platforms are included in Appendix C.

The conductivity for the aluminum walls of the devices has been measured to be  $1.125 \cdot 10^7 (\Omega m)^{-1}$  and was performed by measuring the quality factor of the fundamental mode of the empty cylindrical platform (i.e. there are no internal objects or stirrer present inside the cylinder), which is a circular cavity resonator (see Appendix A for more information), neglecting small imperfections such as seams or weld lines. The fundamental mode Q was measured by examining the fundamental



Figure 3.1: Single segment of the cylindrical enclosed space platform.

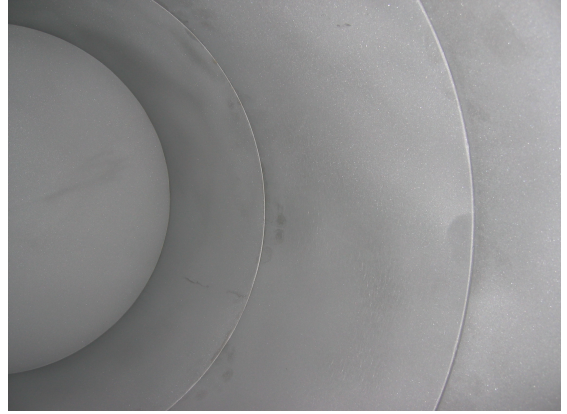


Figure 3.2: Several seams on the inside of the cylindrical enclosed space platform.

mode resonance using a vector network analyzer and the  $S_{11}$  plot on a Smith chart. Any resonant mode in a cavity resonator sufficiently separated from adjacent modes forms a loop on the Smith chart, and reference [101] details a method by which a resonant mode  $Q$  can be extracted from a  $S_{11}$  Smith chart response, removing any loading effects that the antenna may have on the overall measured  $Q$ . Using this method, a quality factor of 22208 was measured for the  $TE_{111}$  mode (fundamental mode) of the cylindrical platform, and using the quality factor for an arbitrary TE mode triplet [102], the conductivity can be found:  $1.125 \cdot 10^7 (\Omega m)^{-1}$ . Since the materials of the conical platform are the same, the conductivity is assumed to be the same for both platforms.

## 3.2 Sparse and Dense Internal Object Populations

In the previous section, it was discussed that the cylindrical and conical empirical platforms were designed so that they were as close to the ideal cylindrical or conical geometry as possible so that spaces could be measured and analyzed using separable techniques (as in Appendix A and Appendix B). However, such a pristine environment is not representative of a realistic reverberant enclosure, which typically is a non-uniform geometry containing objects of random shapes, sizes, materials and locations. Measurements are therefore required in emulated environments that con-

tain an assortment of objects whose shapes, sizes and location could be considered random. A random distribution of internal objects creates a non-regular enclosure shape and has the effect of randomizing the distribution of the eigenfrequencies in the reverberant space, as discussed in Chapter 2, effectively lifting the degeneracies and repelling otherwise degenerate modes apart from one another. Repelling otherwise degenerate or near degenerate modes from one another in an enclosed space is a benevolent effect in that it improves the field homogeneity in the space over frequency.

Furthermore, since measurements are to be taken in a random fashion, an ensemble is required so that richly populated averages can be made and one can feel confident that no one single snapshot of the channel gives a misleading characterization of the channel properties. Towards this end, several different mechanisms were employed to obtain rich ensembles, and one such mechanism, shown in Fig. 3.3, consists of two sections of 5.00 inch diameter steel duct-work—each approx. 9.00 inches long—which have been attached to a vertical metal rod connected to a rotation stage. This rotation stage can be attached to the bottom surface of either the conical or cylindrical enclosure, and is computer controlled so that a large number of measurements can be made in an automated fashion, controlled by a connected laboratory computer. As this metallic object assembly is rotated around its circumference, the eigenfunctions inside the reverberant enclosure are quasi-randomly varied, on account of the changing boundary conditions; however, the fundamental properties of the enclosure—i.e. its volume, surface area, average conductivity, etc.—remain unchanged. From the point of view of a pair of communication devices located inside the enclosure, the rotation of these objects changes the channel seen from transmitter to receiver, and could be considered to be a similar technique to moving the transmitter and receiver themselves randomly throughout the space. The latter technique, however, would be extremely tedious, and does not offer a method that is readily available to investigators to measure a large ensemble of channel instantiations (requiring automation techniques) while maintaining an antenna position at the wall (recalling that enclosed space communications devices must necessarily be attached to a surface, and the assumption is made in this work that this surface is metal). Unless otherwise stated, all forthcoming measurements in the enclosed space platforms will be made over several different positions of the stirrer or objects.

The duct sections shown in Fig. 3.3 are large enough to sufficiently change the eigenfunctions in the reverberant cavities yet are small enough—as compared to the volume of each space—to still consider each internal environment to be sparsely populated (i.e. few internal objects and

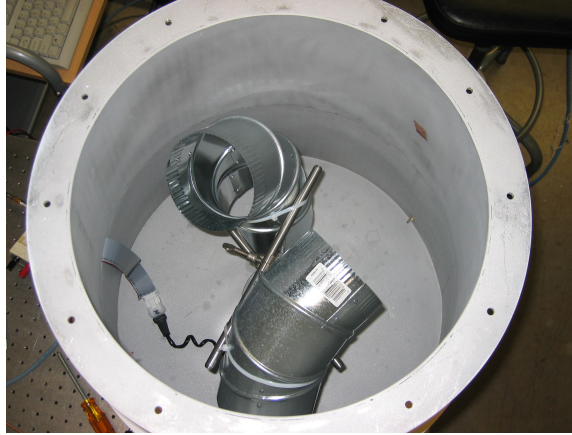


Figure 3.3: Stirring platform mounted to rotation stage to achieve a statistic ensemble for the sparsely populated internal configurations.

a large amount of open volume). Furthermore, the ends of the duct sections are not covered, and thereby do not appreciably reduce the volume of the enclosed space. An analogous technique, called mode stirring, is used to randomly change the boundary conditions and thus the eigenfunctions in reverberation chambers. Adopting this name for the enclosed space platform measurements, the nomenclature used in this document will refer to sparsely populated internal environments equivalently as stirred environments. These stirred, sparse configurations allow investigators to explore the generalized scenario of spaces that do not enclose a high density of contained objects. To contrast the situation of a sparse population of internal objects, a second class of mechanisms that is used to randomly vary the fields in a densely populated reverberant enclosure is employed and two instantiations of these mechanisms are shown in Figs. 3.4 and 3.5. These figures show two assemblies of objects that are placed inside the 2 and 5 section cylindrical and conical enclosures to accomplish the similar goal of randomly changing the eigenfunctions of the reverberant enclosure across the  $360^\circ$  rotation of the assembly. Both the cylindrical and conical environments keep the bottom 2 sections of each respective device when reducing from the full 5 section configuration. Different than the sparsely populated internal environment presented by the aforementioned stirring mechanism, these object assemblies present a dense population of internal objects to the cylindrical and conical enclosures. This experimental configuration, which features cylindrical galvanized steel duct sections and rectangular aluminum project boxes of various sizes that have been sealed to shut out external fields using aluminum foil and tape, is designed to represent enclosed spaces

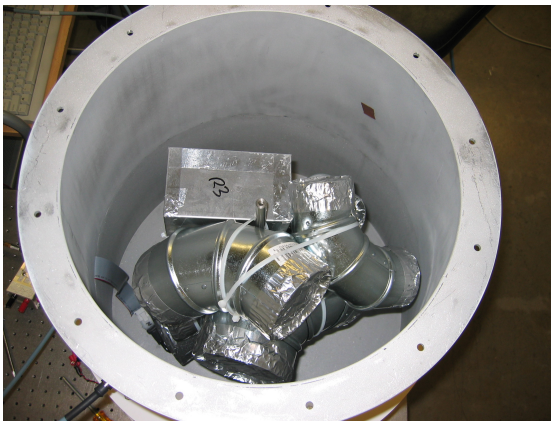


Figure 3.4: Assembly of objects used in the 2 section cylinder and cone platforms mounted to the rotation stage for the densely populated internal configurations.

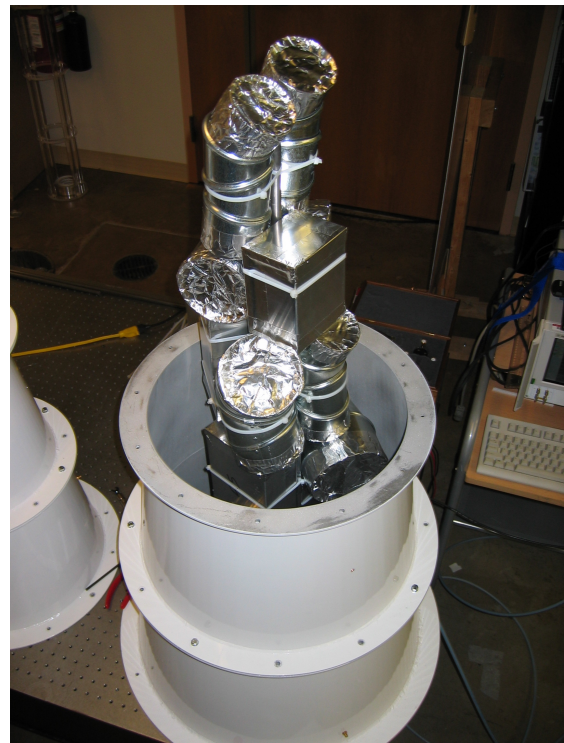


Figure 3.5: Assembly of objects used in the 5 section cylinder and cone platforms mounted to the rotation stage for the densely populated internal configurations.



| CYLINDER | Empty                |                     | Stirrer              |                     | Objects              |                     |
|----------|----------------------|---------------------|----------------------|---------------------|----------------------|---------------------|
| Sections | Volume               | Surface             | Volume               | Surface             | Volume               | Surface             |
| 2        | $0.0917 \text{ m}^3$ | $1.131 \text{ m}^2$ | $0.0917 \text{ m}^3$ | $1.662 \text{ m}^2$ | $0.0777 \text{ m}^3$ | $1.807 \text{ m}^2$ |
| 5        | $0.2294 \text{ m}^3$ | $2.335 \text{ m}^2$ | $0.2294 \text{ m}^3$ | $2.866 \text{ m}^2$ | $0.2055 \text{ m}^3$ | $3.448 \text{ m}^2$ |

Table 3.2: Enclosed free volume and surface area for cylindrical platform in empty, stirred and object-filled configurations.

| CONE     | Empty                |                     | Stirrer              |                     | Objects              |                     |
|----------|----------------------|---------------------|----------------------|---------------------|----------------------|---------------------|
| Sections | Volume               | Surface             | Volume               | Surface             | Volume               | Surface             |
| 2        | $0.1062 \text{ m}^3$ | $1.479 \text{ m}^2$ | $0.1062 \text{ m}^3$ | $1.904 \text{ m}^2$ | $0.0921 \text{ m}^3$ | $2.155 \text{ m}^2$ |
| 5        | $0.1540 \text{ m}^3$ | $1.879 \text{ m}^2$ | $0.1540 \text{ m}^3$ | $2.305 \text{ m}^2$ | $0.1302 \text{ m}^3$ | $2.992 \text{ m}^2$ |

Table 3.3: Enclosed free volume and surface area for conical platform in empty, stirred and object-filled configurations.

that have very little free enclosed volume, i.e. a high density of internal objects contained within the reverberant space that reduce the volume in which fields are allowed to develop. Measurements made using both the stirrer and object assembly will allow a comparison between measurements made in sparsely and densely populated representative enclosed space environments, and the channel measurements presented in Chapters 4-6 will explore and contrast the channel characteristics native to both environments. Tables 3.2 and 3.3 list the free enclosed volume (i.e. the volume in the enclosure that is exposed to electromagnetic fields) and total surface area (including the surface area of the enclosed objects) for the empty, stirred and object-filled cylindrical and conical enclosure configurations. These volumes and surface areas will be used in Chapters 4-6 to characterize the quality factor and other metrics of the enclosed space channel.

### 3.3 Empirical Approaches

Channel measurements have been made using a variety of techniques in the study of other channel environments. A technique that excites the channel with RF impulses and measures the power delay profile of the channel on a scope via an RF detector is used, for example, in [103] to measure the outdoor channel, in [18] to measure the indoor channel and in [34, 35, 104] to measure the ultra-wideband communications channel. This approach allows investigators to characterize the power delay profile using metrics such as its moments, leading to mean excess delay and the RMS

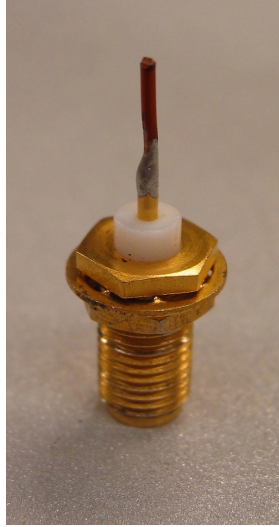


Figure 3.6: Probe antenna with SMA connector used in the reverberant platforms.

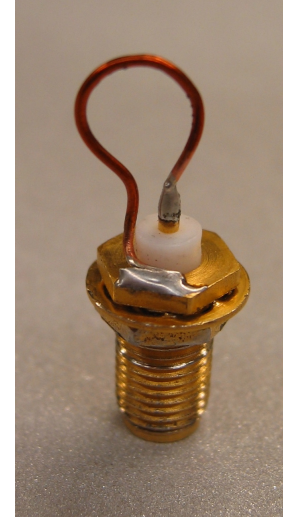


Figure 3.7: Loop antenna with SMA connector used in the reverberant platforms.

delay spread. If the power delay profile were to be split into several bins along the delay time axis, the statistics of multipath arrivals in these bins can also be explored using this pulsed experimental method. A similar method to measure the power delay profile in a channel uses a sliding correlator to receive the transmission of a pseudorandom sequence through the channel. This method has the advantage that it can make high resolution time domain measurements using moderate bandwidth components, which contrasts the RF pulse measurement system that requires high bandwidth components to receive very short duration (high resolution) pulses. The sliding correlator method has been used, for instance, to measure the outdoor channel in [7, 8, 9] and in the indoor channel in [20].

Instead of measuring the power delay profile of a channel directly in the time domain using pulses or pseudorandom sequences, measurements can be made in the frequency domain using a vector network analyzer, which can then be converted into a power delay profile using the inverse Fourier transform. This technique has the added benefit that coherence bandwidths (e.g. 50% or 90%) in the measured channel can be directly found from the correlation functions of the measured channel transfer functions. Channel measurements using a vector network analyzer have been made, for instance, in the indoor channel in [19, 21, 22] and in the channel seen by using heating, ventilation and air-conditioning ducts as radio distribution networks in [49, 50, 51].

Measurements in the enclosed space radio channel, presented in this report are made both in



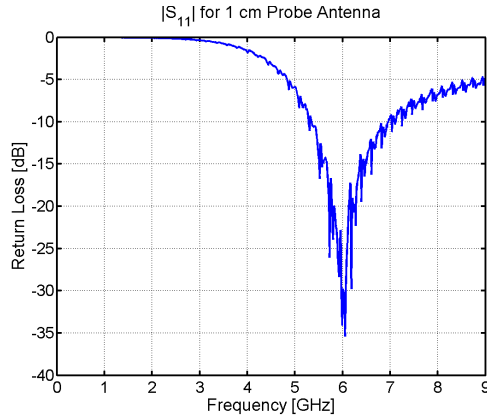


Figure 3.8: Return loss for the 1 cm probe antenna measured in a single open-ended cylindrical segment using phase-stable VNA cables.

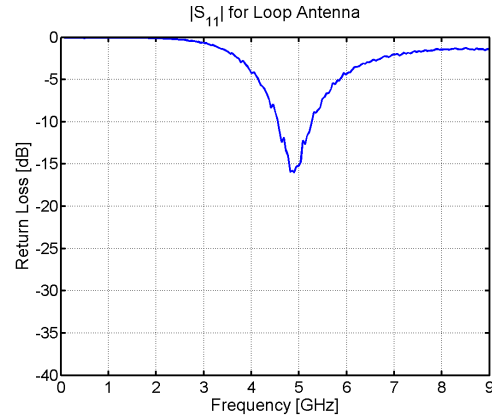


Figure 3.9: Return loss for the loop antenna measured in a single open-ended cylindrical segment using phase-stable VNA cables.

the frequency domain using a vector network analyzer and in the time domain using an RF pulse generator and an RF detector and descriptions of each experimental setup will be follow shortly. Excitations of the enclosed space environments are accomplished by inserting two small identical antennas through the walls of the conical or cylindrical enclosures to excite and sense the channel response. Two different types of antenna are used for measurements: A probe antenna shown in Fig. 3.6 and a loop antenna shown in Fig. 3.7. Since a enclosed space communications device must be affixed to a structure assumed herein to be metallic), the antennas are inserted through the surface of the enclosure walls, allowing the SMA connector of the antenna to be interfaced to the appropriate channel sounding devices, external to the platforms. Signals are thus coupled into and out of the platforms in a manner that does not lead to energy leakage from any other apertures created for RF cables. Small antennas were chosen because antennas on miniature communications devices are required to be small and potentially not frequency matched over the very large range of possible frequency operation. As can be seen from Figs. 3.8 and 3.9, the probe and loop antennas are indeed poorly matched over most of the range of measurements to 9.0 GHz (limited by the VNA—see below). The return loss of the loop and probe antennas shown in these figures is measured with the antenna inserted in a single segment of the cylindrical platform, but this segment is not attached to any other adjacent segments and has no end-caps. In this fashion, the antennas are measured in the presence of a ground-plane (the walls of the cylindrical segment), but do not display the effects of the enclosure's resonant modes in the measured return loss.

### 3.3.1 Frequency Domain using Vector Network Analyzer

Frequency domain measurements in the enclosed space platforms are made using an Agilent 8358A vector network analyzer with time domain option (built-in inverse Fourier transform). The VNA is connected to the enclosures using two phase-stable cables (LA-290 from Semflex, Inc.) so that the VNA calibrations are preserved. A system diagram using the VNA is shown in Fig. 3.10. This analyzer has a range from 300 kHz to 9.0 GHz and a maximum number of points of 1601. A single frequency sweep that would include the fundamental resonance of the platforms (approx. 400 MHz), e.g. from 200MHz to 9.0 GHz, would give a frequency resolution of 5.50 MHz in the frequency domain and a span in the time domain (after the inverse Fourier transform) of 181.8 ns. The highly reverberant environment of an enclosed space would lead one to suspect that coherence bandwidths that are considerably less than 5.50 MHz and RMS delay spreads that are considerable longer than 181.8 ns could be reasonably anticipated. For this reason, measurements using the VNA are broken-up into several segments or bands, obtaining frequency and time domain spans and resolutions that are adequate for measuring the enclosed space channel, and a summary of the parameters used in the vector network analyzer measurements is shown in Table 3.4. In transforming data from the frequency to the time domain, the VNA applies a Kaiser window function to reduce the sidelobe levels in the channel impulse response, and provides the  $\beta$ -parameter of the Kaiser window as a option to the user. A  $\beta$  value of zero corresponds to a rectangular window, effectively deactivating the Kaiser windowing function; the maximum  $\beta$  value available to the user is  $\beta = 13$ . To obtain an adequate time-resolution/sidelobe tradeoff, a Kaiser parameter of  $\beta = 9$  was chosen. Finally, due to the poor match of the antennas to the  $50\Omega$  port impedance of the analyzer and cables, the signals measured by the analyzer can be particularly small, especially in the frequency regions near the fundamental resonance. For this reason, a small IF bandwidth of 1.00 kHz on the analyzer was chosen to effectively lower the noise floor of the measurements. The measurements taken using the vector network analyzer provide both the channel frequency response and the channel impulse response, from which the frequency correlation functions yield the measured coherence bandwidths and impulse response produces the power delay profile and its moments, and thus the measured mean excess delay and RMS delay spread.

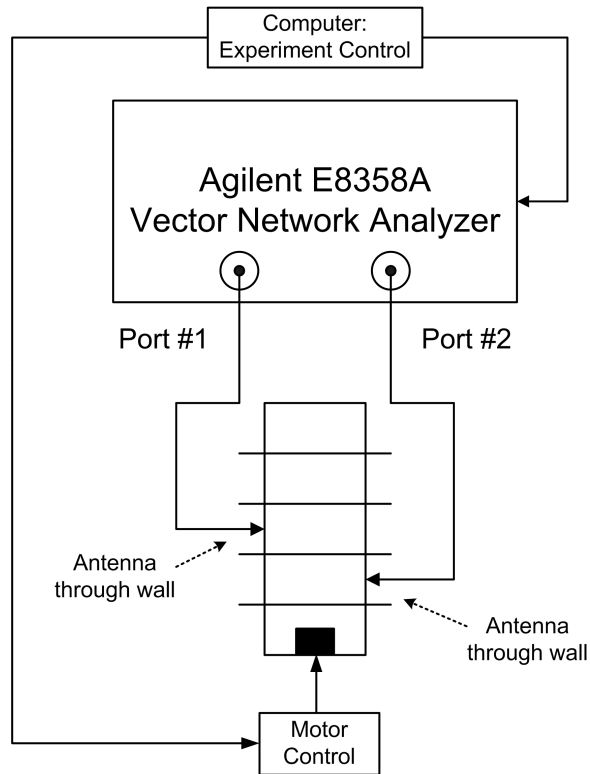


Figure 3.10: Functional diagram for the vector network analyzer measurement system.

| PARAMETER                    | VALUE             |
|------------------------------|-------------------|
| Number of Measurement Bands: | 44                |
| Total Frequency Span:        | 200 MHz - 9.0 GHz |
| Band Frequency Span:         | 200 MHz           |
| Frequency Resolution:        | 125 kHz           |
| Band Time Span:              | 8.00 $\mu$ s      |
| Time Resolution:             | 5.00 ns           |
| IF Bandwidth:                | 1.00 kHz          |
| Kaiser Beta:                 | 9                 |

Table 3.4: Network Analyzer Configuration Parameters.

### 3.3.2 Time Domain using RF Pulse Generator and RF Detector

While the measurements taken with the VNA provide a very complete set of information about the channel being characterized, a considerable limitation is that the analyzer only functions to 9.0 GHz. Measurements above 9.0 GHz are desirable to gain a better understanding of the channel over an even larger range of frequency. Furthermore, the vector network analyzer is an expensive and somewhat cumbersome device, and it would be a benefit not to require such a powerful measurement tool to gain an understanding of any given enclosed space environment. Motivated by both the desire to perform measurements at higher frequencies and also to demonstrate the feasibility of a setup that reduces the expense and complexity of the measurement devices themselves, measurements were performed in the enclosed space environments using a RF pulse generator and RF detector connected to a digital scope. As shown in Fig. 3.11, an Agilent 8251A RF signal generator is used to excite the enclosed space channel and an Agilent 8473C RF detector was used to measure the strength of the RF signal vs. time, recorded on a digital scope. Measurements are made at 40 different frequencies from 500 MHz - 20.0 GHz, at intervals of 500 MHz. Unlike the measurements made by investigators in the indoor, outdoor and ultra-wideband indoor channels discussed previously, which measured the channel impulse response, the time domain measurements presented here measure the channel step response. As will be discussed in Chapter 4, a measurement of the rate of energy decay in the enclosed space can be obtained from the step response of the channel, which is directly related to the channel quality factor, and Chapter 5 further illustrates a connection between the enclosed space quality factor and the dispersion parameters (i.e. mean excess delay, RMS delay spread, coherence bandwidth). The parameters for the time domain setup are summarized in Table 3.5, the most crucial of which is the intrinsic time constant of the setup, or time it takes for the system to respond ( $e^{-1}$  normalized amplitude change) to an RF pulse step when the channel being measured has no dispersion (i.e. a cable between the pulse generator and detector). This intrinsic response time limits the smallest time in which a measured channel can decay and still be detected by the measurement system. To accurately measure the channel step response, it is important that the width of the RF pulses used in these step response experiments are then chosen so that the channel fully rises or falls before the next falling or rising pulse edge.

Even though measurements in the time domain are made using a precision signal generator and

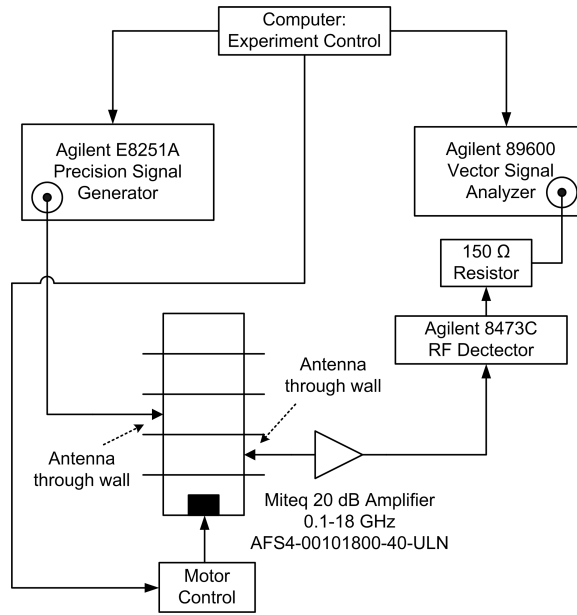


Figure 3.11: Functional diagram for the pulsed-RF measurement system.

| PARAMETER                          | VALUE              |
|------------------------------------|--------------------|
| Number of Measurement Frequencies: | 40                 |
| Total Frequency Span:              | 500 MHz - 20.0 GHz |
| Frequency Interval:                | 500 MHz            |
| Pulse width:                       | 4 $\mu$ s          |
| Pulse interval:                    | 8 $\mu$ s          |
| Intrinsic Time Constant:           | 50 ns              |

Table 3.5: Pulse Generator/RF Detector Configuration Parameters.

digital scope, one can envision a measurement system that uses an inexpensive voltage controlled oscillator and simple detection circuitry to measure the step response of the channel. Such an evolution from expensive laboratory equipment to inexpensive and easily portable devices is desirable if measurements are to be made in the enclosed space channel in the field, i.e. in an aircraft wing that cannot be brought into a well-equipped laboratory. A handheld device that can be placed into an arbitrary enclosed space environment would combine the functions of channel excitation, detection, step response slope-estimation, dispersion parameter readout and storage, all in one simple package. A similar evolution for measurements in the frequency domain is not as clear, since a vector network analyzer requires a complex design and components that provide precision phase information with which to perform the inverse Fourier transform.

### 3.4 Regions of Operation

Two general regions of operation can be identified inside a reverberant enclosed space: An undermoded region and an overmoded region. Intuitively, these two regions correspond to the relative number of modes that exist in a fixed bandwidth (i.e. the smoothed mode density—See Chapter 2), relative to the average half-power bandwidths of the modes. When modes are spaced on the average much further than their average half-power bandwidths, then individual resonant peaks are readily identifiable, and the reverberant space is said to be undermoded. In contrast, if the average spacing between modes is smaller than the average mode resonant width, no single mode can be identified and the reverberant space is said to be overmoded:

$$\frac{f}{Q_{net}(f)} < \bar{D}_0^{-1} = \left[ \frac{8\pi V f^2}{v^3} \right]^{-1} \Rightarrow \text{Undermoded} \quad (3.1)$$

$$\frac{f}{Q_{net}(f)} > \bar{D}_0^{-1} = \left[ \frac{8\pi V f^2}{v^3} \right]^{-1} \Rightarrow \text{Overmoded}, \quad (3.2)$$

where  $Q_{net}$  is the net quality factor of an arbitrarily shaped resonant space and  $\bar{D}_0$  is the smooth mode density of an arbitrarily shaped resonant space as defined in Chapter 2. If every other source of loss was insignificant as compared to the wall losses, then  $Q_{net} = Q_c$  (where  $Q_c$  is the composite quality factor—see (2.46)) and the previous expressions can be solved for  $f$  to define the undermoded and overmoded regions in terms of the constitutive parameters:

$$f < \left[ \frac{3c^3}{16S} \sqrt{\frac{\sigma\mu_0}{\pi\mu_r}} \right]^{\frac{2}{5}} \Rightarrow \text{Undermoded} \quad (3.3)$$

$$f > \left[ \frac{3c^3}{16S} \sqrt{\frac{\sigma\mu_0}{\pi\mu_r}} \right]^{\frac{2}{5}} \Rightarrow \text{Overmoded}. \quad (3.4)$$

The frequency at which these two regions meet, i.e. the boundary of the undermoded and overmoded regions is called the Rayleigh frequency and causes the following condition (expressed in terms of the net quality factor,  $Q_{net}$ ) to be true:

$$\frac{f}{Q_{net}(f)} = \left[ \frac{8\pi V f^2}{v^3} \right]^{-1} \bigg|_{f=f_{\text{Rayleigh}}} \quad (3.5)$$

If  $Q_{net} = Q_c$  (i.e. losses other than wall losses are insignificant), the Rayleigh frequency is written in terms of the constitutive parameters as [81]

$$f_{Rayleigh} = \left[ \frac{3c^3}{16S} \sqrt{\frac{\sigma\mu_0}{\pi\mu_r}} \right]^{\frac{2}{5}}. \quad (3.6)$$

The Rayleigh frequency represents the point at which the average spacing of the modes and the half-power bandwidth are equal. Note that this boundary between the overmoded and undermoded regions can also be found by solving the specific mode density expression given in (2.47) for  $f$  when  $N_s(f) = 1$ .

A glimpse of the frequency response measured in a single position of the stirrer in the sparsely populated (stirred) cylindrical platform (5 sections) in the undermoded region is shown in Fig. 3.12. One can see that the spectrum is characterized by separated peaks, allowing each mode to be identified by mode index (see Appendix A). For communications to be reliably obtained in this region, a transmitter and receiver pair would need to find one or several of these resonant peaks, adjust the symbol rate and consequently the bandwidth to fit within the half-power width of the peak(s) (assuming no equalization is to be implemented) and track any shifts of the peak due to physical changes in the resonant space or the locations of the transmitter or receiver\*. Not only does this envisioned scenario require a fairly complex implementation of peak tracking for the carrier, but it also suffers the added restriction that the individual mode half-power widths are very narrow in this region (as this region contains some of the narrowest half-power bandwidth resonances), greatly restricting the available symbol rate.

Examining the frequency response measured in a single stirrer position of the same sparsely populated cylindrical cavity in the overmoded region, shown in Fig. 3.13, one can see that no individual resonant peak can be identified, and (3.4) suggests that the fields at any one given frequency are the superposition of several excited modes. These several excited modes provide a form of diversity, improving the field homogeneity that causes a reduction in the dynamic range of the frequency response measured in the channel as the frequency of measurement falls more deeply into the over-

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\*Communications could also be considered in the regions of spectrum between resonant modes in the undermoded regions. Energy in these regions does not propagate (resonate) in the enclosure but decays exponentially over distance. However, if the antennas of a transmitter and receiver were to be sufficiently coupled, communications in these decaying mode regions could be considered. Measurements in these regions with sufficiently large antenna showed considerable channel gains.

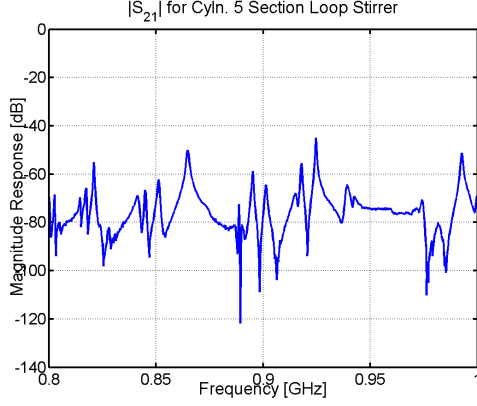


Figure 3.12: Magnitude of the  $S_{21}$  response measured in a single position of the stirring mechanism in the undermoded region of the sparsely populated 5 section cylinder platform, showing individual resonant peaks.

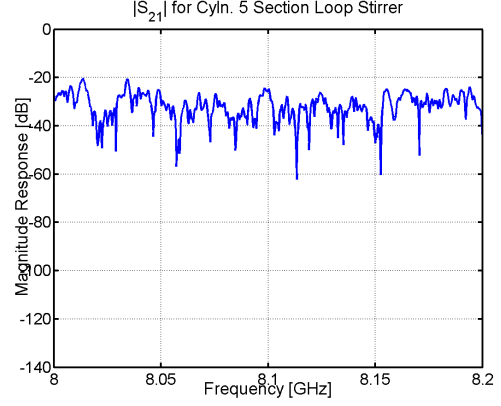


Figure 3.13: Magnitude of the  $S_{21}$  response measured in a single position of the stirring mechanism in the overmoded region of the sparsely populated 5 section cylinder platform, displaying no identifiable individual resonant peaks.

moded region. A reduction in the dynamic range of the frequency response implies that deep nulls (fades) in the channel response are less likely, which is an advantageous trait for a communications environment in which sensors are arbitrarily placed and must use the channel as it is found at that placed location. Chapter 6 demonstrates this decrease in the dynamic range as the enclosed space becomes more overmoded by plotting the measured dynamic range vs. frequency. A further advantage for communications in enclosed spaces, the half-power resonance bandwidths become much larger in the overmoded region, since the half-power widths increase as  $\sqrt{f}$ , as can be seen from (3.2). This increase in the half-power resonance bandwidths suggests that the un-equalized symbol rate may increase in the overmoded region as compared to the undermoded region. Chapter 5 will indeed demonstrate a decreasing trend for the measured dispersion in the channel as the frequency increases, suggesting that the un-equalized symbol rate can increase. Finally, both the derivations of the field statistics and the composite quality factor in reverberant enclosures make the assumption that mode density in the reverberant enclosure is overmoded (see Chapter 2). In this research, we will consider the scenario in which communications are implemented in the overmoded region not only for the reasons of field homogeneity and diversity over frequency and space, but also so that the field statistics and composite quality factor expressions are valid in the reverberant enclosed space channel.



## Chapter 4

# Enclosed Space Quality Factor

Quality factors are well known in several fields of devices, including circuits, antennas and electromagnetic resonators. All of these devices have in common a capacity to store energy; circuits store energy in capacitors and inductors, antennas store energy in the electric and magnetic fields that surround the antenna and electromagnetic resonators store energy in the electric and magnetic fields contained within the cavity enclosure. Each of these devices also includes an energy loss mechanism that are specifically lumped or parasitic element resistances for circuits, radiation and resistive losses in antennas and dielectric and conductive losses in electromagnetic resonators. The previous two quantities along with the radian frequency leads to the general definition of quality factor:

$$Q = \frac{\omega U}{P_l}, \quad (4.1)$$

where  $\omega$  is the radian frequency of system operation,  $U$  is the energy stored in the system, and  $P_l$  is the power lost from the system. One can then recall that  $Q$  is effectively a ratio of the energy stored over time to the energy lost over time. High  $Q$  values indicate that a system maintains its stored energy for long durations of time, and low  $Q$  values indicate a lossier system that dissipates its stored energy more quickly.

Resonant enclosed spaces that are to be used as communications media are effectively energy storage and dissipation systems as well. A transmitter in the enclosed space delivers energy into the system, and a combination of the currents generated in the metallic enclosure walls, radiation from

apertures and power absorption at the receive antenna (among other sources of loss) dissipate energy from the system over time. It is then feasible to discuss the quality factor of a reverberant enclosed space and use it to characterize the energy storage/loss nature of that space. Indeed, the similarity between reverberant enclosed spaces and reverberation chambers has already been discussed, and Chapter 2 presented several well-established models for the quality factor of reverberation chambers.

One can foresee how  $Q$  might be connected to the dispersion properties of the enclosed space channel: High  $Q$  values suggest that an impulse of energy injected into the enclosure is lost slowly over time, precipitating larger dispersion values, whereas low  $Q$  values imply that energy from an impulse is lost quickly, leading to smaller dispersion effects in the channel. A connection between the quality factor and gain can also be foreshadowed: Large  $Q$  values mean less loss and higher channel gains whereas small  $Q$  values suggest less channel gain. The connection between the enclosed space quality factor, dispersion and gain will be explored in more detail in Chapters 5 and 6. In the immediately following sections, the method of describing the decay of energy over time in enclosed space radio channel using an associated quality factor will be presented and measurements of the quality factor over a range of frequencies inside the sparsely and densely configured cylindrical and conical platforms will be presented.

## 4.1 Step and Impulse Responses of Reverberant Enclosures

Measurements of the quality factors of individual modes in a cavity resonator can be made, for instance, using a vector network analyzer to measure the half-power resonant widths of the mode, as was described in Chapter 3 to extract the conductivity of the enclosure walls. Chapter 3 also stated that communications are better suited in the overmoded region for reasons of better frequency diversity and field homogeneity. Since in the overmoded region a CW signal excites any modes simultaneously, measurements of an overmoded reverberant enclosed space therefore cannot identify any single resonant peak from which to extract a half-power bandwidth. To obtain a measurement of the net quality factor,  $Q_{net}$ , which describes the energy storage and loss properties of the aggregate of excited modes, an examination of the manner in which energy is dissipated from the the enclosure is needed.

Let  $U(t)$  be the energy contained in an arbitrary enclosure as a function of time and  $P_{net}$  be the net power that is lost from the enclosure, owing to any arbitrary loss mechanism. The following differential describes the change of enclosure energy vs. time:

$$dU = -P_{net}dt, \quad (4.2)$$

and using the previous definition of the quality factor,  $Q_{net} = \omega U/P_{net}$ , the resulting differential,

$$dU = -\frac{\omega U}{Q_{net}}dt, \quad (4.3)$$

can be solved with the initial condition ( $U(t = 0) = U_0$ ) to obtain the following result:

$$U_s(t) = U_0 \exp\left[-\frac{\omega}{Q_{net}}t\right], \quad (4.4)$$

which is the turn-off energy step response of the energy contained in the enclosure. In the above expressions,  $Q_{net}$  is assumed to be locally constant over a localized bandwidth centered about the considered frequency,  $\omega$ , and is also assumed to be large, such that  $Q/\omega \gg 1/\omega$ . This energy step response can be related to the step response of the power induced in a receiving antenna inserted into the cavity by considering the effective area of the receiving antenna,  $A_e = \lambda^2/8\pi$  (which includes a polarization mismatch factor of 1/2):

$$P_{ant} = A_e \frac{U_0 \nu}{V} \exp\left[-\frac{\omega}{Q_{net}}t\right] = \frac{\lambda^2 m}{8\pi} \frac{U_0 \nu}{V} \exp\left[-\frac{\omega}{Q_{net}}t\right], \quad (4.5)$$

where  $U_0$  is the initial energy contained in the enclosure of volume  $V$ ,  $\nu$  is the speed of light in the medium and  $m$  is the antenna/load impedance mismatch factor. From (4.5), it is evident that the *step power delay profile* (i.e. the magnitude squared channel step response) seen in the load connected to the receiving antenna is proportional to  $e^{-\omega t/Q_{net}}$ , making the *magnitude step delay profile* proportional to  $e^{-\omega t/2Q_{net}}$ . Since the impulse response of a system is the time derivative of the step response, the *magnitude impulse delay profile* of the signal induced in the receiver load is also proportional to  $e^{-\omega t/2Q_{net}}$ , implying that the *power delay profile* (i.e. magnitude square of the channel impulse response) of that signal is proportional to  $e^{-\omega t/Q_{net}}$ . Using this knowledge, it seems

clear that the net quality factor inside a reverberant enclosed space can be estimated by assuming that the measured power delay profile or measured step power delay profile follow an exponential decay ( $e^{-\alpha t}$ ) and estimating the parameter of that decay,  $\alpha$ . As discussed in Chapter 3 the vector network analyzer and inverse Fourier transform can be used to measure the channel impulse power delay profile, and the RF signal generator and detector can likewise be used to measure the channel step power delay profile. Once the channel impulse and step power delay profiles are obtained, the exponential factors of each response can be estimated using a fit algorithm such as that found in the MATLAB®\* toolbox, extracting an empirical Q factor for each measurement band across frequency. This technique of measuring the quality factor of a reverberant enclosure was used by investigators with good results in [105] and [97] to measure the net quality factor of several reverberation chambers.

Figure 4.1 shows an example of an average power delay profile (ensemble averaged across the 60 rotational positions of the internal objects) measured using the network analyzer and the beset exponential fit parameter (obtained using MATLAB®), both plotted on the logarithmic ordinate. Figure 4.2 shows the average step power delay profile measured using the RF pulse generator and detector, and the slope estimate found using MATLAB®. The reasonably linear behavior of both of these curves on the logarithmic ordinate suggests that both the impulse response and the step response measured in the enclosure are indeed largely exponential in nature. Both measurement techniques, however, show a deviation from an exponential behavior (linear on the logarithmic ordinate) at the end of the profile. While the reason for this deviation is unknown, the slope estimation algorithm utilizes only the early values of the profile to estimate the Q and produces good results. The reader should also remember that these power delay profiles do not represent any individual power delay profiles between two points in the enclosure, but rather show the ensemble average response of numerous delay profiles in that space. A large experimental campaign using both of the measurement techniques described above was thus pursued to explore the behavior of the net quality factor in sparsely and densely populated internal configurations in both the 2 and 5 section cylindrical and conical enclosed spaces. Though Figs. 4.1 and 4.2 are only two of a large number of measured impulse and step delay profiles, it was observed by the investigators that the exponential behavior demonstrated in these plots was largely consistent over all configurations and frequency

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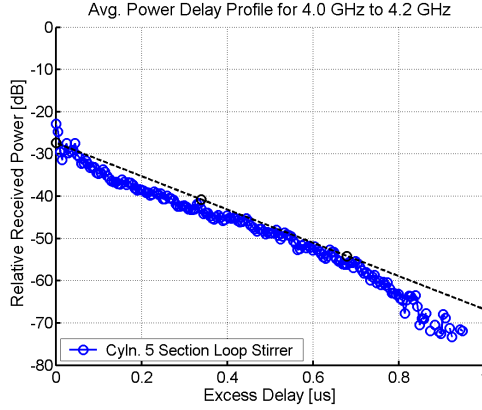


Figure 4.1: Average impulse power delay profile (ensemble averaged over the 60 rotational positions) measured at 4.1 GHz in the 5 section stirred cylinder using the vector network analyzer. Dashed: Best fit estimate given by MATLAB®.

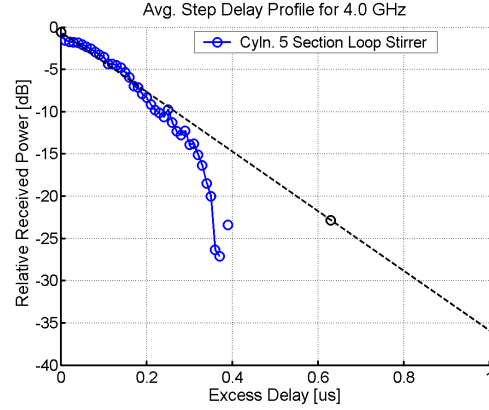


Figure 4.2: Average step power delay profile (ensemble averaged over the 60 rotational positions) measured at 4.0 GHz in the 5 section stirred cylinder using the pulsed measurement system. Dashed: Best fit estimate given by MATLAB®.

bands. Results from this measurement campaign will be presented and discussed presently.

## 4.2 Quality Factor Measurements to 9.0 GHz using a Vector Network Analyzer

Using the setup and techniques described in Chapter 3, the vector network analyzer was used to collect measurements in the cylindrical and conical enclosed space platforms. To explore both densely populated internal environments (i.e. many internal objects and little free volume) and sparsely populated internal environments, the presented results include measurement using both the metallic stirrer assembly and the metallic object assembly, introduced in Chapter 3.

### 4.2.1 Quality Factor in Sparse Internal Configurations

Sparsely populated internal environments are those that have values of fractional free volume inside the enclosed space close to one, where the fractional free volume is defined as the fraction of enclosure volume that is not consumed with internal objects, i.e.

$$v_{free} = \frac{V_{enclosure} - V_{objects}}{V_{enclosure}}. \quad (4.6)$$

One can see from the above expression that as  $v_{free} \rightarrow 1$ , the volume taken by enclosed objects becomes small, and in the limit, the enclosure contains no internal volume-consuming objects. To study sparsely populated internal environments, the conical and cylindrical enclosed space platforms are configured with the stirrer described in Chapter 3 connected to the rotation stage at the bottom of each platform, shown in Fig. 3.3 and are measured using the vector network analyzer from 200 MHz to 9.0 GHz, in 200 MHz bands. Notice that the stirrer consumes a relatively small amount of internal volume in the enclosures<sup>†</sup>. The inverse Fourier transform of each band frequency response yields the channel impulse response in that band, and an ensemble average (across all rotational positions of the stirrer/object assembly) of the magnitude squared of each impulse response produces the power delay profile. The power delay profile nominally demonstrates an exponential behavior whose slope on the logarithmic ordinate is proportional to the composite quality factor of that band. The quality factor measured vs. frequency in this manner in the 2 and 5 section stirred cylindrical and conical platforms using the loop antenna are shown in Figs. 4.3 and 4.4, respectively. The dashed line in both plots corresponds to the best fit  $f^{1/2}$  trend line for each data set. Notice that each of these data sets display a  $f^{1/2}$  frequency behavior with a fairly good fit, which is consistent with the behavior of the composite quality factor derived for an arbitrarily shaped reverberation chamber, as given in (2.46), assuming that all other sources of loss (e.g. antenna, absorber, apertures, etc.) are small in comparison to the composite quality factor. Equation (2.46) can be derived by first assuming that an ensemble of plane waves exist in the reverberant space and then averaging the loss contributions from reflections at the walls across all values of incidence angles [89, 90]. A critical assumption that is made in the derivation of this relation is that the angular arrival of these plane waves at a given point in the space is isotropic, i.e. distributed uniformly across the spherical coordinate system centered at that point. In the sparsely populated enclosed space (i.e. the stirred configuration in these experiments), this assumption is fairly reasonable, and the  $f^{1/2}$  behavior of the measured quality factors supports this assumption and the validity of (2.46) in sparsely populated reverberant enclosed spaces.

Notice also from (2.46) that the quality factor is proportional to the ratio  $V/S$ . Examining

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<sup>†</sup>These cylindrical duct segments could be viewed as reverberant cavities in their own right, each with a fundamental resonance of 1.532 GHz. Since these cavities have open ends, they are coupled into the main cavity of the cylindrical or conical platform, and the energy decay profile measured in the larger enclosure will include the energy decay profiles associated with these smaller cavities [97, 106].

Figs. 4.3 and 4.4, it is clear that the 5 section enclosures exhibit an overall larger quality factor response than the 2 section enclosures. The ratio of the  $V/S$  ratio in the 2 section cylinder to the  $V/S$  ratio in the 5 section cylinder is calculated (via Table 3.2) to be 0.69, and the identical ratio for the 2 and 5 section cone (see Table 3.3) is calculated to be 0.83. A visual examination of Figs. 4.3 and 4.4 shows that these ratios also fit the ratio of the quality factors measured in the 5 and 2 section cylinder and cone, indicating once again that the relation given in (2.46) holds well for sparse internal configurations and that the composite quality factor is proportional to the enclosure volume-to-surface area ratio. Furthermore, recall that (2.46) is independent of the specific shape of the enclosure (given that the enclosure is sufficiently overmoded) and only depends on the volume and surface area of that enclosure. To demonstrate this geometry-independence property, consider the ratio of  $V/S$  for the 5 section stirred conical platform to  $V/S$  for the 5 section stirred cylindrical platform. This ratio is 0.83 and a visual comparison of the ratio of the quality factors measured in these two configurations (shown in Figs. 4.3 and 4.4) agrees with this result fairly well.

To explore the dependence of the enclosed space quality factor on the type of antenna used, Figs. 4.5 and 4.6 compare the measured quality factor in the cylindrical and conical 5 section enclosures measured using the loop antenna and 1 cm long probe antenna. While the probe antenna in both the cylinder and cone does display a noticeable decrease in the measured quality factor, the overall behavior is very similar. The decrease in the quality factor can be explained by considering the antenna quality factor for the probe and loop antennas in the 5 section cylindrical and conical platforms, as shown in Figs. 4.7 and 4.8. These antenna quality factors are derived using (2.49) (assuming that the efficiency factors of both antennas are equal to one,  $\eta_a = 1$ ). Notice from these figures that the antenna quality factor for the probe antenna is considerably lower above 5.0 GHz than the quality factor for the loop antenna. Now, recall from Chapter 2 that the measured net quality factor is a harmonic average of the composite quality factor and the antenna quality factor, i.e. quality factor combine like resistors in parallel [97]:  $Q_{net}^{-1} = Q_c^{-1} + Q_{ant}^{-1}$ . Owing to this parallel combination of quality factors, the notable decrease in the probe antenna quality factor causes a measurable decrease in the measured (net) quality factor. It is also important to note once again that the assumption of an isotropic distribution of plane waves in the enclosed reverberant environment causes the antenna average directive gain (averaged over all possible angles of arrival) to be equal to one. Thus, the difference in the directive gain patterns of the probe and loop antennas does not

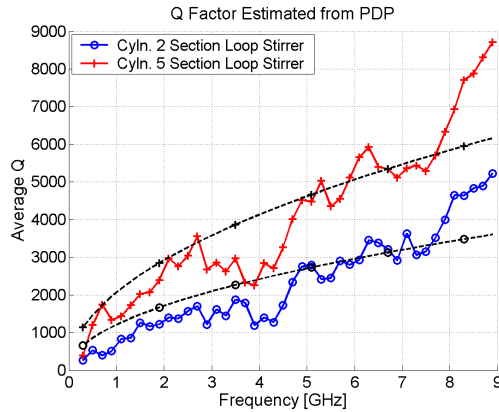


Figure 4.3: Comparison of the quality factor estimated from the power delay profile in the 2 and 5 section cylindrical platform using the loop antenna and stirrer. Dashed: Best  $f^{\frac{1}{2}}$  fit curve as estimated by MATLAB®.

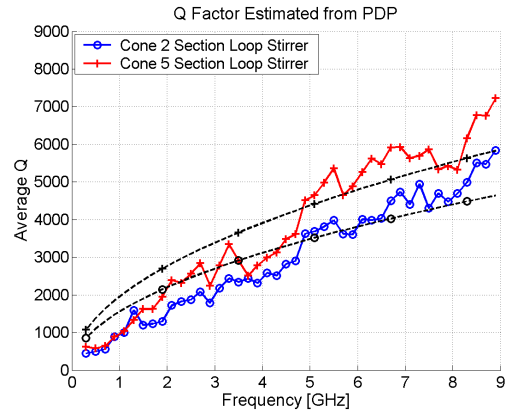


Figure 4.4: Comparison of the quality factor estimated from the power delay profile in the 2 and 5 section conical platform using the loop antenna and stirrer. Dashed: Best  $f^{\frac{1}{2}}$  fit curve as estimated by MATLAB®.

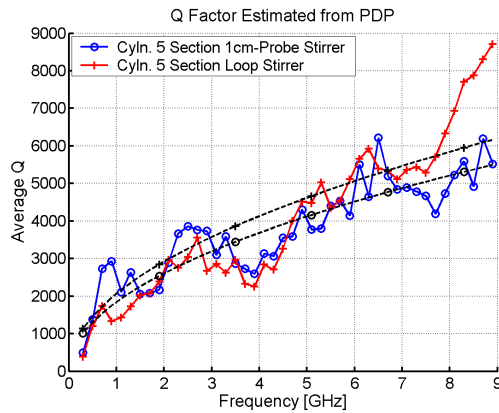


Figure 4.5: Comparison of the quality factor estimated from the power delay profile in the 5 section cylindrical platform using the loop and 1 cm probe antennas and stirrer. Dashed: Best  $f^{\frac{1}{2}}$  fit curve as estimated by MATLAB®.

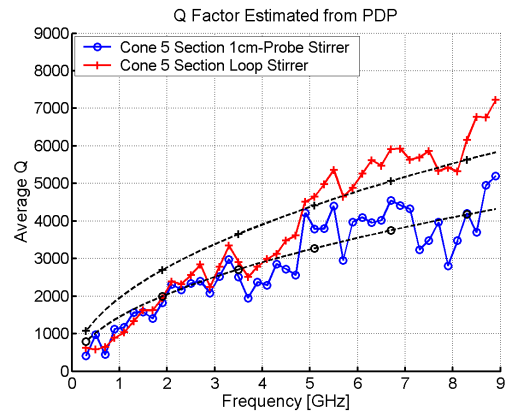


Figure 4.6: Comparison of the quality factor estimated from the power delay profile in the 5 section conical platform using the loop and 1 cm probe antennas and stirrer. Dashed: Best  $f^{\frac{1}{2}}$  fit curve as estimated by MATLAB®.



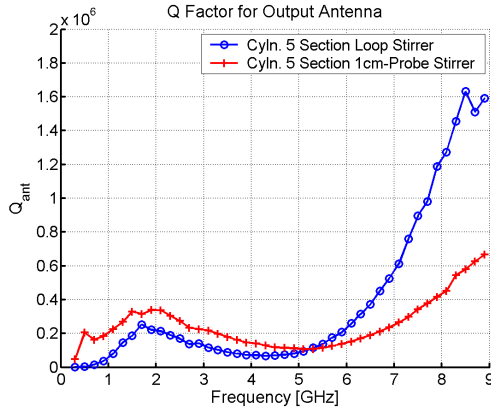


Figure 4.7: Comparison of the antenna quality factors of the the loop and 1 cm probe antennas derived from the  $S_{11}$  of each antenna in the stirred 5 section cylindrical platform.

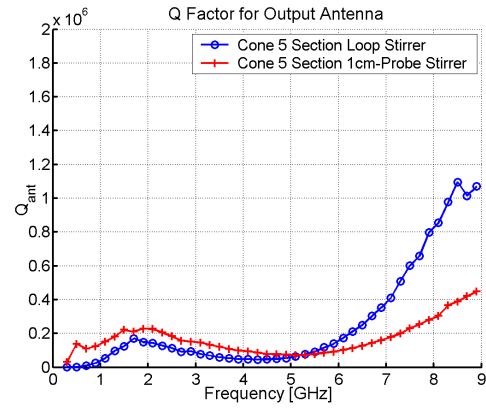


Figure 4.8: Comparison of the antenna quality factors of the the loop and 1 cm probe antennas derived from the  $S_{11}$  of each antenna in the stirred 5 section conical platform.

affect the antenna quality factor; only the antenna impedance match and antenna efficiency affects the antenna quality factor.

#### 4.2.2 Quality Factor in Dense Internal Configuration

The opposite of sparsely populated reverberant enclosures are densely populated reverberant enclosures, or spaces that leave little free volume inside the enclosure walls. In terms of the fractional free volume defined in (4.6), densely populated internal environments are characterized by fractional free volume values that approach zero:  $v_{free} \rightarrow 0$ . To study densely populated enclosures, the object assemblies shown in Figs. 3.4 and 3.5 are placed inside the cylindrical and conical enclosure and rotated to obtain a measurement ensemble. Tables 3.2 and 3.3 list the free volume and total surface area for each configuration in the 2 and 5 section cone and cylinder; one can see from both the aforementioned figures and tables that the free volume in the enclosure has decreased. Measurements proceed in the same fashion as discussed previously for the sparse configurations, using the vector network analyzer and inverse Fourier transform to measure the composite quality factor for each frequency band within each physical configuration. Figures 4.9 and 4.10 show the measured quality factors for the 2 and 5 section cylindrical and conical enclosures using the loop antenna under densely populated conditions. One can clearly see a decrease in the measured quality factors as compared to the sparsely populated environments, which is consistent with the intuition given by the relation in (2.46), since the free volume has decreased and the total surface area—a sum of both

the enclosure walls and the objects—has increased.

The effect of the antenna used on the measured composite quality factor is explored in the densely populated environments in Figs. 4.11 and 4.12, for the 5 section cylindrical and conical platforms, respectively. One can see that the choice of antenna (between the small diameter loop antennas and the short probe antenna mounted at the wall of the enclosure) has not significantly influenced the measured quality factor in the enclosures, showing an even smaller change than the identical comparison in the sparsely configured enclosures in Figs. 4.11 and 4.12. To understand why this change is even smaller than the change in the sparse configurations, consider the quality factors of the probe and loop antennas for the 5 section cylindrical and conical object-filled enclosures, shown in Figs. 4.13 and 4.14, respectively. Just as in the sparse configurations, the quality factor for the probe antenna in the densely populated configurations is smaller than the quality factor of the loop antenna; however, note that the quality factors for both antennas in the dense configuration are not significantly smaller than the corresponding antenna quality factors for the sparse configuration. Yet, as already identified, the measured quality factors in the densely populated configurations are considerably smaller than their sparse counterparts, which implies that the decrease in the measured quality factor is due to the considerable decrease in the composite quality factor (due to wall losses), and not the antenna quality factor. The smaller composite quality factors in the dense configuration dominate the harmonic average of the composite and antenna quality factors (i.e.  $Q_{net}^{-1} = Q_c^{-1} + Q_{ant}^{-1}$ ) in a more profound manner than in the sparse configurations, thus making the change in net quality factor due to the antenna quality factor difference between the probe and loop antennas less pronounced than in the densely populated configurations. Said another way, the difference in the combined resistance when a 5 k $\Omega$  resistor is placed in parallel separately with a 500 k $\Omega$  resistor and a 1 M $\Omega$  resistor is less pronounced than the difference in the combined resistance when a 10 k $\Omega$  resistance is placed in parallel separately with a 500 k $\Omega$  resistor and a 1 M $\Omega$  resistor.

### 4.2.3 Discussion

The quality factor measured in the sparsely populated enclosure configurations demonstrates a good match to a  $f^{1/2}$  behavior, which is harmonious in the context of the relationship in (2.46), since the assumption of an isotropic distribution of plane waves is reasonable for sparsely populated enclo-

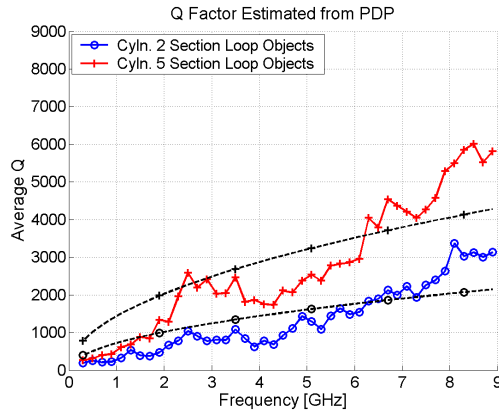


Figure 4.9: Comparison of the quality factor estimated from the power delay profile in the 2 and 5 section cylindrical platform using the loop antenna and objects. Dashed: Best  $f^{\frac{1}{2}}$  fit curve as estimated by MATLAB®.

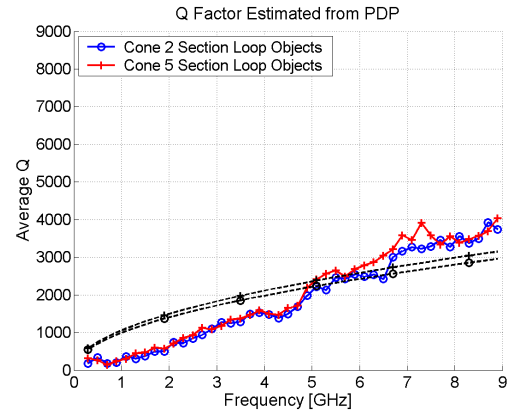


Figure 4.10: Comparison of the quality factor estimated from the power delay profile in the 2 and 5 section conical platform using the loop antenna and objects. Dashed: Best  $f^{\frac{1}{2}}$  fit curve as estimated by MATLAB®.

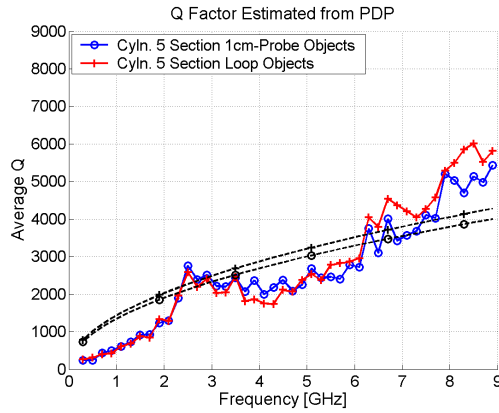


Figure 4.11: Comparison of the quality factor estimated from the power delay profile in the 5 section cylindrical platform using the loop and 1 cm probe antennas and objects. Dashed: Best  $f^{\frac{1}{2}}$  fit curve as estimated by MATLAB®.

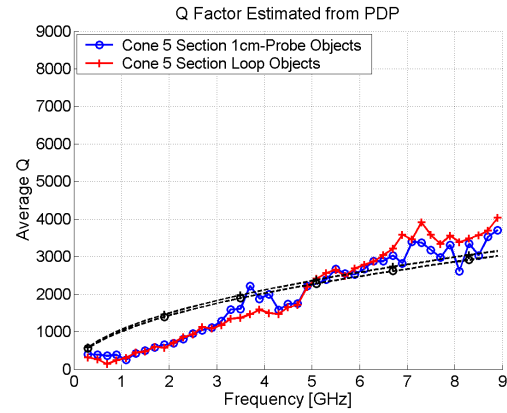


Figure 4.12: Comparison of the quality factor estimated from the power delay profile in the 5 section conical platform using the loop and 1 cm probe antennas and objects. Dashed: Best  $f^{\frac{1}{2}}$  fit curve as estimated by MATLAB®.

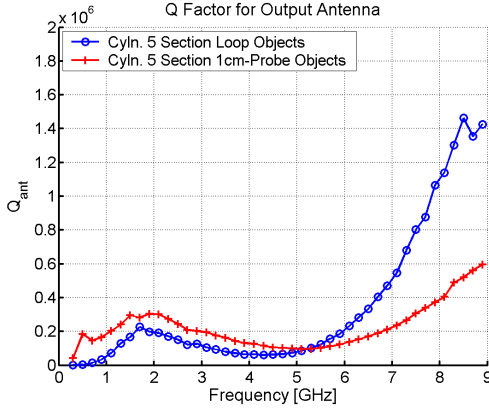


Figure 4.13: Comparison of the antenna quality factors of the the loop and 1 cm probe antennas derived from the  $S_{11}$  of each antenna, measured outside of the enclosures, in the object-filled 5 section cylindrical platform.

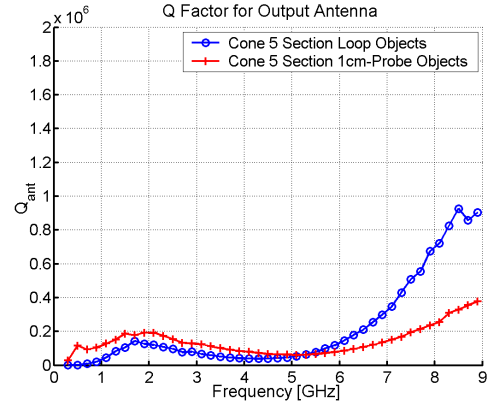


Figure 4.14: Comparison of the antenna quality factors of the the loop and 1 cm probe antennas derived from the  $S_{11}$  of each antenna, measured outside of the enclosures, in the object-filled 5 section conical platform.

tures. It is not clear, however, that the assumption of an isotropic distribution of plane waves is necessarily a good assumption in the scenario of densely populated environments. When objects are placed inside the cylindrical and conical platforms, it would seem that the arrival of plane waves would be biased to directions parallel to the main axis of each platform, since the center of each platform is consumed with the inserted objects. In that case, the assumption of an isotropic plane wave arrival could be less valid, potentially leading to a deviation from the behavior suggested by (2.46). Consider the quality factor measurements in the 2 and 5 section conical enclosures, shown in Fig. 4.10. The dashed line representing the best  $f^{1/2}$  fit does not match the data nearly as well as the  $f^{1/2}$  fit line for the sparsely configured conical environment, shown in Fig. 4.4. It is not decisively clear that this deviation is necessarily due to a violation of the isotropic plane wave distribution assumption; however, one should keep in mind this fundamental assumption that supports the derivation of (2.46) when attempting to apply it to densely populated reverberant enclosures.

In addition to discussing the isotropy of wave arrivals, an alternative explanation of this discrepancy can be offered. When two metal surfaces are placed in parallel and close to one another (such as the objects are in parallel to the enclosure walls), boundary conditions force the tangential component of the electric field to be zero on each surface. Fields with wavelengths that are on the order of or longer than the gap width created by the surfaces will not be sufficiently developed in this gap, and modes at those corresponding frequencies will not resonate in the gap. Without these

resonant modes, the effective number of modes which are excited by a CW signal could drop to such a level that the space in the gap is locally undermoded, even though other regions in the enclosure away from the gap may be overmoded. Since the derivation of (2.46) is predicated upon an overmoded condition throughout the enclosure, configurations which produce narrow gaps such as those considered in this chapter may not be accurately modeled using the relationship given in (2.46).

It was shown in the previous paragraphs that the antenna quality factor had a noticeable effect on net measured quality factor of the enclosed space. However, the effect of the antenna on the measured net quality factor of the enclosed space will, in general, become dramatically less pronounced as frequency increases since the antenna quality factor is proportional to  $f^3$ , compared to the nominal  $f^{1/2}$  behavior of the composite quality factor. Thus, in the highly overmoded regions of reverberant enclosed spaces where the operation frequency becomes very large, the effect of the antenna on the measured net quality factor will become negligible, regardless of the impedance match that the antenna has with its paired receiver.

### 4.3 Discrepancy between Measured and Predicted Quality Factor

The expression for the composite quality factor inside an overmoded reverberant space is given in (2.46) and was shown to be valid for any arbitrary geometry, given that the assumption of isotropic plane wave distribution is valid for that enclosure. The conductivity of the metal used in the manufacture of the cylindrical and conical platforms was extracted using the measured quality factor of  $TE_{111}$  mode in the empty cylindrical resonator (description given in Chapter 3). A conductivity of  $1.125 \cdot 10^7 (\Omega m)^{-1}$  was measured for the aluminum of the enclosures, which is within the range of values for aluminum published in the literature. Inserting this conductivity value, the volume and surface area for the empty 5 section cylinder (from Table 3.2),  $\mu_r = 1$  and the range of frequency values into (2.46), the composite quality factor predicted using the constitutive parameters for the cylindrical and conical enclosures can be found. The net quality factor also needs to include the loading due to both antennas, using a modified version of the expression in (2.49) for the transmit and receive antennas. This modification is given in (6.3) (calculated in Chapter 6 for the purpose of modeling gain) and is required since the antennas are located at the walls of the reverberant en-

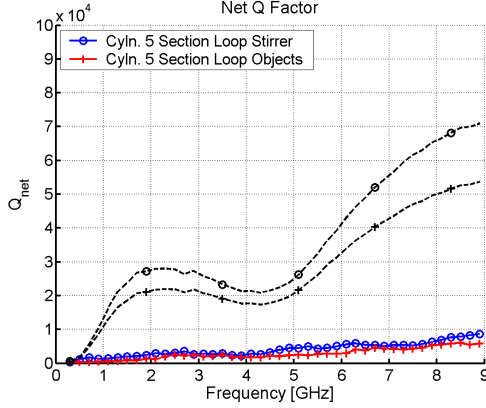


Figure 4.15: Net quality factor measured by estimating the slope of the power delay profile (solid) compared to the net quality factor predicted using the conductivity of the walls and the loop antenna quality factor (dashed) in the 5 section cylinder for both the stirred and object-filled configurations.

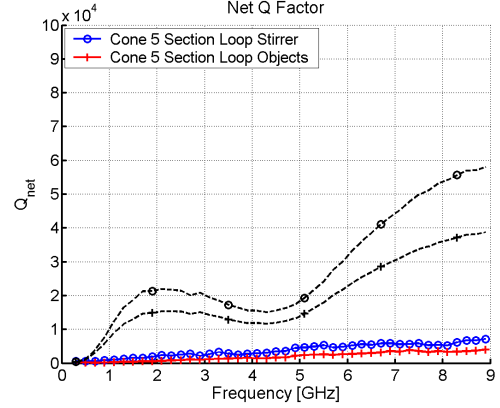


Figure 4.16: Net quality factor measured by estimating the slope of the power delay profile (solid) compared to the net quality factor predicted using the conductivity of the walls and the loop antenna quality factor (dashed) in the 5 section cone for both the stirred and object-filled configurations.

closure where the energy density is twice the energy density away from the walls [89]. Inserting the volume, speed of light in the medium and  $|S_{11}|$  for each antenna into the expression in (6.3), the complete net quality factor is thus  $Q_{net}^{-1} = Q_{TX}^{-1} + Q_{RX}^{-1} + Q_c^{-1}$ , where  $Q_{TX} = Q_{RX}$  as the same antenna is used for both the transmit and receive antennas. This ideal net quality factor can be compared to the quality factor estimated from the slope of the power delay profile in each band, and Figs. 4.15 and 4.16 compare the ideal net quality factors (dashed lines) to the estimated quality factors (solid lines) for the 5 section cylindrical and conical platforms in the sparsely populated (stirrers) and densely populated (objects) configurations.

One can immediately notice that the ideal net quality factor calculated from the constitutive parameters (i.e. conductivity) is considerably larger than the estimated net quality factors, for both devices and in both configurations. The reason for this discrepancy is not presently understood; however, this behavior has been observed by others in the reverberation chamber literature [96]. The particular mechanism that causes the net quality factor in these measurements to deviate from the predicted values is not known, but can be explored by considering the derivation of composite quality factor, given in (2.46). This expression is predicated on an isotropic distribution of plane waves inside the reverberant space, and it has been shown to closely describe the composite quality factor inside overmoded reverberation chambers (e.g. as in [97]). However, if a reverberant enclosure were not sufficiently overmoded and/or shaped in such a way that a rigorous isotropic

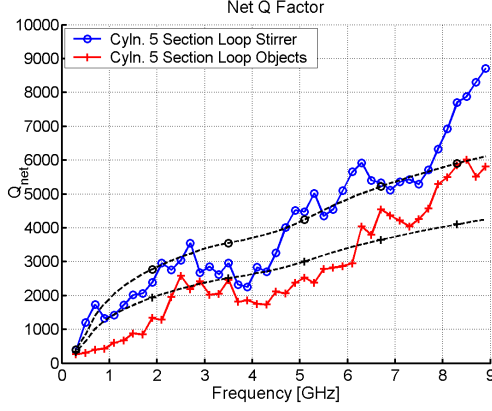


Figure 4.17: Net quality factor measured by estimating the slope of the power delay profile (solid) compared to the net quality factor predicted using the *effective conductivity* of the walls and the loop antenna quality factor (dashed) in the 5 section cylinder for both the stirred and object-filled configurations.

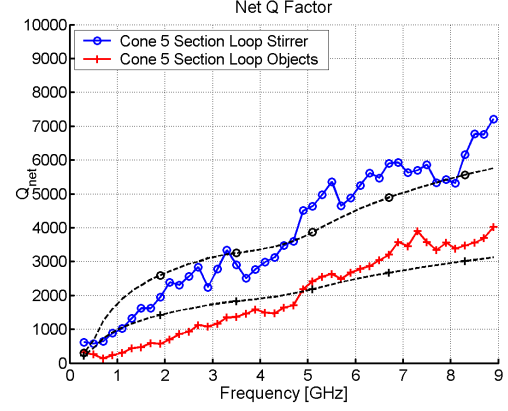


Figure 4.18: Net quality factor measured by estimating the slope of the power delay profile (solid) compared to the net quality factor predicted using the *effective conductivity* of the walls and the loop antenna quality factor (dashed) in the 5 section cone for both the stirred and object-filled configurations.

plane wave assumption becomes less valid, the actual composite quality factor may be less than as given by the expression in (2.46). When an object assembly is loaded into the enclosure (shown in Figs. 3.4 and 3.5) it seems plausible to suspect that a rigorous isotropic plane wave distribution is necessarily accurate, except perhaps under extremely overmoded conditions. However, as will be presented in Chapter 6, the specific mode density is on average  $N_s = 20$  over all configurations at 9.0 GHz—which is significantly overmoded, but not extremely overmoded.

Instead of using the conductivity extracted from the empty cylindrical resonator fundamental mode ( $TE_{111}$ ) to predict the net quality factor, the effective conductivity,  $\sigma_{eff}$ —which is empirically obtained using the best-fit curves seen in Figs. 4.3-4.6 and Figs. 4.9-4.12 and (2.46) (including the appropriate values for the volume and surface area from Tables 3.2 and 3.3)—can be used as the value of conductivity for predicting the net quality factor. As observed in Figs. 4.17 and 4.18, the use of this effective conductivity produces a much better fit between the predicted net quality factor (dashed lines) and the measured net quality factor (solid lines). The effective conductivity values used in Figs. 4.17 and 4.18 are as follows:  $7.50 \cdot 10^4 (\Omega m)^{-1}$  and  $6.52 \cdot 10^4 (\Omega m)^{-1}$  for the 5 section cylinder in the stirred and object-filled configurations, respectively, and  $9.63 \cdot 10^4 (\Omega m)^{-1}$  and  $6.62 \cdot 10^4 (\Omega m)^{-1}$  for the 5 section cone in the stirred and object-filled configurations, respectively. One can immediately see that these effective conductivity values are considerably less than the conductivity of  $1.125 \cdot 10^7 (\Omega m)^{-1}$  measured using the fundamental mode. While the reason

for this difference is not presently known, these effective conductivity values do incorporate any unaccounted losses in the system (e.g. losses due to the seams, roughened wall surfaces, bending of the walls, etc.) that are not already included in the measurement of the conductivity using the fundamental mode. In other words, losses may be present in the system that affect a large number of modes, but may not necessarily affect the fundamental; therefore, such losses might not be considered by the conductivity measurement that uses the fundamental mode quality factor. It would be very enabling if this discrepancy in the net quality factor could be resolved, as a prediction of the net quality factor would allow communications designers to make *a priori* predictions (using knowledge of the enclosure volume, surface area and conductivity) of the communications potential (i.e. dispersion and gain) of the enclosed space communications channel, before making any measurements. The connection between the net quality factor and the dispersion and gain in the enclosed space channel will be explored in Chapters 5 and 6.

#### **4.4 Quality Factor Measurements to 20.0 GHz using a RF Pulse Generator and Detector**

The vector network analyzer is a highly precise, very powerful measurement instrument that can typically be very costly and bulky especially when the frequency of measurement approaches the millimeter wave range. While the net quality factor of a reverberant enclosed space is very readily measured using a network analyzer and the inverse Fourier transform to obtain the power impulse delay profile and estimate its slope, the net quality factor can equivalently be found by measuring the slope of the power step delay profile, or the response of the cavity to a RF pulse of energy having been turned-off after a long duration of being turned-on (as motivated in the first section of this chapter). Chapter 3 describes the empirical setup that was used to measure the power step delay profile in the enclosed space channel environments, and one will recall that it is essentially comprised of an RF pulse generator, an RF detector and a scope to capture the delay profile. Even though the RF pulse generator used in the laboratory is a precision device—costing a considerable sum of money to ensure very low carrier drift and phase noise—measurements of the delay profile require neither such low levels of phase noise, nor such small values of carrier drift, since measuring the step delay profile is phase insensitive (the RF detector measures carrier envelope) and takes place



over a very short duration in time (several  $\mu s$  for one measurement), and is thus immune to carrier drift. As such, a much less costly and compact method would be to use a simple voltage controlled oscillator in a closed loop feedback configuration with a simple timing circuit to duty-cycle the oscillator output and create the pulse trains necessary to excite and measure the step delay profile of the reverberant channel. The output of the square-law RF detector would be connected to a simple A/D and stored in memory for subsequent processing or perhaps passed to slope estimating circuitry directly. The estimate of the net enclosure quality factor can be estimated from the measured power step delay profile, and using the theory presented in Chapters 5 and 6, such a device would then be able to predict the dispersion and gain within the channel as well.

An additional benefit, which is specific to the equipment utilized to make measurements for this study, is the expanded frequency range realized by the RF pulse generator (20 GHz, maximum) over the network analyzer (9.0 GHz, maximum). Measurements to 20 GHz enables investigators to explore the measured net quality factor of the reverberant spaces over a much larger frequency range than measurements of the quality factor with the network analyzer. Figures 4.19 and 4.20 show the estimated quality factors measured using the RF pulse generator/detector setup from 500 MHz to 20.0 GHz in the 2 and 5 section cylindrical platforms, comparing the stirred and object-filled configurations. The dashed lines in the figures represent the estimated quality factors measured using the vector network analyzer to 9.0 GHz. One can see that the match between the two measurement methods is good. Figures 4.21 and 4.22 show the same measurements for the 2 and 5 section conical platforms, comparing the stirred and object-filled configurations.

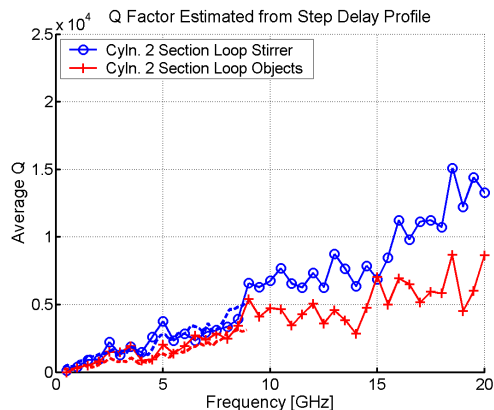


Figure 4.19: Quality factor measured using the RF signal generator/detector setup in the 2 section cylindrical platform using the stirrer and internal objects with the loop antenna. Dashed: Quality factor for identical setup measured using vector network analyzer.

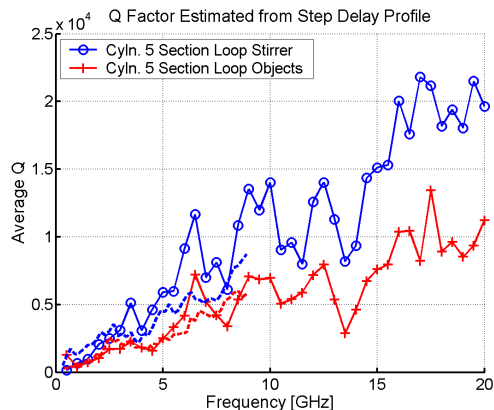


Figure 4.20: Quality factor measured using the RF signal generator/detector setup in the 5 section cylindrical platform using the stirrer and internal objects with the loop antenna. Dashed: Quality factor for identical setup measured using vector network analyzer.

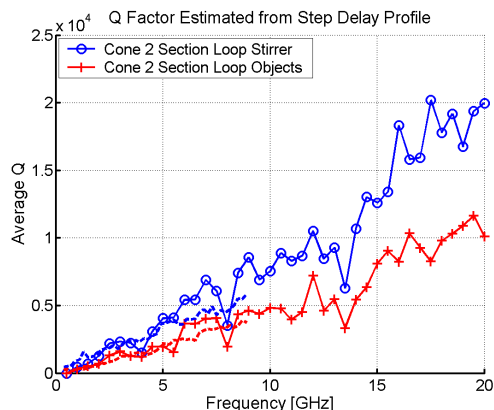


Figure 4.21: Quality factor measured using the RF signal generator/detector setup in the 2 section conical platform using the stirrer and internal objects with the loop antenna. Dashed: Quality factor for identical setup measured using vector network analyzer.

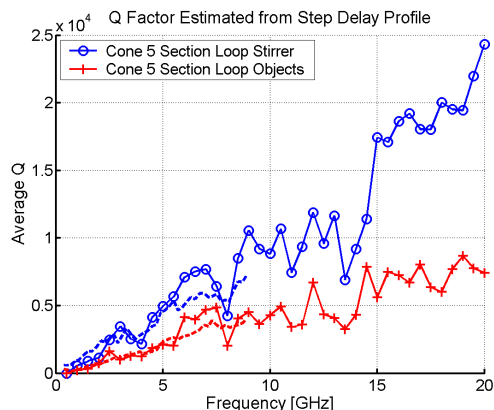


Figure 4.22: Quality factor measured using the RF signal generator/detector setup in the 5 section conical platform using the stirrer and internal objects with the loop antenna. Dashed: Quality factor for identical setup measured using vector network analyzer.

## **Chapter 5**

# **Dispersion in the Enclosed Space Radio Channel**

Dispersion is a phenomena in which a transmitted signal is replicated, or dispersed, over time or frequency before its arrival at the receiver and is common to most practical wireless channels. The outdoor wireless channel experiences time dispersion arising from the transmitted signal reflecting off buildings, vehicles and other structures before arriving in multiple copies spread over time at the receiver. The indoor wireless channel experiences the same effect due to reflections off walls, furniture and people. Wireless communications utilizing heating ventilation and air conditioning (HVAC) ductwork as in-building waveguides experiences dispersion in time from reflections off the metallic end caps of the ductwork and other structures as well as from the well-known inter- and intra-modal dispersion effects of metal waveguides. With the exception of intra-modal dispersion, all of the other mentioned types of dispersion are also known as multipath propagation, and the effect of multipath propagation on communications in all of these channels is the same: Inter-symbol interference can arise due to the presence of dispersion if the symbol periods of the deployed communications system are small enough such that copies of one transmitted symbol arrive during the subsequent symbol period, thereby causing interference. This interference must be resolved using an equalizer or another form of multipath mitigation so that reliable communications can be realized in the dispersive channel being considered.

Reverberant enclosed spaces also experience dispersion, which is caused by the transmitted sig-

nal bouncing, or reverberating, off the highly reflective walls enclosing the space as well as the objects contained within that space. A characterization of dispersion in the enclosed space communications channel is crucial information for the architects of enclosed space RF communication systems so that those designers know whether multipath mitigation techniques such as equalization are required to obtain reliable communications at the desired symbol rates. Dispersion has been measured in other radio channels using the first and second moments of the power delay profile—yielding the mean excess delay and the RMS delay spread—and the autocorrelation function of the channel frequency transfer function that provides the 50% and 90% coherence bandwidths. These metrics can also be applied to the enclosed space radio channel to measure and model the dispersion inside reverberant spaces. In what follows, empirical measurements of RMS delay spread, mean excess delay and the coherence bandwidths taken in the enclosed spaces platforms will be presented, and a model that connects the average of each of these parameters to the net quality factor of the enclosed space (presented in Chapter 4) will be introduced and compared to the empirical measurements. *The model for each of these parameters predicts the average value of dispersion that would be seen by a large number of receivers located randomly throughout the enclosure—or equivalently, the average dispersion seen by a single receiver if it were average the dispersion observed over a large number or random positions in the enclosure.* The presented results will illustrate that the net quality factor in the enclosed space is a central parameter that can be used to accurately model and predict the reverberant enclosed space channel dispersion.

## 5.1 Dispersion in the Time Domain

The RMS delay spread,  $\sigma_\tau$ , and mean excess delay,  $\bar{\tau}$ , dispersion parameters are derived from the first and second moment of a channel's power delay profile,  $p(t)$ :

$$\sigma_\tau = \sqrt{\bar{\tau}^2 - \bar{\tau}^2}, \quad (5.1)$$

$$\text{where } \bar{\tau} = \frac{\int_0^\infty (t - t_0) p(t - t_0) dt}{\int_0^\infty p(t - t_0) dt} \quad (5.2)$$

$$\text{and } \bar{\tau}^2 = \frac{\int_0^\infty (t - t_0)^2 p(t - t_0) dt}{\int_0^\infty p(t - t_0) dt}. \quad (5.3)$$

Chapter 4 motivated with theory and showed with empirical results that the power delay profile of an overmoded reverberant space decays with an exponential profile, characterized by the exponential parameter,  $\omega/Q$ , and measurements for the net quality factor in several different configurations of the enclosed space platforms were presented. Saleh and Valenzuela also made the assumption that the power delay profile in the indoor propagation channel follows an exponential behavior [18]. Similarly, Holloway, et. al., have suggested using an exponential decay behavior to model the power delay profile in reverberant environments such as ships and metal warehouses [107]. On account of this theoretically derived and experimentally confirmed exponential behavior, let  $p(t) = p_0 \exp(-\omega t/Q)$ , and substitute in for  $p(t)$  in (5.2) and (5.3). The following steps are taken to find the first moment and hence the mean excess delay, using integration by parts:

$$\bar{\tau} = \frac{\int_0^a t p_0 \exp\left(\frac{-t\omega}{Q}\right) dt}{\int_0^a p_0 \exp\left(\frac{-t\omega}{Q}\right) dt} \quad (5.4)$$

$$= \frac{-t \frac{Q}{\omega} \exp\left(-\frac{\omega}{Q}t\right)\Big|_0^a - \int_0^a -\frac{Q}{\omega} \exp\left(\frac{-t\omega}{Q}\right) dt}{-\frac{Q}{\omega} \exp\left(-\frac{\omega}{Q}t\right)\Big|_0^a} \quad (5.5)$$

$$= \frac{\left(\frac{Q}{\omega}\right)^2 \ln(\alpha)\alpha + \left(\frac{Q}{\omega}\right)^2 (1 - \alpha)}{\frac{Q}{\omega}(1 - \alpha)} \quad (5.6)$$

$$= \frac{Q(\alpha \ln(\alpha) - \alpha + 1)}{\omega(1 - \alpha)}. \quad (5.7)$$

Note that the upper limit of integration has been changed to  $a = -Q \ln(\alpha)/\omega$ , where  $\alpha$  is the normalized power level (normalized to the maximum power,  $p_0$ ) through which the integration is performed. The parameter  $10 \log(\alpha)$  is sometimes referred as the threshold for dispersion calculations, and represents a receiver's finite dynamic range. A similar calculation using integration by parts can

be executed to find the second moment,  $\overline{\tau^2}$ , and the RMS delay spread,  $\sigma_\tau$ :

$$\overline{\tau^2} = \frac{\int_0^a t^2 p_0 \exp\left(\frac{-t\omega}{Q}\right) dt}{\int_0^a p_0 \exp\left(\frac{-t\omega}{Q}\right) dt} \quad (5.8)$$

$$= \frac{-t^2 \frac{Q}{\omega} \exp\left(-\frac{\omega}{Q}t\right)\Big|_0^a - 2 \int_0^a -t \frac{Q}{\omega} \exp\left(\frac{-t\omega}{Q}\right) dt}{-\frac{Q}{\omega} \exp\left(-\frac{\omega}{Q}t\right)\Big|_0^a} \quad (5.9)$$

$$= \frac{-\left(\frac{Q}{\omega}\right)^3 \ln^2(\alpha)\alpha + 2\left(\frac{Q}{\omega}\right)^3 (\ln(\alpha) - \alpha + 1)}{\frac{Q}{\omega}(1 - \alpha)} \quad (5.10)$$

$$= \frac{Q^2 (2\alpha \ln(\alpha) - 2\alpha + 2 - \alpha (\ln(\alpha))^2)}{\omega^2 (1 - \alpha)} \quad (5.11)$$

$$\Rightarrow \sigma_\tau = \sqrt{\overline{\tau^2} - \bar{\tau}^2} = \frac{Q}{\omega} \sqrt{\frac{(2\alpha \ln(\alpha) - 2\alpha + 2 - \alpha (\ln(\alpha))^2)}{(1 - \alpha)} - \frac{(\alpha \ln(\alpha) - \alpha + 1)^2}{(1 - \alpha)^2}}. \quad (5.12)$$

Notice that the above results describe the RMS delay spread and the mean excess delay in terms of only the operation frequency and net quality factor of the enclosure. The results also show that both the RMS delay spread and the mean excess delay are proportional to the enclosure quality factor. As this quality factor increases, the time dispersion parameters also increase, which is harmonious since an increase of both the RMS delay spread and the mean excess delay imply that energy is held within the reverberant space for longer durations of time. It is important to remember that these models for the RMS delay spread and the mean excess delay do not represent the exact RMS delay spread between any two transmit and receive points in the enclosed space, but rather represent the average predicted RMS delay spread and mean excess delay over all points in the enclosure.

### 5.1.1 Empirical Results for RMS Delay Spread

The empirical RMS delay spread can be measured by taking the inverse Fourier transform of the frequency response measured at every rotation of the objects or stirrer for each of the frequency bands and finding the moments of each associated magnitude-squared impulse response,  $|h(t - t_0)|^2$ . Evaluating the RMS delay spread at each of the rotation positions allows investigators to explore the variation of the RMS delay spread about its average over the several different instantiations of the enclosed space channel\*. The RMS delay spread values in each frequency band can be

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\*Evaluating the RMS delay spread of each magnitude squared channel impulse response and taking the average is a distinct method from the method that first finds an ensemble average magnitude-squared impulse response—i.e. the

averaged over all rotational positions to obtain an average RMS delay spread curve vs. frequency. Figures 5.1 and 5.2 show this average RMS delay spread curve vs. frequency for the 5 section cylindrical and conical enclosed space platforms, respectively, measured using the loop antenna. In both plots, the dashed lines present the predicted RMS delay spread using the relation in (5.12) and the quality factor, estimated from the slope of the power delay profile as described in Chapter 4. A very good match is observed between the predicted and empirically measured RMS delay spread, validating the approach and expressions for the RMS delay spread in (5.12). Even though the time dispersion model only defines a single exponentially decaying oscillatory mode, such an excellent match between the model predictions and empirical measurements indicates that the model assumption is capable of accurately representing the RMS delay spread due to the excitation of the several excited modes by a signal in the overmoded region. Figures 5.1 and 5.2 compare the 5 section cylindrical and conical platforms in the sparsely populated (stirred) and densely populated (object-filled) configurations. It is clear that the presence of a dense population of internal objects causes the delay spread to decrease. This effect can be explained by recognizing that in the situation where there are more objects, a ray propagating round-trip through the reverberant space will on average experience a larger number of lossy reflections than a round-trip ray in a sparsely populated space, causing the energy contained in propagating rays to decrease at an increased rate.

Figures 5.3 and 5.4 compare RMS delay spread measured in two different enclosure volumes (2 and 5 sections) for the cylinder and cone, respectively. It is clear from these figures that the decrease in volume has also reduced the RMS delay spread in the environment, an effect that can be explained in the following way. Recall that the RMS delay spread has been shown to be proportional to the net quality factor of the reverberant space (via (5.12)), which if other losses (i.e. antenna, aperture, absorbers, etc.) can be considered small, is proportional to the composite quality factor, given in (2.46). Since the composite quality factor is also proportional to volume, when the volume of an enclosed space decreases, so will the dispersion measured by the RMS delay spread. Furthermore, a reduction in the quality factor and therefore the RMS delay spread resulting from a decrease in volume is an alternative manner in which to explain the decrease in delay spread between the sparse and dense configurations. The objects in the dense configuration have completely-connected

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power delay profile—and then calculates the RMS delay spread. Both methods have been used in the literature; for example, the first method is used in [17] and the second is used in [103].

conducting facets enclosing an internal empty volume that is isolated from the fields of the main reverberant enclosure. These micro-volumes reduce the volume of the macro-enclosure, thereby reducing the large enclosure's net quality factor and RMS delay spread.

To examine the effect on the RMS delay spread of changing the antenna used, measurements were also made in the 5 section cylinder with the 1 cm probe antenna, in both the sparsely populated (stirred) and densely populated (object-filled) configurations. Figure 5.5 shows the comparison in RMS delay spread measured using the probe and loop antennas in the stirred 5 section cylindrical platform, and Fig. 5.6, likewise shows the same comparison for the object filled environment. The changes due to the antennas used to excite and sense the reverberant space are small, particularly for the object-filled space. This difference between the object-filled and stirred space can be explained by recalling the discussion of the effect of antenna quality factor on the measured net quality factor in Chapter 4. In the densely populated configurations, the composite quality factor (associated with the losses from induced surface currents) becomes considerably smaller than the composite quality factor in the sparsely populated configurations, making any changes in the much larger antenna quality factor less pronounced.

Figures 5.7 and 5.8 show the individually measured values of the RMS delay spread in the 5 section cylindrical platform in sparse and dense configurations using the loop antenna, taken at each rotational position of the stirrer/object-assembly and plotted with the predicted RMS delay spread based on the measured net quality factor (dashed line). Both of these figures qualitatively show that the RMS delay spread has a small variance as compared to the predicted value of the data set, especially at high frequencies. This small variation suggests that the average value of RMS delay spread that is predicted via the quality factor and the expression given in (5.12) will be strongly representative of the actual RMS delay spread value that a receiver placed in an arbitrary position in an enclosed space environment might experience. Similarly small values of the variance of the RMS delay spread have also been observed for the conical platform measurements.



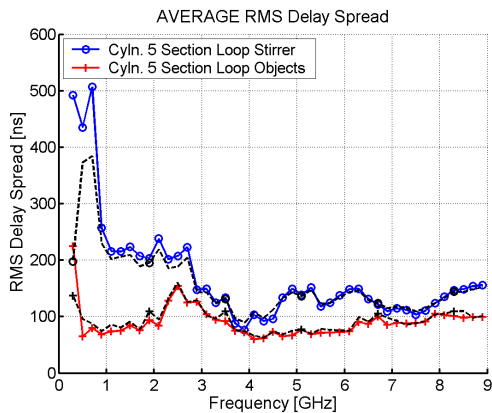


Figure 5.1: Average RMS delay spread measured in the 5 section cylindrical platform using the loop antenna, comparing the sparsely populated stirred environment with the densely populated object environment. Solid: Measured using vector network analyzer. Dashed: Predicted using the net quality factor.

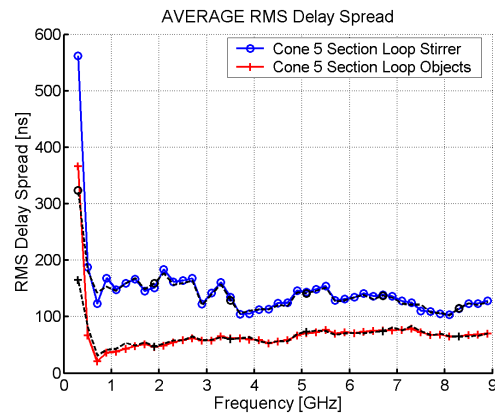


Figure 5.2: Average RMS delay spread measured in the 5 section conical platform using the loop antenna, comparing the sparsely populated stirred environment with the densely populated object environment. Solid: Measured using vector network analyzer. Dashed: Predicted using the net quality factor.

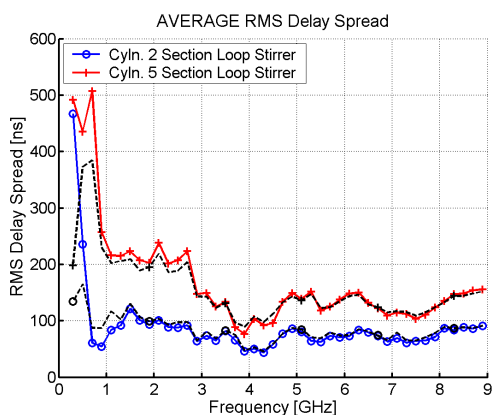


Figure 5.3: Average RMS delay spread measured in the stirred cylindrical platform using the loop antenna, comparing the 2 and 5 section configurations. Solid: Measured using vector network analyzer. Dashed: Predicted using the net quality factor.

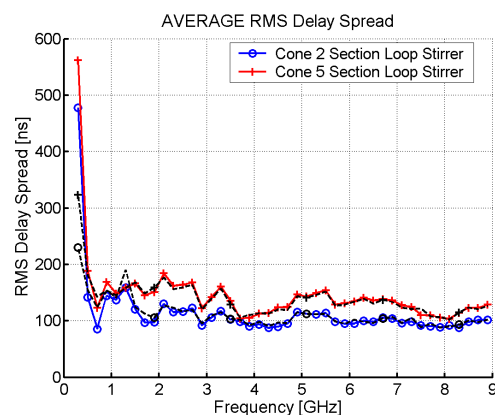


Figure 5.4: Average RMS delay spread measured in the stirred conical platform using the loop antenna, comparing the 2 and 5 section configurations. Solid: Measured using vector network analyzer. Dashed: Predicted using the net quality factor.

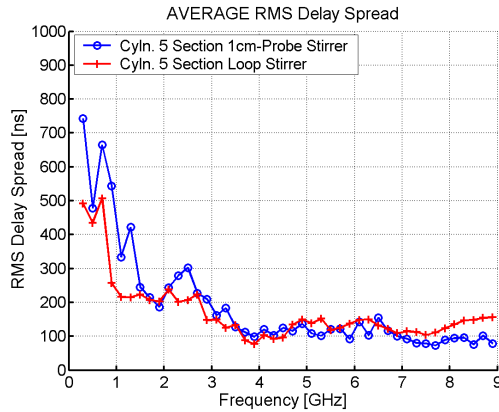


Figure 5.5: Average RMS delay spread measured in the stirred 5 section cylindrical platform, comparing the response due to the loop and probe antenna configurations.

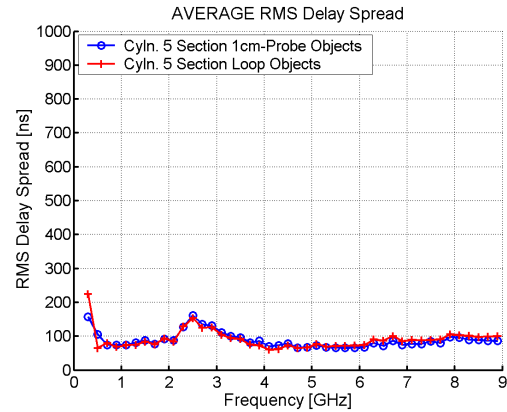


Figure 5.6: Average RMS delay spread measured in the 5 section object filled cylindrical platform, comparing the response due to the loop and probe antenna configurations.

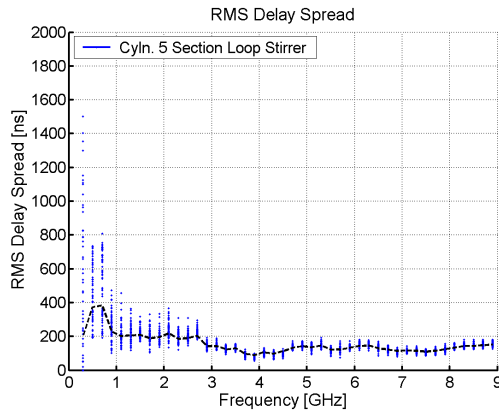


Figure 5.7: RMS delay spread measured in each stirrer position in the 5 section cylindrical platform using the loop antenna in the sparsely populated stirred environment. Dots: Measured using vector network analyzer. Dashed: Predicted using the net quality factor.

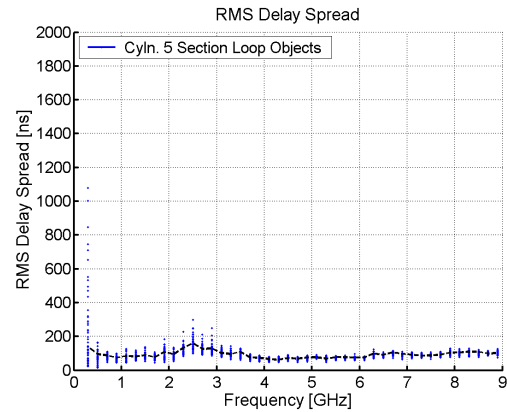


Figure 5.8: RMS delay spread measured in each position of the object assembly in the 5 section cylindrical platform using the loop antenna in the densely populated object-filled environment. Dots: Measured using vector network analyzer. Dashed: Predicted using the net quality factor.

### 5.1.2 Empirical Results for Mean Excess Delay

The expression relating the net quality factor of a reverberant enclosed space to the mean excess delay was derived in the previous section and is found in closed-form in (5.7). As already established, the mean excess delay is proportional to the net quality factor of the enclosed space, and the same was found for the RMS delay spread. Figures 5.9 and 5.10 show the empirically measured mean excess delay versus frequency, averaged over all rotations of the stirrer/objects, for the 5 section cylinder and cone, respectively, using the loop antenna. The predictions for the mean excess delay derived from the measured net quality factor in the corresponding configurations are shown using the dashed lines in both figures. One notices from both figures that the predicted mean excess delay for both the cylinder and cone tend to be higher than the measured mean excess delay, obtained by finding the empirical first moment of the channel's power delay profile. The overestimate of the predicted mean excess delay appears to be higher for the cylinder than for the cone, however. Aside from this discrepancy, the predictions appear to follow the measured results well, and the comparison between the mean excess delay in the sparse and dense internal population configurations shows that the addition of objects and the resulting additional ray-reflections causes the mean excess delay to decrease, just as for the RMS delay spread.

Figures 5.11 and 5.12 compare the 2 and 5 section volumes for the sparse configuration for the cylindrical and conical platforms, respectively, and show a decrease in the mean excess delay corresponding to the decrease in the net quality factor and enclosure volume (as presented in Chapter 4). Once again, the predictions for the mean excess delay obtained from the estimated quality factor are an overestimate to the average mean excess delay. Figures 5.13 and 5.14 are plots of the mean excess delay calculated at every rotational position of the stirrer or object-assembly and show, nevertheless, that the predicted mean excess delay appears to be reasonably representative of the empirically measured mean excess delay. This estimate of the mean excess delay remains to be a useful modeling mechanism for the enclosed space radio channel, as it continues to give system architects a valid, albeit conservative, estimate of the channel dispersion. Furthermore, Figs. 5.13 and 5.14 show that the low variance (compared to the predicted value) of the mean excess delay values measured within each frequency segment in both figures indicates that the prediction given by the quality factor via (5.7) is indeed a good representation of the actual mean excess delay as

would be seen by a receiver in the channel. This small variance of the measured values as compared to the predictions has also been observed in the conical platform measurements.

The results for the mean excess delay comparing the 1 cm probe antenna and the loop antenna, shown for the sparsely and densely populated configurations in Figs. 5.15 and 5.16 respectively, show a behavior very similar to the comparison of antenna types for the RMS delay spread. Recall Figs. 3.8 and 3.9, which show the  $|S_{11}|$  for the probe and loop antennas, and note that the probe antenna is better matched at frequencies greater than 6.0 GHz. Since the  $|S_{11}|$  is a term in the antenna quality factor, given in (2.49), the antenna quality factor for the probe antenna is noticeably smaller than for the loop antenna above 6.0 GHz, and this will cause the net quality factor, and hence, the mean excess delay to behave differently in this range of frequencies, particularly for the sparse internal configurations.

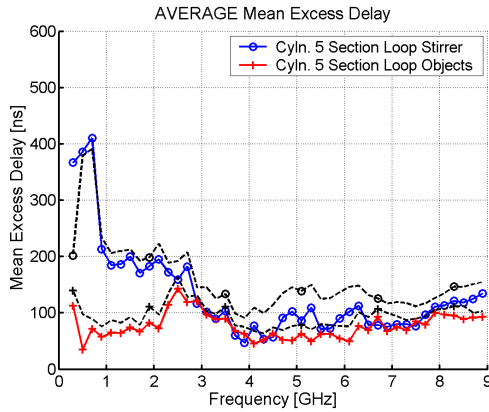


Figure 5.9: Average mean excess delay measured in the 5 section cylindrical platform measured using the loop antenna, comparing the stirred and object-filled configurations. Solid: Measured using vector network analyzer. Dashed: Predicted using the net quality factor.

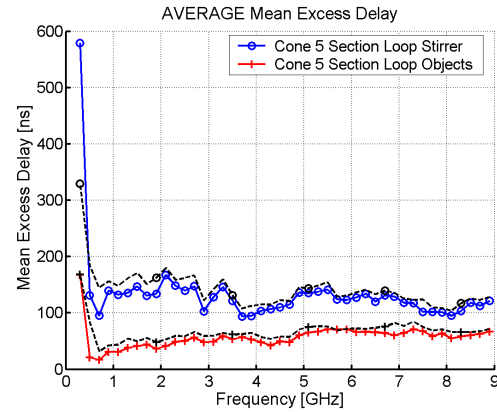


Figure 5.10: Average mean excess delay measured in the 5 section conical platform measured using the loop antenna, comparing the stirred and object-filled configurations. Solid: Measured using vector network analyzer. Dashed: Predicted using the net quality factor.

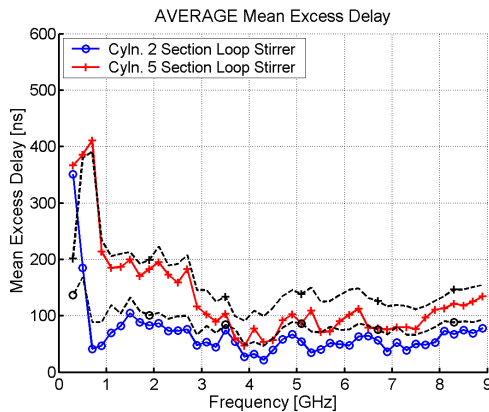


Figure 5.11: Average mean excess delay measured in the stirred cylindrical platform using the loop antenna, comparing the 2 and 5 section configurations. Solid: Measured using vector network analyzer. Dashed: Predicted using the net quality factor.

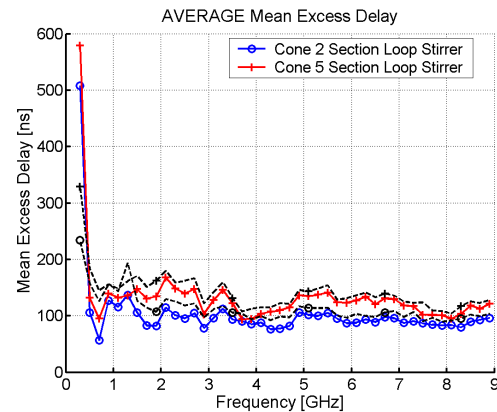


Figure 5.12: Average mean excess delay measured in the stirred conical platform using the loop antenna, comparing the 2 and 5 section configurations. Solid: Measured using vector network analyzer. Dashed: Predicted using the net quality factor.

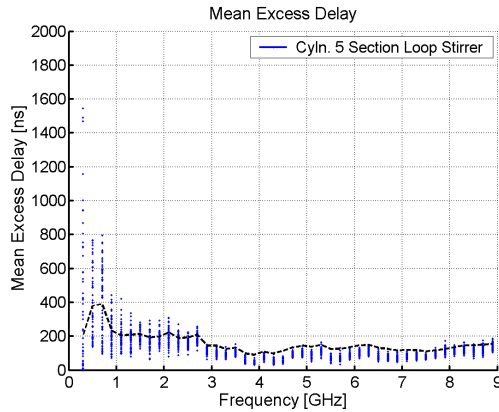


Figure 5.13: Mean excess delay measured in each stirrer position in the 5 section cylindrical platform using the loop antenna in the sparsely populated stirred environment. Dots: Measured using vector network analyzer. Dashed: Predicted using the net quality factor.

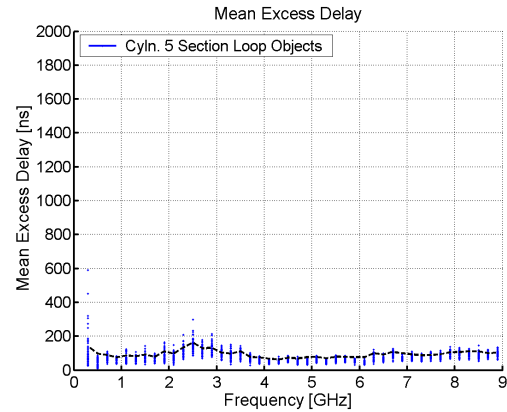


Figure 5.14: Mean excess delay measured in each position of the object assembly in the 5 section cylindrical platform using the loop antenna in the densely populated object-filled environment. Dots: Measured using vector network analyzer. Dashed: Predicted using the net quality factor.

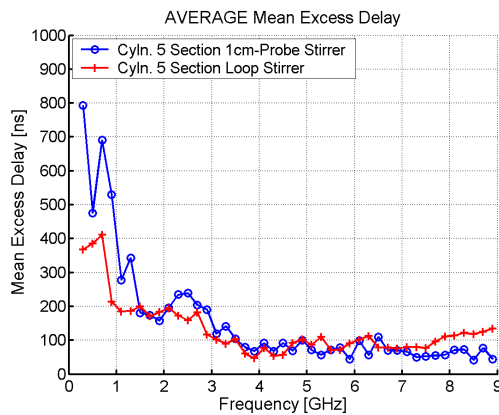


Figure 5.15: Average mean excess delay measured in the stirred 5 section cylindrical platform, comparing the response due to the loop and probe antenna configurations.

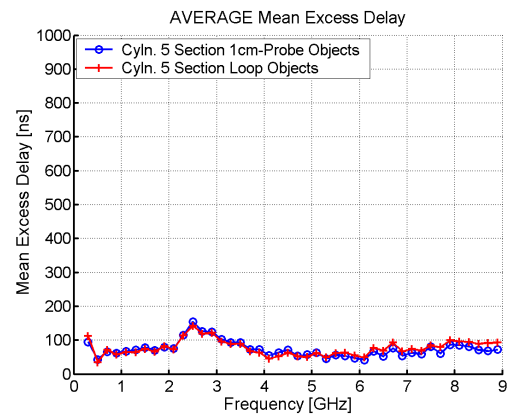


Figure 5.16: Average mean excess delay measured in the 5 section object filled cylindrical platform, comparing the response due to the loop and probe antenna configurations.

### 5.1.3 Discussion

The above results show that the model described by the relationships in (5.7) and (5.12) can accurately predict the time dispersion parameters of RMS delay spread and mean excess delay. The match between the predicted and actual results for the RMS delay spread is very good, suggesting that if designers have the choice between utilizing the RMS delay spread or the mean excess delay, the RMS delay spread appears to yield results that more closely reflect the average behavior of the RMS delay spread. The model prediction for the mean excess delay appears to provide a conservative estimate for the actual mean excess delay in the channel, and this fact should be kept in mind when designers apply the model of (5.7) to engineering problems. The discrepancy in the predictive model and the actual behavior of the mean excess delay could be explained by considering the phenomena of reverberation in the enclosed space more closely. When a reverberant space is excited with an electromagnetic signal, before complete reverberation occurs—a condition characterized by a steady-state homogenous energy distribution over the entire enclosure volume—the signal must first propagate throughout the enclosure. This transition to reverberation consumes a finite duration in time and has been previously identified in the electromagnetic communications channels to take on average between  $8l_c/\nu$  and  $10l_c/\nu$ , where  $l_c = 4V/S$  is the characteristic length of the enclosure,  $\nu$  is the speed of energy propagation in the medium,  $V$  is the enclosure volume and  $S$  is the surface area of the boundaries containing the enclosure [107]. During this transition time window, the power delay profile seen at the antenna terminals that observes the decay of energy contained in the enclosure is not exponential in nature, but takes on the following form:

$$p(t) = \begin{cases} 1 & t = 0 \\ \frac{\gamma^n}{4n^2}, & n = \frac{t}{t_c} + \frac{1}{2} \quad t > 0 \end{cases}, \quad (5.13)$$

where  $t_c = 2l_c/\nu = 8V/\nu S$  is the characteristic time of the enclosure and  $\gamma$  is the average power reflection coefficient of the enclosure walls [107]. After this time window, the exponential profile is accurate, and the power delay profile measured by placing a receiving antenna within the enclosure, which has already reduced in amplitude from its maximum, will continue to be considered in the dispersion calculations until the threshold compared to the maximum is met (i.e. the parameter  $\alpha$ ).

However, if the energy decay during the pre-reverberation period is faster than an exponential

decay, the power delay profile will arrive at the chosen threshold value sooner than what the entirely exponential model suggests. Consider the 5 section cylindrical stirred platform, measured with the loop antenna, whose mean excess delay is shown in Fig. 5.9. From Table 3.2, the characteristic length for this configuration is  $l_c = 4 \cdot 0.2294m^3 / 2.866m^2 = 0.3202m$ , which makes the characteristic time  $t_c = 2l_c/v = 2.134$  ns. Reverberation then richly occurs after approx. 4.5 characteristic times, or after 9.605 ns for this data set. Using (5.13) by letting  $t = 4.5t_c$  and assuming for simplicity that  $\gamma = 1$ , we find that the power delay profile at the onset of rich reverberation has a value of -20 dB. Comparing this to the exponential model, choosing an example frequency of 7.0 GHz at which this dataset has a estimated Q of 5000,  $\exp(-2 \cdot \pi \cdot 7.0 \cdot 10^9 \cdot 9.605 \cdot 10^{-9} / 5000) = 0.9190$  or -0.3669 dB. According to this model, it is clear that the power delay profile in this pre-reverberation window falls-off more quickly than the exponential profile would suggest, and this difference in profile behavior during the pre-reverberation time suggests that measured mean excess delay would indeed be less than what was predicted by the exponential model for mean excess delay. The exponential model, however, does yet predict a useful upper bound on the measured mean excess delay, as depicted in Figs. 5.13 and 5.14.

While the difference in pre-reverberation profile certainly affects the mean of the profile (first moment), it is not clear that it would as noticeably affect the mean-removed spread of the profile (second central moment). The RMS delay spread is the square root of the profile's second *central* moment, and since the data presented in the chapter shows a very good match between the entirely exponential model-based predictions and the measurements of RMS delay spread, it appears that the delay spread is resilient to this difference between modeled profile and actual profile behavior.

## 5.2 Dispersion in the Frequency Domain

Dispersion in the frequency domain is measured by considering the likelihood of the spectrum envelope at two frequencies in the vicinity of one another to fade dependently, i.e. when one fades in amplitude, the other is also likely to experience a fade. This likelihood is measured by the frequency correlation function, defined as follows:

$$\Phi(\Delta\omega) = \frac{\int_{-\infty}^{\infty} H(\omega)H^*(\omega + \Delta\omega)d\omega}{\int_{-\infty}^{\infty} H(\omega)H^*(\omega)d\omega}, \quad (5.14)$$



where  $H(\omega)$  is the frequency transfer function of the channel in question. The value of  $\Delta\omega$  at which  $\Phi(\Delta\omega)$  falls below a value  $x$  for the first time ( $\Phi(\Delta\omega = 0)=1$ ) is referred to as the  $x\%$  coherence bandwidth. For instance, when  $\Phi(\Delta\omega) = 0.5$  for the first time, the value of  $\Delta\omega$  is known as the 50% coherence bandwidth.

While it is natural in the time domain to choose the power delay profile to be the exponential function since that is the manner in which energy decays in the enclosure, a similar choice is not as intuitive in the frequency domain. It has been shown in other channel studies that the coherence bandwidth and the RMS delay spread behave reciprocally [108], similar to the reciprocal nature of an arbitrary signal's decay time in the time domain and its bandwidth in the frequency domain. A reasonable, yet not as intuitive, approach would be to let the spectrum in the frequency domain (for the purposes of the frequency autocorrelation function) be the Fourier transform of the exponential function in the time domain. That is, if the power delay profile is  $p(t) = p_0 \exp(-\omega_0 t/Q)$ , then its impulse response is  $g(t) = g_0 \exp(-\omega_0 t/2Q)$ , which has the following Fourier transform:

$$g(t) = g_0 \exp(-\omega_0 t/2Q) \longleftrightarrow \frac{g_0}{\omega_0/2Q + j\omega} = G(\omega). \quad (5.15)$$

This result can be placed in the frequency correlation function in (5.14) to find the frequency correlation which will be used to calculate the coherence bandwidth. Even though the frequency correlation function is empirically evaluated in the frequency band of operation, e.g. a band centered at 5.0 GHz, the spectrum  $G(\omega)$  shown above is located at baseband, but multiplying the time domain signal by  $\exp(j\omega_0 t)$  would cause the entire spectrum to be centered at  $\omega_0$ . However, this shift in frequency does not affect the frequency correlation function and so the spectrum shown above can be left at baseband for calculations. Notice that the spectrum  $G(\omega)$  is an even function in magnitude and an odd function in phase, and so  $G(-\omega) = G^*(\omega)$ . Also, recall that the magnitude of any autocorrelation function is an even function. These two properties allow the magnitude of the frequency autocorrelation function in (5.14) (replacing  $H(\omega)$  with  $G(\omega)$ ), to be written as a convolution:

$$|\Phi(\Delta\omega)| = \left| \frac{\int_{-\infty}^{\infty} G(\omega') G(\Delta\omega - \omega') d\omega'}{\int_{-\infty}^{\infty} G(\omega') G(-\omega') d\omega'} \right|. \quad (5.16)$$

The numerator of (5.16) can be solved easily by transforming the convolution to the time domain,

multiplying and transforming back to the frequency domain:

$$\int_{-\infty}^{\infty} G(\omega')G(\Delta\omega - \omega')d\omega' \longleftrightarrow g^2(t) = g_0^2 \exp(-\omega_0 t/Q) \longleftrightarrow \frac{g_0^2}{\omega_0/Q + j\Delta\omega}. \quad (5.17)$$

The denominator of (5.16) is the same as the numerator with  $\Delta f = 0$ ; the new frequency correlation function can be thus be written as follows, considering only the magnitude correlation function:

$$|\Phi(\Delta\omega)| = \left| \frac{\frac{g_0^2}{\omega_0/Q + j\Delta\omega}}{\frac{g_0^2}{\omega_0/Q}} \right| = \left| \frac{\omega_0/Q}{\omega_0/Q + j\Delta\omega} \right| = \frac{\omega_0/Q}{\sqrt{(\omega_0/Q)^2 + (\Delta\omega)^2}}. \quad (5.18)$$

The expression in (5.18) can then be solved for  $|\Phi(\Delta\omega)| = 0.5$  and  $|\Phi(\Delta\omega)| = 0.9$  to find the 50% and 90% coherence bandwidth estimates:

$$\Delta\omega_{50} = \sqrt{3} \frac{\omega_0}{Q}, \quad (5.19)$$

$$\Delta\omega_{90} = \frac{1}{2} \frac{\omega_0}{Q}, \quad (5.20)$$

which can also be written in terms of non-radian frequency ( $\omega_0 = 2\pi f$ ):

$$\Delta f_{50} = \sqrt{3} \frac{f}{Q}, \quad (5.21)$$

$$\Delta f_{90} = \frac{1}{2} \frac{f}{Q}. \quad (5.22)$$

Note that both of these coherence bandwidth estimates are functions of only the frequency and quality factor of the enclosed space and are analogous to the closed form expressions for the RMS delay spread and the mean excess delay in the sense that once an estimate of the enclosed space quality factor is obtained, estimates for the 50% and 90% coherence bandwidths can be found. One can observe from these expressions that the coherence bandwidths and the quality factor are inversely proportional. If the quality factor increases, the the coherence bandwidths decrease, which agrees with the intuition developed from the time domain dispersion estimates and the reciprocal relationship between the time and frequency dispersion parameters.

### 5.2.1 Empirical Results for 90% Coherence Bandwidth

As was done for the time domain dispersion parameters, the predictions for the 50% and 90% coherence bandwidths are compared to the measured coherence bandwidth, measured by calculating the empirical autocorrelation of the channel frequency transfer function in each of the 44 measured frequency bands and at each of the 60 rotational measurement positions. The coherence bandwidth is then ensemble averaged over the 60 positions to obtain an average coherence bandwidth versus frequency curve. Figures 5.17 and 5.18 compare the average 90% coherence bandwidth measured in the sparsely and densely configured internal environments of the 5 section cylindrical and conical platforms, respectively, using the loop antenna. The plots identify a very good match between the empirically measured data (solid curves) and the predicted values (dashed curves), using (5.22) and the estimated quality factors presented for the identical configuration in Chapter 4. The figures indicate that as the internal environment becomes more densely populated, the 90% coherence bandwidth increases, suggesting that larger signal bandwidths can be accommodated in the enclosed space channel without experiencing frequency selective fading. Figures 5.19 and 5.20 compare the measured 90% coherence bandwidth from the 2 and 5 section configurations of the sparsely populated (stirred) cylindrical and conical platforms, respectively, again demonstrating a very good match between the predictions and measured values. One can see from the figures that a decrease in the volume-to-surface area ratio causes the coherence bandwidth to increase for both the cylinder and cone environments (recalling that the coherence bandwidth is inversely proportional to the net quality factor, which is proportional to volume and inversely proportional to surface area), though the increase is more pronounced in the cylinder environment. This difference can be explained by considering the percent change in the volume-to-surface area ratio from the 5 section to 2 section stirred cylinder and likewise from the 5 section to 2 section cone. From Tables 3.2 and 3.3, it is calculated that the change in  $V/S$  for the cylinder is 31.0% whereas the change in  $V/S$  for the cone is only 16.6%. The smaller change for the cone can be attributed to the fact that the 3 sections that are removed from the cone are the upper sections, which contain the smallest volume and surface area, whereas each section of the cylinder contributes an equal amount of volume and surface area to the enclosure. The increase in the coherence bandwidth experienced by adding internal objects can also be explained by a change in the  $V/S$  ratio: Internal objects reduce the free volume in the enclosure

that fields can occupy and increase the available surface area on which dissipative currents can be generated. Both of these changes cause the composite quality factor to decrease (recall equation (2.46)), and correspondingly cause the coherence bandwidth to increase (via (5.22)).

Figures 5.21 and 5.22 compare the 90% coherence bandwidth measured using the loop antenna and the 1 cm probe antenna in the 5 section cylinder, using both the stirred and object-filled configurations. While the response using the two antennas is qualitatively similar, the 90% coherence bandwidth is affected by the difference in antenna quality factors between the loop and the probe antennas, given in Figs. 4.7 and 4.13, which differ due to distinct  $|S_{11}|$  responses measured for each antenna (see Figs. 3.8 and 3.9). This effect was also seen in the RMS delay spread and the mean excess delay, further showing that the antennas' impedance match can change the dispersion measured by those antennas in the channel (via the antenna quality factor and hence the net quality factor). Note however, that the only parameter that affects the quality factor and therefore the dispersion is the impedance match of the antenna; the type of antenna used, its gain and pattern do not influence the antenna quality factor.

Figures 5.23 and 5.24 show the measured values of the 90% coherence bandwidth in the 5 section cylindrical platform in sparse and dense configurations using the loop antenna, taken at each rotational position of the stirrer/object-assembly and plotted with the predicted 90% coherence bandwidth curve based on the measured net quality factor (dashed line). These figures show in a qualitative fashion that the measured 90% coherence bandwidth has a small variance as compared to the predicted value for the data set. As is the case for the RMS delay spread and the mean excess delay, this small variation suggests that the average value of the 90% coherence bandwidth that is predicted via the quality factor and the expression given in (5.22) will be strongly representative of the actual 90% coherence bandwidth value that a receiver placed in an arbitrary position in an enclosed space environment might experience. This small variance in the measured values as compared to the predicted value has also been observed in the conical platform measurements.

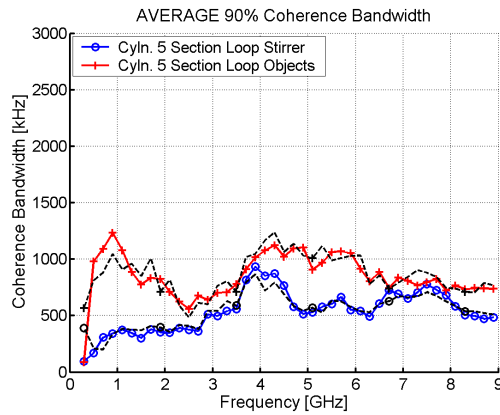


Figure 5.17: Average 90% coherence bandwidth measured in the 5 section cylindrical platform using the loop antenna, comparing the sparsely populated stirred environment with the densely populated object environment. Solid: Measured using vector network analyzer. Dashed: Predicted using the net quality factor.

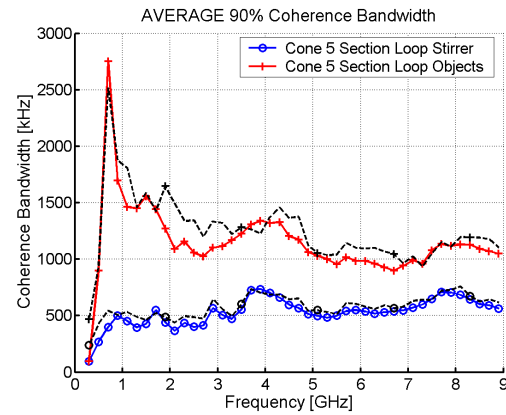


Figure 5.18: Average 90% coherence bandwidth measured in the 5 section conical platform using the loop antenna, comparing the sparsely populated stirred environment with the densely populated object environment. Solid: Measured using vector network analyzer. Dashed: Predicted using the net quality factor.

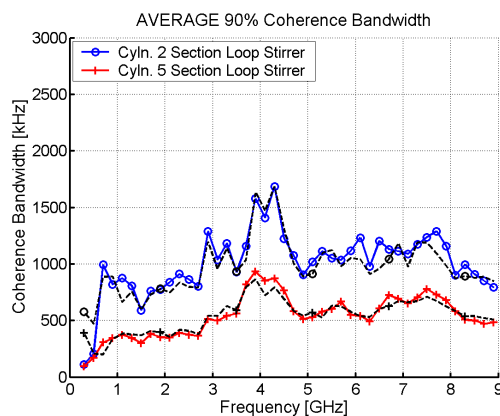


Figure 5.19: Average 90% coherence bandwidth measured in the stirred cylindrical platform using the loop antenna, comparing the 2 and 5 section configurations. Solid: Measured using vector network analyzer. Dashed: Predicted using the net quality factor.

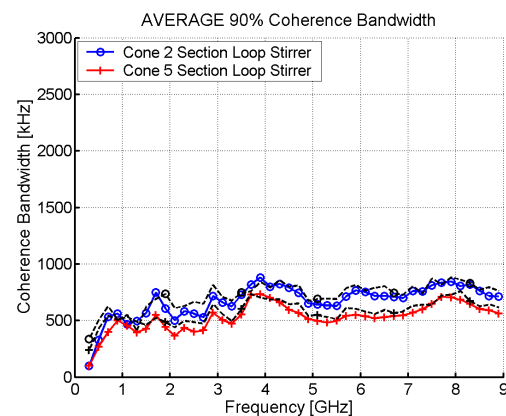


Figure 5.20: Average 90% coherence bandwidth measured in the stirred conical platform using the loop antenna, comparing the 2 and 5 section configurations. Solid: Measured using vector network analyzer. Dashed: Predicted using the net quality factor.

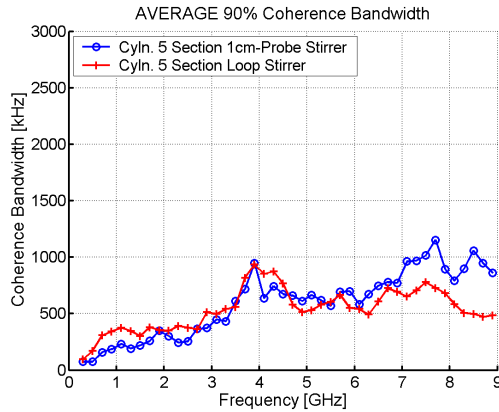


Figure 5.21: Average 90% coherence bandwidth measured in the stirred 5 section cylindrical platform, comparing the response due to the loop and probe antenna configurations.

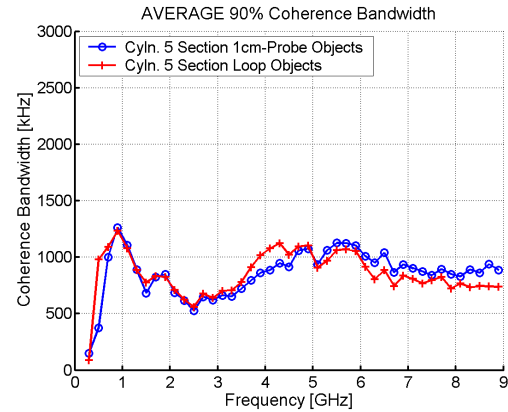


Figure 5.22: Average 90% coherence bandwidth measured in the 5 section object-filled cylindrical platform, comparing the response due to the loop and probe antenna configurations.

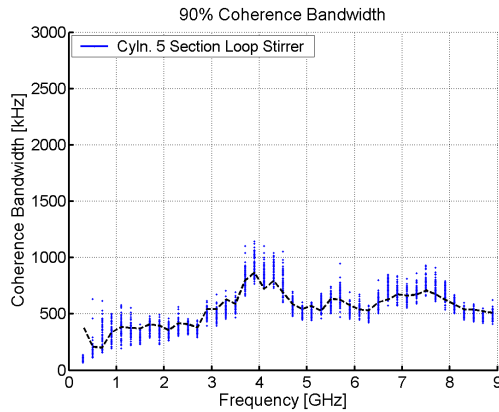


Figure 5.23: 90% coherence bandwidth measured in each stirrer position in the 5 section cylindrical platform using the loop antenna in the sparsely populated stirred environment. Dots: Measured using vector network analyzer. Dashed: Predicted using the net quality factor.

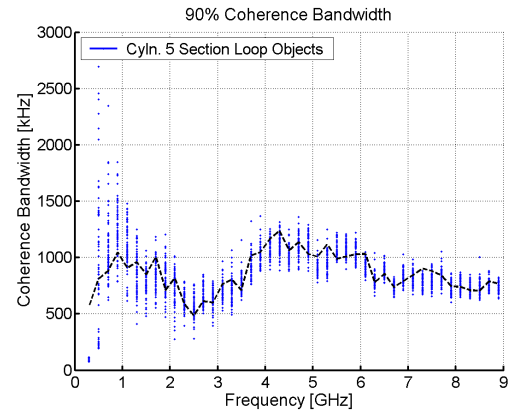


Figure 5.24: 90% coherence bandwidth measured in each position of the object assembly in the 5 section cylindrical platform using the loop antenna in the densely populated object-filled environment. Dots: Measured using vector network analyzer. Dashed: Predicted using the net quality factor.

### 5.2.2 Empirical Results for 50% Coherence Bandwidth

Figures 5.25 and 5.26 show the average measured 50% coherence bandwidth for the 5 section cylindrical and conical platforms using the loop antenna, and both figures compare the coherence bandwidth measured in the sparse (stirred) and dense (object-filled) internal population configurations. Measurements of the 50% coherence bandwidth are made in the same fashion as the 90% coherence bandwidth, presented in the previous section. The predictions for the 50% coherence bandwidth, governed by (5.21), are shown in both plots using dashed lines. While the match between the predictions and the empirically measured values for the conical environment is good, the match of the predicted values with the the average measured 50% coherence bandwidth for the cylindrical section is not as satisfactory. From Figs. 5.27 and 5.28—which are the scatter plots of the individual coherence bandwidth measurements at each of the 60 rotational positions in the 5 section cylindrical platform in both sparse and dense configurations—it seems that the 50% coherence bandwidth predictions (dashed lines) tend to underestimate the 50% coherence bandwidth. Figures 5.31 and 5.32, which compare the 2 and 5 section stirred environments for the cylinder and cone, respectively, display a similar effect: The predictions adequately describe the average response for the conical platform, yet they tend to underestimate the 50 % coherence bandwidth in the cylindrical platform. It is not decisively clear what physical reasoning captures this phenomena, though it should be noted that the antenna used (loop antenna) has a null in the  $|S_{11}|$  function in the vicinity of the locations where the 50% coherence bandwidth experiences a large local variance in its measured values (upwardly spread), which causes the average values to increase. Note that if this upward spread did not exist, the match between the average of the measured values and the predicted curves would be much better. To contrast this situatuion, Figs. 5.29 and 5.30 show the scatter plots for the 50% coherence bandwidth measurements at each rotational position of the stirrer/objects-assembly in the sparsely and densely configured 5 section conical platform. One can clearly see that the match between the measurements and the predicted average 50% coherence bandwidth is better than in the 5 section cylindrical platform and display a fairly constrained variance as compared to the predicted value. Nonetheless, all of these results indicate that the predictions obtained using (5.21) provide a good representation of the actual 50% coherence bandwidth that would be observed by a receiver in the enclosed space channel.

With the exception of Fig. 5.25, the aforementioned figures all demonstrate the same effect seen in the 90% coherence bandwidth where the coherence bandwidth increases as the enclosure V/S ratio decreases, either from the introduction of objects in the space (less free volume and more surfaces), or the reduction of the overall enclosure dimensions. This increase in coherence bandwidth is consistent with the inversely proportional relationship that the coherence bandwidth has with the enclosure net quality factor, which is proportional to the free volume and inversely proportional to the total surface area.

Figures 5.33 and 5.34 compare the loop and probe antennas used to measure the 5 section cylinder in both the sparsely and densely populated internal configurations. Both figures show that the antennas chosen do not cause a substantial change in the 50% coherence bandwidths measured; however, a similar effect is seen in these figures as was also seen for the analogous figures comparing the 90% coherence bandwidths measured using the probe vs. the loop antenna (Figs. 5.21 and 5.22). The 50% coherence bandwidth measured using the loop antenna clearly shows a decrease with respect to the probe antenna above 6.0 GHz (Fig. 5.33), and this effect is more pronounced in the sparsely populated configurations than it is in their densely populated counterparts (Fig. 5.34). As it was explained for the RMS delay spread and the mean excess delay, this difference can be attributed to the difference in the quality factor of the two antennas, shown in Figs. 4.7 and 4.13.



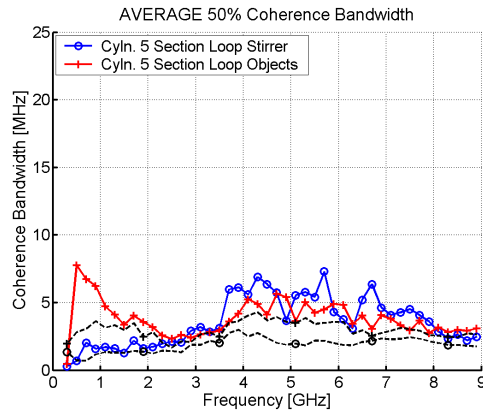


Figure 5.25: Average 50% coherence bandwidth measured in the 5 section cylindrical platform using the loop antenna, comparing the sparsely populated stirred environment with the densely populated object environment. Solid: Measured using vector network analyzer. Dashed: Predicted using the net quality factor.

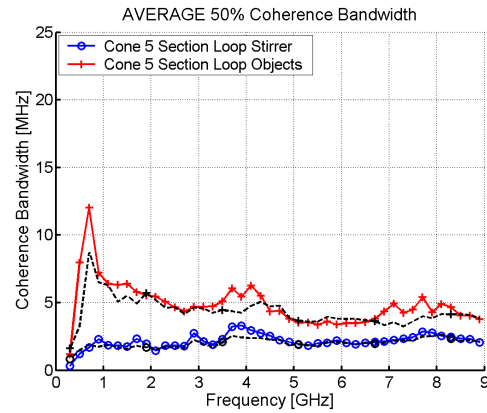


Figure 5.26: Average 50% coherence bandwidth measured in the 5 section conical platform using the loop antenna, comparing the sparsely populated stirred environment with the densely populated object environment. Solid: Measured using vector network analyzer. Dashed: Predicted using the net quality factor.

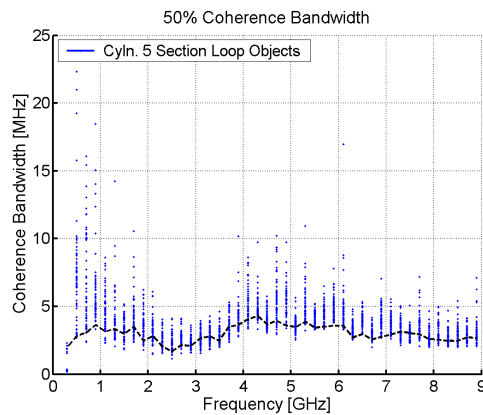


Figure 5.27: 50% coherence bandwidth measured in each position of the object assembly in the 5 section cylindrical platform with objects using the loop antenna. Dots: Measured using vector network analyzer. Dashed: Predicted from net quality factor.

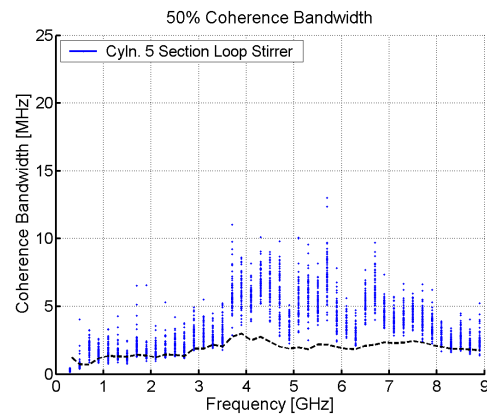


Figure 5.28: 50% coherence bandwidth measured in each stirrer position in the 5 section cylindrical platform with the stirrer using the loop antenna. Dots: Measured using vector network analyzer. Dashed: Predicted from net quality factor.

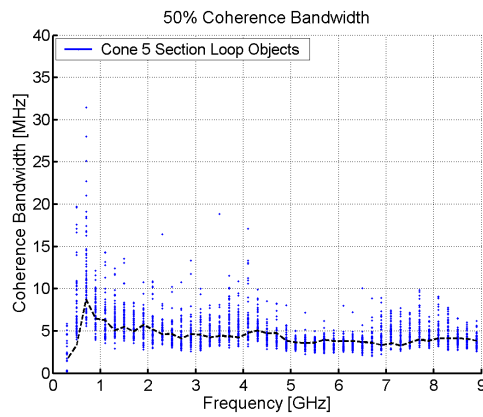


Figure 5.29: 50% coherence bandwidth measured in each position of the object assembly in the 5 section conical platform with objects using the loop antenna. Dots: Measured using vector network analyzer. Dashed: Predicted from net quality factor.

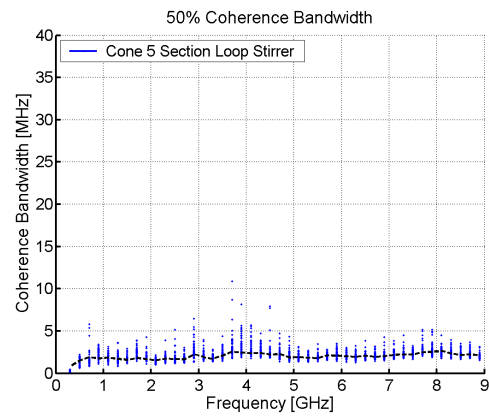


Figure 5.30: 50% coherence bandwidth measured in each stirrer position in the 5 section conical platform with the stirrer using the loop antenna. Dots: Measured using vector network analyzer. Dashed: Predicted from net quality factor.

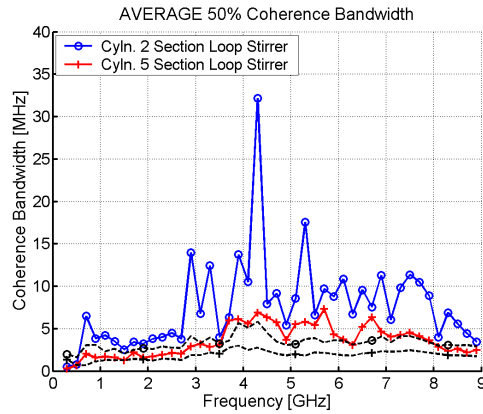


Figure 5.31: Average 50% coherence bandwidth measured in the stirred cylindrical platform using the loop antenna, comparing the 2 and 5 section configurations. Solid: Measured using vector network analyzer. Dashed: Predicted using the net quality factor.

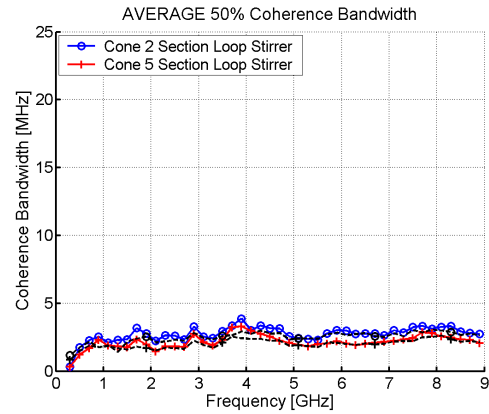


Figure 5.32: Average 50% coherence bandwidth measured in the stirred conical platform using the loop antenna, comparing the 2 and 5 section configurations. Solid: Measured using vector network analyzer. Dashed: Predicted using the net quality factor.

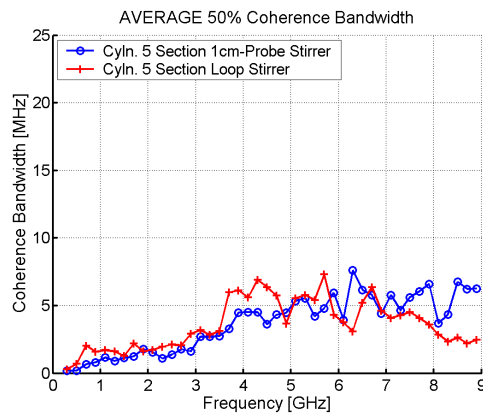


Figure 5.33: Average 50% coherence bandwidth measured in the stirred 5 section cylindrical platform, comparing the response due to the loop and probe antenna configurations.

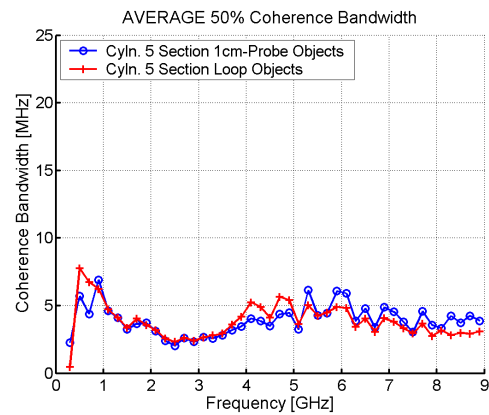


Figure 5.34: Average 50% coherence bandwidth measured in the 5 section object-filled cylindrical platform, comparing the response due to the loop and probe antenna configurations.

### 5.2.3 Discussion

It is interesting to note that while the average response of both the RMS delay spread and the 90% coherence bandwidth are very accurately predicted using (5.12) and (5.22), the predictions given for the mean excess delay in (5.7) and the 50% coherence bandwidth in (5.21) do not match the average responses of those dispersion parameters very well for some of the measurement configurations. It was discussed in Section 5.1.3 that the discrepancy seen in the mean excess delay could be caused by a non-exponential behavior of the power delay profile in the pre-reverberation period. The departure from this purely exponential behavior of the power delay profile is only significant for the very early portions of the power delay profile, (i.e. the first 10 ns of an exponential profile with a time constant  $> 100$  ns). Recall that the development of the predictive model for the coherence bandwidths in this section established that the frequency autocorrelation function is a Fourier transform pair with the square of the channel impulse response, i.e. the power delay profile. The time-frequency dual nature of the power delay profile and the frequency autocorrelation function is entirely natural: As the decay time of the power delay profile in time becomes shorter, the time dispersion parameters get smaller, and through the duality, it is expected that the width frequency autocorrelation function will increase, thereby making the coherence bandwidths larger. The rapid change occurring in a very short pre-reverberation period of the delay profile constitutes a high frequency variation in the profile, and given the time-frequency duality already established, higher frequency components would be expected to appear in the frequency autocorrelation function. However, the comparably slow changes of the overall exponentially-shaped power delay profile remain unchanged, suggesting that the low-frequency components of the frequency autocorrelation function remain largely unchanged. Recall that the frequency autocorrelation function equals one at  $\Delta f = 0$ . It then seems feasible to suggest that the addition of these higher frequency components causes a larger effect on the 50% coherence bandwidth (i.e. the value in frequency where the frequency correlation function equals 0.5), than on the 90% coherence bandwidth. The low frequency components—and the point where the frequency autocorrelation function equals 0.9—remain largely unchanged, yet the addition of the high frequency components could cause the 0.5 point to move further away from  $f = 0$ , causing the 50% coherence bandwidth to increase. An interesting observation relevant to this suggestion can be made by considering the mean excess delay and the 50% coherence bandwidths measured in the

5 section cone (see Figs. 5.10 and 5.26) and comparing the match seen between the predicted and measured dispersion parameters to the analogous results for the 5 section cylinder (see Figs. 5.9 and 5.25). One can see that the match for both of these parameters in the cone is fairly good, despite the discussion of the pre-reverberation behavior, but is poor in the cylinder. The reason why the cone is more immune to the suspected pre-reverberation effect than the cylinder is not evident; however, the fact that cylinder does show the effect for both the mean excess delay and the 50% coherence bandwidth—but the cone does not for either parameter—is suggestive, and could support the theory that the 50% coherence bandwidth endures an increase as a result of the pre-reverberation behavior.

### 5.3 Functional Relationship between the RMS delay spread and the 50% Coherence Bandwidth

It is well known in other channels that the RMS delay spread and 50% coherence bandwidth are inversely proportional to one another [108]. The exact functional relation between these two dispersion parameters is dependent on the channel being considered, as it depends on the specific character of a channels power delay profile. The functional relationship between the RMS delay spread and the 50% coherence bandwidth is easily obtained for the enclosed space radio channel, since both the RMS delay spread and the 50% coherence bandwidth are functions of the enclosure quality factor. To obtain the precise expression for the RMS delay spread,  $\sigma_\tau$ , in terms of the 50% coherence bandwidth, (5.21) is solved for  $Q$  and placed into (5.12) to find the following result:

$$\sigma_\tau = \frac{\sqrt{3}}{2\pi\Delta f_{50}} \sqrt{\frac{(2\alpha \ln(\alpha) - 2\alpha + 2 - \alpha(\ln(\alpha))^2)}{(1-\alpha)} - \frac{(\alpha \ln(\alpha) - \alpha + 1)^2}{(1-\alpha)^2}}, \quad (5.23)$$

where  $\Delta f_{50}$  is the 50% coherence bandwidth and  $\alpha$  is the power delay profile threshold as described previously. The expression under the radical in (5.23) is simply a constant in terms of  $\alpha$  that can take on values between zero (for  $\alpha = 1$ , corresponding to a threshold equal to the largest value of the power delay profile) and one (for  $\alpha = 0$ , corresponding to the case of no threshold). For  $\alpha < 0.01$  (corresponding to thresholds of -20 dB or less), the value taken on by the radical in (5.23) is close to one (e.g. 0.8852 for  $\alpha = 0.01$  and 0.9758 for  $\alpha = 0.001$ ). Thresholds of -20 dB or less are typical, and if the radical were assumed to be equal to one, the following simplified expression can

be found, which relates the RMS delay spread with the 50% coherence bandwidth:

$$\sigma_\tau = \frac{\sqrt{3}}{2\pi\Delta f_{50}}. \quad (5.24)$$

This expression can be inverted to obtain a simplified expression for the 50% coherence bandwidth in terms of the RMS delay spread:

$$\Delta f_{50} = \frac{\sqrt{3}}{2\pi\sigma_\tau}. \quad (5.25)$$

Note in the above expressions that the relationship between RMS delay spread and 50% coherence bandwidth is constant and is not dependent on frequency. It can also be noted that analogous relationships can be derived between any of the dispersion parameters (e.g. mean excess delay in terms of the 90% coherence bandwidth, etc.).

## 5.4 Using Q to Estimate Dispersion to 20.0 GHz

In the previous two sections, relationships were derived that relate the four considered dispersion parameters to the enclosure net quality factor. Employing the experimental setup described in Chapter 3, which measures the quality factor by using an RF pulse generator and detector to estimate the slope of the turn-off power step delay profile, the net quality factor in the several enclosed space configurations can be measured to 20.0 GHz, and the results for these quality factor measurements were presented in Section 4.4. When the values from these quality factor measurements are placed into the derived dispersion relationships, estimates for the dispersion parameters can be generated to 20.0 GHz, without the direct measurement of either the power delay profile or the frequency transfer function. This method has the advantage in that it does not require a vector network analyzer or any other complex, expensive device to sound the channel response, but can obtain estimates of the enclosed space channel dispersion (in the time and frequency domain) by using a simple, pulsed RF and detection scheme. Results for the RMS delay spread, mean excess delay, and the 90% and 50% coherence bandwidths, estimated to 20.0 GHz, are presented in the following sections.

### 5.4.1 RMS Delay Spread

Equation (5.12) establishes the connection between the net quality factor and the RMS delay spread estimated for the enclosed space. Figures 5.35 and 5.36 show the estimated RMS delay spread for the 2 and 5 section cylindrical platforms, comparing the sparse and dense populations, measured with the loop antenna. Figures 5.37 and 5.38 show the RMS delay spread measured in the 2 and 5 section conical platforms using the loop antenna and compare the dense and sparse configurations. Consistent with the measurements made using the vector network analyzer, the RMS delay spread measured in the dense configuration is smaller than the the delay spread measured in the sparse configuration, and the delay spread is greater for the configurations that involve larger volumes.

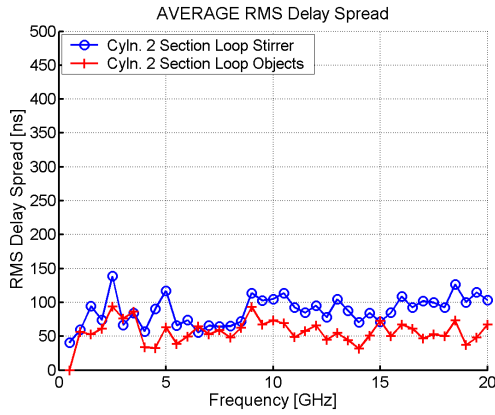


Figure 5.35: Average RMS delay spread measured with the RF pulse generator/detector empirical setup in the 2 section object-filled and stirred cylindrical platform using the loop antenna.

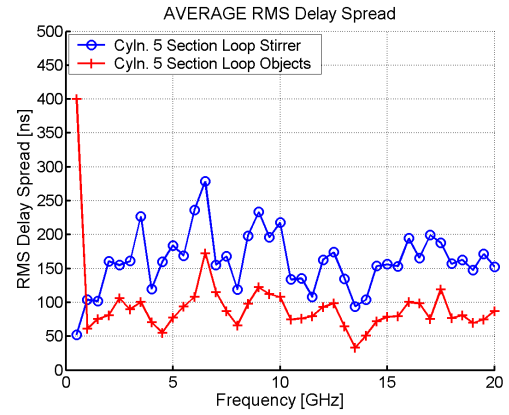


Figure 5.36: Average RMS delay spread measured with the RF pulse generator/detector empirical setup in the 5 section object-filled and stirred cylindrical platform using the loop antenna.

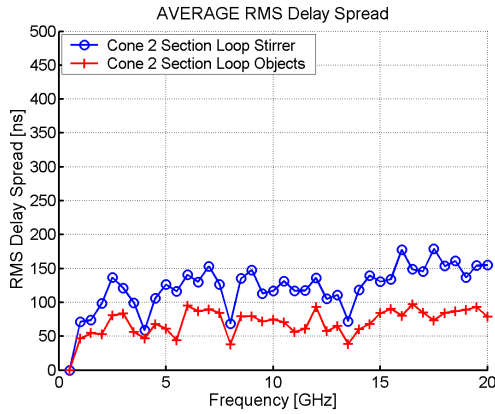


Figure 5.37: Average RMS delay spread measured with the RF pulse generator/detector empirical setup in the 2 section object-filled and stirred conical platform using the loop antenna.

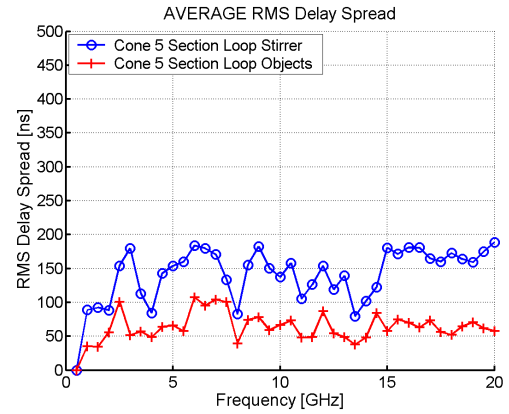


Figure 5.38: Average RMS delay spread measured with the RF pulse generator/detector empirical setup in the 5 section object-filled and stirred conical platform using the loop antenna.



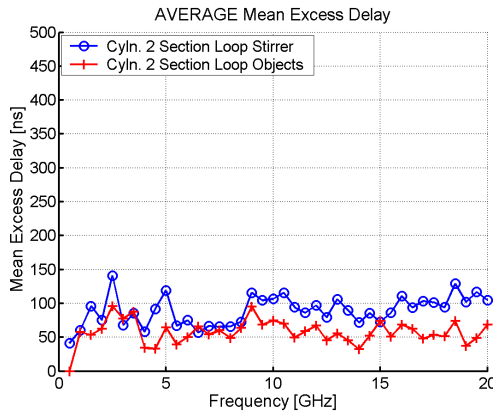


Figure 5.39: Average mean excess delay measured with the RF pulse generator/detector empirical setup in the 2 section object-filled and stirred cylindrical platform using the loop antenna.

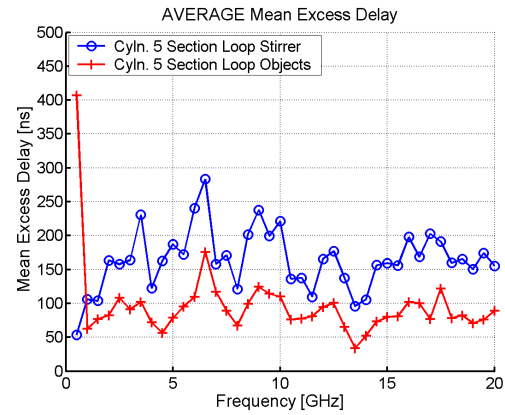


Figure 5.40: Average mean excess delay measured with the RF pulse generator/detector empirical setup in the 5 section object-filled and stirred cylindrical platform using the loop antenna.

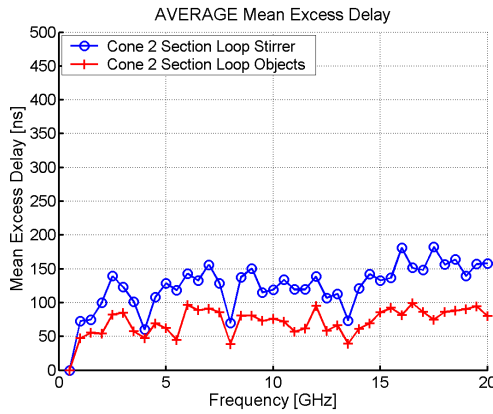


Figure 5.41: Average mean excess delay measured with the RF pulse generator/detector empirical setup in the 2 section object-filled and stirred conical platform using the loop antenna.

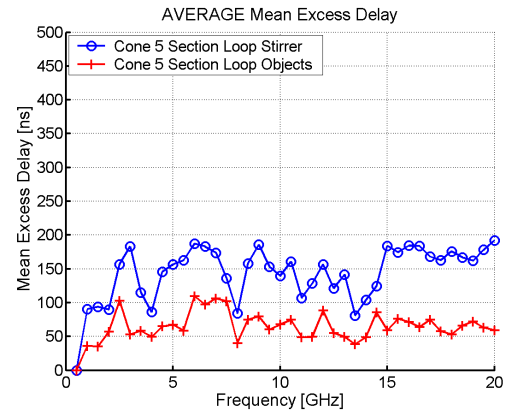


Figure 5.42: Average mean excess delay measured with the RF pulse generator/detector empirical setup in the 5 section object-filled and stirred conical platform using the loop antenna.

### 5.4.2 Mean Excess Delay

Using (5.7) and the quality factor data from Section 4.4, Figs. 5.39 and 5.39 were generated to show the estimated mean excess delay in the 2 and 5 section cylindrical platform, comparing the sparse and dense population responses. Figures 5.41 and 5.42 show the similar estimates for the conical platform. The behavior of these mean excess delay estimates is comparable to the behavior of the mean excess delay measured using the network analyzer.

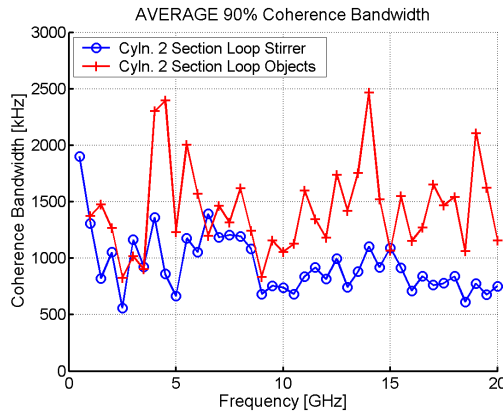


Figure 5.43: Average 90% coherence bandwidth measured with the RF pulse generator/detector empirical setup in the 2 section object-filled and stirred cylindrical platform using the loop antenna.

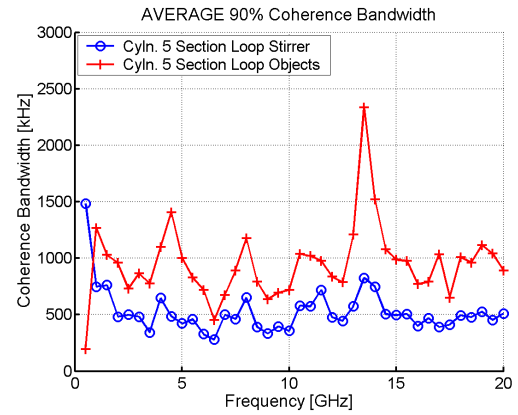


Figure 5.44: Average 90% coherence bandwidth measured with the RF pulse generator/detector empirical setup in the 5 section object-filled and stirred cylindrical platform using the loop antenna.

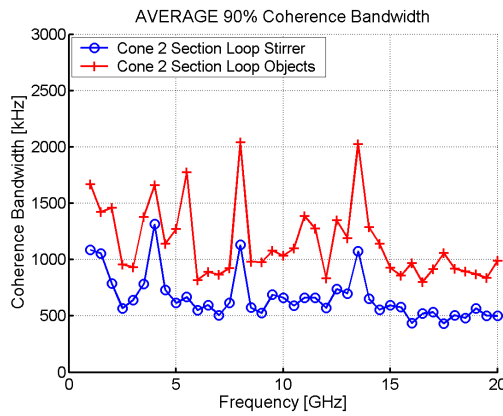


Figure 5.45: Average 90% coherence bandwidth measured with the RF pulse generator/detector empirical setup in the 2 section object-filled and stirred conical platform using the loop antenna.

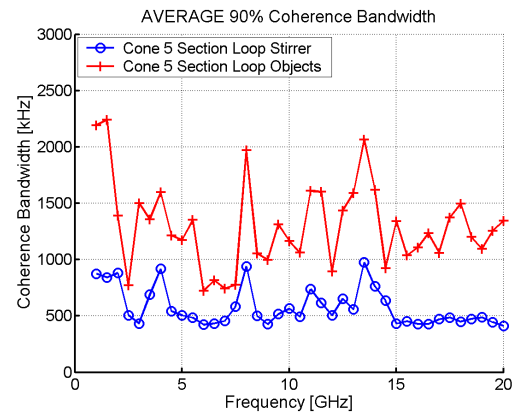


Figure 5.46: Average 90% coherence bandwidth measured with the RF pulse generator/detector empirical setup in the 5 section object-filled and stirred conical platform using the loop antenna.

### 5.4.3 90% Coherence Bandwidth

Figures 5.43 and 5.44 are generated by inserting the quality factor data from Section 4.4 into (5.22), and show the estimated 90% coherence bandwidth to 20.0 GHz for the 2 and 5 section cylindrical platform in the sparse and dense population configurations. Figures 5.45 and 5.46 show the same estimate for the conical enclosed space platform. The behavior of these data is consistent with expected effects for the 90% coherence bandwidth, as the size of the enclosure and population density change.

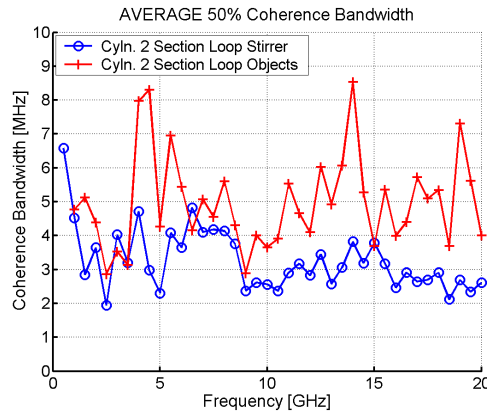


Figure 5.47: Average 50% coherence bandwidth measured with the RF pulse generator/detector empirical setup in the 2 section object-filled and stirred cylindrical platform using the loop antenna.

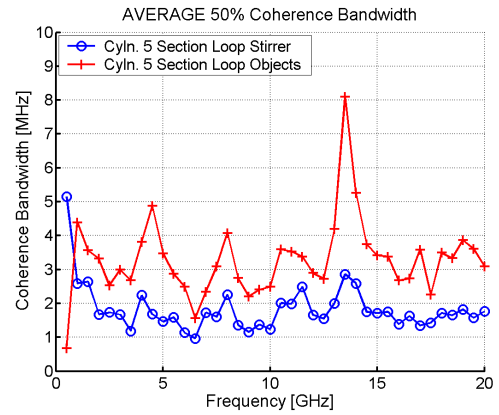


Figure 5.48: Average 50% coherence bandwidth measured with the RF pulse generator/detector empirical setup in the 5 section object-filled and stirred cylindrical platform using the loop antenna.

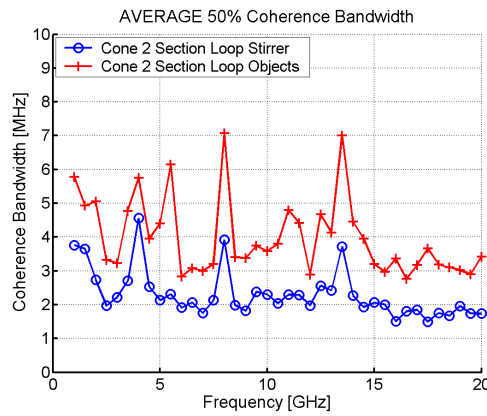


Figure 5.49: Average 50% coherence bandwidth measured with the RF pulse generator/detector empirical setup in the 2 section object-filled and stirred conical platform using the loop antenna.

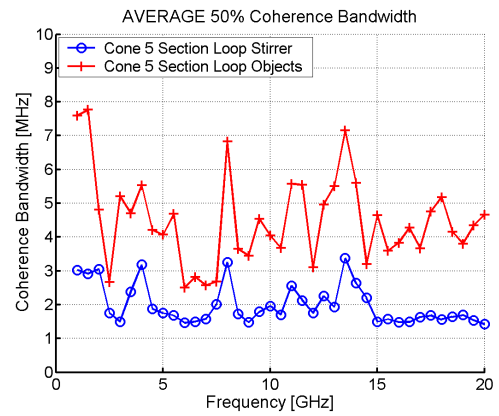


Figure 5.50: Average 50% coherence bandwidth measured with the RF pulse generator/detector empirical setup in the 5 section object-filled and stirred conical platform using the loop antenna.

#### 5.4.4 50% Coherence Bandwidth

Figures 5.47 and 5.48 show the estimated 50% coherence bandwidth to 20.0 GHz for the 2 and 5 section cylindrical platform, measured in the sparse and dense population configurations, and were generated by inserting the quality factor data presented in Section 4.4 into (5.21). Figures 5.49 and 5.50 show the analogous data for the conical platform. All sets of data behave in a manner consistent with the behavior for the coherence bandwidth discussed earlier in this chapter.

## Chapter 6

# Gain of the Enclosed Space Radio Channel

Gain is an important description for every radio channel, and the expected value of gain in any channel tells designers how to design the transmitters and receivers of a communications system so that an adequate link condition exists to sustain the desired or optimal physical layer data rate. The gain can vary from channel to channel; the gain of an indoor communications channel can be as strong as -40 dB or more and as weak as -90 dB or less, whereas a satellite communications channel can exhibit gains as small as -120 dB. Practitioners of the enclosed space radio channel will seek an initial estimate of the gain in a given reverberant enclosure to so that system engineering decisions can be made. Dynamic range is also a concern for any studied channel as it tells designers over what range the gain can be expected to change. Knowledge of this range is important, for instance, to the design of automatic gain control (AGC) circuits, which are responsible for ensuring that the signal at the input of a receiver front-end is within a specified limit.

As outlined in Chapter 2, extensive studies have been conducted for both the average gain (e.g. [85, 90, 96, 97, 100]) and the statistical distribution of the power gain (e.g. [85, 90, 97]) in a reverberation chamber. Reverberation chambers and reverberant enclosed spaces share many electromagnetic similarities and as a result, some of the behaviors identified for the reverberation chamber can be explored for application in enclosed spaces. In the following sections, the results for the gain in a reverberation chamber will be applied to the enclosed space channel to model the gain of

the enclosure, with good results. The statistics for the power gain will also be used to predict the dynamic range of a reverberant enclosed space channel.

## 6.1 Modeling Gain

Chapter 5 studied the dispersive nature of the wireless communications channel of reverberant enclosed spaces and found that all of the dispersion parameter average values (i.e. RMS delay spread, mean excess delay, 50% and 90% coherence bandwidth) are connected via closed-form expressions to the net quality factor of the reverberant enclosure. A similar result was obtained for the gain of a reverberation chamber in [96, 100], recognizing that the gain of a reverberation chamber—a ratio of the power out of the chamber to the power into the chamber—is also a ratio of the chamber net quality factor to the receiving antenna quality factor:

$$G = \frac{P_{ant}}{P_{net}} = \frac{\frac{\omega U}{P_{net}}}{\frac{\omega U}{P_{ant}}} = \frac{Q_{net}}{Q_{ant}}, \quad (6.1)$$

where  $P_{net}$  is the net power delivered to the chamber (which by power conservation is equal to the net power removed from the chamber by the walls, absorbers, antennas, etc.),  $P_{ant}$  is the power removed from the chamber by the receiving antenna,  $U$  is the energy contained by the chamber, and  $\omega$  is the radian frequency of operation. The quality factor of an antenna in the reverberant space,  $Q_{ant}$ , is found using knowledge of the volume of the space, the radian frequency of operation, the antenna efficiency and the  $S_{11}$  of the antenna used (see the expression in (2.49)). If the only other source of loss in the reverberant space is the due to the wall currents, the expression for the composite quality factor of the several excited modes, (2.46) and the expression for the antenna quality factor can be harmonically averaged to yield the net quality factor, which produces an expression for the chamber gain using only geometric and constitutive parameters—presented in (2.52). However, if the wall currents and antenna losses are not the only source of loss in the system, gain must be estimated using the expression in (6.1), inserting the  $Q_{net}$  derived or measured for the system in question.

The expression for the antenna quality factor given in (2.49) has been derived assuming that the antenna has been placed away from the walls in the enclosure. However, as mentioned in Chapter 2, it has been shown in the literature pertaining to reverberation chambers that the non-zero fields at

the enclosure walls contain twice the energy as the fields in the central portion of the chamber (i.e. (2.22)-(2.25)). As such, the power sensed by an antenna at the walls will be twice the power sensed by an antenna located away from the walls:

$$P_{ant,walls} = \frac{U_{walls}\nu}{V} A_e = 2 \frac{U\nu}{V} \frac{\lambda^2 m \eta_a}{8\pi} = 2P_{ant}, \quad (6.2)$$

where  $U_{walls}$  is the apparent energy contained in the enclosure as perceived by the energy density at the walls, and  $\nu$  is the speed of light in the medium. Recalling the definition of quality factor,  $Q = \omega U/P$ , the quality factor of the antenna placed at the wall can be written as follows:

$$Q_{ant,walls} = \frac{8\pi^2 V}{\lambda^2 m \eta_a}, \quad (6.3)$$

where  $m = 1 - |S_{11}|^2$  is the impedance mismatch factor,  $\eta_a$  is the antenna efficiency,  $V$  is the enclosure volume and  $\lambda$  is the wavelength. This expression must be used for  $Q_{ant}$  in (6.1) when the receiving antenna is placed at the wall of the enclosure. Note that this expression for the antenna quality factor is one-half the magnitude of the antenna quality factor derived in (2.49) for an antenna placed away from the walls.

The quality factors presented in Chapter 4 by estimating the slope of the power delay profile, measured using the vector network analyzer and inverse Fourier transform, are indeed empirical net quality factors and can be inserted into (6.1) with the antenna quality factor to obtain an estimate for the power gain in the enclosure. It should be noted that this prediction of the gain is taken in the CW limit of signal bandwidth. This estimate of enclosure gain can be compared to the empirically measured power gain of the channel,  $G' = |S_{21}|^2/(1 - |S_{11}|^2)$ . The denominator of the previous expression accounts for the power reflected from the transmit antenna back to the source, as the antenna is poorly matched across the measured frequencies. Such a comparison is made in the following figures of gain data measured in the cylindrical and conical enclosed space platforms.

### 6.1.1 Empirical Results for Gain

Figure 6.1 compares the gain measured (via the  $S_{21}$ ) for the sparsely populated 2 and 5 section cylindrical platforms using the loop antenna to the gain estimated by (6.1) (and the net quality

factors presented in Chapter 4) in the identical configurations. It is clear that the match above 3.0 GHz is very good, showing that the relationship presented in (6.1) can very accurately predict the power gain of the enclosed space channel, given knowledge of the net quality factor and the antenna quality factor. Figure 6.2 makes the same comparison for the densely populated 2 and 5 section cylinder, using the loop antenna, showing a match that is almost as good as the sparse population result. Figures 6.3 and 6.4 show the gain measured and predicted in the same configurations of the cylinder as Figs. 6.1 and 6.2, but use the probe antenna instead of the loop antenna. One can see that the predicted values match the measured values for power gain almost as well as the results in the previous figures, yet the predictions are still within a few dB of the measurements and are therefore good. The jagged behavior of the measured curves below 3.0 GHz is the result of the antennas, which are electrically very short, causing the  $|S_{11}|$  to temporarily rise above 0 dB (on the order of the measurement precision of the analyzer). Nevertheless, the match above 3.0 GHz is good for the data presented, validating the model that uses the net quality factor to predict the enclosure gain.

It is clear that the predicted values do not match the measured values well below 3.0 GHz (neglecting the jagged behavior of the curves discussed previously). This frequency value is not coincidentally very near the Rayleigh frequency of approx. 3.0 GHz (the boundary between overmoded and undermoded regions) for all the measured data sets. When the reverberant enclosure becomes undermoded, the assumption of an isotropic plane wave distribution is no longer valid, and the average directive gain of the antenna can no longer be assumed to be equal to one (see Section 2.2.3). When this assumption becomes faulty, the expression for  $Q_{ant}$  is no longer valid, and the predictions using (6.1) will no longer be accurate. Accordingly, the predictions made using the net and antenna quality factor presented in this section are valid only for the overmoded regions of reverberant enclosed spaces.

Figures 6.5 and 6.6 show the measured power gain compared with the predicted power gain for the 2 and 5 section conical platform, using the loop antenna, in the sparse and dense population configurations, respectively. The predictions in these figures are larger than the measurements by several dB, unlike the analogous measurements and predictions for the cylindrical platform. The reason for this discrepancy is not clear, though these predictions are still within several dB of the actual measured results and would still give system architects a good idea of what kind of gain to expect in such a reverberant communications channel. Figures 6.7 and 6.8 show the same measure-

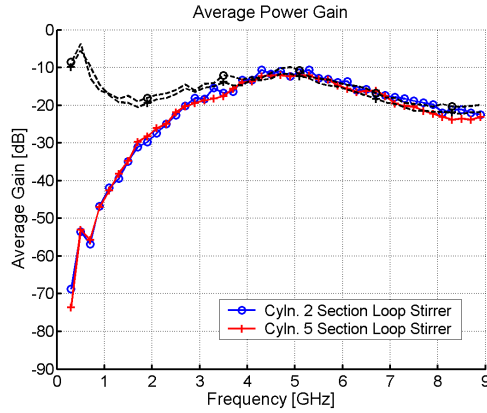


Figure 6.1: Average power gain derived from the measured  $S_{21}$  and  $S_{11}$  in the 2 and 5 section stirred cylindrical platform using the loop antenna. Solid: Measured using VNA, Dashed: Predicted from the estimated  $Q$  and the antenna  $Q$ .

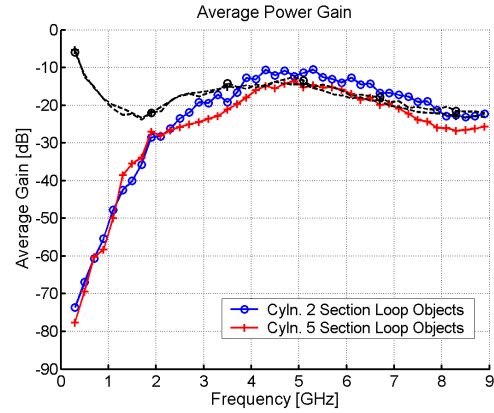


Figure 6.2: Average power gain derived from the measured  $S_{21}$  and  $S_{11}$  in the 2 and 5 section cylindrical platform with objects using the loop antenna. Solid: Measured using VNA, Dashed: Predicted from the estimated  $Q$  and the antenna  $Q$ .

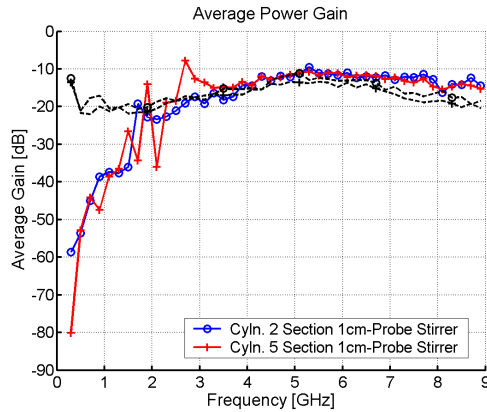


Figure 6.3: Average power gain derived from the measured  $S_{21}$  and  $S_{11}$  in the 2 and 5 section stirred conical platform using the 1 cm probe antenna. Solid: Measured using VNA, Dashed: Predicted from the estimated  $Q$  and the antenna  $Q$ .

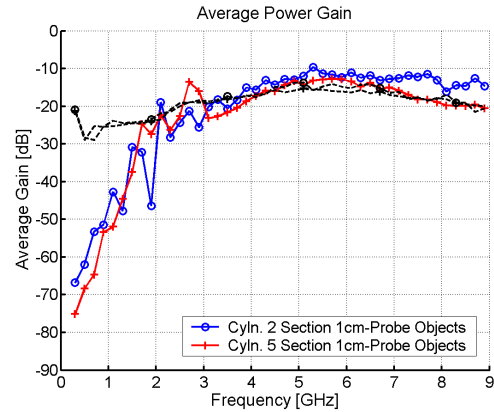


Figure 6.4: Average power gain derived from the measured  $S_{21}$  and  $S_{11}$  in the 2 and 5 section cylindrical platform with objects using the 1 cm probe antenna. Solid: Measured using VNA, Dashed: Predicted from the estimated  $Q$  and the antenna  $Q$ .



ments in the cone, using the 1 cm probe antenna instead of the loop antenna. One can see that the match between the predicted and measured power gain values for these data sets is fairly good.

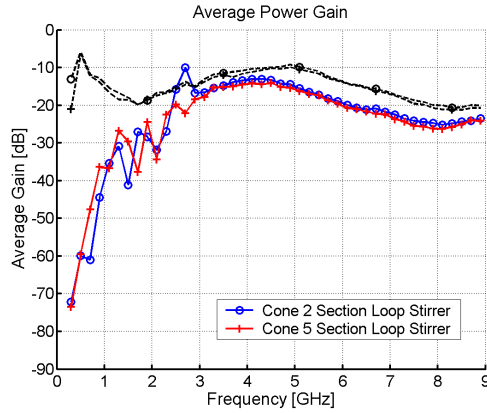


Figure 6.5: Average power gain derived from the measured  $S_{21}$  and  $S_{11}$  in the 2 and 5 section stirred conical platform using the loop antenna. Solid: Measured using VNA, Dashed: Predicted from the estimated  $Q$  and the antenna  $Q$ .

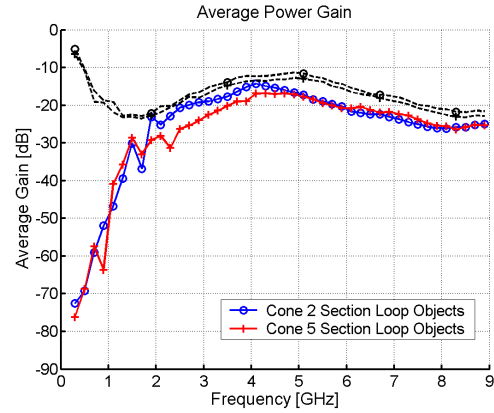


Figure 6.6: Average power gain derived from the measured  $S_{21}$  and  $S_{11}$  in the 2 and 5 section stirred conical platform using the loop antenna. Solid: Measured using VNA, Dashed: Predicted from the estimated  $Q$  and the antenna  $Q$ .

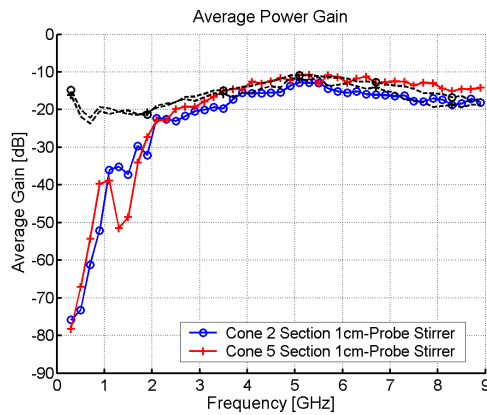


Figure 6.7: Average power gain derived from the measured  $S_{21}$  and  $S_{11}$  in the 2 and 5 section stirred conical platform using the 1 cm probe antenna. Solid: Measured using VNA, Dashed: Predicted from the estimated  $Q$  and the antenna  $Q$ .

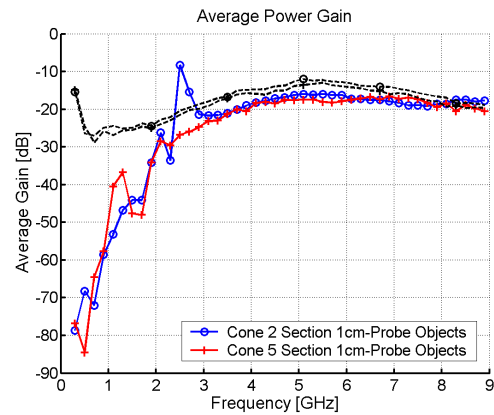


Figure 6.8: Average power gain derived from the measured  $S_{21}$  and  $S_{11}$  in the 2 and 5 section conical platform with objects using the 1 cm probe antenna. Solid: Measured using VNA, Dashed: Predicted from the estimated  $Q$  and the antenna  $Q$ .

### 6.1.2 Discussion

The results of comparing the predicted gain to the measured gain presented in the previous plots show that the quality factor is also a central parameter to modeling the gain of a reverberant enclosed space, just as it was shown that the quality factor is central to modeling enclosed space dispersion. The quality factor thus becomes a powerful tool in modeling the behavior of the reverberant enclosed space radio channel, and it should be clear to the reader that to know the net quality factor of an enclosed space is to know the behavior of its communications channel.

The above results show that the gain of the enclosed space radio channel is very large in comparison to the gains seen in some other common radio channels (i.e. the indoor or outdoor channels). Results from these experiments show gains on the order of -20 dB, where gains in the indoor and outdoor channel can be of the order of -100 dB, a difference of eight orders of magnitude. This result is highly convenient for the enclosed space communications scenario and requires that communications nodes operate on very restricted power budgets, resulting from a miniature form factor that does not allow abundant space for energy storage. The large gain of the enclosed space radio channel allows designers to save energy by not only reducing the transmit power—and perhaps eliminating the energy-costly power amplifier altogether—but also by reducing the potentially large bias currents required by a receiver's low-noise amplifier to realize the sensitivity required to receive the small signal amplitudes expected in other channels. Such steps would reduce the power required by both the transmit and receive functions of transceiver nodes, and would allow architects to more closely realize the goal of miniature communications devices that require only infrequent energy replenishment or perhaps no replenishment at all.

Furthermore, it should be noted that the phenomena of shadowing, common to the outdoor and indoor radio channels, does not exist in kind in the reverberant enclosed space channel. An underlying assumption for reverberant enclosures is that the energy density is uniformly distributed, leading to a uniform average received power distributed across the entire enclosure volume. As such, the lack of a line of sight and significant secondary propagation paths (i.e. shadowing) between a transmitter and a receiver does not imply that the received signal amplitude will decrease, as it does in the indoor and outdoor channels.

### 6.1.3 Engineering Tradeoff: Gain vs. Dispersion

Now that it is clear that the net quality factor of an enclosed space connects not only all of the dispersion parameters, but also the gain, an important observation needs to be made. Consider what happens when the quality factor increases. Equation (6.1) shows that the gain increases, and (5.7) and (5.12) indicate that the RMS delay spread and mean excess delay also increase, while (5.21) and (5.22) demonstrate that the coherence bandwidths decrease. The opposite is true for if the quality factor decreases: The RMS delay spread and the mean excess delay decrease, the coherence bandwidths increase and the gain decreases. The engineering tradeoff between gain and dispersion in the enclosed space channel is thus motivated. By virtue of the connection of the dispersion parameters and the gain to the quality factor of the enclosure, it is clear that if the gain increases, so does the dispersion, and conversely, if the dispersion decreases, so too will the gain. Knowledge of this tradeoff could allow system designers to manipulate an enclosed space environment to provide the gain and dispersion characteristics the best suit their application, perhaps by introducing absorbent material to the space, reducing the net quality factor of that space. The absorbent material could be in the form of a lossy paint, dielectrics with a significant loss tangent, RF absorber material, etc. Apertures that allow energy to radiate from the enclosed space also present a source of loss to the system, causing the net quality factor to decrease, and likewise the gain and dispersion would also decrease. Results in this chapter have shown that the gain of the enclosed space channel can be considerably large as compared to other channels; accordingly, the dispersion in the channel can be undesirably large. It is possible that some of this gain can be sacrificed (and may not be needed if a COTS device designed for use in channels with much smaller gain is deployed in the enclosed space channel) by adding loss to the enclosure, thereby also reducing some of the undesirable dispersion. Incorporating such types of loss into the a reverberant space has been considered at length in [97]. Conversely, if a particular communications scheme employed between a transmitter and receiver in the enclosed space channel is capable of handling more dispersion in the channel than it already sees (perhaps due to the use of an equalizer), then it is conceivable that communications could be improved by increasing the gain (and therefore dispersion) by decreasing loss in the channel. The additional gain could allow a greater symbol rate or perhaps a more challenging modulation scheme to be used, both causing an increase in the data rate seen between the transmitter and receiver.

## 6.2 Modeling Dynamic Range

Knowledge of the average gain in the reverberant enclosed space radio channel provided by the theory presented in Section 6.1 allows radio system designers to make critical decisions about the transmitter power and receiver sensitivity so that a reliable link condition can be obtained. These results present an average predicted gain for the enclosed space channel in the CW-limit of signal bandwidth, but do not offer any indication of by how much the gain can vary as a local function of frequency or as a function of spacial position. That is, once a link has been established between a transmitter and receiver based on the average predicted gain value, by how much could one expect that value to deviate from the average? Knowledge of this variance would allow designers to build a certain dynamic range into the receiver architecture, via an AGC circuit or perhaps the ADC dynamic range, and the consequence to not making accommodations for these gain dynamics could be an undesirable and unnecessary loss of the radio link between two communication nodes.

### 6.2.1 Empirical Results for Power Gain Statistics

The statistics of the signal power distribution inside resonant cavities and reverberation chambers have been studied fairly extensively; the published results of some of these studies are presented in Sections 2.1.3 and 2.2.1, respectively. In short, it was found that the statistics of the power gain in the limit of a highly overmoded resonant cavity or reverberation chamber are exponentially distributed. The dynamic range of the power gain of a radio channel can be determined if the statistics of the power gain distribution are known; accordingly, the knowledge of an exponential distribution of power gain can be used to predict the dynamic range of the reverberant enclosed space communications channel. The  $\Gamma$ -distribution presented in (2.13) gives the distribution of the magnitude-squared field statistics (i.e. power gain) in a resonant cavity of arbitrary geometry as a function of the specific mode density, given in (2.18) [69]. Recall that the specific mode density is a measure of how many modes on average are excited by a CW tone in the resonant enclosure and is an increasing function with increasing frequency and volume and a decreasing function with increasing quality factor. In the limit that the specific mode density becomes large, the  $\Gamma$ -distribution for the power statistics becomes the exponential distribution [69]. To explore the behavior of the power gain statistics measured in the enclosed space platforms, the empirically measured specific

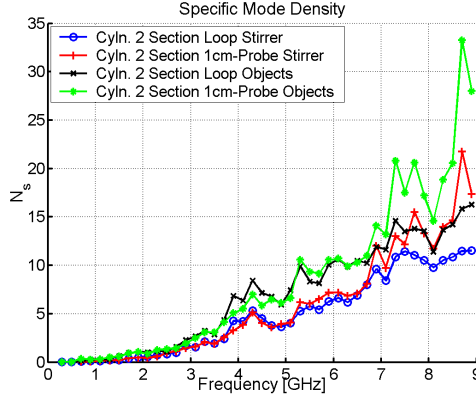


Figure 6.9: Measured specific mode density in the 2 section cylindrical platform.

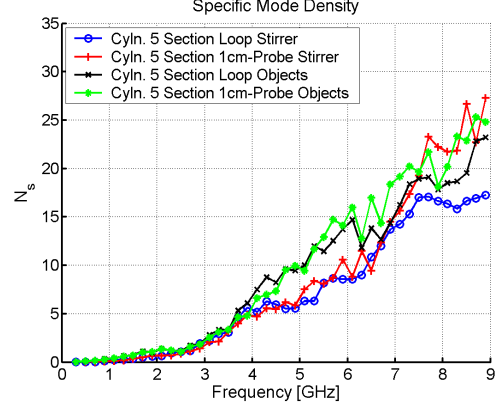


Figure 6.10: Measured specific mode density in the 5 section cylindrical platform.

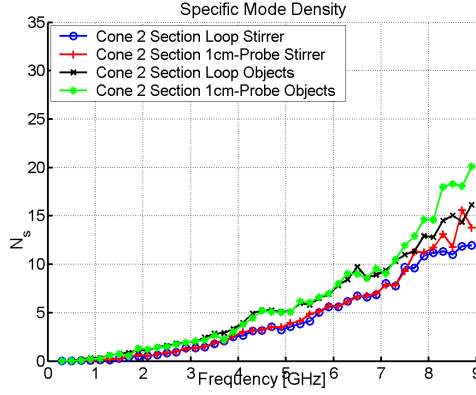


Figure 6.11: Measured specific mode density in the 2 section conical platform.

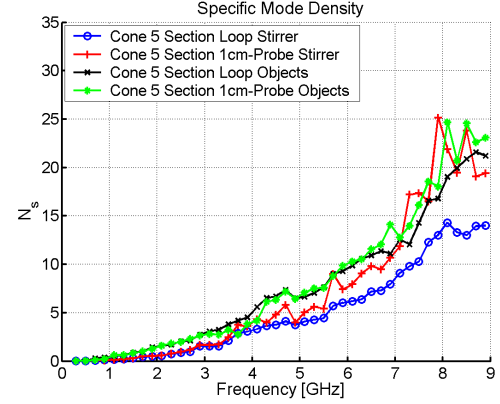


Figure 6.12: Measured specific mode density in the 5 section conical platform.

mode density should first be examined. From (2.18), the net quality factor of the enclosure and its enclosed volume are required to obtain an estimate of the specific mode density as a function of frequency. Using the volumes presented in Tables 3.2 and 3.3 and the net quality factors measured in Chapter 4, the specific mode densities estimated for the several configurations of the cylindrical platform are shown in Figs. 6.9 and 6.10 as well as the conical platform in Figs. 6.11 and 6.12. As expected, the curves show an increasing function with increasing frequency and demonstrate generally higher mode densities for the larger volumes, compared to the smaller volumes. Also notice that the curves have all passed through  $N_s = 1$  by  $f = 3.0$  GHz, showing that 3.0 GHz is a good rough approximate for the Rayleigh frequency (i.e. the boundary between the overmoded and undermoded regions of the enclosures) for all the geometries considered.

The aforementioned plots of specific mode density suggest that the distribution of the empirically measured power statistics should tend towards the exponential distribution for frequencies well above 3.0 GHz. To explore and validate this anticipated result, Komolgorov-Smirnov (K-S) goodness-of-fit tests [109] were run on the measured power gain values (using the uncorrected  $|S_{21}|^2$  values) for the same configurations presented in the mode density plots. The K-S tests are run at each of the 801 frequency points in each of the 44 frequency bands, utilizing the 60 different rotational position measurements as the ensemble for the fit test. Figures 6.13 and 6.14 show the results of these K-S tests for the 2 and 5 section cylindrical configurations, showing the percentage of the 801 tests that pass the K-S test in each band, plotted as a function of the estimated specific mode density. Figures 6.15 and 6.16 show the analogous results for the 2 and 5 section conical platform. A feature that is common to all of the configurations is the rapid increase in the K-S pass-rate as the specific mode density increases from one that suggests the power gain ensembles take on an increasingly exponential distribution—a behavior that is consistent with the  $\Gamma$ -distribution statistics of Section 2.1.3. Another common behavior to all of the configurations is the decreasing pass-rate for the probe antenna after an initial maxima near 100% as the mode density increases. The physical reasoning for this distinct behavior for the probe antenna data is not currently understood. The pass-rates measured in the conical platform using the loop antenna show very good pass-rates for high mode density values, indicating that the power gain distribution is largely exponential in these configurations. However, measurements using the loop antenna in the cylindrical platform do not demonstrate nearly as high a pass-rate as for the conical platform configurations, the cause of which is currently unknown.

### 6.2.2 Deriving Dynamic Range from Power Gain Statistics

The K-S pass rates for most of the configurations are greater than 50%, suggesting that measurements conform to exponential statistics most of the time. While the reason has not been determined as to why some of the K-S tests (i.e. the probe antenna at large mode densities) do not display high pass rates, assuming an exponential distribution may indeed be a good *approximation* for the purposes of modeling the dynamic range seen by communication nodes placed on the enclosure wall. Making this approximation assumption, one can consider the cumulative distribution function (CDF) for the exponential distribution and the values of the abscissa between which an arbitrary

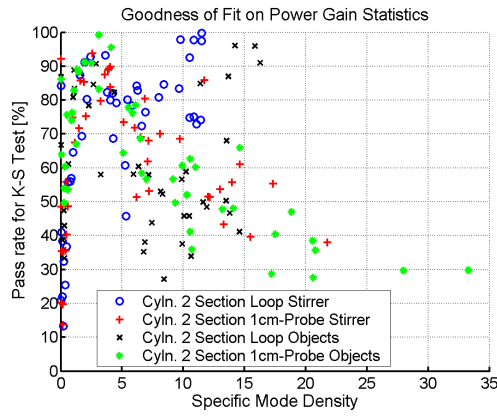


Figure 6.13: Komolgorov-Smirnov goodness of fit test results for the empirical power gain data measured in the 2 section cylindrical platform.

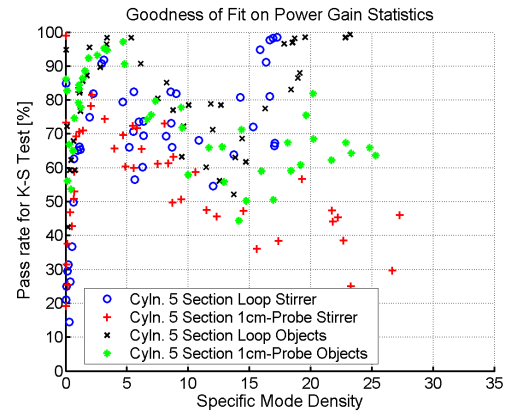


Figure 6.14: Komolgorov-Smirnov goodness of fit test results for the empirical power gain data measured in the 5 section cylindrical platform.

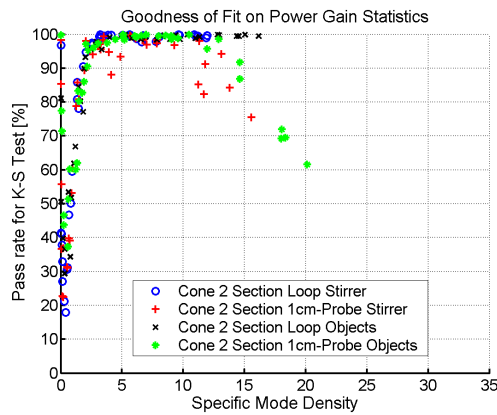


Figure 6.15: Komolgorov-Smirnov goodness of fit test results for the empirical power gain data measured in the 2 section conical platform.

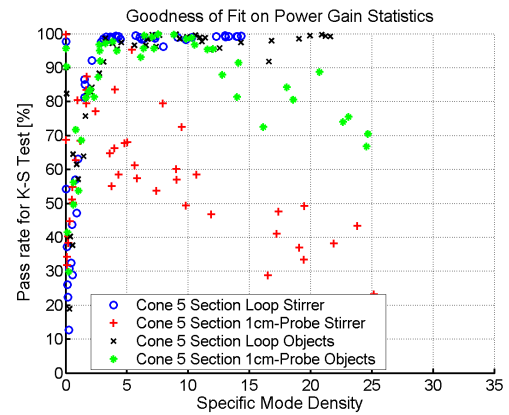


Figure 6.16: Komolgorov-Smirnov goodness of fit test results for the empirical power gain data measured in the 5 section conical platform.



percentage of points lie, centered about the median value. The analytical CDF for the exponential distribution is given as

$$F(x) = 1 - e^{-x/\mu}, \quad (6.4)$$

where  $\mu$  is the mean value of the distribution (i.e. first moment) and is sometimes referred to as the exponential parameter. An arbitrary percentage of points equal to  $100 \cdot \gamma$ , centered about the median value, are then contained in the following bounds:

$$x_1 < x < x_2, \quad (6.5)$$

$$x_1 = -\mu \ln \left( \frac{1 + \gamma}{2} \right), \quad (6.6)$$

$$x_2 = -\mu \ln \left( \frac{1 - \gamma}{2} \right). \quad (6.7)$$

Since the abscissa of the cumulative power distribution function is actually a power level,  $100 \cdot \gamma$  percent of the power sample points are greater than the power level  $x_1$  but less than the power level  $x_2$ . The ratio  $x_2/x_1$  is then dynamic range of power gain that contains  $100 \cdot \gamma$  of the power gain samples centered about the median, and can be concisely written as follows, in a logarithmic form:

$$DR = 10 \cdot \log_{10} \left( \frac{x_2}{x_1} \right) = 10 \cdot \log_{10} \left( \frac{\ln[(1 - \gamma)/2]}{\ln[(1 + \gamma)/2]} \right). \quad (6.8)$$

Notice that (6.8) is not a function of the mean of the exponential distribution,  $\mu$ . This result shows that if the exponential distribution is a reasonable approximate to the empirical power gain distribution, the dynamic range centered about the median power value will be independent of the mean power gain (studied in the previous sections of this chapter). Designers of wireless communications devices in the enclosed space radio channel could thus use (6.8) to predict the dynamic range that would describe  $100 \cdot \gamma$  percent of the power values inside of an arbitrary reverberant enclosed space, and this dynamic range *would not change with variations in the average power gain*. Table 6.1 shows the numerical results for the dynamic range given by (6.8) for several values of  $\gamma$ .

| $100 \cdot \gamma\%$ | Dynamic Range |
|----------------------|---------------|
| 90.0%                | 17.66 dB      |
| 95.0%                | 21.63 dB      |
| 99.0%                | 30.24 dB      |
| 99.9%                | 41.82 dB      |

Table 6.1: Dynamic range containing a various percentage of power gain values.

### 6.2.3 Empirical Results for Dynamic Range

Using the measured power gain data (from  $|S_{21}|^2$ ) for the several configurations, empirical dynamic ranges centered about the median power gain value can be found that contain any arbitrary percentage of the measured power gain points. For instance, the empirical dynamic range that contains 90% of the measured power gain points centered about the median power gain value is called the 90% dynamic range. Figures 6.17 through 6.20 show the 90% dynamic range measured in the various configurations; the dynamic range predicted from the exponential distribution approximation is 17.66 dB (from Table 6.1) and is shown in the figures using a dashed line. One can clearly see that as frequency increases (and therefore the specific mode density also increases), the measured 90% dynamic range tends towards the dynamic range derived by assuming the power distribution is exponential. In fact, at high frequencies the match is very good—within a few dB. Figures 6.21 through 6.24 show the 95% dynamic range measured in the various configurations, and these results show a similar good fit at high frequencies to the exponential-assumption 95% dynamic range. Figures 6.25 through 6.28 show the measure 99% dynamic range, and these results likewise show a good fit at high frequencies between the measured and exponential-derived dynamic range. The 99.9% dynamic range measurements are shown in Figs. 6.29 through 6.32, and it can be seen that the exponential distribution assumption gives a dynamic range prediction that tends towards an upper bound on the measured dynamic range at high frequencies, underscoring the fact that the empirical power gain values are not perfectly exponential in their statistics.

However, it is clear from these figures comparing the measured dynamic range to the exponential-predicted dynamic range that despite this slight deviation in the actual statistical distribution from a perfect exponential distribution, the analytically-derived dynamic range predictions assuming exponential statistics serve as a very good prediction for the actual dynamic range in the highly overmoded regions of operation, and these predictions are independent of the mean value of the power

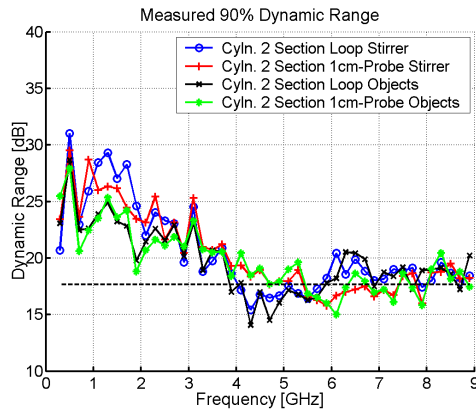


Figure 6.17: Dynamic range containing 90% of power gain samples measured in the 2 section cylinder. Dashed line is the 90% dynamic range assuming power gain is exponentially distributed.

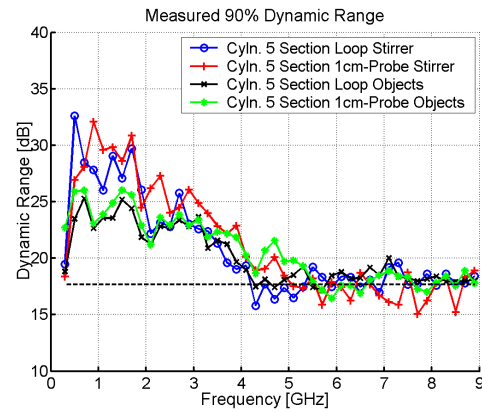


Figure 6.18: Dynamic range containing 90% of power gain samples measured in the 5 section cylinder. Dashed line is the 90% dynamic range assuming power gain is exponentially distributed.

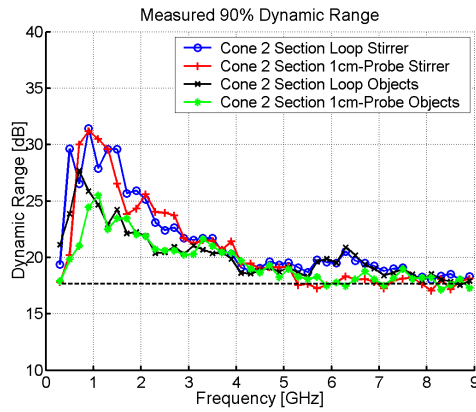


Figure 6.19: Dynamic range containing 90% of power gain samples measured in the 2 section cone. Dashed line is the 90% dynamic range assuming power gain is exponentially distributed.

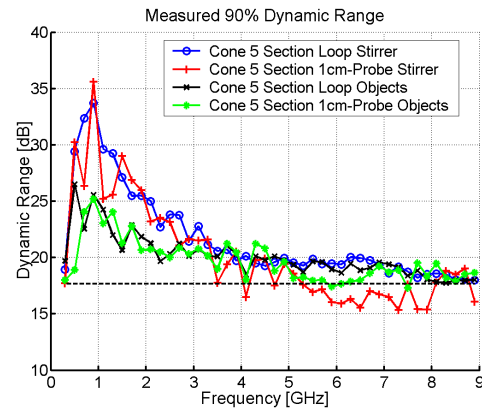


Figure 6.20: Dynamic range containing 90% of power gain samples measured in the 5 section cone. Dashed line is the 90% dynamic range assuming power gain is exponentially distributed.

gain. Table 6.1 thus gives the dynamic range estimates for the enclosed space radio channel when the enclosure is rigorously overmoded, but these estimates do not depend on the average gain in the particular channel. Knowledge of this independence is very useful for designers of physical layer communications systems for use in reverberant enclosed spaces.

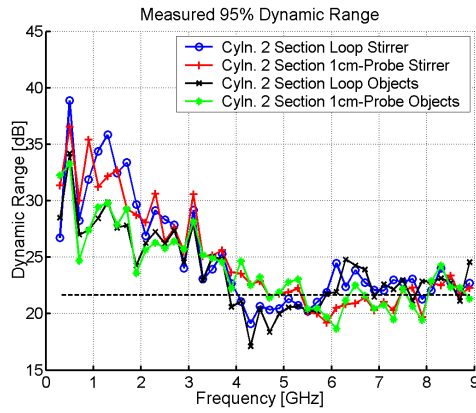


Figure 6.21: Dynamic range containing 95% of power gain samples measured in the 2 section cylinder. Dashed line is the 95% dynamic range assuming power gain is exponentially distributed.

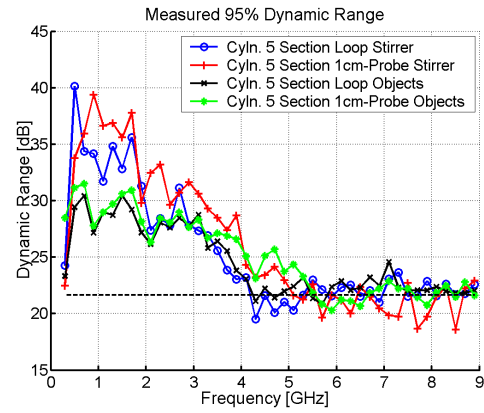


Figure 6.22: Dynamic range containing 95% of power gain samples measured in the 5 section cylinder. Dashed line is the 95% dynamic range assuming power gain is exponentially distributed.

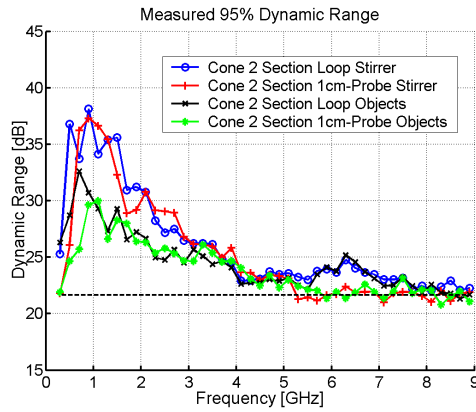


Figure 6.23: Dynamic range containing 95% of power gain samples measured in the 2 section cone. Dashed line is the 95% dynamic range assuming power gain is exponentially distributed.

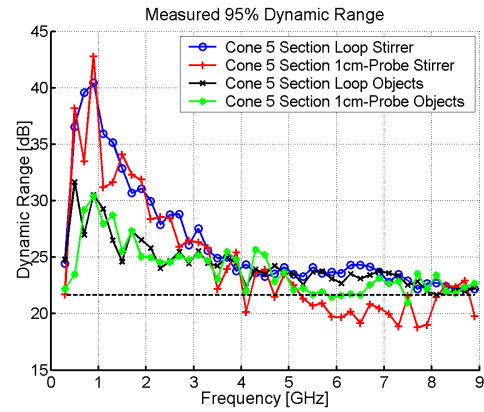


Figure 6.24: Dynamic range containing 95% of power gain samples measured in the 5 section cone. Dashed line is the 95% dynamic range assuming power gain is exponentially distributed.

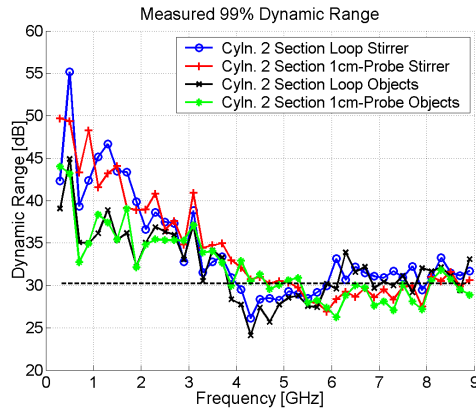


Figure 6.25: Dynamic range containing 99% of power gain samples measured in the 2 section cylinder. Dashed line is the 99% dynamic range assuming power gain is exponentially distributed.

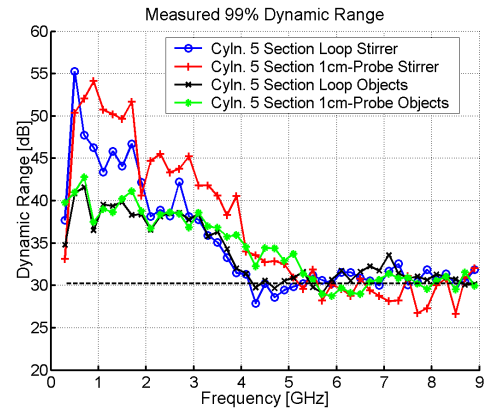


Figure 6.26: Dynamic range containing 99% of power gain samples measured in the 5 section cylinder. Dashed line is the 99% dynamic range assuming power gain is exponentially distributed.

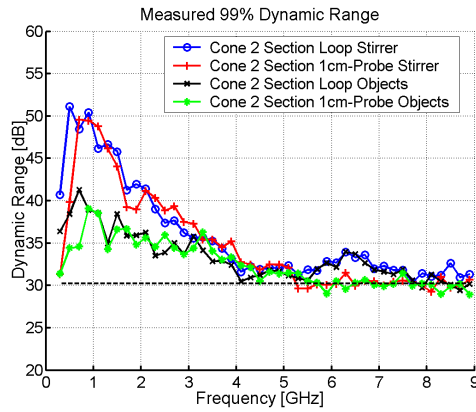


Figure 6.27: Dynamic range containing 99% of power gain samples measured in the 2 section cone. Dashed line is the 99% dynamic range assuming power gain is exponentially distributed.

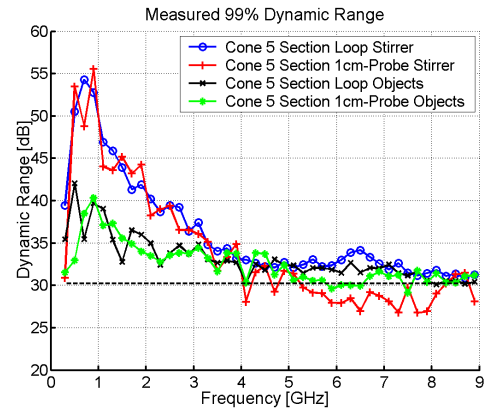


Figure 6.28: Dynamic range containing 99% of power gain samples measured in the 5 section cone. Dashed line is the 99% dynamic range assuming power gain is exponentially distributed.

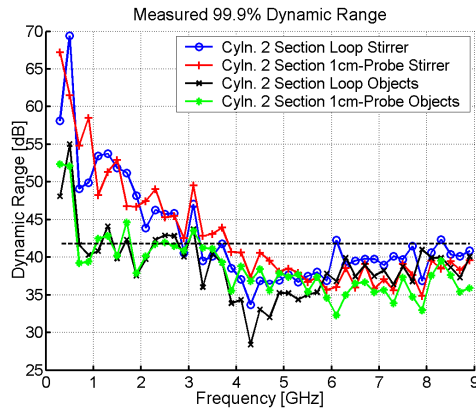


Figure 6.29: Dynamic range containing 99.9% of power gain samples measured in the 2 section cylinder. Dashed line is the 99.9% dynamic range assuming power gain is exponentially distributed.

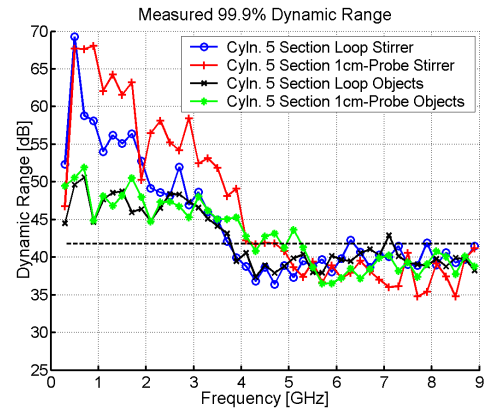


Figure 6.30: Dynamic range containing 99.9% of power gain samples measured in the 5 section cylinder. Dashed line is the 99.9% dynamic range assuming power gain is exponentially distributed.

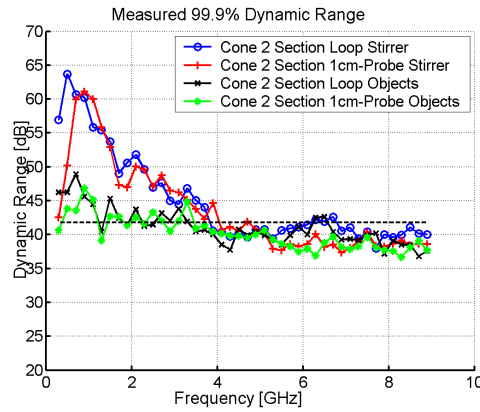


Figure 6.31: Dynamic range containing 99.9% of power gain samples measured in the 2 section cone. Dashed line is the 99.9% dynamic range assuming power gain is exponentially distributed.

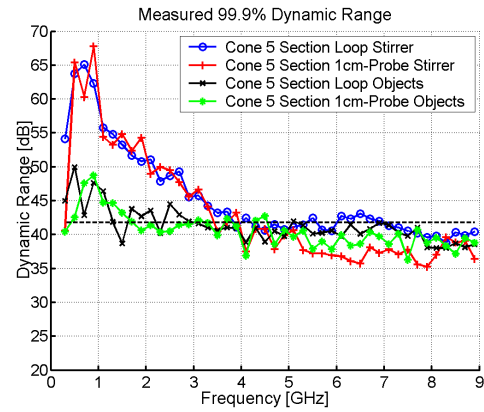


Figure 6.32: Dynamic range containing 99.9% of power gain samples measured in the 5 section cone. Dashed line is the 99.9% dynamic range assuming power gain is exponentially distributed.

## Chapter 7

# Dispersion-Constrained Communications in the Enclosed Space Channel

Chapter 5 motivated and discussed the theoretical description of dispersion in the enclosed space radio channel, as well as presented measurements of the RMS delay spread, mean excess delay and the 50% and 90% coherence bandwidths in the cylindrical and conical platforms. These dispersion measurements allow investigators to contrast the enclosed space radio channel with other channels, such as the indoor and outdoor channels. For instance, the outdoor channel can typically see RMS delay spreads on the order of 1-5  $\mu s$  [8, 9, 10] and the indoor office-environment channel can often see delay spreads on the order of 10-100 ns [17, 23, 24, 27]. The delay spread measurements made in Chapter 5 showed delay spreads of 100-300 ns, indicating that the dispersion seen in the enclosed space channel is larger than the delay spreads often seen in the indoor channel, smaller than the outdoor channel and very similar to the HVAC channel. Similar values of the RMS delay spread have been observed in the open-office building/factory environments [29] and in mine tunnels [42]. This comparative knowledge of the delay spread in the context of other common radio channels assists radio system architects in designing viable enclosed space communications systems that may utilize COTS devices developed for other channel environments, perhaps with some modification. However, knowledge of the dispersion alone cannot entirely predict how different

modulation schemes will perform in the enclosed space channel, since the properties of the channel embodied by the power delay profile and its associated statistics are dependent upon the specific channel being considered. A method that can be used to achieve some insight into the performance of various modulation schemes in the presence of a given radio channel's dispersion is to excite that channel with real communications signals and evaluate the performance of each evaluated signaling scheme in dispersion-constrained channel conditions. Testing in dispersion-constrained conditions means that the performance degradation of an arbitrary signal transmitted through the channel is due largely to the distortion caused by dispersion (i.e. inter-symbol interference), and is not significantly limited by the amount of signal power seen at the receiver.

The following sections will present the results of communications performance measurements in the enclosed space channel using a variety of different single-carrier signaling schemes, including phase-shift keying, quadrature amplitude modulation and minimum shift keying. Results of measurements using the multicarrier modulation scheme of orthogonal frequency division multiplexing (OFDM) as defined in the IEEE 802.11a standard will also be presented and discussed.

## 7.1 Experimental Apparatus and Methodology

As discussed previously, this body of work focuses on communications in the overmoded region of operation, since communications in the undermoded region would be bandwidth limited due to the very narrow half-power widths of the widely separated resonant modes. Furthermore, complex carrier tracking would be required in the undermoded region to find and maintain the spectral location for a transmitted signal in the resonant bandwidth of a given mode, which could move due to reconfiguration or motion in the enclosed space. Excitation of enclosed spaces using signals in the overmoded frequency region (i.e. above 3.0 GHz) allows for an arbitrary placement of the transmitted spectrum, since at any one frequency in the overmoded region, multiple resonant modes will be excited by the transmitted signal, providing a form of frequency diversity in the channel. This implicit diversity is important, as a transmitter-receiver pair placed in an arbitrary location in an enclosed space is required to operate in the channel as it is found.

To excite the enclosed space platforms with real signals, an Agilent E4433A digital signal generator is used to generate a variety of signal types and is capable of selecting a carrier frequency



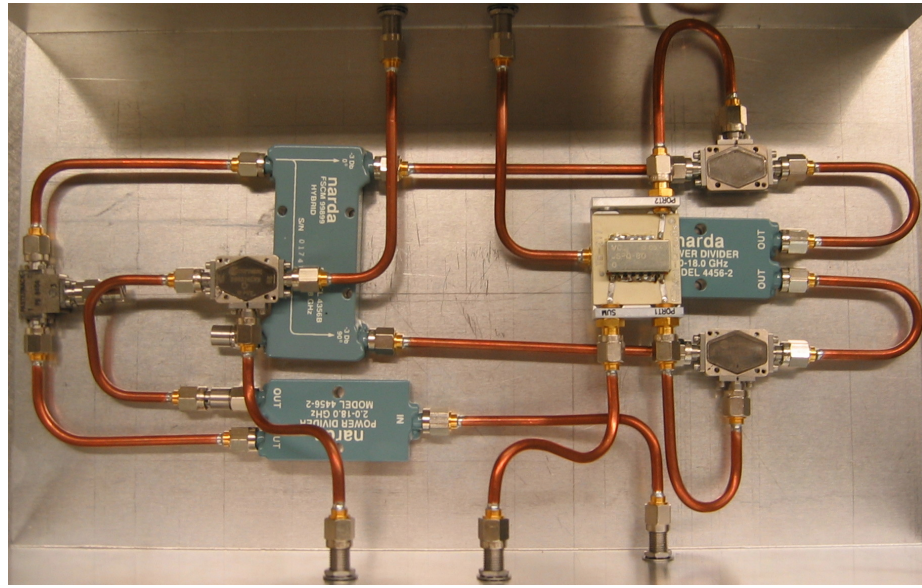


Figure 7.1: Single-sideband up- and down-converter used to shift the modulated signals generated by the digital signal generator at an IF of 20 MHz to an RF carrier between 3.0 GHz and 18.0 GHz for transmission into the channel and to downconvert the RF signal received from the channel to an IF of 20 MHz for analysis by the vector signal analyzer.

between 100 kHz and 4.0 GHz. The received signals are analyzed using an Agilent 89600 Vector Signal Analyzer that has an instantaneous bandwidth of 40 MHz, located at baseband. Since the transmitted signals need to be centered at frequencies greater than 3.0 GHz (i.e. the overmoded region) and due to the 4.0 GHz limitation of the digital signal generator, the signals generated by this device need to be up-converted to a higher range of frequencies. Furthermore, the vector signal analyzer requires that signals be provided to it nominally at an intermediate frequency (IF) of 20.0 MHz, and down-conversion from the transmission frequency is thus required. Ideally, measurements well into the overmoded region are desired to explore the response of the channel to signal excitation across as large a frequency range as possible.

To accomplish this goal, a single-sideband up- and down-converter was designed and implemented using discrete RF components, which is shown in Fig. 7.1. This device uses a quadrature mixing technique to shift an IF signal that is generated between 10 and 80 MHz to an RF signal centered between 2.0 and 18.0 GHz in such a manner that only the upper spectrum image ( $f_{LO} + f_{IF}$ ) is selected. A single mixer is then used to down-convert the received RF signal back to the identical IF frequency from which it was up-converted. Figure 7.2 shows a schematic block diagram of

the up-/down-converter system (as connected for this measurement campaign). An additional RF precision signal generator (an Agilent 8251A precision signal generator) is used to provide a local oscillator (LO) signal to the mixers for up- and down-conversion. Since the LO and IF signals going to the two up-conversion mixers must be exactly  $90^\circ$  out of phase, care was taken to ensure that the cables used to connect the 2-18 GHz hybrid to the up-conversion mixers are the same length, as was likewise done for the cables used to connect the 10-80 MHz hybrid to the same mixers. A circulator whose third port is terminated in a matched load (i.e. functioning as an isolator) is placed between the up- and down-conversion sections of the system to prevent the IF signals from coupling through the IF and LO ports of the mixers and the two signal ports of the 2-18 GHz power divider, which could provide an alternate path for the transmitted signal to arrive at the receiver (vector signal analyzer) without passing through the channel. A second circulator is placed at the port of the transmit antenna to compensate for the poor match of that antenna to the characteristic impedance of the devices used in the converter system (i.e.  $50\ \Omega$ ). Measurements of the transmitted signal from the up-converter using a spectrum analyzer show that the selected upper ( $f_{LO} + f_{IF}$ ) spectral image is 20 dB stronger than the rejected lower image, indicating that the system is operating sufficiently.

The methodology that is used to characterize the various communications signals in the dispersion-constrained enclosed space radio channel will now be described. As previously mentioned, to test in the dispersion-constrained conditions is to transmit with a sufficient amount of power in such a way that the communications performance obtained is not limited by the power received from the channel. That is, in the absence of dispersion, enough energy is transmitted in each symbol such that communications are achieved with a performance above a desired threshold. In a given channel with a known RMS delay spread or coherence bandwidth, as the transmitted symbol duration of a given signaling scheme is decreased from a duration much longer than the delay spread to a duration much shorter than the delay spread—or equivalently from a bandwidth much smaller than the coherence bandwidth to a bandwidth much larger than the coherence bandwidth—the received signal will show the effects of an increasing amount of distortion from inter-symbol interference (ISI), or equivalently, from frequency-selective fading. This signal distortion will manifest itself in a decreased effective signal-to-noise ratio (SNR), as measured by the error vector magnitude (EVM) of the received signal using a vector signal analyzer (VSA). A VSA functions in the following manner: When a particular symbol, represented by a point on the complex (I-Q) signal plane,

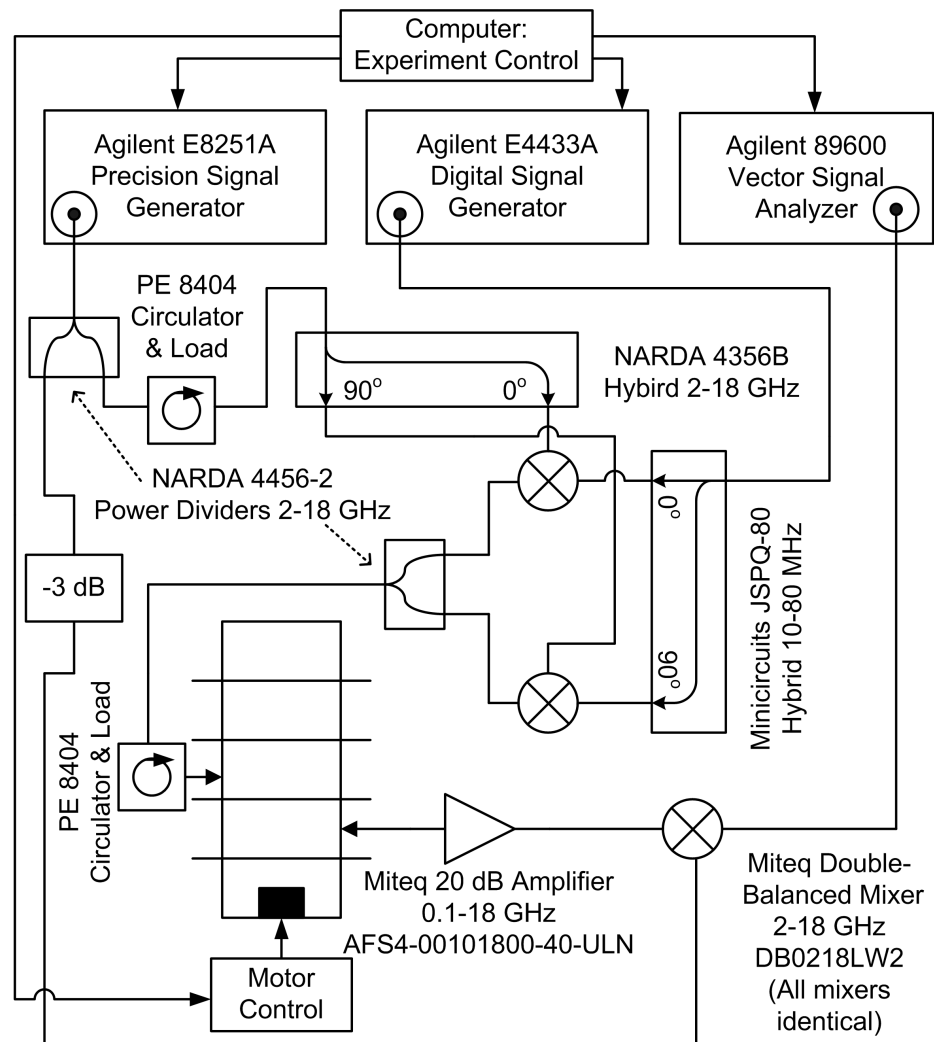


Figure 7.2: Schematic diagram of the single-sideband up- and down-converter.

is presented with interference from one or more adjacent symbols (represented by distinct points on the I-Q plane), the resulting distorted signal seen by the receiver will be a vector sum of the two or more complex points on the I-Q plane, corresponding to each replica of the transmitted symbol(s) received in the given symbol duration. Each given digital modulation scheme has a discrete number of nominal points on the I-Q plane, corresponding to each allowable symbol of the given scheme, and the vector sum of the original symbol plus all of the interfering symbols will cause this point to deviate from the nominal location. As shown in Fig. 7.3, the vector that connects the nominal symbol location with the actual measured symbol location on the I-Q plane is referred to as the *error vector* whose length is called the *error vector magnitude*, which is related to the SNR as follows [2]:

$$\text{SNR} = -20 \cdot \log_{10}(\text{EVM}/100\%) \quad (7.1)$$

The SNR represented in (7.1) would be more accurately called a signal to distortion ratio, but the use of SNR will be continued for simplicity. The SNR required for a given bit error rate (BER) performance is dependent on the chosen modulation scheme, and a comparison over several different modulation schemes with the same target BER is a goal of these characterizations. The BER, i.e. the probability of bit error ( $P_b$ ), can be written as a function of the SNR for a given modulation scheme using well-known mathematical relationships, but in the derivation of these relationships an assumption is made about the statistical distribution of the noise received with the signal. Additive white Gaussian noise (AWGN) is a typical assumption used for the derivation of these functional relations, since in distortion-less channels the predominant noise source is from thermal noise, which has a Gaussian distribution and whose power spectral density is flat (i.e. white). Since the actual distribution of the distortion noise seen in the enclosed space radio channel is not known—and would be very difficult to adequately characterize—the distribution of the distortion noise will be simply modeled as Gaussian. As such, the BER/SNR relationships mentioned previously will be used to connect a measured SNR to a desired BER, bearing in mind that all results will be approximate due to the uncertainty in the underlying distribution of the distortion noise.

Proceeding with this limitation in mind, single-carrier communications experiments are performed for a given modulation scheme in the enclosed space platforms as follows: 1) The object-

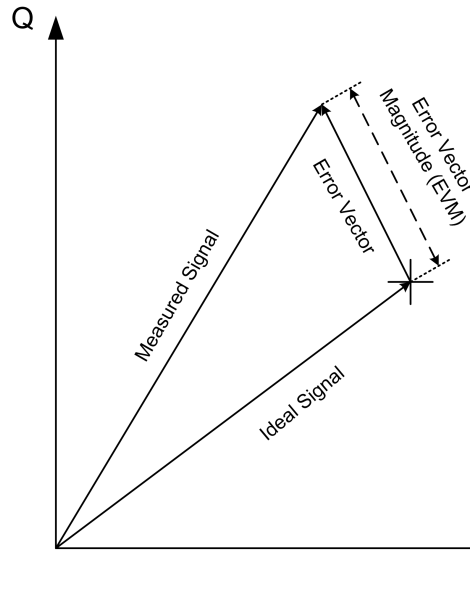


Figure 7.3: I-Q plane with illustration of the error vector magnitude measured between an ideal and a measured symbol [2].

assembly/stirrer located inside the platform is rotated to one of eight selected positions, separated by  $45^\circ$ ; 2) the carrier frequency is chosen starting at 3.0 GHz, continuing to 18.0 GHz, in 1.875 GHz increments; 3) the symbol rate is selected from the following set, in the listed order [kbps] {10, 20, 50, 100, 200, 500, 700, 1000, 1500, 2000, 2500, 3000, 3500, 4000, 5000, 6000, 7000, 8000, 9000, 10000}. Step #3 is performed for each iteration of step #2, and step #2 is likewise performed for each iteration of step #1. To ensure that all symbol rates and modulation formats are tested using an equal amount of energy per bit, the transmit power from the digital signal generator is scaled proportionally with the symbol rate and the number of bits per symbol. For instance, the output power of the digital signal generator is -11.76 dBm at a symbol rate of 10 Msps and -41.76 dBm at a symbol rate of 10 ksps for the BPSK format (1 bit per symbol); accordingly, the output power for 64-QAM (6 bits per symbol) is -3.98 dBm for 10 Msps and -33.97 dBm for 10 ksps. At each symbol rate, the vector signal analyzer calculates the RMS error vector magnitude of 1000 received symbols for the chosen modulation. This value is then recorded, and the iterative measurement process continues, for a total of 1440 different communications experiments for each chosen modulation scheme in each configuration. As motivated previously, as the symbol rate increases the EVM will also increase as a result of increasing ISI, causing the SNR to decrease and the AWGN-equivalent

BER to increase, eventually becoming larger than some chosen threshold. For the measurements presented in this chapter, a BER threshold of  $10^{-5}$  will be used. These measurements will provide investigators with the maximum bit rate for which a BER of  $10^{-5}$  or less is maintained in the given configuration of the enclosed space platforms for each tested modulation format. For simplicity, measurements are made only in the cylindrical platform using the loop antenna, and include both sparse and dense internal object populations.

Communications signaling is tested from the phase shift keying (PSK), quadrature amplitude modulation (QAM) and minimum shift keying (MSK) families; a root-raised cosine filter ( $\alpha = 0.35$ ) is applied with the PSK and QAM modulation formats, and a Gaussian filter ( $BT = 0.300$  and  $0.589$ ) is applied to the MSK modulation experiments, creating a GMSK modulation format. Binary phase shift keying (BPSK), quadrature phase shift keying (QPSK) and 8-PSK are tested from the PSK family, as these modulation formats (particularly BPSK and QPSK) can be very easily formed and are among the simplest modulation schemes. 16-QAM and 64-QAM are tested from the quadrature amplitude modulation family, allowing investigators to explore two more complex modulation formats in the enclosed space channel. Finally, two different GMSK schemes are chosen both to provide one more modulation format with which to characterize the channel performance and to explore the behavior of the very accessible GMSK modulation format, which is easily formed using an FM modulator and a Gaussian baseband filter.

Before the system diagramed in Fig. 7.2 can be used to reliably transmit signals into and receive signals from the channel, test characterizations are first required to ensure that the system is capable of providing the kind of measurement results desired. If testing is to be done in the dispersion-constrained channel, then in the absence of dispersion but with a similar value of channel gain, a SNR that provides a BER less than  $10^{-5}$  should be maintained using the measurement system for the investigated modulation formats. One can see from Figs. 6.1 and 6.2 that the gain measured in the 5 section cylinder with objects or stirrer above 3.0 GHz is approximately -25 dB, at minimum. As such, a cable with 25 dB of attenuation inserted in-line is substituted for the transmit and receive antennas, as shown in Fig. 7.2. This experimental channel will display a similar value of attenuation as the enclosed space channel but will display no dispersion. Three different modulation formats—QPSK, 64-QAM and 0.3 GMSK, one from each of the three investigated families—is tested in this attenuated cable channel in the fashion described above, and the SNR derived from the EVM via

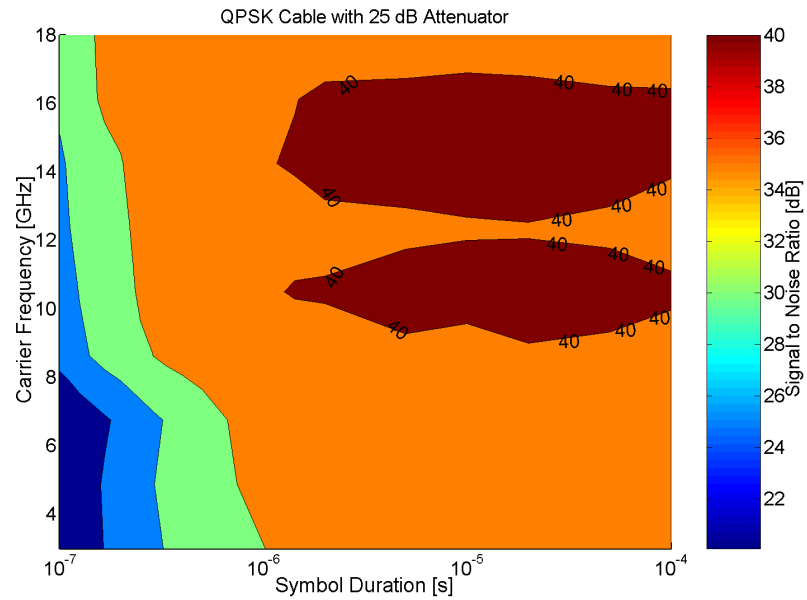


Figure 7.4: Signal to noise ratio measured using root-raised cosine filtered QPSK in a cable with 25 dB of attenuation. The threshold SNR for a BER of  $10^{-5}$  for QPSK is 11.3 dB.

(7.1) is plotted vs. carrier frequency and symbol time using a color contour plot in Figs. 7.4, 7.5 and 7.6, respectively.

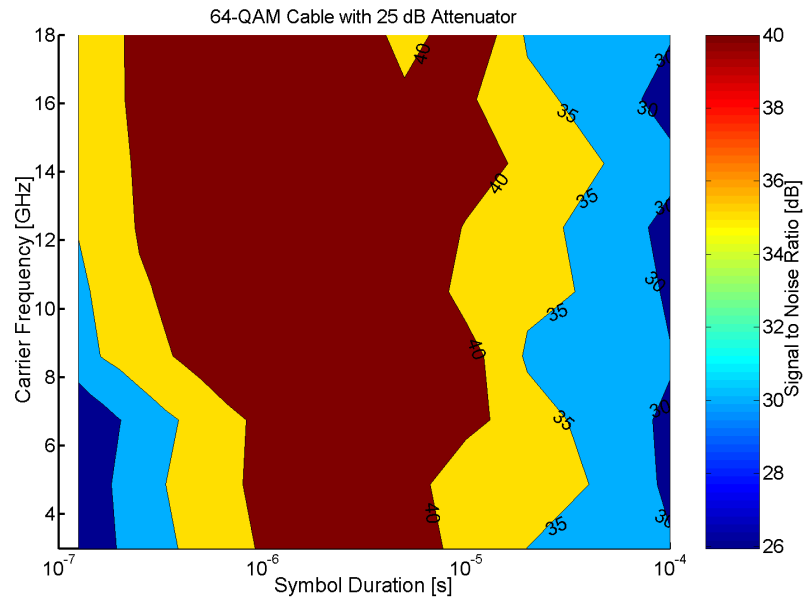


Figure 7.5: Signal to noise ratio measured using root-raised cosine filtered 64-QAM in a cable with 25 dB of attenuation. The threshold SNR for a BER of  $10^{-5}$  for 64-QAM is 24.3 dB.

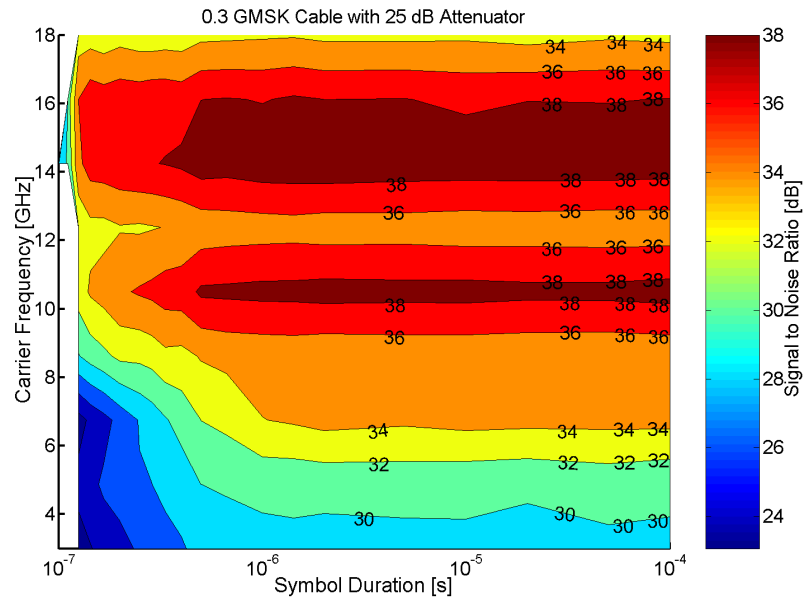


Figure 7.6: Signal to noise ratio measured using 0.3 GMSK in a cable with 25 dB of attenuation. The threshold SNR for a BER of  $10^{-5}$  for 0.3 GMSK is 18.3 dB.



One can see from these figures that the SNR measured using the vector signal analyzer is above the threshold SNR required to maintain a BER of  $10^{-5}$  or less for all carrier frequencies and all bit rates using all three modulation formats, which are specifically 11.3 dB for QPSK, 24.3 dB for 64-QAM and 18.3 dB for 0.3 GMSK, as listed in the captions. Since these SNR values are all greater than the required SNR for each respective modulation to maintain a  $10^{-5}$  bit error rate, the test communications system shown in Fig. 7.2 is indeed operating with a sufficient amount of received power and overall system fidelity in the absence of dispersion. This implies that once the system is connected to the dispersive enclosed space platform with a similar channel gain as the attenuated cable, any loss in SNR that causes the BER to increase above  $10^{-5}$  will necessarily be due to the added dispersion. Therefore, as the symbol rate is increased, there should be a rate beyond which the SNR falls below the  $10^{-5}$  BER threshold for each modulation scheme, providing investigators with the maximum bit rate in the channel using each modulation listed above, as desired.

## 7.2 Single Carrier Modulation Schemes

The experimental methodology described above has been used to measure the received SNR vs. symbol time and carrier frequency for the BPSK, QPSK, 8-PSK, 16-QAM, 64-QAM, 0.589 GMSK and 0.3 GMSK modulation formats in the 5 section cylindrical platform using the loop antennas in both the sparse and dense internal population configurations. Using these SNR data sets, the maximum symbol rate at each carrier frequency will be derived for each modulation scheme using the functions that relate the probability of bit error to the signal to noise ratio. This maximum symbol rate yields the maximum signal bandwidth for each modulation format, given the specific pulse shaping filter used for each modulation. The maximum signal bandwidth for each modulation will then be compared to the 50% coherence bandwidth of the channel—estimated to 20.0 GHz using the step response method described in Chapter 3—to give an insight into how much bandwidth a signal can maintain in comparison to the coherence bandwidth of a channel before distortion from frequency selective fading prevents reliable communications without an equalizer. At the end of this section, the minimum symbol time obtained while maintaining a  $10^{-5}$  BER for each of the investigated modulations will be plotted together with the RMS delay spread estimated to 20.0 GHz in the channel. Finally, the maximum bit rate obtained using each modulation format while

maintaining a  $10^{-5}$  BER will be plotted together for comparison.

### 7.2.1 Phase Shift Keying

Each of the phase shift keying modulation formats is transmitted with a root-raised cosine (RRC) pulse-shaping filter ( $\alpha = 0.35$ ). The bandwidth of a RRC filtered signal is related to its symbol rate or symbol duration by the following relationship:

$$B = R_s(1 + \alpha) = \frac{1 + \alpha}{T_s}, \quad (7.2)$$

where  $B$  is the signal bandwidth,  $R_s$  is the symbol rate and  $T_s$  is the symbol duration. Once the maximum symbol rate is obtained using the prescribed method, this relationship will yield the maximum bandwidth of the signal, for comparison with the 50% coherence bandwidth. Another relationship that is important to all of the PSK modulations is the relationship of SNR, represented here as  $\gamma$ , to  $E_b/N_0$ , the energy per bit ( $E_b$ ) to noise power spectral density ( $N_0$ ) ratio:

$$\gamma = \frac{P_s}{P_n} = \frac{E_b}{N_0} \frac{L}{B} = \frac{E_b}{N_0} \frac{L}{1 + \alpha}, \quad (7.3)$$

where  $P_s$  is the signal power,  $P_n$  is the noise power, and  $L$  is the number of bits per symbol. All of the functions that will be presented below use the well-known Q error function, which is defined as [110]:

$$Q(z) = \frac{1}{\sqrt{2\pi}} \int_z^\infty e^{-y^2/2} dy. \quad (7.4)$$

### BPSK

The probability of bit error (i.e. the BER) for a coherently demodulated BPSK signal in the AWGN channel, assuming matched-filter detection is given [110] as

$$P_{b,\text{BPSK}} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right), \quad (7.5)$$

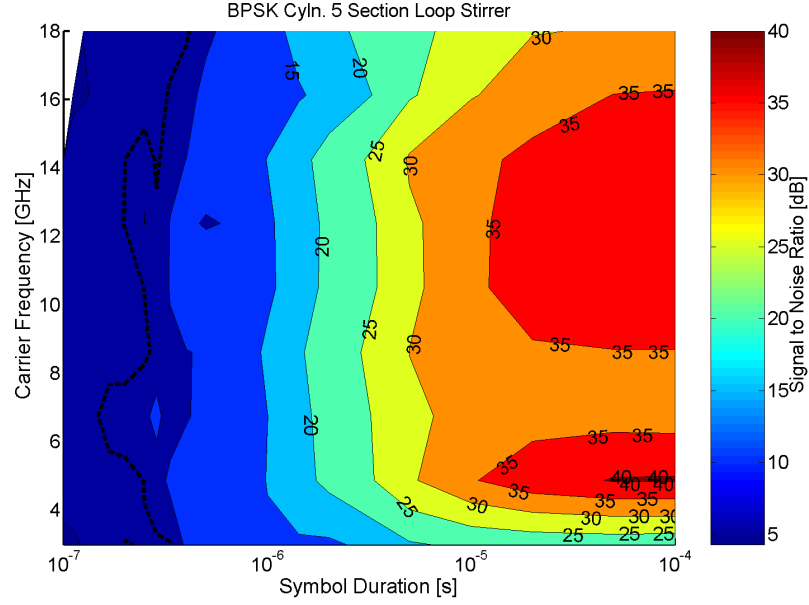


Figure 7.7: Signal to noise ratio of transmitted RRC-BPSK waveform measured in the sparsely populated 5 section cylindrical platform using the loop antenna. The minimum SNR threshold required to maintain a BER of  $10^{-5}$  is 8.28 dB and is indicated by the black dashed line.

which can be written in terms of the SNR,  $\gamma$ , using (7.3):

$$P_{b,\text{BPSK}} = Q\left(\sqrt{2\gamma(1+\alpha)}\right), \quad (7.6)$$

where  $L = 1$  (one bit per symbol in BPSK). Solving this equation for  $P_{b,\text{BPSK}} = 10^{-5}$ , one finds a threshold SNR of  $\gamma_T = 8.28$  dB. Figure 7.7 shows the SNR contour plot vs. carrier frequency and symbol duration and the dashed line on the left side of the plot represents the threshold SNR of 8.28 dB required for  $10^{-5}$  BER BPSK operation in the 5 section cylinder in a sparse object configuration. Figure 7.8 shows the same results for the dense internal object configuration. The figures plot the average of the measured SNR values calculated across all values of stirrer/object rotation. As evidenced by the presence of the threshold SNR (dashed contour), there is a minimum symbol duration (maximum symbol rate) beyond which the BER requirement is no longer satisfied for both physical configurations. The symbol duration at the point where the SNR falls below 8.28 dB is used to find the maximum signal bandwidth of the RRC BPSK signal with a BER of  $10^{-5}$ , and this maximum signal bandwidth is plotted in Figs. 7.9 and 7.10 for the 5 section sparse and dense cylindrical platforms, respectively. The dots in the figures represent the maximum signal

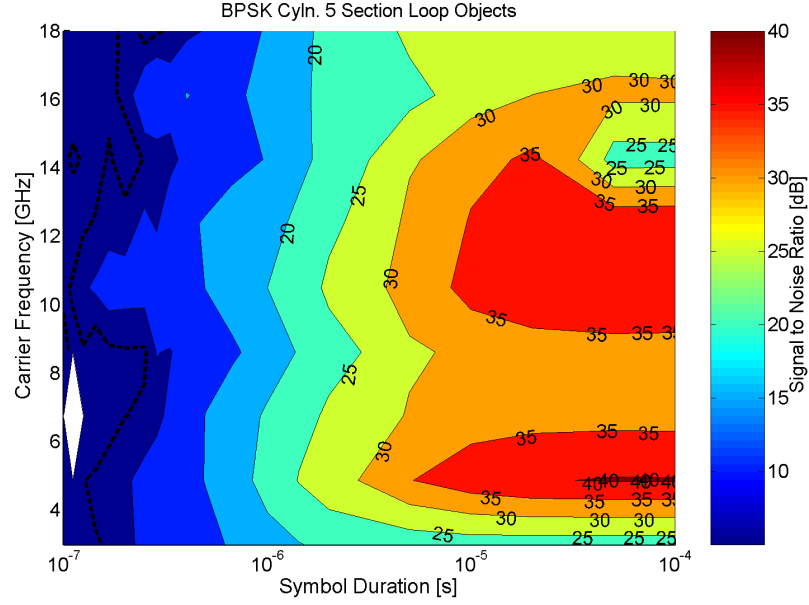


Figure 7.8: Signal to noise ratio of transmitted RRC-BPSK waveform measured in the densely populated 5 section cylindrical platform using the loop antenna. The minimum SNR threshold required to maintain a BER of  $10^{-5}$  is 8.28 dB and is indicated by the black dashed line.

bandwidth for each of the experiments across rotation, the solid line represents the average of the individual experiments at each frequency, and the dashed line is the 50% coherence bandwidth estimated for the channel. Interestingly, the bandwidth of the RRC BPSK signal appears to be greater than the 50% coherence bandwidth for many of the experimental points. BPSK is a very simple modulation scheme, and it seems intuitive that it would be tolerant of frequency selective distortion from the channel in such a manner that the signal bandwidth may exceed the coherence bandwidth on average.

### QPSK

The probability of bit error for a coherently demodulated QPSK signal in the AWGN channel, assuming matched-filter detection is given [110] as

$$P_{b,\text{QPSK}} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right), \quad (7.7)$$

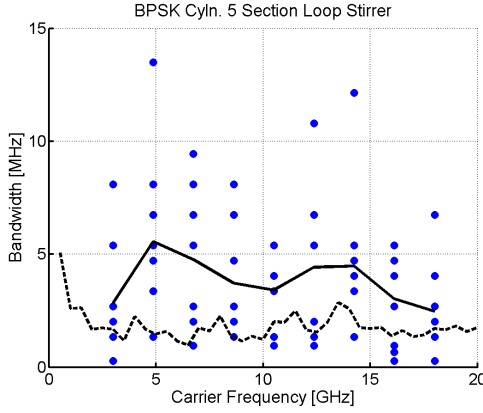


Figure 7.9: Maximum signal bandwidth transmitted using a RRC-BPSK signal while maintaining a  $10^{-5}$  bit error rate in the sparsely populated 5 section cylindrical platform. Dots: Measured data points over rotational positions, Solid: Average of measured data points, Dashed: 50% coherence bandwidth estimated from the quality factor measured to 20.0 GHz.

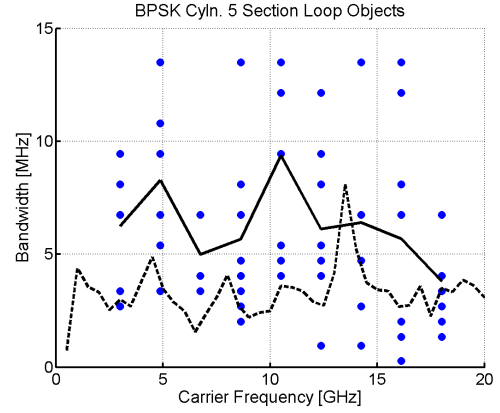


Figure 7.10: Maximum signal bandwidth transmitted using a RRC-BPSK signal while maintaining a  $10^{-5}$  bit error rate in the densely populated 5 section cylindrical platform. Dots: Measured data points over rotational positions, Solid: Average of measured data points, Dashed: 50% coherence bandwidth estimated from the quality factor measured to 20.0 GHz.

which can be written in terms of the SNR,  $\gamma$ , using (7.3):

$$P_{b,QPSK} = Q\left(\sqrt{\gamma(1+\alpha)}\right), \quad (7.8)$$

where  $L = 2$ , since there are two bits for each symbol in the QPSK constellation. These relationships can be used to find the  $\gamma_T = 11.3$  dB SNR required to maintain a BER of  $10^{-5}$ . Figures 7.11 and 7.12 plot the average measured SNR vs. carrier frequency and symbol duration and the dashed contour shows the 11.3 dB SNR on the left side of the plots. As was observed for the BPSK experiments, there are clearly symbol rates that will not support the desired BER performance in these measured channels. Using this minimum symbol duration to find the maximum signal bandwidth, Figs. 7.13 and 7.14 plot the maximum signal bandwidth measured in the 5 section cylinder under dense and sparse object configurations, along with the estimated 50% coherence bandwidth in the channel. One can observe that the maximum signal bandwidth is higher for the densely populated channel—as it was for the BPSK experiments—confirming that there is less dispersion in the densely populated configuration, allowing faster symbol rates and wider signal bandwidths.

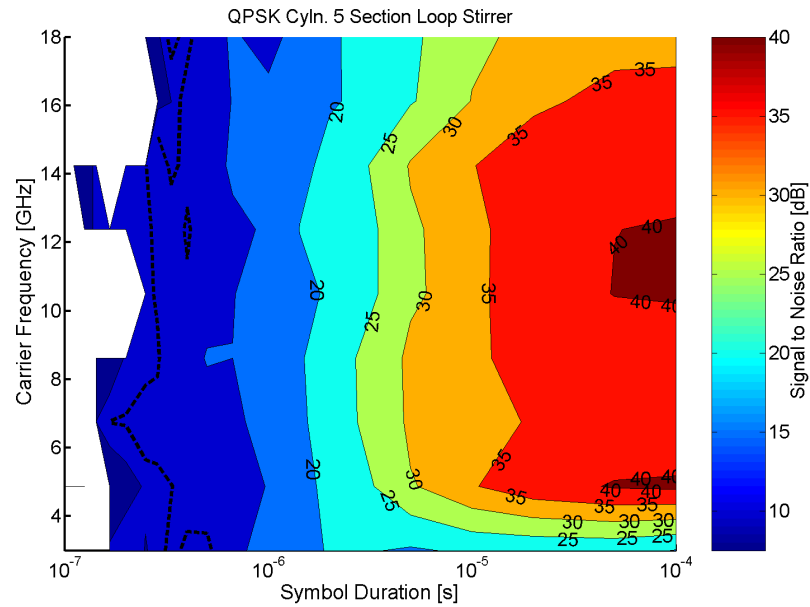


Figure 7.11: Signal to noise ratio of transmitted RRC-QPSK waveform measured in the sparsely populated 5 section cylindrical platform using the loop antenna. The minimum SNR threshold required to maintain a BER of  $10^{-5}$  is 11.3 dB and is indicated by the black dashed line.

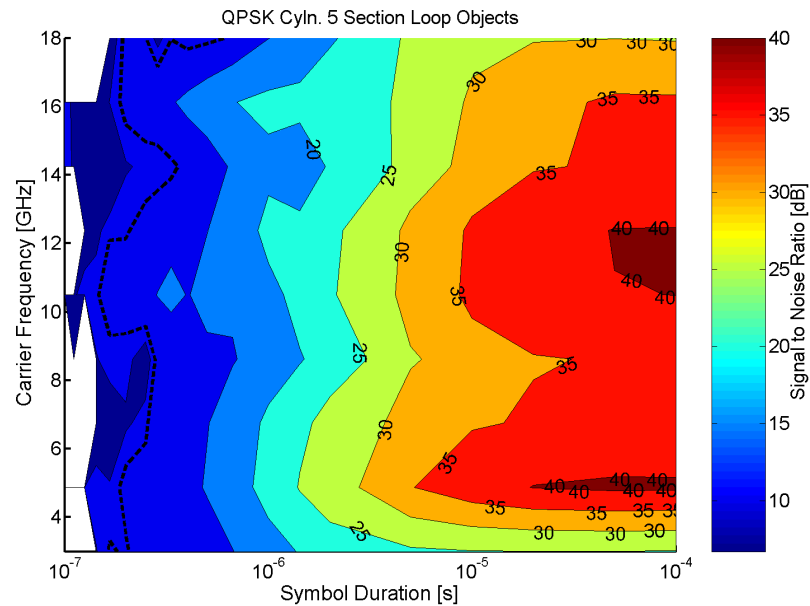


Figure 7.12: Signal to noise ratio of transmitted RRC-QPSK waveform measured in the densely populated 5 section cylindrical platform using the loop antenna. The minimum SNR threshold required to maintain a BER of  $10^{-5}$  is 11.3 dB and is indicated by the black dashed line.

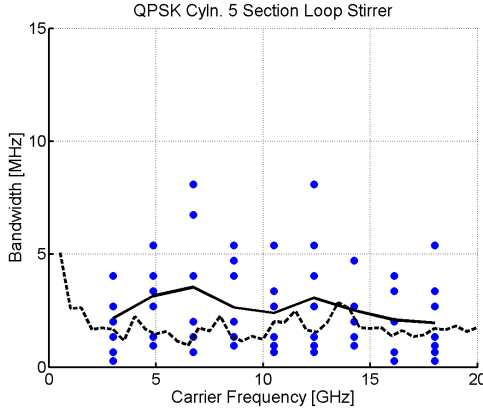


Figure 7.13: Maximum signal bandwidth transmitted using a RRC-QPSK signal while maintaining a  $10^{-5}$  bit error rate in the sparsely populated 5 section cylindrical platform. Dots: Measured data points over rotational positions, Solid: Average of measured data points, Dashed: 50% coherence bandwidth estimated from the quality factor measured to 20.0 GHz.

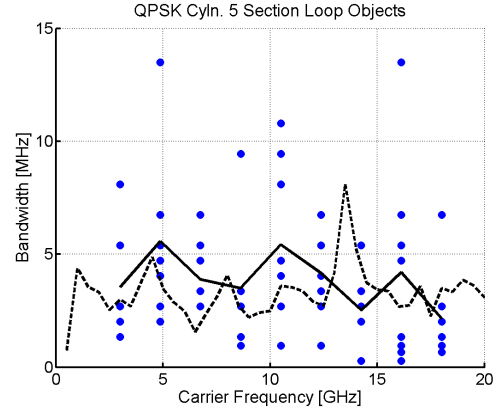


Figure 7.14: Maximum signal bandwidth transmitted using a RRC-QPSK signal while maintaining a  $10^{-5}$  bit error rate in the densely populated 5 section cylindrical platform. Dots: Measured data points over rotational positions, Solid: Average of measured data points, Dashed: 50% coherence bandwidth estimated from the quality factor measured to 20.0 GHz.

### 8-PSK

The probability of bit error for a coherently demodulated 8-PSK signal in the AWGN channel, assuming matched-filter detection is given [110] as

$$P_{b,8\text{-PSK}} \approx \frac{2}{\log_2 M} Q \left( \sqrt{\frac{2E_b \log_2 M}{N_0}} \sin \left( \frac{\pi}{M} \right) \right), \quad (7.9)$$

where  $M = 2^L$  is the number of constellation points ( $M = 8$ ). Using (7.3) the previous expression can be written in terms of the SNR,  $\gamma$ :

$$P_{b,8\text{-PSK}} \approx \frac{2}{\log_2 M} Q \left( \sqrt{2\gamma(1 + \alpha)} \sin \left( \frac{\pi}{M} \right) \right), \quad (7.10)$$

where  $L = 3$ , since there are three bits for each symbol in the 8-PSK constellation. The above expressions can be used to find the  $\gamma_T = 16.4$  dB SNR required to maintain a BER of  $10^{-5}$ . Figures 7.15 and 7.16 show the average SNR measured in the 5 section cylindrical platform in both sparse and dense object configurations using a RRC 8-PSK modulation format. Both plots confirm that the experiment is able to excite the channel with symbol rates that produce received signal SNR values both greater than and less than the required 16.4 dB, showing that communications are successfully

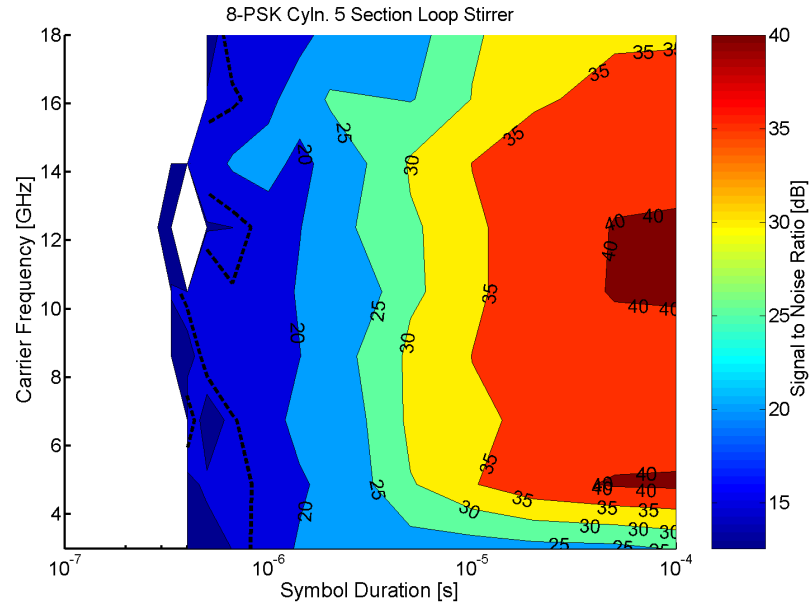


Figure 7.15: Signal to noise ratio of transmitted RRC 8-PSK waveform measured in the sparsely populated 5 section cylindrical platform using the loop antenna. The minimum SNR threshold required to maintain a BER of  $10^{-5}$  is 16.4 dB and is indicated by the black dashed line.

explored in the dispersion-constrained channel. The maximum signal bandwidth is derived in a likewise fashion as for the BPSK and QPSK trials and is plotted for the 8-PSK measurements of the 5 section cylindrical platform in sparse and dense object configurations in Figs. 7.17 and 7.18. The maximum signal bandwidth obtained in each of the physical configurations for 8-PSK is the closest to the 50% coherence bandwidth of the three PSK modulations explored; this observation matches intuition, since 8-PSK is the most challenging of the three PSK modulations explored.



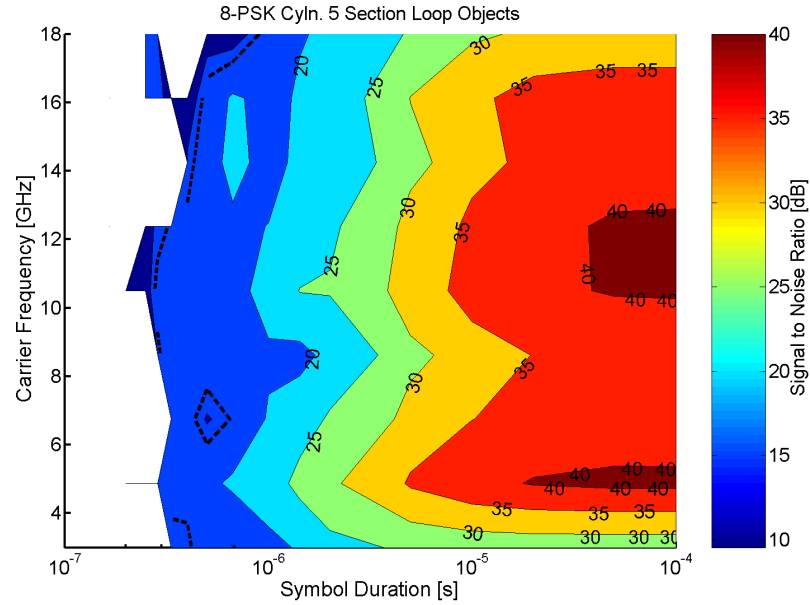


Figure 7.16: Signal to noise ratio of transmitted RRC 8-PSK waveform measured in the densely populated 5 section cylindrical platform using the loop antenna. The minimum SNR threshold required to maintain a BER of  $10^{-5}$  is 16.4 dB and is indicated by the black dashed line.

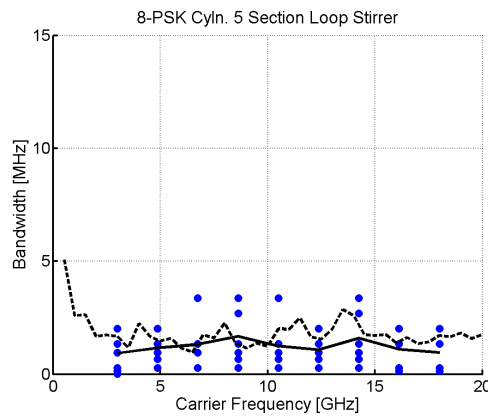


Figure 7.17: Maximum signal bandwidth transmitted using a RRC 8-PSK signal while maintaining a  $10^{-5}$  bit error rate in the sparsely populated 5 section cylindrical platform. Dots: Measured data points over rotational positions, Solid: Average of measured data points, Dashed: 50% coherence bandwidth estimated from the quality factor measured to 20.0 GHz.

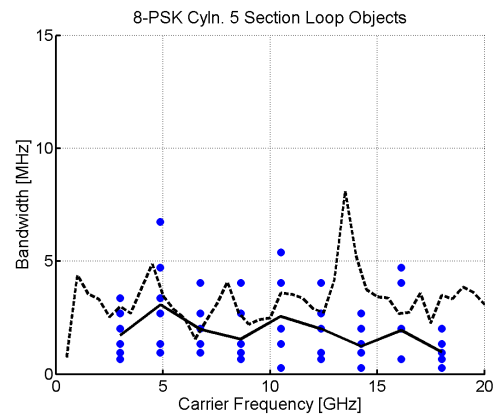


Figure 7.18: Maximum signal bandwidth transmitted using a RRC 8-PSK signal while maintaining a  $10^{-5}$  bit error rate in the densely populated 5 section cylindrical platform. Dots: Measured data points over rotational positions, Solid: Average of measured data points, Dashed: 50% coherence bandwidth estimated from the quality factor measured to 20.0 GHz.

### 7.2.2 Quadrature Amplitude Modulation

Each of the quadrature amplitude modulation formats is filtered with a root-raised cosine pulse-shaping filter with  $\alpha = 0.35$ , whose signal bandwidth is related to the symbol rate and duration by

$$B = R_s(1 + \alpha) = \frac{1 + \alpha}{T_s}, \quad (7.11)$$

as it is for the PSK modulations previously presented. The SNR and the  $E_{av}/N_0$  (average symbol energy to noise power spectral density) ratio can be similarly related as in the PSK modulations:

$$\gamma = \frac{P_s}{P_n} = \frac{E_{av} R_s}{N_0 B} = \frac{E_{av}}{N_0} \frac{1}{1 + \alpha}. \quad (7.12)$$

Two QAM modulation formats are explored below: 16-QAM and 64-QAM.

#### 16-QAM

The probability of bit error for a coherently demodulated 16-QAM signal in the AWGN channel using matched filter detection is given by the following expression [111]:

$$P_{b,16\text{-QAM}} \approx \frac{4}{\log_2 M} \left(1 - \frac{1}{\sqrt{M}}\right) Q \left( \sqrt{\frac{3E_{av}}{N_0(M-1)}} \right), \quad (7.13)$$

where  $M = 2^L$  is the number of constellation points ( $M = 16$ ) and  $E_{av}$  is the average symbol energy.

The previous expression can be written in terms of the SNR,  $\gamma$ , using (7.12):

$$P_{b,16\text{-QAM}} \approx \frac{4}{\log_2 M} \left(1 - \frac{1}{\sqrt{M}}\right) Q \left( \sqrt{\frac{3\gamma(1 + \alpha)}{(M-1)}} \right). \quad (7.14)$$

Solving this expression for the SNR that ensures a  $10^{-5}$  bit error rate finds that  $\gamma_T = 18.2$  dB of signal to noise ratio is required. Figures 7.19 and 7.20 show the average measured signal to noise ratio for the 5 section sparsely and densely populated cylindrical platform, where the dashed line on the left side of the plot represents the 18.2 dB threshold SNR. The decrease in the SNR at long symbol durations is not believed to be an effect of the channel, since it also appears in the cable-

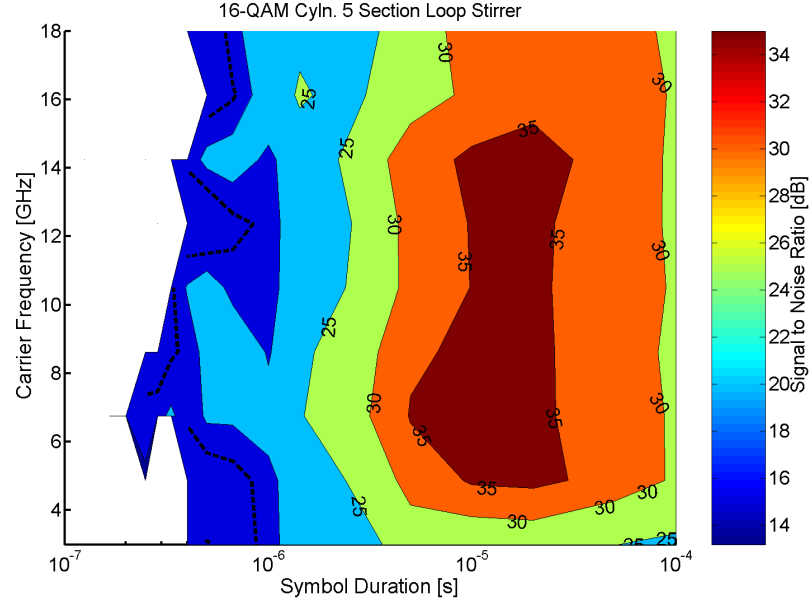


Figure 7.19: Signal to noise ratio of transmitted RRC 16-QAM waveform measured in the sparsely populated 5 section cylindrical platform using the loop antenna. The minimum SNR threshold required to maintain a BER of  $10^{-5}$  is 18.2 dB and is indicated by the black dashed line.

channel QAM experiments presented previously. The cause of this behavior, however, is unknown.

The minimum symbol duration that preserves a SNR of 18.2 dB in the above SNR plots gives the maximum 16-QAM signal bandwidth that is supported by the channel and is shown in Figs. 7.21 and 7.22. These plots show that the maximum 16-QAM bandwidth is very comparable to the estimated 50% coherence bandwidth, very similar to the 8-PSK modulation format. As expected, the maximum signal bandwidth in the densely populated environments is greater than the maximum signal bandwidth in the sparsely populated environments.

### 64-QAM

The probability of bit error for a coherently demodulated 64-QAM signal in the AWGN channel using matched filter detection is given by the following expression [111]:

$$P_{b,64\text{-QAM}} \approx \frac{4}{\log_2 M} \left( 1 - \frac{1}{\sqrt{M}} \right) Q \left( \sqrt{\frac{3E_{av}}{N_0(M-1)}} \right), \quad (7.15)$$

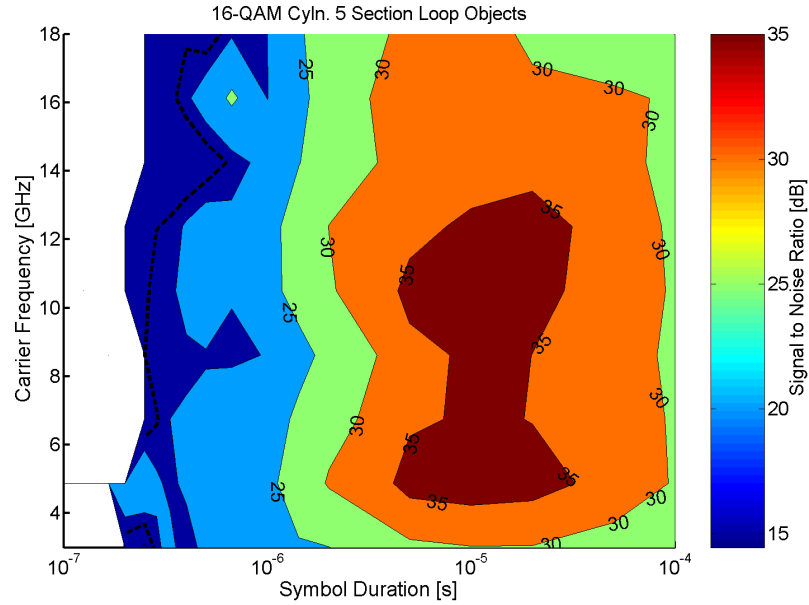


Figure 7.20: Signal to noise ratio of transmitted RRC 16-QAM waveform measured in the densely populated 5 section cylindrical platform using the loop antenna. The minimum SNR threshold required to maintain a BER of  $10^{-5}$  is 18.2 dB and is indicated by the black dashed line.

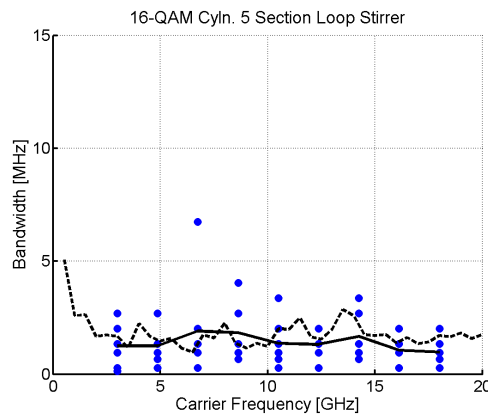


Figure 7.21: Maximum signal bandwidth transmitted using a RRC 16-QAM signal while maintaining a  $10^{-5}$  bit error rate in the sparsely populated 5 section cylindrical platform. Dots: Measured data points over rotational positions, Solid: Average of measured data points, Dashed: 50% coherence bandwidth estimated from the quality factor measured to 20.0 GHz.

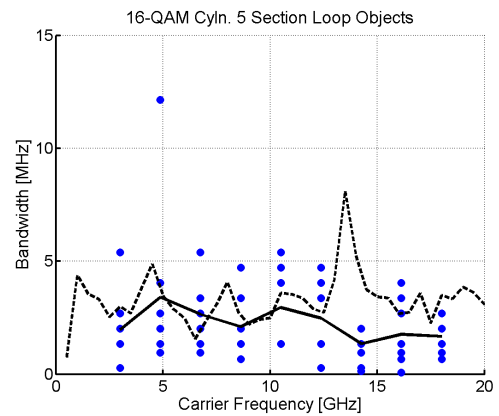


Figure 7.22: Maximum signal bandwidth transmitted using a RRC 16-QAM signal while maintaining a  $10^{-5}$  bit error rate in the densely populated 5 section cylindrical platform. Dots: Measured data points over rotational positions, Solid: Average of measured data points, Dashed: 50% coherence bandwidth estimated from the quality factor measured to 20.0 GHz.

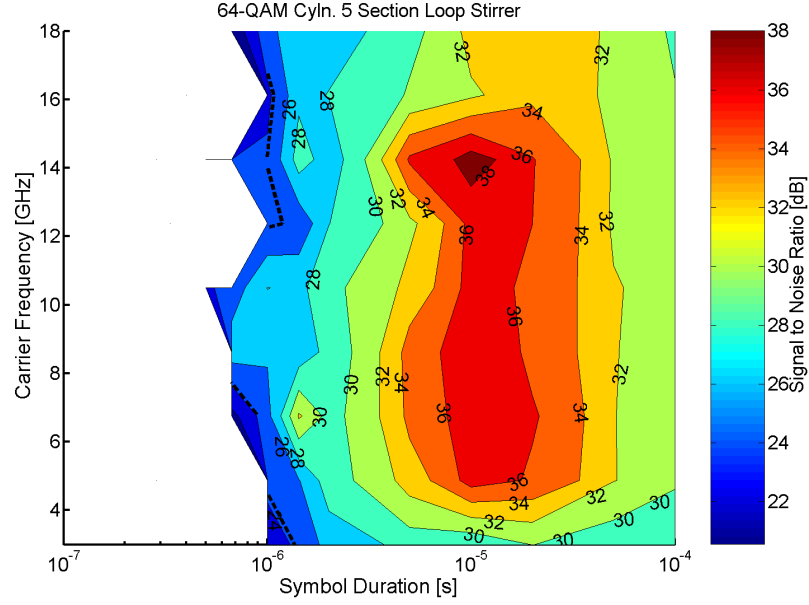


Figure 7.23: Signal to noise ratio of transmitted RRC 64-QAM waveform measured in the sparsely populated 5 section cylindrical platform using the loop antenna. The minimum SNR threshold required to maintain a BER of  $10^{-5}$  is 24.3 dB and is indicated by the black dashed line.

where  $M = 2^L$  is the number of constellation points ( $M = 64$ ) and  $E_{av}$  is the average symbol energy.

The previous expression can be written in terms of the SNR,  $\gamma$ , using (7.12):

$$P_{b,64\text{-QAM}} \approx \frac{4}{\log_2 M} \left( 1 - \frac{1}{\sqrt{M}} \right) Q \left( \sqrt{\frac{3\gamma(1+\alpha)}{(M-1)}} \right). \quad (7.16)$$

This expression is once again solved assuming a BER of  $10^{-5}$  to find that a threshold SNR of  $\gamma_T = 24.3$  dB is required to obtain the desired performance. Figures 7.23 and 7.24 show the contour plots of the average SNR measured in the channel using the RRC 64-QAM modulation format, and a threshold SNR of 24.3 dB is represented by the dashed contour on the left side of the plot. The maximum 64-QAM signal bandwidth supported by the channel is plotted in Figs. 7.25 and 7.26, and one can immediately see that the average maximum signal bandwidths (solid lines) for both the sparse and dense internal object configurations are notably less than the 50% coherence bandwidths estimated in each channel. Intuitively, this makes good sense, as 64-QAM is the most challenging modulation explored in these experiments, making it the most vulnerable to frequency selective distortion (i.e. ISI) in the channel.

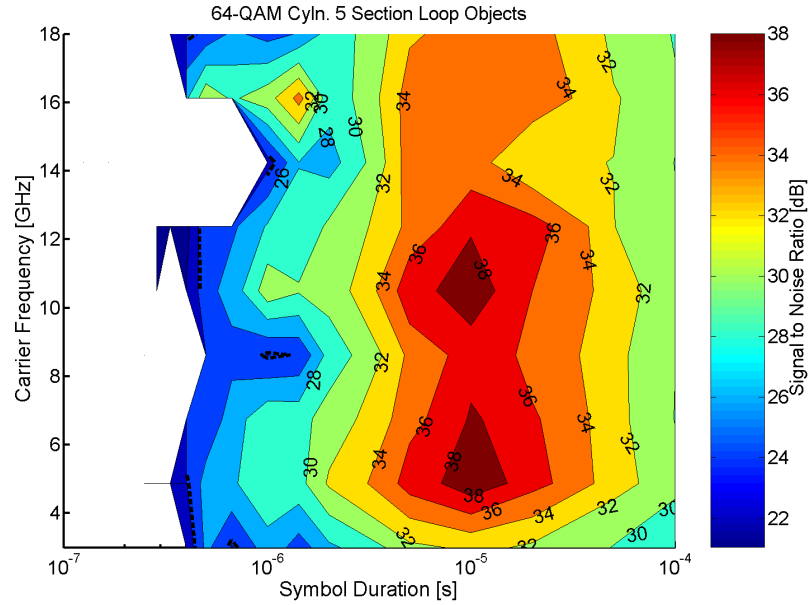


Figure 7.24: Signal to noise ratio of transmitted RRC 64-QAM waveform measured in the densely populated 5 section cylindrical platform using the loop antenna. The minimum SNR threshold required to maintain a BER of  $10^{-5}$  is 24.3 dB and is indicated by the black dashed line.

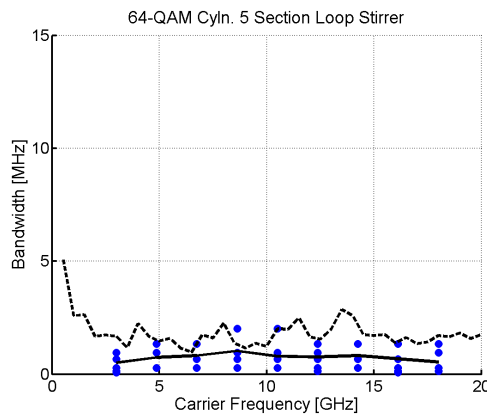


Figure 7.25: Maximum signal bandwidth transmitted using a RRC 64-QAM signal while maintaining a  $10^{-5}$  bit error rate in the sparsely populated 5 section cylindrical platform. Dots: Measured data points over rotational positions, Solid: Average of measured data points, Dashed: 50% coherence bandwidth estimated from the quality factor measured to 20.0 GHz.

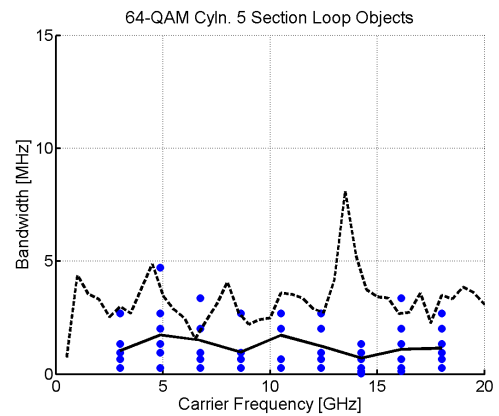


Figure 7.26: Maximum signal bandwidth transmitted using a RRC 64-QAM signal while maintaining a  $10^{-5}$  bit error rate in the densely populated 5 section cylindrical platform. Dots: Measured data points over rotational positions, Solid: Average of measured data points, Dashed: 50% coherence bandwidth estimated from the quality factor measured to 20.0 GHz.

### 7.2.3 Frequency Shift Keying

To further contrast the performance of phase shift keying and quadrature amplitude modulation in the enclosed space radio channel, frequency shift keying has additionally been explored. Specifically, minimum shift keying has been used with a Gaussian pulse shaping filter, creating Gaussian minimum shift keying (GMSK). The GSM (Global System for Mobile [telecommunications]) standard, for instance, uses 0.3 GMSK. The first number designator refers to the BT product (bandwidth-time product) of the Gaussian pulse shaping filter, where B is the 3 dB baseband bandwidth of the pulse shaping filter and T is the symbol duration. The GSM standard specifies  $BT = 0.3$  for the Gaussian pulse shaping filter, and this BT product has accordingly been chosen for the performance tests described in this section. A second BT product,  $BT = 0.589$ , has been chosen in these experiments to contrast the  $BT = 0.3$  measurements. Unlike root-raised cosine pulse shaping—which is known as a Nyquist filter since it does not create ISI in the process of filtering the transmitted symbols—Gaussian pulse shaping filters do inherently create ISI in the process of filtering the transmitted symbols. Since the ISI created becomes more severe as the BT product becomes smaller, experiments using  $BT = 0.589$  should exhibit less inherent ISI than the experiments using  $BT = 0.3$ . In fact, it was shown in [112] that the ISI resulting from pulse shaping is minimum at  $BT = 0.5887$ , where the change in the required  $E_b/N_0$  is only 0.14 dB as compared to the case of no pulse shaping (i.e.  $BT \rightarrow \infty$ ) [108].

The bit error probability for GMSK is given as [113]

$$P_{b,\text{GMSK}} = Q\left(\sqrt{\frac{2\psi E_b}{N_0}}\right), \quad (7.17)$$

where  $Q(\cdot)$  is the Q error function used in the previous sections,  $E_b$  is the energy per bit and  $N_0$  is the noise power spectral density;  $\psi = 0.450$  for  $BT = 0.300$  and  $\psi = 0.485$  for  $BT = 0.5$  [113]. This expression can be written in terms of the SNR,  $\gamma$ , as follows:

$$P_{b,\text{GMSK}} = Q\left(\sqrt{2\psi\gamma \cdot BT}\right), \quad (7.18)$$

since

$$\gamma = \frac{E_b}{N_0} \frac{R_b}{B} = \frac{E_b}{N_0} \frac{1}{BT}. \quad (7.19)$$

This expression for the bit error rate will be used in the following presentation of results to find the threshold SNR that is required to maintain a bit error rate of  $10^{-5}$ .

### 0.589 GMSK

Using the expression shown above with  $\psi = 0.485$  (since this is the closest value of  $\psi$  known for  $BT = 0.589$ ), a threshold SNR of  $\gamma_T = 15.0$  dB is required in order to maintain a BER of  $10^{-5}$ . The average signal to noise ratio has been measured in the same fashion as presented for the PSK and QAM modulation formats and is plotted vs. carrier frequency and symbol duration in Figs. 7.27 and 7.28 for the 5 section cylinder in sparse and dense internal object configurations, respectively. The dashed contour on the left side of the plots shows the threshold SNR of 15.0 dB required to maintain the desired bit error rate. The successful measurement of signals on both sides of this threshold contour show that the experiments were able to fully examine communications in the dispersion-constrained channel, as desired. As was the case for the QAM formats, measurements at long symbol durations and at certain frequencies (e.g. 3.0 GHz in this case) experience a decrease in the measured signal to noise ratio. The cause of this effect is once again unknown; however, measurements at higher symbol rates clearly do exceed the SNR threshold, and these measurements are used to explore the maximum symbol rate for GMSK in the channel. The maximum bandwidth measured in each of the object/stirrer positions is plotted in Figs. 7.29 and 7.30 for the sparse and dense internal configurations, respectively. The solid line in these plots shows the average maximum signal bandwidth, and the dashed line represents the estimated 50% coherence bandwidth in the channel. It is evident from these plots that the average maximum signal bandwidth (while maintaining a BER of  $10^{-5}$  or less) is very comparable in value to the 50% coherence bandwidth, as was the case for the 16-QAM and 8-PSK modulation formats.



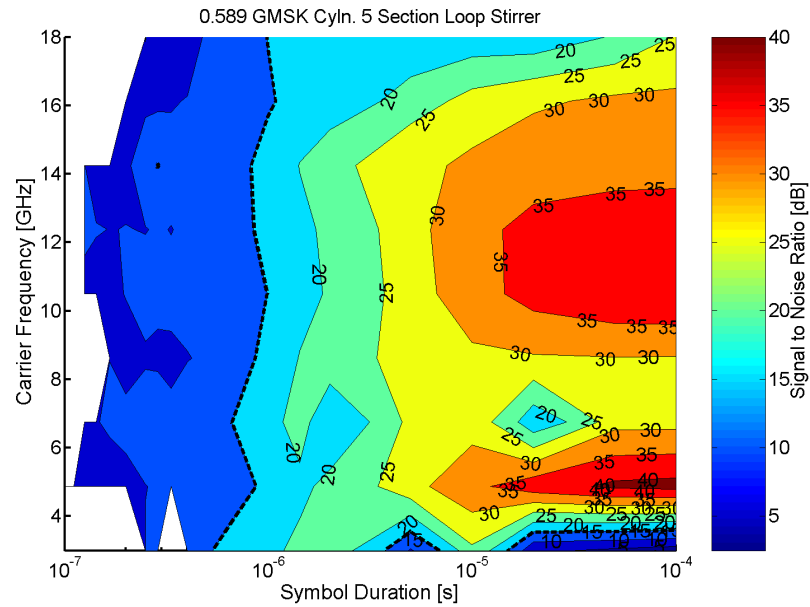


Figure 7.27: Signal to noise ratio of transmitted Gaussian filtered 0.589 GMSK waveform measured in the sparsely populated 5 section cylindrical platform using the loop antenna. The minimum SNR threshold required to maintain a BER of  $10^{-5}$  is 15.0 dB and is indicated by the black dashed line.

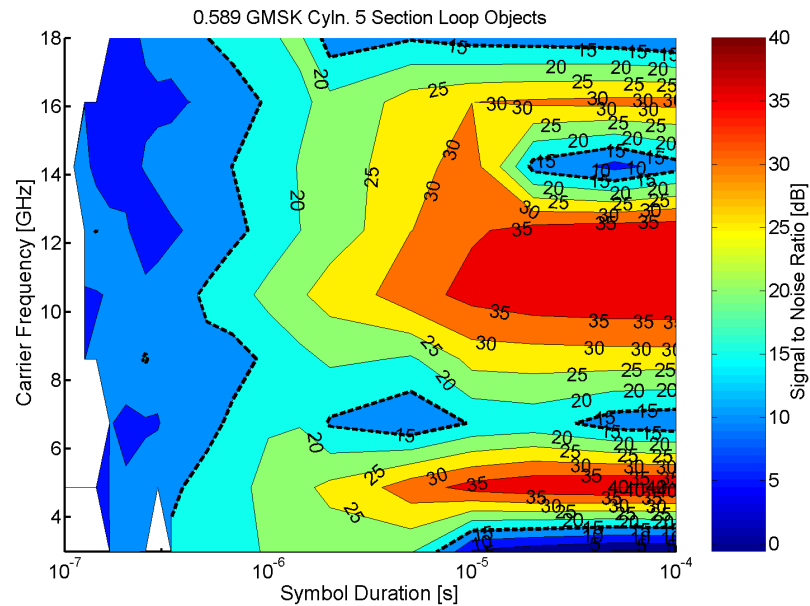


Figure 7.28: Signal to noise ratio of transmitted Gaussian filtered 0.589 GMSK waveform measured in the densely populated 5 section cylindrical platform using the loop antenna. The minimum SNR threshold required to maintain a BER of  $10^{-5}$  is 15.0 dB and is indicated by the black dashed line.

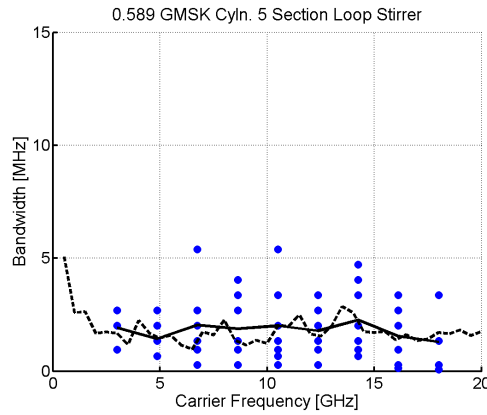


Figure 7.29: Maximum signal bandwidth transmitted using a Gaussian filtered 0.589 GMSK signal while maintaining a  $10^{-5}$  bit error rate in the sparsely populated 5 section cylindrical platform. Dots: Measured data points over rotational positions, Solid: Average of measured data points, Dashed: 50% coherence bandwidth estimated from the quality factor measured to 20.0 GHz.

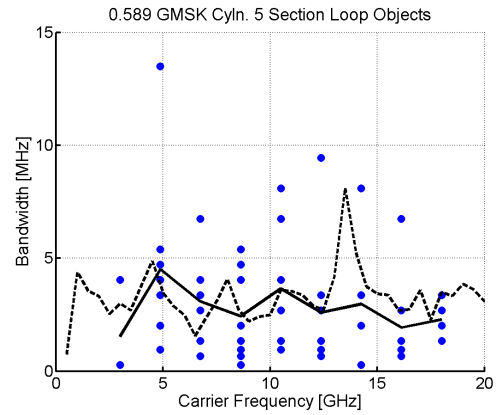


Figure 7.30: Maximum signal bandwidth transmitted using a Gaussian filtered 0.589 GMSK signal while maintaining a  $10^{-5}$  bit error rate in the densely populated 5 section cylindrical platform. Dots: Measured data points over rotational positions, Solid: Average of measured data points, Dashed: 50% coherence bandwidth estimated from the quality factor measured to 20.0 GHz.

### 0.3 GMSK

The expression for the GMSK probability of error in terms of the SNR, given previously, can be used to find the threshold SNR equal to  $\gamma_T = 18.3$  dB that is required to maintain a BER of  $10^{-5}$  (using  $\psi = 0.450$ , which corresponds to  $BT = 0.3$ ). Figures 7.31 and 7.32 show the measured SNR vs. carrier frequency and symbol duration, with the threshold SNR represented using a dashed contour on the left side of both plots. These plots show that measurements were indeed made in the dispersion-constrained channel, as the SNR values to the left of the dashed contour do not meet the 18.3 dB threshold required for  $10^{-5}$  BER operation. One also notices that the SNR values measured in the dense object populations are generally higher than the SNR values measured in the sparse object configurations, indicative of the reduction in dispersion that is characteristic for the more densely populated spaces. As pointed out for the 0.589 GMSK SNR plots, measurements at certain frequencies for the long symbol durations experience an unexplained drop in the SNR; therefore, the SNR measurements that lie just above the threshold SNR at shorter symbol durations (left side of the SNR contour plots) are used to characterize the maximum achievable symbol rate for 0.3 GMSK in the channel. The maximum signal bandwidth for 0.3 GMSK is plotted for the sparse and dense populations in Figs. 7.33 and 7.34. Similar to the 0.589 GMSK results, the maximum

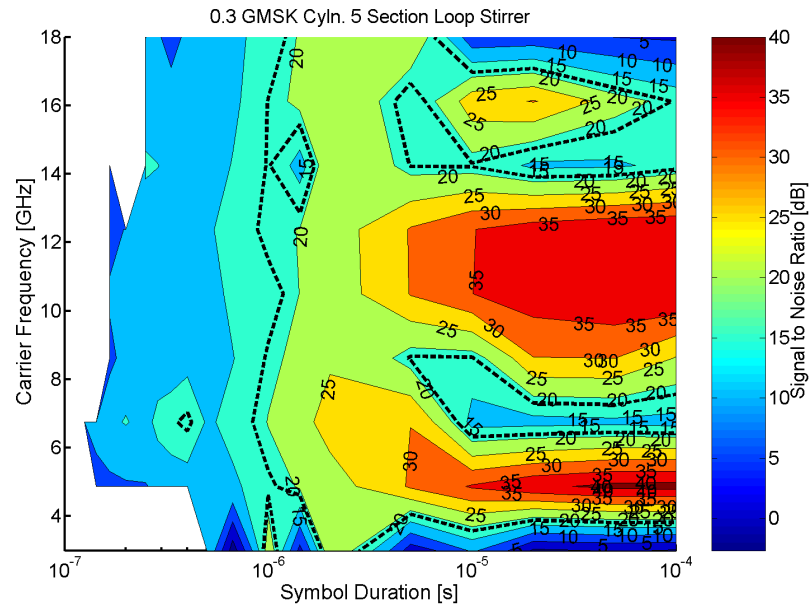


Figure 7.31: Signal to noise ratio of transmitted Gaussian filtered 0.3 GMSK waveform measured in the sparsely populated 5 section cylindrical platform using the loop antenna. The minimum SNR threshold required to maintain a BER of  $10^{-5}$  is 18.3 dB and is indicated by the black dashed line.

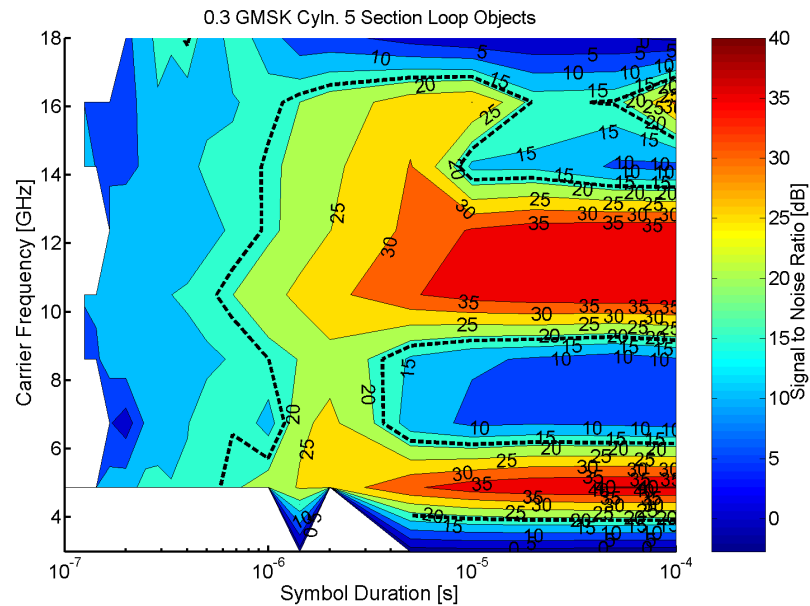


Figure 7.32: Signal to noise ratio of transmitted Gaussian filtered 0.3 GMSK waveform measured in the densely populated 5 section cylindrical platform using the loop antenna. The minimum SNR threshold required to maintain a BER of  $10^{-5}$  is 18.3 dB and is indicated by the black dashed line.

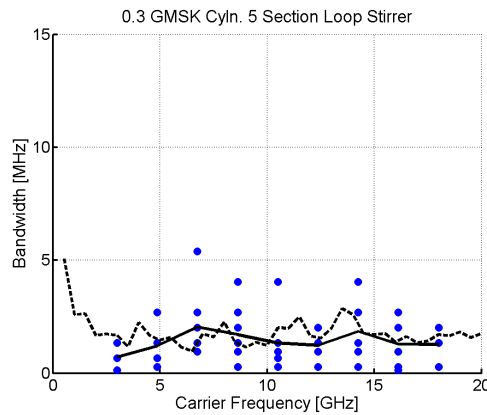


Figure 7.33: Maximum signal bandwidth transmitted using a Gaussian filtered 0.3 GMSK signal while maintaining a  $10^{-5}$  bit error rate in the sparsely populated 5 section cylindrical platform. Dots: Measured data points over rotational positions, Solid: Average of measured data points, Dashed: 50% coherence bandwidth estimated from the quality factor measured to 20.0 GHz.

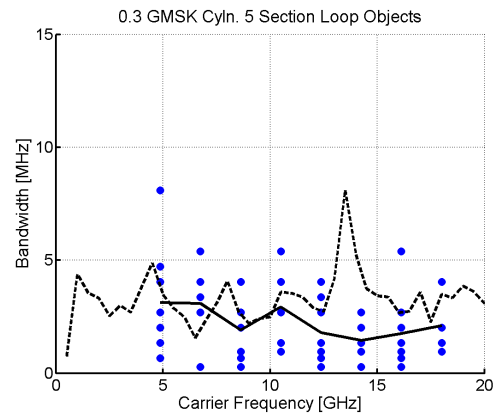


Figure 7.34: Maximum signal bandwidth transmitted using a Gaussian filtered 0.3 GMSK signal while maintaining a  $10^{-5}$  bit error rate in the densely populated 5 section cylindrical platform. Dots: Measured data points over rotational positions, Solid: Average of measured data points, Dashed: 50% coherence bandwidth estimated from the quality factor measured to 20.0 GHz.

bandwidth of the 0.3 GMSK signals is very close to the 50% coherence bandwidth estimated in the channel. However, comparing the 0.3 GMSK and 0.589 GMSK maximum bandwidth results shows that the 0.3 GMSK maximum bandwidth is less than the same measurements for 0.589 GMSK, which is expected, since a smaller BT product in Gaussian pulse shaping filters is known to cause more intrinsic ISI than larger BT products. Communications system designers should keep this behavior in mind when selecting a BT value for use with a GMSK modulation scheme.

### 7.2.4 Discussion

In the previous sections, results for the maximum bandwidth of the several examined modulation schemes were presented separately, comparing the bandwidth of each scheme to the 50% coherence bandwidth estimated for the channel. A comparison could similarly be made between the minimum symbol duration obtained while still maintaining a  $10^{-5}$  bit error rate for each modulation and the RMS delay spread estimated in the channel to 20.0 GHz. Figures 7.35 and 7.36 plot the average minimum symbol duration for each of the examined modulation formats in the 5 section sparsely and densely populated cylindrical platform, and the dashed line in both figures represents the RMS delay spread measured in the corresponding channel. Both figures show, as expected, that the most simple modulation formats (i.e. BPSK, QPSK) achieve the smallest symbol durations, and the most challenging modulation formats (i.e. 16-QAM, 64-QAM) require the longest symbol durations. This behavior is entirely consistent with the intuition that the more challenging modulation formats are more sensitive to ISI than less challenging schemes. It is interesting to note that while both the GMSK formats have the same number of bits per symbol as BPSK, the GMSK formats are unable to obtain symbol durations as small as BPSK, on average. In both Fig. 7.35 and Fig. 7.36, the RMS delay spread is approximately one order of magnitude smaller than the symbol duration averaged over all modulation formats. If the 8-PSK format were roughly considered to be a representative average, then less challenging formats such as BPSK and QPSK are less than one order larger than the RMS delay spread, and more challenging formats, such as 16-QAM and 64-QAM are more than one order larger than the RMS delay spread. However, the order of magnitude heuristic used in other channel environments (e.g. indoor, outdoor) appears to hold well, over the average of the considered modulations, for the enclosed space channel: The pan-modulation average minimum symbol duration is approximately ten times the RMS delay spread in the channel.

While the symbol time is an insightful method with which to explore the relative performance of the several examined modulation schemes, it does not account for the actual bit rates that can be obtained in the channel, which is a crucial parameter for wireless sensor network designers [4, 5, 46]. Accordingly, the maximum bit rate obtained from the minimum symbol duration while maintaining a bit error rate of  $10^{-5}$  or less for each modulation scheme is plotted in Figs. 7.37 and 7.38 for the 5 section sparsely and densely configured platforms, respectively. Several insights could

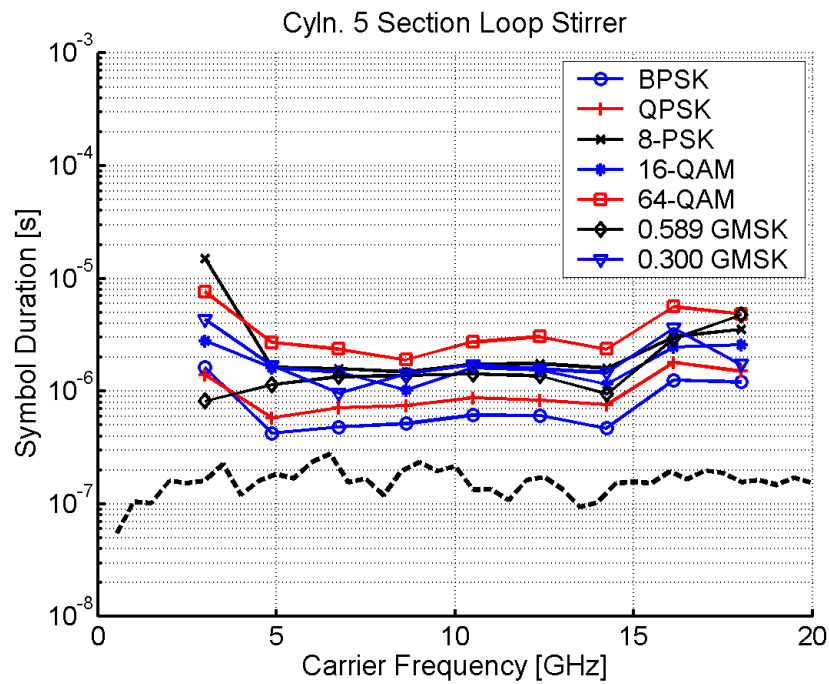


Figure 7.35: Minimum symbol time measured in the sparsely populated 5 section cylindrical platform using the loop antenna while maintaining a  $10^{-5}$  bit error rate. Dashed: RMS delay spread estimated from the quality factor measured to 20.0 GHz.

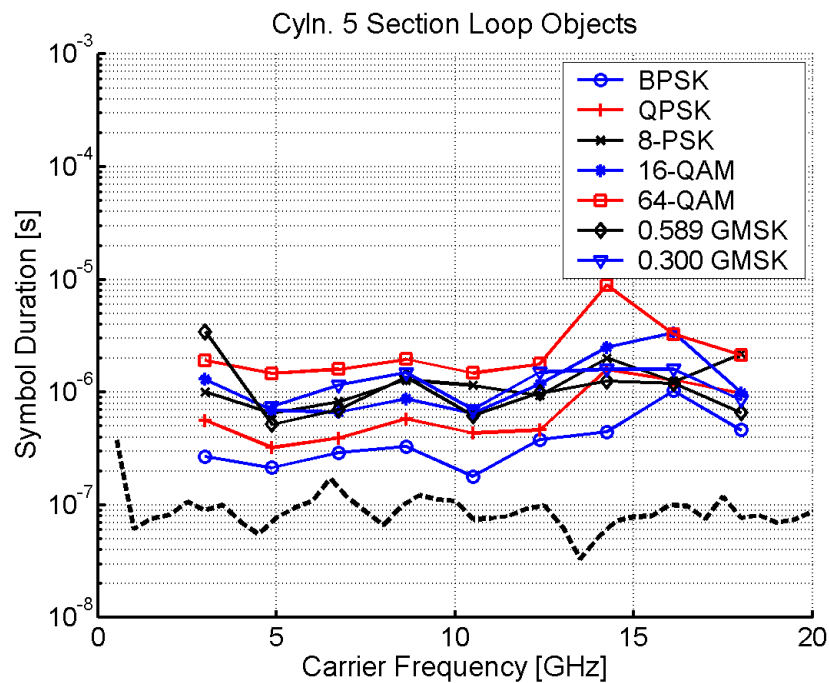


Figure 7.36: Minimum symbol time measured in the densely populated 5 section cylindrical platform using the loop antenna while maintaining a  $10^{-5}$  bit error rate. Dashed: RMS delay spread estimated from the quality factor measured to 20.0 GHz.

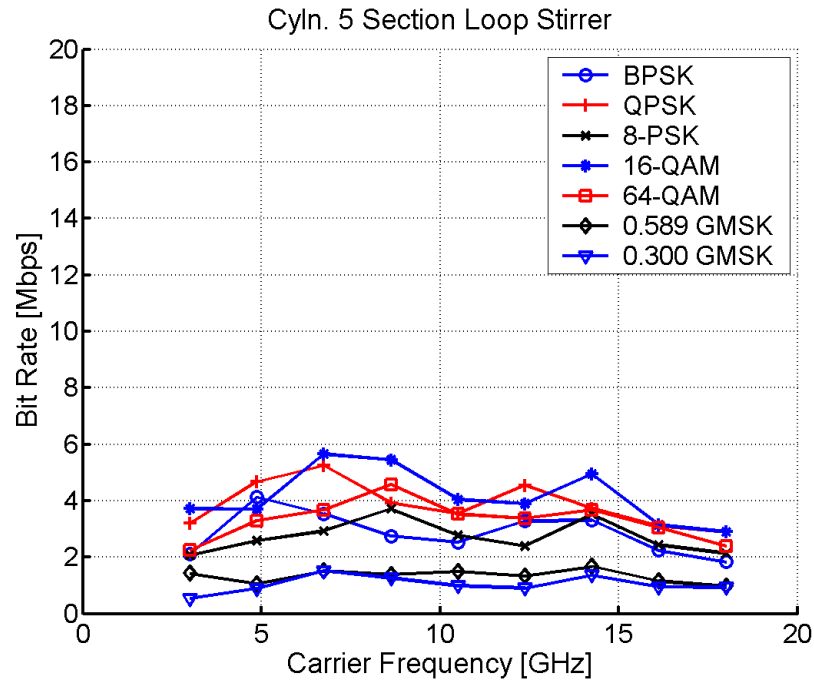


Figure 7.37: Maximum bit rate measured in the sparsely populated 5 section cylindrical platform using the loop antenna while maintaining a  $10^{-5}$  bit error rate.

be identified from these figures. It was observed in the minimum symbol time plots of Figs. 7.35 and Fig. 7.36 that in increasing order of complexity from the least challenging to the most challenging formats, the minimum symbol duration on average increased, agreeing with intuition. However, as the modulation became more challenging, the number of bits per symbol also increased. The results of the average maximum bit rate plots of Figs. 7.37 and 7.38 show that there is an optimum point in the tradeoff between maximum symbol rate and number of bits per symbol. In both experimental channels, 16-QAM is the top performer, even though it is not most bandwidth efficient modulation format used. QPSK and 64-QAM are the next best performers. BPSK—despite its simplicity—is not a top performer as measured by bit rate, and this effect is likely due to the fact that BPSK conveys only one bit of information in each symbol. Thus, if designers are attempting to obtain a maximum single carrier bit rate, a mid-grade modulation format such as 16-QAM may be the best choice.

One can also notice that the GMSK modulation formats are consistently the poorest performers in the enclosed space radio channel, suggesting that this may not be the best suited modulation format for use in the channel without equalization. For instance, the SNR thresholds for a  $10^{-5}$

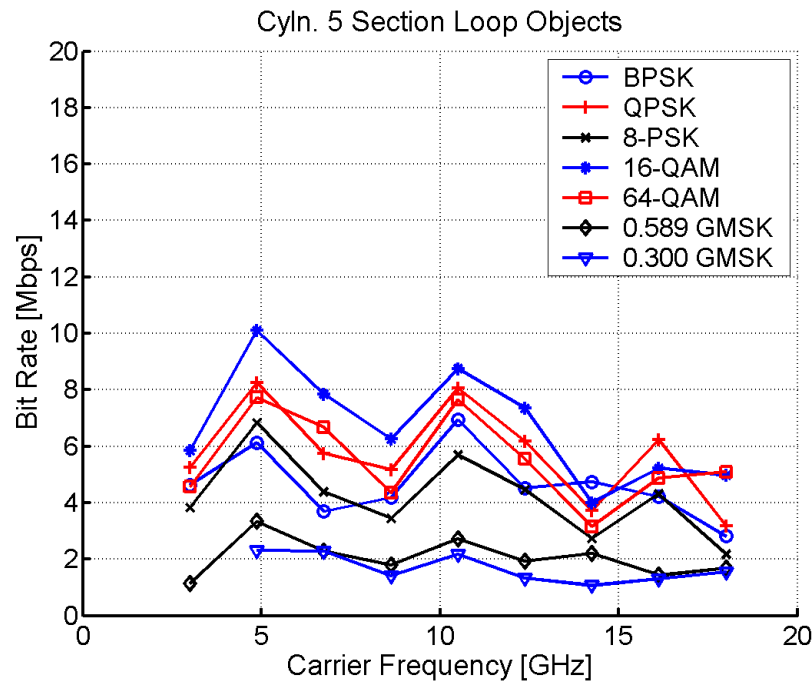


Figure 7.38: Maximum bit rate measured in the densely populated 5 section cylindrical platform using the loop antenna while maintaining a  $10^{-5}$  bit error rate.

BER using 16-QAM and 0.3 GMSK are nearly identical—18.2 dB for 16-QAM and 18.3 dB for 0.3 GMSK—but 16-QAM achieve nearly four times the maximum bit rate of 0.3 GMSK. This poorer performance could possibly be attributed to the fact that the Gaussian pulse shaping filter inherently adds ISI to the transmitted signal, compounding the ISI contributed by the dispersive channel. Nevertheless, notice that both of the GMSK formats tested are able to achieve a maximum bit rate on the order of 1-2 Mbps; if this rate is sufficient to communications network designers, GMSK could once again look attractive due to its simplicity in transmitter and receiver design and implementation.

One can clearly see that the reduction of dispersion in the densely populated environments as compared to the sparsely populated environments allows the average maximum bit rate for all modulations to increase by roughly the same factor by which the dispersion decreased (a factor of 2 in both 50% coherence bandwidth and RMS delay spread in this case). This knowledge is very valuable when modifications of a space are being considered. These results suggest that if a given space supports a maximum bit rate that is too restrictive for the intended application, a reduction in the channel dispersion by a some factor  $D$  (reducing the RMS delay spread by  $D$  or increasing the



50% coherence bandwidth by  $D$ ) will roughly increase the maximum bit rate in the space by a factor of  $D$ . As discussed previously, such a reduction in dispersion can be realized by adding absorbent material to the space, opening an aperture for energy to radiate away from the space or perhaps by adding a coat of lossy paint to the surfaces.

Finally, the results show that the average maximum bit rates measured in these results (shown in Figs. 7.37 and 7.38) are on the order of several megabits per second. Since many instrumentation and sensor networks require bit rates from 10 kbps to 1 Mbps, the bit rates of several megabits per second suggested by these results should be quite sufficient for many instrumentation and sensor network applications.

### 7.3 Multiple Carrier Modulation Schemes

Single carrier modulation schemes that do not implement equalization are limited in signal bandwidth such that the transmitted bandwidth is often on the order of, or less than, the coherence bandwidth of the channel. This effect was certainly observed in the previous section in which several single carrier modulation formats were examined experimentally. If the data rates obtained using these single carrier schemes are not sufficient for a given application, designers could obtain more bandwidth by transmitting data on more than one carrier with a bandwidth comparable to or smaller than the channel coherence bandwidth. Each of the individual transmitted narrowband signals, each with its own distinct carrier, would nominally experience flat fading (i.e. little distortion due to ISI), and the aggregate data rate obtained over all the individual transmitted signals could be quite considerable. Such transmission schemes are generally known as multicarrier modulation schemes, and one particular method by which a multicarrier transmission can be created is called orthogonal frequency division multiplexing (OFDM). The OFDM method uses an inverse Fourier transform to create the aggregate of carriers, with each carrier transmitting a distinct stream of data. If any one or group of these carriers experiences a flat fade that erodes those carriers' bit error rates, substantial data throughput can still be achieved on those other carriers that have not experienced a fade. Furthermore, since the aggregate bandwidth of the entire spectrum—including all of the carriers—will be considerably larger than the coherence bandwidth, it is unlikely that the entire set of carriers will experience a simultaneous degradation, thereby implementing a form of frequency diversity.

| Modulation | Coding Rate | Data Rate |
|------------|-------------|-----------|
| BPSK       | 1/2         | 6 Mbps    |
| BPSK       | 3/4         | 9 Mbps    |
| QPSK       | 1/2         | 12 Mbps   |
| QPSK       | 3/4         | 18 Mbps   |
| 16-QAM     | 1/2         | 24 Mbps   |
| 16-QAM     | 3/4         | 36 Mbps   |
| 64-QAM     | 2/3         | 48 Mbps   |
| 64-QAM     | 3/4         | 54 Mbps   |

Table 7.1: Modulation, coding rate and information data rate implemented in the IEEE 802.11a wireless standard [3].

The IEEE 802.11a wireless LAN communications standard implements the OFDM scheme with 64 subcarriers in region of spectrum between 5.0 and 6.0 GHz, in several different bands. Of those 64 carriers, 12 are used as guard band and 4 are used as pilot signals; the remaining 48 subcarriers are used for data transmission [3]. Each of the subcarriers is 312.5 kHz wide, which is considerably smaller than the coherence bandwidths found in the indoor wireless channel from 5.0 to 6.0 GHz, and four different modulations can be used on the 48 subcarriers (all subcarriers use the same chosen modulation): BPSK, QPSK, 16-QAM and 64-QAM [3]. The 802.11a standard implements a symbol rate of 250 ksps and uses a convolutional coding and interleaving scheme over the 48 subcarriers to obtain an information data rate given by Table 7.1. It is clear that the effect of having multiple carriers transmitting data in parallel on all 48 subcarriers can realize some very substantial data rates in a dispersive wireless channel. Similar good results were found using multicarrier communication techniques in the heating, ventilation and air conditioning (HVAC) duct channel, which can display RMS delay spreads on the order of 100 ns [54].

Now consider the coherence bandwidths measured in the enclosed space wireless channel. Chapter 5 presented measured results of the 50% coherence bandwidth on average between 1 and 5 MHz—bandwidths that are considerably larger than the 312.5 kHz bandwidth of the subcarriers used in the 802.11a wireless standard. As a result, it seems plausible that a 802.11a wireless transmission could perform very well in the dispersive enclosed space radio channel, as measured by the throughput data rate obtained. Motivated by this possibility, 802.11a signals generated by the digital signal generator were transmitted through the enclosed space channel using the same experimental platform as was used for the single carrier experiments of this chapter, depicted in Fig. 7.2.

The vector signal analyzer once again measures the RMS error vector magnitude of the received symbols of all of the subcarriers, from which a signal to noise ratio can be obtained. For each of the modulation formats used in the 802.11a standard (i.e. BPSK, QPSK, 16-QAM or 64-QAM), the measured SNR can be compared to the threshold SNR required to maintain a  $10^{-5}$  bit error rate. Accordingly, Fig. 7.39 plots the measured SNR results of transmitting an 802.11a signal into the sparsely populated 5 section cylinder using the loop antennas, examining all four modulation formats. The points represent the average SNR measured at each center frequency (from 3.0 GHz to 18.0 GHz in 500 MHz steps) and averaged over each rotational position (20 total) of the internal mode stirrer. The solid lines represent the threshold SNR required to obtain a  $10^{-5}$  bit error rate for each of the modulation formats implemented by the standard. A transmit power of approximately -16 dBm is used for these experiments. The results show that there is more than adequate SNR to operate using the BPSK and QPSK formats, and the 16-QAM format often experiences an adequate SNR. The measured results are not, however, sufficient for operation using the 64-QAM format. Nonetheless, it seems that throughput bit rates as high as 36 Mbps using the 16-QAM format with the  $\frac{3}{4}$ -rate convolutional code (see Table 7.1) are possible in this sparsely configured enclosed space channel, which displays RMS delay spreads on the order of 200-300 ns.

Figure 7.40 shows the results of transmitting the 802.11a signal into the densely populated 5 section cylinder using the loop antennas and measuring the received SNR vs. center frequency, averaged across the object assembly's rotational position. These results indicate that the measured SNR is very adequate for operation using the BPSK, QPSK and 16-QAM formats, and may be sufficient for operation using the 64-QAM standard, given a transmit power of approximately -18 dBm (at the output of the up-converter). Data rates are therefore possible at the full 54 Mbps using 64-QAM at the  $\frac{3}{4}$ -rate convolutional code in this densely populated enclosed space channel with an RMS delay spread on the order of 100-200 ns. Both 36 Mbps and 54 Mbps are considerably larger than the bit rates on the order of 5 Mbps that were observed using the single carrier modulation schemes, and it seems that 802.11a and other multicarrier modulation formats could be considerable tools with which designers of wireless sensor networks in enclosed space environments could increase the data rates achieved between wireless network nodes. This increase, however, comes at a price, since the OFDM format requires an implementation of the inverse Fourier transform, which is typically done using digital signal processing hardware. This added expense in terms of both

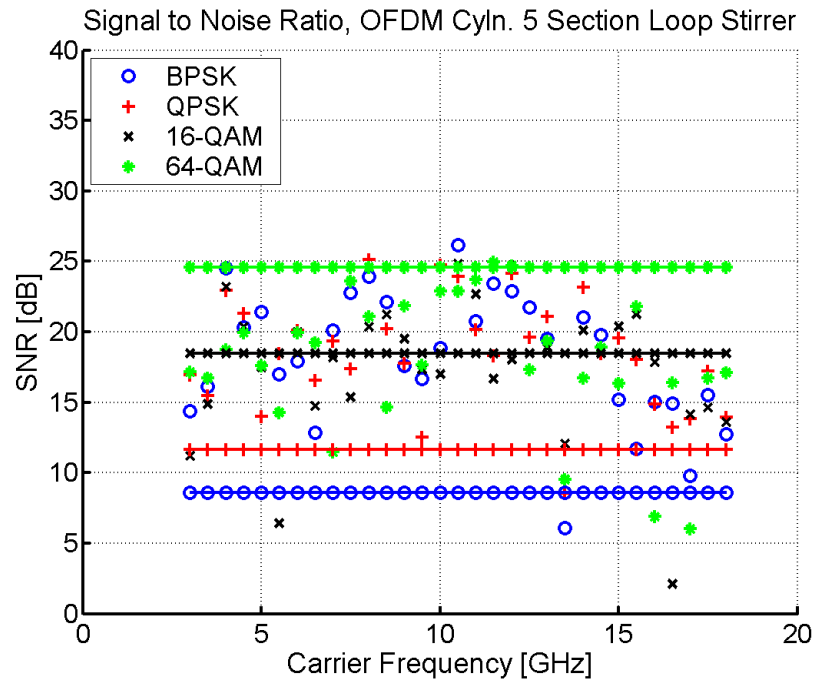


Figure 7.39: Signal to noise ratio measured in the sparsely populated 5 section cylindrical platform using the loop antenna for the four different constitutive modulations of the 802.11a OFDM scheme from 3.0 to 18.0 GHz. Symbols: Measured SNR for each of the modulation formats tested. Solid: Threshold SNR required to maintain a BER of  $10^{-5}$ .

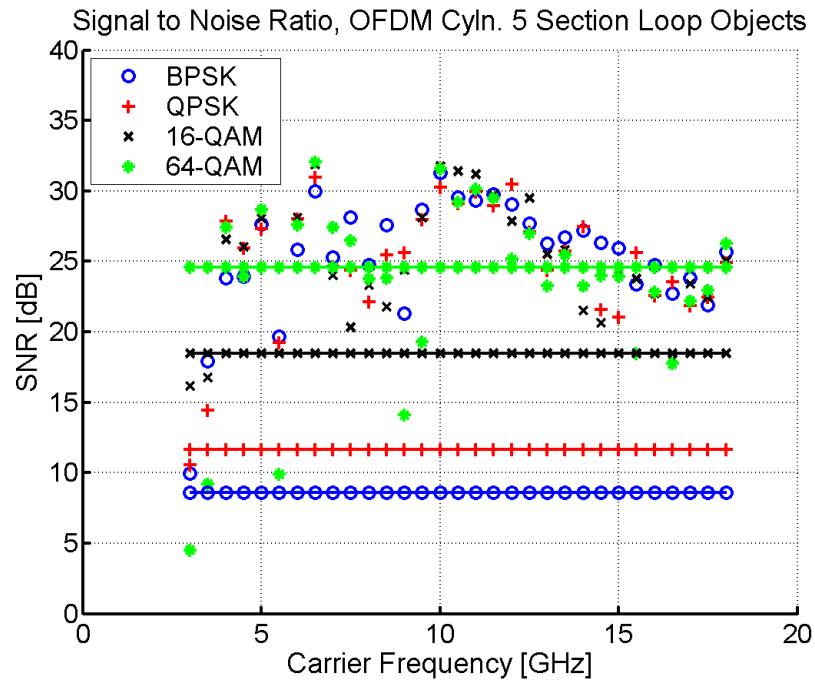


Figure 7.40: Signal to noise ratio measured in the densely populated 5 section cylindrical platform using the loop antenna for the four different constitutive modulations of the 802.11a OFDM scheme from 3.0 to 18.0 GHz. Symbols: Measured SNR for each of the modulation formats tested. Solid: Threshold SNR required to maintain a BER of  $10^{-5}$ .

development costs and node energy consumption may not provide an adequate tradeoff in a wireless sensor network where inexpensive and low-power nodes are paramount.

## Chapter 8

# Conclusions and Future Work

Enclosed space environments do indeed provide a viable channel for wireless communications. As stated in the introduction, *the principle goal of the research presented in this dissertation is to devise a generally applicable channel model based on physical principles to predict the channel dispersion and gain, given an arbitrary enclosed space environment.* The dispersion and gain of the enclosed space radio channel have been considered at length through their mutual connection to the enclosure quality factor. The behavior of the dynamic range of the channel has also been characterized and described. The results described in this thesis provide the first-known description of the enclosed space channel dispersion and gain, and this description has been empirically verified in representative environments, at length. Even though the experiments conducted in this work focused on enclosed space environments of cylindrical and conical geometry, the theory used to describe and model the results have no such geometric restriction, and investigators believe that the theory should hold well for other geometries of enclosed space environments. Signal communications experiments have provided an insight into the communications potential of the enclosed space radio channel, showing some promising single-carrier and multicarrier data rates. The highlights of the results discussed in this dissertation are presented below, discussing the practical implications of the contributions, where appropriate. After this summary of the key contributions and their implications, a summary of the steps required to characterize an arbitrary enclosed space will be given. Finally, a discussion of potential future work and relevant insights will be presented.

## 8.1 Summary of Contributions

→ **Average RMS delay spreads of 100-300 ns observed.**

These average values of RMS delay spread measured in the enclosed space channel environments explored in this work are greater than RMS delay spreads typically found within the indoor office and home radio channel (10-100 ns on average [17, 23, 24, 27]) and smaller than RMS delay spreads often observed in the outdoor radio channel (1-5  $\mu$ s on average [8, 9, 10]). Measurements of the RMS delay spread in factory and open indoor environments has been identified to be on the order of 100 ns [29]. While the 100-300 ns average RMS delay spreads exhibited by the measured enclosed space channel environments may not be a challenge for wireless RF devices designed to endure the delay spreads native to the outdoor mobile environment, they could present a challenge to wireless RF devices intended for indoor environments and investigations of any specific devices would be advisable; however, there is no evidence to suggest a certain failure of such devices to withstand these delay spreads. The reader should bear in mind that the delay spreads measured in this body of work are somewhat associated with the overall volume and surface area of the enclosed space platforms used in this work (see Chapter 3 for descriptions of the space). Spaces that are much larger than these platforms will likely show larger RMS delay spreads than those measured inside the platforms pertaining to this work. However, spaces with dimensions on the same order of the platforms used in this work contained by metallic walls without any other significant source of loss (e.g. large apertures, absorbing internal materials, etc.) are expected to display similar RMS delay spreads. Sparsely populated internal object environments display higher RMS delay spread values than environments with dense internal object populations.

→ **Average 50% coherence bandwidths of 1-5 MHz observed.**

These values of coherence bandwidth are greater than the corresponding measurements in the outdoor mobile propagation channel that display average 50% coherence bandwidths on the order of 50-200 kHz [10, 114] and less than the 50% coherence bandwidths measured in the indoor propagation channel that display on average 5-20 MHz bandwidths [115, 116]. The measurements in the enclosed space radio channel once again show that devices that are designed to operate in the outdoor environment are likely to work well in the enclosed



space radio channel from a dispersion perspective, but devices designed for an indoor propagation channel require some caution, as they may have been designed to operate in channels that support larger signal bandwidths before frequency selective fading (distortion) become significant. As the overall volume of the enclosure increases and/or the number and arrangement of internal objects become more sparse, the 50% coherence bandwidth will decrease, supporting smaller narrowband signal bandwidths without distortion in the channel.

→ **Power gain values between -10dB and -30 dB observed.**

Results show that the power gain in the enclosed space radio channel is substantially larger than the indoor or outdoor communications channel, which can often have channel gains smaller than -90 dB for the indoor channel and less than -100 dB for the outdoor mobile channel. In comparison, the gains of the enclosed space radio channel are massive, and if deployed in an enclosed space environment, it is likely that the front-end of any RF receiver communicating with a transmitter that is designed for the indoor or outdoor radio environment will be overwhelmed with the amount of power received. Two of several possible outcomes include the saturation of the receiver's low-noise amplifier or railing of the receiver's automatic gain control (AGC) circuit, and designers of wireless sensor and instrumentation networks for use in reverberant enclosures should take steps to prevent this outcome, perhaps by reducing the transmitted power, or by attenuating the received signal of a COTS device being considered for use in such an environment. It is convenient, however, that such a large power gain is observed in this channel. Wireless sensor and instrumentation networks are required to consume little energy resources. Such large gains allow designers to consider, for instance, wireless transceiver designs that do not have transmit-side power amplifiers (PAs) or have reduced bias currents on the receive-side low-noise amplifiers (LNAs). Both the PA and the LNA typically consume large amounts of energy and reducing their energy consumption and/or eliminating these devices altogether could dramatically reduce the stored energy requirements at each transceiver node.

→ **Dispersion given by enclosure net quality factor.**

The average dispersion of the enclosed space radio channel—as measured by the RMS delay spread, the mean excess delay, and the 50% and 90% coherence (correlation) bandwidths—

has been analytically and empirically shown to be directly related to the quality factor associated with a given enclosed space environment. Specifically, the average RMS delay spread and average mean excess delay are directly proportional to the net quality factor (see Chapter 5) of the enclosed space, and the average coherence bandwidths are inversely proportional to the net quality factor. Since these estimates predict the average value of dispersion in a given channel, they represent the average dispersion observed by a large number of communications nodes located randomly throughout the enclosure, or equivalently, the average value of dispersion that a single communication node might experience if it were placed in a large number of random positions in the enclosure. Reverberation chamber theory (presented in Chapter 2) has shown that as the volume of a reverberant enclosure increases, so does its net quality factor and hence its dispersion. Conversely, decreases in volume will cause a decrease in the net quality factor, decreasing the dispersion seen in the channel. As the volume of the enclosed space becomes increasingly consumed with the volume of internal objects, the net quality factor will decrease, causing a decrease in the channel dispersion. As a result, sparsely configured enclosures (i.e. enclosures with only a small percentage of the contained volume consumed by the objects' volume) will be generally more dispersive than densely configured enclosures (i.e. enclosures in which a large percentage of the enclosed volume is consumed by enclosed objects). Decreases in the conductivity of the metallic enclosure surfaces and/or object surfaces will cause the quality factor to decrease—as will any other source of loss, such as apertures, or absorbing material located inside the enclosure—thereby causing the dispersion to decrease. Measurements of the dispersion parameters and estimates of the same parameters from the enclosure net quality factor are shown in this work to be very well matched, and the small variance of the measured dispersion parameters (across the rotational positions of the internal objects/stirrer) as compared to the estimates for the same parameters indicates that the predictions are strongly representative of the actual dispersion that a receiver might experience in the enclosed space channel.

→ **Gain given by enclosure net quality factor.**

The gain of the enclosed space radio channel is directly proportional to the net quality factor of the enclosure, a result that is well-known in the theory of reverberation chambers. As

discussed in Chapter 6, an estimate for the net quality factor of an enclosure together with knowledge of the antenna quality factor (which requires an estimate of the enclosure free volume and the antenna  $S_{11}$ ) allows the gain in the channel to be estimated. These estimates closely match the empirically measured power gains (using the  $S_{21}$ ) in the enclosed space platforms in this work, allowing knowledge of the enclosure quality factor to provide estimates for not only the channel dispersion but also the channel gain.

→ **Dispersion and gain tradeoff via enclosure quality factor.**

Since the enclosed space channel dispersion and gain are both related to the net enclosure quality factor, a fundamental tradeoff is defined. As the enclosure quality factor increases, the gain increases, as does the channel dispersion. Conversely, as the quality factor decreases, the channel dispersion decreases, but so does the channel gain. This tradeoff gives the architects of wireless sensor and instrumentation networks in the enclosed space channel a valuable tool with which apply subtle changes and achieve considerable results. For example, it is shown in Chapter 6 that the gain of the enclosed space channel is quite large as compared to other radio channels. The dispersion is also large in comparison to a channel such as the indoor channel. By modifying the space in a non-obtrusive way (e.g. by painting some or all of the internal surfaces with a lossy paint), designers are able to reduce the dispersion in the channel while reducing what was perhaps more than sufficient gain without significantly modifying the original space. The gain may still be quite large, yet the dispersion might be reduced to levels consummate with indoor channels, all without appreciably altering the space.

→ **Dynamic range is bounded and independent of average gain.**

The dynamic range of the enclosed space radio channel was measured and characterized in Chapter 6 and was shown to have a bounded behavior independent of the mean gain of the channel. The dynamic range of the power gain describes the variation that the power gain experiences in the channel; it is a description of the difference between the peaks and valleys in the channel frequency transfer function. For instance, it was shown that 95% of the power gain points measured in a given channel (measured over different points in space) lie within a 21.63 dB range, centered about the median value, independent of whatever the mean value happens to be. These results hold in the limit of large specific mode density (i.e.

the number of electromagnetic modes excited by a CW signal at a given frequency), which increases continuously as frequency increases, volume increases or quality factor decreases. This bounded dynamic range is very useful information to a physical layer radio designer, as it limits the range over which an input automatic gain control (AGC) or analog to digital converter (ADC) must operate in order to reliably maintain a physical layer link between wireless sensor nodes.

→ **Overmoded regions better suited for communications than undermoded regions.**

The overmoded regions of operation are named as such since a CW signal at any point in frequency in the overmoded region excites more than one electromagnetic mode. In this region, the half-power bandwidths of the individual modes are wider than the average frequency spacing between the modes, causing the modes' spectra to overlap such that no single resonant peak belonging to an individual mode can be identified. This contrasts the characteristics of the undermoded region, in which the resonant peaks associated with individual resonant modes are clearly identifiable. If the undermoded region were to be used for communications, not only would a transmitter/receiver pair be required to find and track the location of an arbitrary resonant peak (which could move due to changes in the environment), but the pair would also be restricted to the narrow half-power bandwidth of the mode, which can be on the order of 100 kHz for reverberant spaces. Chapter 3 discusses these two regions in more detail and shows a channel frequency response in both regions. The overmoded region has the additional advantage that a transmitted signal can be arbitrarily placed across frequency in the region, and that signal is unlikely to find a power gain between transmitter and receiver outside of a certain range, i.e. the dynamic range (studied and characterized in Chapter 6). Conversely, the undermoded region has decaying-mode bands between resonant peaks that have extremely low power gains since energy in these frequency bands does not resonate in the space but rather decays exponentially between the transmitter and the receiver. This causes the dynamic range in the undermoded region to be very large as compared to the overmoded region, and it is unlikely that a receiver would be able to track such large changes in gain. However, it is interesting to consider communications in these decaying-mode regions using larger antennas that would couple a transmitter and receiver together in

a non-propagating channel. Nonetheless, designers need to find the boundary between the undermoded and overmoded regions of operation. The boundary between these two regions is known as the Rayleigh frequency, and a discussion of regions of operation in Section 3.4 relates this boundary to the net quality factor and estimated volume of the enclosed space.

→ **Arbitrary channel environments quickly and simply estimated using RF pulses and detector.**

The quality factor of an enclosed space can be very simply estimated using RF pulses transmitted at the frequency of interest and detected using a simple square law detector, as described in Chapter 3. The quality factor is measured by examining the decay rate of the turn-off or turn-on step response of the channel, as measured on the detector, when the channel is excited with such pulses. Since the detector measures the envelope of the RF signal only, the RF source is neither required to be extremely phase stable nor is it required to have very small drift values, allowing inexpensive voltage controlled oscillator designs to be considered. Furthermore, the duty cycle of the RF pulses is required to be much longer than the response time of the channel (so it is fully settled), requiring unchallenging pulse repetition rates on the order of several microseconds or more. These rates are easily achieved using straightforward timing circuitry. Inexpensive approaches can thus be used to estimate the enclosed space net quality factor, from which the channel gain and dispersion directly follow, as shown in this work.

→ **Single-carrier data rates up to 5 Mbps obtainable.**

Narrowband single carrier data transmission without equalization were demonstrated in Chapter 7 using real digital signals transmitted in the enclosed space platforms. While the results confirmed the intuition that more challenging modulations such as QAM-64 are more vulnerable to the effects of inter-symbol interference than less challenging modulations such as BPSK, the results also show that there is a middle ground where a mid-grade modulation format such as QAM-16 achieves the best overall bit rate. In this sense, the QAM-16 modulation format has enough bits per symbol to achieve higher data rates, but the format is not so complex that it is the most vulnerable to distortion. Comparing the data rates achieved in the sparse and dense internal object populations, one notices that the dense configuration achieves

approximately twice the average maximum data rate as the sparse configuration, roughly corresponding to the factor-of-two difference in the corresponding RMS delay spreads of the sparse and dense configurations. Though these results are somewhat specific to the 100-300 ns delay spreads measured in this channel, similar single-carrier data rates could be anticipated for other similar enclosed space channels. Data rates anywhere from 10 kbps to 1 Mbps are often required for sensor and instrumentation networks, suggesting that the average 5 Mbps rates are sufficient for wireless operation of these networks.

→ **Multicarrier modulation dramatically increases data rates.**

As discussed in Chapter 7, it seems clear that the IEEE 802.11a standard (which uses an OFDM modulation scheme) is well suited to operate in enclosed space radio environments whose dimensions are similar to the experimental platforms used in these investigations (see Chapter 3). The bandwidth of each subcarrier is 312.5 kHz, which is less than the average single carrier bandwidths reliably transmitted through the channel and considerably less than the measured average 50% coherence bandwidths of 1-5 MHz. Utilization of the 802.11a standard could allow data rates as large as 36 to 54 Mbps, depending on the actual delay spread of the enclosure. These data rates come with the price of the OFDM scheme used in 802.11a, which requires digital signal processing hardware to implement an inverse Fourier transform. Such hardware can carry high development costs as well as high energy consumption costs, which may not be well-suited for inexpensive, low power communications nodes.

## 8.2 Summary of Steps to Characterize an Arbitrary Enclosure

Chapters 5 and 6 showed that the average channel dispersion (i.e. RMS delay spread, mean excess delay, 50% and 90% coherence bandwidths) seen by a receiver (averaged over a large ensemble of different locations that a wireless transmitter and receiver pair could be located throughout an enclosure of arbitrary shape) and the average power gain seen between a transmitter and receiver can be predicted using knowledge of the intended frequency of operation and an estimate of the enclosure net quality factor. Chapter 3 described an experimental setup that uses an RF pulse generator to simply estimate the net quality factor of an arbitrary enclosed space, and the results of using such an experimental technique to measure the quality factor were given in Chapter 4. Using the

previously mentioned knowledge, a summary of steps required to estimate the dispersion and gain of a RF wireless channel inside a reverberant enclosure is presented as follows:

**1. Estimate the enclosure quality factor.**

Investigators would first be required to implement or acquire a simple channel sounding device that excites the channel with RF pulses and measures the decay of the energy in the channel (e.g. using a VCO and square-law detector) over the interested range of frequencies. The slope of the energy decay profile gives the net quality factor of the enclosure, and an ensemble average of quality factor measurements should be made over a variety of locations in the enclosure so that a representative average quality factor is obtained.

**2. Locate the start of the overmoded region.**

Before the dispersion and gain of the channel can be estimated, investigators need to identify the boundary between the undermoded and overmoded regions of the enclosure (called the Rayleigh frequency and was considered in Section 3.4). Recall that the overmoded region is better suited for enclosed space communications on account of the implicit frequency diversity granted by the overmoded conditions. Also recall that the expressions derived for the dispersion and gain in the enclosed space channel are only rigorously valid in the overmoded region. The expression in (3.5) allows investigators to find the Rayleigh frequency, using the estimated net quality factor (obtained from the slope of the energy decay profile), the frequency of operation and an estimate of the free volume inside the enclosure. The overmoded region includes those frequencies which are larger than the Rayleigh frequency.

**3. Estimate the dispersion and gain using the derived expressions.**

This average quality factor estimated from the slope of the energy decay is used in the derived expressions to find estimates for the channel dispersion and gain via (5.12) for the RMS delay spread, (5.7) for the mean excess delay, (5.21) for the 50% coherence bandwidth, (5.22) for the 90% coherence bandwidth and (6.1) for the gain. Estimating the enclosure gain additionally requires an estimate of the receive antenna quality factor, which is given in (6.3) for an antenna located near the enclosure wall or other metallic boundary; if the receiver antenna is located far away from a metallic boundary, the expression in (2.49) should instead be used. Both of these expressions for the antenna Q additionally require an estimate of the encl-

sure volume and knowledge of the antenna impedance match, which can be measured using a network analyzer (via the  $|S_{11}|$  scattering parameter).

#### 4. **Explore channel performance over frequency.**

Since the exact performance of the channel—as measured by the dispersion and gain parameters—will change over frequency, a characterization of the enclosure radio channel by measuring the quality factor at several frequencies will help investigators to identify the most desirable frequency operation point. The expression for the dispersion and gain in the enclosed space channel generally show that as the frequency increases, the loss due to the enclosure's metal walls will increase, and the gain and dispersion will therefore decrease. As the frequency increases and the point of operation moves more deeply into the overmoded region, the dynamic range of power gain becomes bounded, as described in Chapter 6.

### 8.3 Future Work

Future work studying the enclosed space radio channel that is envisioned by the investigators is detailed below. Where appropriate, insights into the next steps and relevant resources to investigating those steps are included in the discussion.

#### → **Explore the channel across boundaries and through apertures.**

The topics studied in this dissertation have considered communications in enclosed space environments under the condition that all communications nodes are part of the same overall enclosure. In other words, while a line-of-sight condition may not exist within the enclosure, no two nodes are considered to be isolated from one another via a bulkhead or other such metallic boundary. It is quite possible, however, that such a situation exists in a realistic enclosed space environment; an example would be the boundary created between the interior of an aircraft wing and the interior of the fuselage. Other such boundaries could include bulkheads separating different compartments inside a small submarine hull. If these boundaries were perfect electromagnetic boundaries (i.e. no seams or apertures), there would be no coupling of energy between the adjoined resonant spaces, and communications would be unable to take place between the spaces. However, there will almost certainly be seams or apertures



in the boundary between two resonant spaces, i.e. seams in the metal between fasteners or apertures to allow services to be fed between each space. These seams, however small, could allow electromagnetic energy to couple between each space and therefore allow communications between wireless nodes on each side of the boundary. Several questions can be asked with regards to the nature of this modified enclosed space communications channel. What is the modified power gain between communications nodes on either side of the boundary? What is the dispersion seen in the channel between the boundary-separated wireless nodes? The coupling of power between two connected reverberant enclosures has been considered by the reverberation chamber community and is known as *shielding effectiveness* [97, 106]. As the name suggests, the shielding effectiveness measures how well a particular boundary shields energy from a connected enclosure or external environment. By its nature, the shielding effectiveness must also measure the amount of power that couples into a space from an adjacent space or external environment, and this directly relates to the gain that two communications nodes would experience across the boundary. Just as the dispersion in a single reverberant enclosure is related to the net quality factor of that enclosure, it seems reasonable that the dispersion seen between two coupled reverberant enclosures is related to the modified net quality factor in each enclosure that results from the coupling. The studies reported in [97, 106] lend important insight into deriving such a modified net quality factor. As discussed previously, communications are best suited for the overmoded region in frequency, and the boundary between the undermoded and overmoded regions (i.e. the Rayleigh frequency) is a function of the net quality factor and volume of the enclosure. When two reverberation chambers are coupled together, each with a different volume, the boundary between the overmoded and undermoded regions within each space will be different. Thus, it will be important to define a modified Rayleigh frequency that describes the undermoded/overmoded boundary of the coupled enclosures. Bethe did some of the first work considering the coupling of energy through small apertures [117], and an understanding of this work may prove helpful to solving the problem of finding the modified Rayleigh frequency. Coupling through apertures and seams of complex geometry that are commonly found in the environments of aircraft or submarines has also been considered in [118].

→ **Explore antenna diversity and MIMO systems.**

As motivated in the introduction of this report, wireless sensor nodes will be required to operate reliably in the positions in which they are initially placed in the enclosed space. If the communications link between any two nodes was unsatisfactory in their initial placement and the frequency of operation was not allowed to be re-selected (so as to obtain a better channel), the link would effectively be lost between those two nodes. A method that is used in the indoor and outdoor channel to mitigate this risk is spatial diversity using multiple antennas. A similar approach using multiple antennas that is used to improve the capacity of a wireless channel is known as multiple-input, multiple-output (MIMO) techniques. Both spatial diversity and MIMO techniques require the knowledge of the spacial correlation functions of the fields and magnitude-squared fields inside a reverberant enclosed space. Chapter 2 references several of the works that have considered the spatial correlation functions of the fields inside an enclosure and several of the important results have been included in the discussion. In fact, reverberation chambers are currently being used to test the diversity/MIMO performance of wireless products designed for the indoor and outdoor communications channel [73, 74, 75]. Future work that considers spatial diversity or MIMO techniques to enhance the reliability or capacity of communications in reverberant enclosed space radio channel could be very contributive, and the substantial theory in the literature provides a very good starting point.

→ **Examine decaying-mode region communications.**

As mentioned in a footnote in Chapter 3, communications can also be considered in frequency regions in the undermoded region, and/or below the frequency of the enclosure's fundamental mode. Energy does not propagate in these regions but rather decays exponentially over distance. However, it was observed during the campaign of these experiments that the strength of coupling between two antennas with significant length (i.e. using two linear monopole antennas of approx. 5 cm length) can be considerable (i.e. approx. -40 dB) as measured using the vector network analyzer near the fundamental mode (400 MHz for the 5 section cylindrical platform). These significant coupling values could be particularly utilized to obtain communications in spaces whose enclosure volume is so small that communications devices would be required to use high operation frequencies in order to communicate in the overmoded fre-

quency region of the enclosure. Such high operation frequencies may not be available or cost-effective, and decaying-mode communications could offer a viable alternative.

→ **Characterize spaces with larger geometries.**

The environments studied in this dissertation are relatively small, each less than several meters in each dimension. Communications in much larger reverberant enclosures is a realistic scenario and could include rooms in a large ship, rooms within a large submarine, large shipping containers or perhaps interior of the fuselage of a large cargo aircraft. The theory developed and presented in this work could be directly used to predict the behavior of the communications channel found in these spaces. Furthermore, the reverberation chamber theory that describes the composite quality factor as a function of volume (given in (2.46)) is not associated with any specific geometry or volume of a reverberant enclosure; it only requires the assumption that the energy density over the entire enclosure be uniform. As such, an application of the composite quality factor theory to large reverberant enclosures (e.g. rooms in a ship or large submarine, shipping containers, cargo aircraft) is very plausible and could allow an a priori prediction of the dispersion and gain within these spaces when combined with the dispersion and gain theory described in Chapters 5 and 6 of this report. Experimental campaigns within these larger enclosed spaces would be very intriguing to explore.

→ **Explore the capacity limits of the enclosed space channel.**

The results of experimental campaign described and presented in Chapter 7 found the *empirical maximum bit rate* that could be supported by the enclosed space radio channel, given the values of dispersion present in the experimental platforms utilized in the campaign. It would also be interesting to derive the *theoretical maximum bit rate* for the enclosed space radio channel. This theoretical description would give a information capacity limit for the enclosed space radio channel, taking as inputs the channel dispersion (e.g. via RMS delay spread) and the power received from the channel (i.e. from the channel power gain descriptions) and would yield as an output the maximum bit rate for a chosen modulation scheme. A theoretical maximum bit rate for single carrier modulation schemes would require knowledge of the aggregate noise distribution, including noise from thermal sources (i.e. AWGN) and noise from multipath reception (i.e. inter-symbol interference). This aggregate noise distribution

could be derived from the description of the power delay profile (i.e. exponential profile) and the statistics of the individual multipath amplitude and phase values. A theoretical maximum bit rate for multicarrier modulation schemes used in the enclosed space radio channel could also be obtained using a similar approach as investigators did when deriving the maximum bit rate for a multicarrier modulation scheme in the HVAC radio channel using M-QAM on all of its subcarriers (where the assumption is made that the subcarrier bandwidth is less than the channel coherence bandwidth) [46].

## Appendix A

# Eigenfunctions and Resonances of the Regular Cavity Resonators

The solutions of the electric and magnetic fields in the rectangular, cylindrical and spherical resonators represent a fundamental solution set for each of the three corresponding coordinate systems. Accordingly, the fields for these fundamental cavity resonators and their resonant frequencies will be solved and presented in the following discussion.

Maxwell's equations in a source-free environment,

$$\nabla \times \bar{E} = -j\omega\mu\bar{H} \quad (\text{A.1})$$

$$\nabla \times \bar{H} = j\omega\epsilon\bar{E} \quad (\text{A.2})$$

$$\nabla \cdot \bar{E} = 0 \quad (\text{A.3})$$

$$\nabla \cdot \bar{H} = 0, \quad (\text{A.4})$$

can be combined using vector potentials to form the vector Helmholtz equations [102, eq. 3-81],

$$\nabla^2 \bar{F} + k^2 \bar{F} = 0 \quad (\text{A.5})$$

$$\nabla^2 \bar{A} + k^2 \bar{A} = 0, \quad (\text{A.6})$$

where  $F$  is the electric vector potential and  $A$  is the magnetic vector potential, and the relationship

between the vector potentials and electric and magnetic field vectors is as follows [102, eq. 3-83]:

$$\bar{E} = -\nabla \times \bar{F} - j\omega\mu\bar{A} + \frac{1}{j\omega\epsilon}\nabla(\nabla \cdot \bar{A}) \quad (\text{A.7})$$

$$\bar{H} = \nabla \times \bar{A} - j\omega\epsilon\bar{F} + \frac{1}{j\omega\mu}\nabla(\nabla \cdot \bar{F}). \quad (\text{A.8})$$

Depending on the coordinate system in question, the  $\nabla^2$  can be expanded in the scalar Helmholtz equation (a single component  $\psi$  of the vector Helmholtz equation (A.6)), given as follows:

$$\nabla^2\psi + k^2\psi = 0. \quad (\text{A.9})$$

A product solution of the form  $\psi(u_1, u_2, u_3) = U_1(u_1)U_2(u_2)U_3(u_3)$  is then assumed, where  $u_1$ ,  $u_2$  and  $u_3$  are the coordinates in the specified coordinate system, and substitution of this solution into (A.9) and dividing by the solution itself produces a separable differential equation in three variables. The solution of this differential equation is called the *mode function*, and is specific to each polarization (TE or TM) for each coordinate system. This mode function can be substituted into the vector potential equations in (A.7) and (A.8) to obtain tentative field descriptions for the coordinate system under consideration. To obtain complete field and resonant frequency solutions, perfectly conducting walls ( $\sigma \rightarrow \infty$ ) are assumed, and boundary conditions are thus imposed at the walls:

$$\hat{n} \times \bar{E} = 0 \quad (\text{A.10})$$

$$\hat{n} \cdot \bar{H} = 0, \quad (\text{A.11})$$

where  $\hat{n}$  is a normal vector to the surface of the wall. The eigenvalues of the member functions for each coordinate dimension of the mode function are chosen to satisfy the boundary conditions specified above. The eigenvalues can therefore take only a specific set of values, and these values determine—along with the dispersion relations and field expressions derived from above—the specific eigenfunctions and eigenfrequencies of the cavity in question. In what follows, the rectangular, cylindrical and spherical cavity solutions corresponding to the same coordinate systems are presented. A conical cavity resonator is a special version of a spherical resonator and has solutions

in the spherical coordinate system; the solutions for this resonator are presented in Appendix B.

## A.1 Rectangular Cavity Resonators

For rectangular coordinates, substituting the assumed solution  $\psi(x, y, z) = X(x)Y(y)Z(z)$  into (A.9) and dividing by the same assumed solution gives [102, eq. 4-3]

$$\frac{1}{X(x)} \frac{d^2 X(x)}{dx^2} + \frac{1}{Y(y)} \frac{d^2 Y(y)}{dy^2} + \frac{1}{Z(z)} \frac{d^2 Z(z)}{dz^2} + k^2 = 0. \quad (\text{A.12})$$

Solution of this separable differential equation is straightforward, and the solutions are the harmonic functions in all dimensions:

$$\frac{d^2 X(x)}{dx^2} + k_x^2 X(x) = 0 \quad (\text{A.13})$$

$$\frac{d^2 Y(y)}{dy^2} + k_y^2 Y(y) = 0 \quad (\text{A.14})$$

$$\frac{d^2 Z(z)}{dz^2} + k_z^2 Z(z) = 0, \quad (\text{A.15})$$

whose separation constants lead to the dispersion relation in the rectangular resonator:

$$k^2 = k_x^2 + k_y^2 + k_z^2. \quad (\text{A.16})$$

Application of the boundary conditions leads to the mode functions for the TM and TE polarizations in a rectangular cavity of dimensions  $a$ ,  $b$  and  $d$  in the  $x$ ,  $y$  and  $z$  directions (as shown in Fig. A.1), respectively [102, eqs. 4-37 & 4-39]:

$$\psi_{mnp}^{TM}(x, y, z) = \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \cos \frac{p\pi z}{d} \quad (\text{A.17})$$

$$\psi_{mnp}^{TE}(x, y, z) = \cos \frac{m\pi x}{a} \cos \frac{n\pi y}{b} \sin \frac{p\pi z}{d} \quad (\text{A.18})$$

The mode function in (A.17) is then substituted into (A.7) and (A.8) with  $\bar{A} = \hat{z}H_0\psi_{mnp}^{TM}(x, y, z)$  and

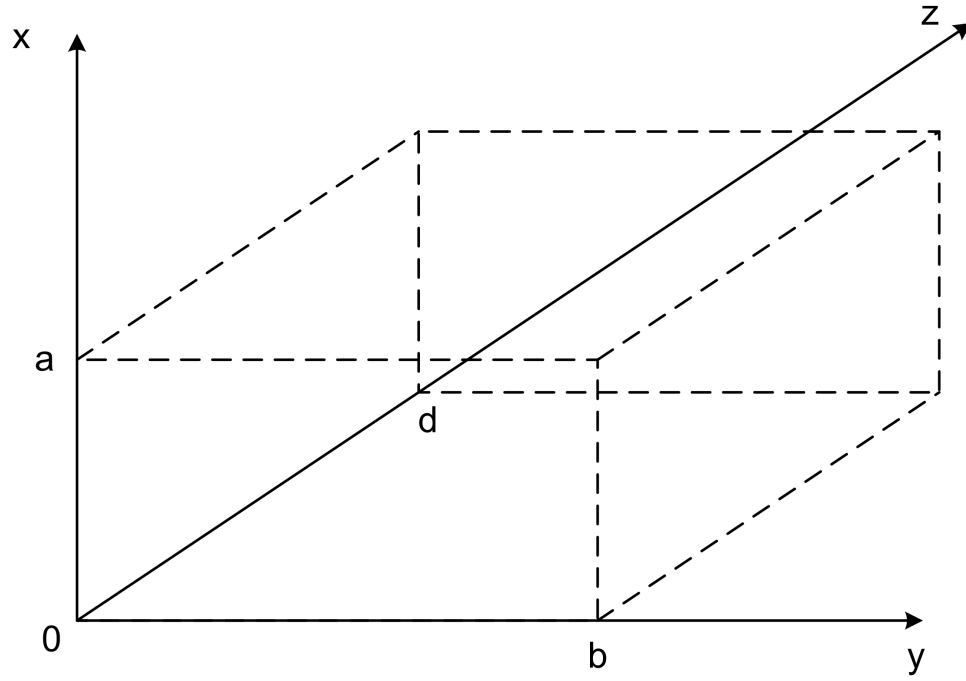


Figure A.1: Geometry of the rectangular cavity resonator shown in the rectangular coordinate system.

$\bar{F} = 0$  to solve for the fields that are TM to the  $\hat{z}$ -direction in the rectangular cavity:

$$E_x = \frac{-H_0}{j\omega\epsilon} \frac{mp\pi^2}{ad} \cos \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \sin \frac{p\pi z}{d} \quad (\text{A.19})$$

$$E_y = \frac{-H_0}{j\omega\epsilon} \frac{np\pi^2}{bd} \sin \frac{m\pi x}{a} \cos \frac{n\pi y}{b} \sin \frac{p\pi z}{d} \quad (\text{A.20})$$

$$E_z = \frac{H_0}{j\omega\epsilon} \left( \omega^2\mu\epsilon - \left( \frac{p\pi}{d} \right)^2 \right) \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \cos \frac{p\pi z}{d} \quad (\text{A.21})$$

$$H_x = H_0 \frac{n\pi}{b} \sin \frac{m\pi x}{a} \cos \frac{n\pi y}{b} \cos \frac{p\pi z}{d} \quad (\text{A.22})$$

$$H_y = -H_0 \frac{m\pi}{a} \cos \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \cos \frac{p\pi z}{d} \quad (\text{A.23})$$

$$H_z = 0. \quad (\text{A.24})$$

Similarly, the fields that are TE to the  $\hat{z}$ -direction are found by substituting the mode function in



(A.18) into (A.7) and (A.8) with  $\bar{F} = \hat{z}E_0\psi_{mnp}^{TE}(x, y, z)$  and  $\bar{A} = 0$ :

$$E_x = E_0 \frac{n\pi}{b} \cos \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \sin \frac{p\pi z}{d} \quad (\text{A.25})$$

$$E_y = -E_0 \frac{m\pi}{a} \sin \frac{m\pi x}{a} \cos \frac{n\pi y}{b} \sin \frac{p\pi z}{d} \quad (\text{A.26})$$

$$E_z = 0 \quad (\text{A.27})$$

$$H_x = \frac{-E_0}{j\omega\mu} \frac{mp\pi^2}{ad} \sin \frac{m\pi x}{a} \cos \frac{n\pi y}{b} \cos \frac{p\pi z}{d} \quad (\text{A.28})$$

$$H_y = \frac{-E_0}{j\omega\mu} \frac{np\pi^2}{bd} \cos \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \cos \frac{p\pi z}{d} \quad (\text{A.29})$$

$$H_z = \frac{E_0}{j\omega\mu} \left( \omega^2 \mu \epsilon - \left( \frac{p\pi}{d} \right)^2 \right) \cos \frac{m\pi x}{a} \cos \frac{n\pi y}{b} \sin \frac{p\pi z}{d}. \quad (\text{A.30})$$

The resonant frequencies of the rectangular cavity are given by the following expression, immediately derived from the dispersion relation  $k^2 = k_x^2 + k_y^2 + k_z^2$  (where  $k_x = m\pi/a$ ,  $k_y = n\pi/b$ , and  $k_z = p\pi/d$ ):

$$f_r(m, n, p) = \frac{\nu}{2\pi} \sqrt{\frac{m\pi}{a} + \frac{n\pi}{b} + \frac{p\pi}{d}}, \quad (\text{A.31})$$

where  $m = 0, 1, \dots$ ;  $n = 0, 1, \dots$ ;  $p = 1, 2, \dots$  for the TE modes ( $m$  and  $n$  cannot both be zero),  $m = 1, 2, \dots$ ;  $n = 1, 2, \dots$ ;  $p = 0, 1, \dots$  for the TM modes, and  $\nu$  is the speed of light in the medium. The fundamental mode for the rectangular cavity whose dimensions meet the criteria that  $a < b < d$  is the  $TE_{011}$  mode. When  $m \neq 0$ ,  $n \neq 0$  and  $p \neq 0$ , the TE and TM modes are degenerate, thereby having identical resonant frequencies, albeit distinct field structure. Since  $m$ ,  $n$  and  $p$  have no upper bound, there are a triply infinite number of resonances supported by each polarization (TE and TM) of the rectangular cavity; this is an important property that is true for any arbitrarily shaped resonator.

## A.2 Cylindrical Cavity Resonators

In a similar fashion to the rectangular coordinate system, the assumed solution,  $\psi(\rho, \phi, z) = R(\rho)\Phi(\phi)Z(z)$ , is substituted into the scalar Helmholtz equation, (A.9); dividing by the assumed solution yields

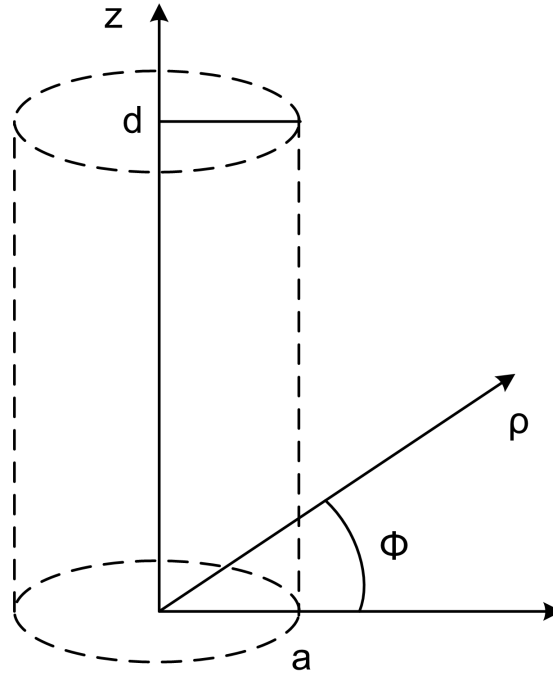


Figure A.2: Geometry of the cylindrical cavity resonator shown in the cylindrical coordinate system.

[102, eqs. 5-1 & 5-2]

$$\frac{1}{\rho R(\rho)} \frac{d}{d\rho} \left( \rho \frac{dR(\rho)}{d\rho} \right) + \frac{1}{\rho^2 \Phi(\phi)} \frac{d^2 \Phi(\phi)}{d\phi^2} + \frac{1}{Z(z)} \frac{d^2 Z(z)}{dz^2} + k^2 = 0. \quad (\text{A.32})$$

Using the separation of variables technique, this differential equation is separated into three individual differential equations [102, eq. 5-7]:

$$\frac{d^2 Z(z)}{dz^2} + k_z^2 Z(z) = 0 \quad (\text{A.33})$$

$$\frac{d^2 \Phi(\phi)}{d\phi^2} + n^2 \Phi(\phi) = 0 \quad (\text{A.34})$$

$$\rho \frac{d}{d\rho} \left( \rho \frac{dR(\rho)}{d\rho} \right) + [(k_\rho \rho)^2 - n^2] R(\rho) = 0, \quad (\text{A.35})$$

which are connected by the dispersion relation for the cylindrical coordinate system:

$$k^2 = k_\rho^2 + k_z^2. \quad (\text{A.36})$$

The solutions of the above differential equations are the Bessel functions in the  $\hat{\rho}$ -dimension,

and the harmonic functions in the  $\hat{\phi}$  and  $\hat{z}$  dimensions. Applying the boundary conditions and using the vector potentials in (A.7) and (A.8), the mode functions for the TM and TE polarizations (respect to the  $\hat{z}$ -direction) of a cylindrical resonator of radius  $a$  and length  $d$  (as shown in Fig. A.2) can be found [102, eqs. 5-53 & 5-54]:

$$\psi_{npq}^{TM}(\rho, \phi, z) = J_n\left(\frac{x_{np}\rho}{a}\right) \begin{Bmatrix} \sin n\phi \\ \cos n\phi \end{Bmatrix} \cos\left(\frac{q\pi z}{d}\right) \quad (\text{A.37})$$

$$\psi_{npq}^{TE}(\rho, \phi, z) = J_n\left(\frac{x'_{np}\rho}{a}\right) \begin{Bmatrix} \sin n\phi \\ \cos n\phi \end{Bmatrix} \sin\left(\frac{q\pi z}{d}\right). \quad (\text{A.38})$$

In the above mode functions,  $J_n(x)$  is the  $n^{\text{th}}$  order Bessel function of the first kind,  $J'_n(x)$  is its derivative and  $x_{np}$  and  $x'_{np}$  are the  $p^{\text{th}}$  roots of the  $n^{\text{th}}$  order Bessel function of the first kind and its derivative, respectively. The sine and cosine functions in the braces are degenerate solutions about the rotational coordinate  $\phi$ . Substituting (A.37) into (A.7) and (A.8), letting  $\bar{A} = \hat{z}H_0\psi_{npq}^{TM}(\rho, \phi, z)$  and  $\bar{F} = 0$ , give the complete field solutions for the TM polarization in the cylindrical resonator:

$$E_\rho = \frac{-H_0}{j\omega\epsilon} \frac{x_{np}q\pi}{ad} J'_n\left(\frac{x_{np}\rho}{a}\right) \begin{Bmatrix} \sin n\phi \\ \cos n\phi \end{Bmatrix} \sin \frac{q\pi z}{d} \quad (\text{A.39})$$

$$E_\phi = \frac{-H_0}{j\omega\epsilon\rho} \frac{nq\pi}{d} J_n\left(\frac{x_{np}\rho}{a}\right) \begin{Bmatrix} \cos n\phi \\ -\sin n\phi \end{Bmatrix} \sin \frac{q\pi z}{d} \quad (\text{A.40})$$

$$E_z = \frac{H_0}{j\omega\epsilon} \left( \omega^2\mu\epsilon - \left(\frac{q\pi}{d}\right)^2 \right) J_n\left(\frac{x_{np}\rho}{a}\right) \begin{Bmatrix} \sin n\phi \\ \cos n\phi \end{Bmatrix} \cos \frac{q\pi z}{d} \quad (\text{A.41})$$

$$H_\rho = H_0 \frac{n}{\rho} J_n\left(\frac{x_{np}\rho}{a}\right) \begin{Bmatrix} \cos n\phi \\ -\sin n\phi \end{Bmatrix} \cos \frac{q\pi z}{d} \quad (\text{A.42})$$

$$H_\phi = H_0 \frac{-x_{np}}{a} J'_n\left(\frac{x_{np}\rho}{a}\right) \begin{Bmatrix} \sin n\phi \\ \cos n\phi \end{Bmatrix} \cos \frac{q\pi z}{d} \quad (\text{A.43})$$

$$H_z = 0. \quad (\text{A.44})$$

Similarly, the solutions for the TE (respect to  $\hat{z}$ ) polarization can be found by substituting (A.38)

into (A.7) and (A.8), letting  $\bar{F} = \hat{z}E_0\psi_{npq}^{TE}(\rho, \phi, z)$  and  $\bar{A} = 0$ :

$$E_\rho = -E_0 \frac{n}{\rho} J_n \left( \frac{x'_{np}\rho}{a} \right) \begin{Bmatrix} \cos n\phi \\ -\sin n\phi \end{Bmatrix} \sin \frac{q\pi z}{d} \quad (\text{A.45})$$

$$E_\phi = E_0 \frac{x'_{np}}{a} J'_n \left( \frac{x'_{np}\rho}{a} \right) \begin{Bmatrix} \sin n\phi \\ \cos n\phi \end{Bmatrix} \sin \frac{q\pi z}{d} \quad (\text{A.46})$$

$$E_z = 0 \quad (\text{A.47})$$

$$H_\rho = \frac{E_0}{j\omega\mu} \frac{x'_{np}q\pi}{ad} J'_n \left( \frac{x'_{np}\rho}{a} \right) \begin{Bmatrix} \sin n\phi \\ \cos n\phi \end{Bmatrix} \cos \frac{q\pi z}{d} \quad (\text{A.48})$$

$$H_\phi = \frac{E_0}{j\omega\mu\rho} \frac{nq\pi}{d} J_n \left( \frac{x'_{np}\rho}{a} \right) \begin{Bmatrix} \cos n\phi \\ -\sin n\phi \end{Bmatrix} \cos \frac{q\pi z}{d} \quad (\text{A.49})$$

$$H_z = \frac{E_0}{j\omega\mu} \left( \omega^2\mu\epsilon - \left( \frac{q\pi}{d} \right)^2 \right) J_n \left( \frac{x'_{np}\rho}{a} \right) \begin{Bmatrix} \sin n\phi \\ \cos n\phi \end{Bmatrix} \sin \frac{q\pi z}{d}. \quad (\text{A.50})$$

$$(\text{A.51})$$

The resonance frequencies of the TM and TE cylindrical resonator modes are found from the dispersion relation,  $k^2 = k_\rho^2 + k_z^2$ , where  $k_\rho = x'_{np}/a$  and  $k_z = q\pi/d$ :

$$f_r^{TM}(n, p, q) = \frac{\nu}{2\pi} \sqrt{\left( \frac{x_{np}}{a} \right)^2 + \left( \frac{q\pi}{d} \right)^2} \quad (\text{A.52})$$

$$f_r^{TE}(n, p, q) = \frac{\nu}{2\pi} \sqrt{\left( \frac{x'_{np}}{a} \right)^2 + \left( \frac{q\pi}{d} \right)^2}. \quad (\text{A.53})$$

Recall that  $\nu$  is the speed of light in the medium filling the cavity. For the TM modes the allowable index values are  $n = 0, 1, 2, \dots$ ;  $p = 1, 2, 3, \dots$ ;  $q = 0, 1, 2, \dots$ , and for the TE modes the allowable values are  $n = 0, 1, 2, \dots$ ;  $p = 1, 2, 3, \dots$ ;  $q = 1, 2, 3, \dots$ . In the situation where  $d \geq 2a$ , the  $TE_{111}$  mode is the fundamental mode of the cavity ( $x'_{11} = 1.8412$ ); if  $d \leq 2a$ , then the  $TM_{010}$  will be the fundamental mode ( $x_{01} = 2.4048$ ).

### A.3 Spherical Cavity Resonators

The spherical coordinate system gives rise to such resonant enclosures as the spherical cavity, the conical cavity and others. The assumed product solution to the scalar Helmholtz equation in spherical coordinates is  $\psi(r, \theta, \phi) = R(r)\Theta(\theta)\Phi(\phi)$  and substitution into (A.9) and division by the assumed

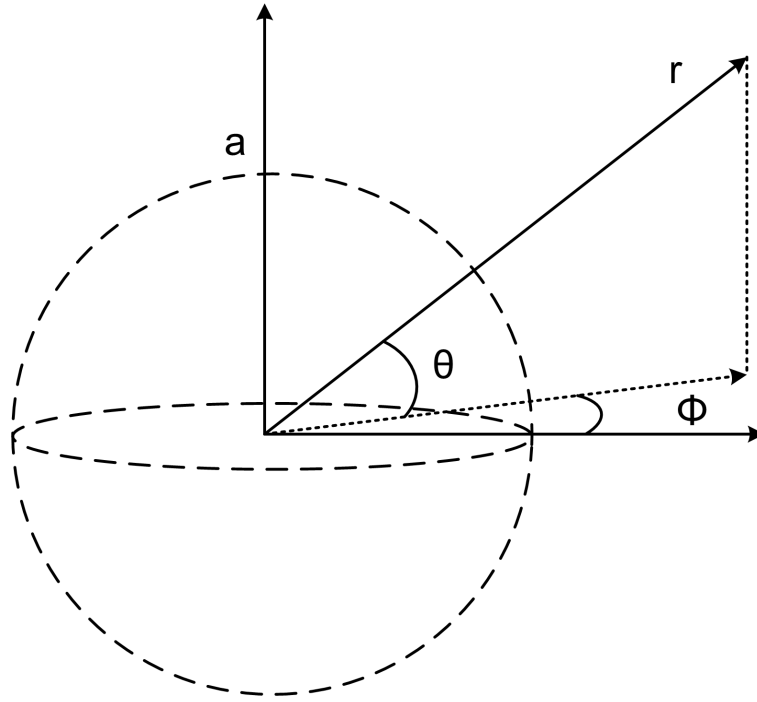


Figure A.3: Geometry of the spherical cavity resonator shown in the spherical coordinate system.

solution gives the following differential equation [102, eqs. 6-1 & 6-2]:

$$\frac{\sin^2 \theta}{R(r)} \frac{d}{dr} \left( r^2 \frac{dR(r)}{dr} \right) + \frac{\sin \theta}{\Theta(\theta)} \frac{d}{d\theta} \left( \sin \theta \frac{d\Theta(\theta)}{d\theta} \right) + \frac{1}{\Phi(\phi)} \frac{d^2 \Phi(\phi)}{d\phi^2} + k^2 r^2 \sin^2 \theta = 0. \quad (\text{A.54})$$

The solution by means of separation of variables can be applied as it was with the rectangular and cylindrical coordinate systems and are found to be

$$\frac{d}{dr} \left( r^2 \frac{dR(r)}{dr} \right) + [(kr)^2 - n(n+1)]R(r) = 0 \quad (\text{A.55})$$

$$\frac{1}{\sin \theta} \frac{d}{d\theta} \left( \sin \theta \frac{d\Theta(\theta)}{d\theta} \right) + \left[ n(n+1) - \frac{m^2}{\sin^2 \theta} \right] \Theta(\theta) = 0 \quad (\text{A.56})$$

$$\frac{d^2 \Phi(\phi)}{d\phi^2} + m^2 \Phi(\phi) = 0. \quad (\text{A.57})$$

These three separated differential equations have solutions of the spherical Bessel functions in the  $\hat{r}$  dimension, the associated Legendre functions in the  $\hat{\theta}$  dimension and the harmonic functions in the  $\hat{\phi}$  dimension. If a spherical cavity with radius  $a$  is considered (as shown in Fig. A.3), the solution to (A.54)—the mode functions for the TM and TE polarizations (transverse to the increasing

direction of  $r$ )—are found by applying the boundary conditions through the evaluation of the vector potentials in (A.7) and (A.8) [102, eqs. 6-30 & 6-33]:

$$\psi_{mnp}^{TM}(r, \theta, \phi) = \hat{J}_n\left(\frac{u'_{np}r}{a}\right) P_n^m(\cos \theta) \begin{Bmatrix} \cos n\phi \\ \sin n\phi \end{Bmatrix} \quad (\text{A.58})$$

$$\psi_{mnp}^{TE}(r, \theta, \phi) = \hat{J}_n\left(\frac{u_{np}r}{a}\right) P_n^m(\cos \theta) \begin{Bmatrix} \cos n\phi \\ \sin n\phi \end{Bmatrix}. \quad (\text{A.59})$$

In the above expressions,  $\hat{J}_n(x)$  is the  $n^{\text{th}}$  integer order alternative spherical Bessel function of the first kind and  $u_{np}$  and  $u'_{np}$  are the  $p^{\text{th}}$  roots of the  $n^{\text{th}}$  order alternative spherical Bessel function of the first kind its first derivative.  $P_n^m(x)$  is the  $m^{\text{th}}$  order,  $n^{\text{th}}$  degree associated Legendre function of the first kind. Substitution of (A.58) into (A.7) and (A.8), letting  $\bar{A} = \hat{r}H_0\psi_{mnp}^{TM}(r, \theta, \phi)$  and letting  $\bar{F} = 0$  leads to the TM solutions for the spherical cavity resonator:

$$E_r = \frac{H_0}{j\omega\epsilon} \left( \omega^2\mu\epsilon + \left(\frac{u'_{np}}{a}\right)^2 \right) \hat{J}_n'\left(\frac{u'_{np}r}{a}\right) P_n^m(\cos \theta) \begin{Bmatrix} \cos m\phi \\ \sin m\phi \end{Bmatrix} \quad (\text{A.60})$$

$$E_\theta = \frac{-H_0}{j\omega\epsilon r} \frac{u'_{np}}{a} \hat{J}_n\left(\frac{u'_{np}r}{a}\right) P_n^m(\cos \theta) \sin \theta \begin{Bmatrix} \cos m\phi \\ \sin m\phi \end{Bmatrix} \quad (\text{A.61})$$

$$E_\phi = \frac{H_0}{j\omega\epsilon r \sin \theta} \frac{u'_{np}m}{a} \hat{J}_n\left(\frac{u'_{np}r}{a}\right) P_n^m(\cos \theta) \begin{Bmatrix} -\sin m\phi \\ \cos m\phi \end{Bmatrix} \quad (\text{A.62})$$

$$H_r = 0 \quad (\text{A.63})$$

$$H_\theta = H_0 \frac{m}{r \sin \theta} \hat{J}_n\left(\frac{u'_{np}r}{a}\right) P_n^m(\cos \theta) \begin{Bmatrix} -\sin m\phi \\ \cos m\phi \end{Bmatrix} \quad (\text{A.64})$$

$$H_\phi = \frac{H_0}{r} \hat{J}_n\left(\frac{u'_{np}r}{a}\right) P_n^m(\cos \theta) \sin \theta \begin{Bmatrix} \cos m\phi \\ \sin m\phi \end{Bmatrix}. \quad (\text{A.65})$$

The solutions for the TE modes in the spherical cavity can be similarly found via substitution of

(A.59) into (A.7) and (A.8), letting  $\bar{F} = \hat{r}E_0\psi_{mnp}^{TE}(r, \theta, \phi)$  and  $\bar{A} = 0$ :

$$E_r = 0 \quad (\text{A.66})$$

$$E_\theta = E_0 \frac{-m}{r \sin \theta} \hat{J}_n \left( \frac{u_{np}r}{a} \right) P_n^m(\cos \theta) \begin{Bmatrix} -\sin m\phi \\ \cos m\phi \end{Bmatrix} \quad (\text{A.67})$$

$$E_\phi = \frac{-E_0}{r} \hat{J}_n \left( \frac{u_{np}r}{a} \right) P_n^m(\cos \theta) \sin \theta \begin{Bmatrix} \cos m\phi \\ \sin m\phi \end{Bmatrix} \quad (\text{A.68})$$

$$H_r = \frac{E_0}{j\omega\mu} \left( \omega^2\mu\epsilon + \left( \frac{u_{np}}{a} \right)^2 \right) \hat{J}_n' \left( \frac{u_{np}r}{a} \right) P_n^m(\cos \theta) \begin{Bmatrix} \cos m\phi \\ \sin m\phi \end{Bmatrix} \quad (\text{A.69})$$

$$H_\theta = \frac{-E_0}{j\omega\mu r} \frac{u_{np}}{a} \hat{J}_n' \left( \frac{u_{np}r}{a} \right) P_n^m(\cos \theta) \sin \theta \begin{Bmatrix} \cos m\phi \\ \sin m\phi \end{Bmatrix} \quad (\text{A.70})$$

$$H_\phi = \frac{E_0}{j\omega\mu r \sin \theta} \frac{u_{np}m}{a} \hat{J}_n' \left( \frac{u_{np}r}{a} \right) P_n^m(\cos \theta) \begin{Bmatrix} -\sin m\phi \\ \cos m\phi \end{Bmatrix}. \quad (\text{A.71})$$

The resonant frequencies of the spherical cavity are given by the following expressions, again utilizing the roots of the spherical Bessel function of the first kind and its derivative,  $u_{np}$  and  $u'_{np}$  [102]:

$$f_r^{TM}(m, n, p) = \frac{v}{2\pi} \frac{u'_{np}}{a} \quad (\text{A.72})$$

$$f_r^{TE}(m, n, p) = \frac{v}{2\pi} \frac{u_{np}}{a}. \quad (\text{A.73})$$

Recall that  $v$  is the speed of light in the medium filling the cavity. The allowable index values for the TM and TE modes are  $m = 0, 1, 2, \dots$ ;  $n = 1, 2, 3, \dots$ ;  $q = 1, 2, 3, \dots$  and  $m \leq n$ , since  $P_n^m(x)$  only exists under those index restrictions. The fundamental mode of the spherical cavity is the  $TM_{m11}$  mode ( $u'_{np} = 2.744$ ), which is infinitely degenerate in the index  $m$  (notice the resonant frequency does not depend on  $m$ ). As it is for the cylindrical cavity, the spherical cavity has an additional degeneracy in the  $\phi$  rotational variable, for all index values, except  $m = 0$ .

## Appendix B

# Eigenfunctions and Resonances of a Truncated Conical Resonator

The solutions for the fields inside a truncated conical metallic cavity resonator are very similar to the solutions of the spherical cavity resonator, whose solutions are well known [102]. One of the key differences is the additional boundary conditions added by the slanted side-walls of the conical cavity resonator and the truncation of the cone at its small end. Solutions for the truncated conical cavity resonator have been previously considered in [119], where investigators pursued the truncated cone resonator to test the conductivity of several super-conducting thin films by measuring the quality factor of the  $TE_{011}$  mode. Solutions for this specific mode were presented, along with charts comparing the resonant frequencies of several mode families as functions of the cone length, width and half-cone angle.

This work will provide the complete field descriptions for the electric and magnetic fields in the transverse electric (TE) and transverse magnetic (TM) polarizations for all modes in the truncated conical cavity resonator and will describe in detail two methods that can be used to find the resonant frequency of an arbitrary mode. One of the methods involves solving the classic boundary value problem, finding the eigenvalues that cause the governing field distribution functions to be zero at the required boundaries. This first method can be quite tedious and requires the use of sophisticated mathematical tools, such as MATLAB<sup>®\*</sup> and Mathematica<sup>®†</sup>, to find the allowable

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\*Product of The MathWorks, Inc., Natick, MA 01760. [www.mathworks.com](http://www.mathworks.com)

†Product of Wolfram Research, Inc., Champaign, IL 61820. [www.wolfram.com](http://www.wolfram.com)



non-integer degrees of the associated Legendre function and to find the eigenvalues of non-integer order spherical Bessel functions. The second method is much simpler, using the WKB method to derive a transcendental equation whose solutions give the resonant frequencies of an arbitrary mode. This second method involves only trigonometric functions and requires only the zeros of the integer-order Bessel functions of the first kind and of its derivative, which are readily available in the form of tables [120]. The resonant frequencies that are obtained from both the Legendre/Bessel eigenvalue method, from the WKB method and from measurements of a physical conical cavity resonator will be compared, and a very close match is observed between these three methods.

## B.1 Geometry

Even though the methods that are used to predict the eigenfrequencies of a conical cavity resonator can be applied to any arbitrary truncated cone resonator, a specific physical resonator will be considered so empirical measurements of the resonant frequencies—found by measuring the frequency transfer function of the resonator using a vector network analyzer—can be compared to the theoretical eigenfrequency predictions. The specific truncated conical cavity resonator used is shown in Fig. B.1 and whose geometric specifications are listed in Table B.1. As the figure shows, the resonator is divided into 5 segments so antenna probes can be easily inserted through the cavity walls, which consist of 0.125 in. aluminum. The end caps are flat, a fact that should be kept in mind throughout the upcoming discussions since the conical resonator is classically modeled in the spherical coordinate system and assumes that the end caps are spherically rounded. The WKB method, however, assumes that the end caps are flat, which agrees with the physical configuration of the resonator. If one were to imagine the small end of the cone extended to a point and allowing that point to be the origin of the spherical coordinate system, with the  $\theta = 0$   $\hat{r}$ -direction parallel to the main axis of the cone, the radial location of the small end cap edges would be located at  $r_a = 0.3983$  m and the edges of the large end cap would likewise be located at  $r_b = 1.938$  m (geometry shown in Fig. B.2). These values will be important when the resonant frequencies of the truncated conical cavity resonator are derived.

| PARAMETER              | VALUE                         |
|------------------------|-------------------------------|
| Axial Length:          | 1.524 m (60 in.)              |
| Small Diameter:        | 0.1143 m (4.5 in.)            |
| Large Diameter:        | 0.5563 m (21.9 in.)           |
| Half-cone Angle:       | 8.25°                         |
| Segments:              | 5                             |
| Material:              | 3.175 mm (0.125 in.) Aluminum |
| Fundamental Resonance: | $TE_{111}$ @ 433 MHz          |

Table B.1: Geometric specifications for the truncated conical cavity resonator.



Figure B.1: Truncated conical cavity resonator, shown with a meter stick for perspective.

## B.2 Boundary Conditions and Mode Functions

The conical cavity resonator has the same underlying field solutions as a spherical resonator, whose mode functions and field solutions were solved for in Appendix A. The solution for the cone begins by assuming that the projected center of the cone is at the origin of the spherical coordinate system as shown in Fig. B.2. The slanted walls of the cone are located at a half-cone angle of  $\theta = \theta_c$ , and the end caps of the cone are located at  $r = a$  and  $r = b$ . Assume for the moment that the end caps are curved in shape so that all points on the end caps are at  $r = a$  or  $r = b$  from the origin, as appropriate. Note that boundary conditions  $\hat{n} \times \vec{E} = 0$  and  $\hat{n} \cdot \vec{H} = 0$  (where  $\hat{n}$  is the normal vector at any point on the cavity walls) must be met both on the outer end caps, the inner end caps and the slanted walls; the latter two boundaries are additional conditions in the conical resonator that do not exist in the spherical resonator. To apply the new boundary conditions, the mode functions derived for the spherical coordinate system are required [102, eqs. 6-30 & 6-33]:

$$\psi_{mnp}^{TM}(r, \theta, \phi) = \hat{B}_\nu \left( \frac{w'_{np} r}{a} \right) P_\nu^m(\cos \theta) \begin{Bmatrix} \cos m\phi \\ \sin m\phi \end{Bmatrix} \quad (\text{B.1})$$

$$\psi_{mnp}^{TE}(r, \theta, \phi) = \hat{B}_\nu \left( \frac{w_{np} r}{a} \right) P_\nu^m(\cos \theta) \begin{Bmatrix} \cos m\phi \\ \sin m\phi \end{Bmatrix}. \quad (\text{B.2})$$

In the above expressions,  $\hat{B}_\nu(x)$  is the  $\nu^{th}$  order spherical Bessel function (used by Schelkunoff [121]), and  $w_{np}$  and  $w'_{np}$  are the  $p^{th}$  roots of the  $n^{th}$  allowable order (given by the set of allowable of  $\nu$ -values) of these spherical Bessel functions and their first derivatives.  $P_\nu^m(x)$  is the  $m^{th}$  order,  $\nu^{th}$  degree associated Legendre function of the first kind. In order to apply the boundary conditions, the field solutions must be produced from the mode functions in (B.1) and (B.2). The electric and magnetic vector potentials, as expressed in (A.7) and (A.8), can be used to express the electric and magnetic fields; the expansion of these field potential functions in the spherical coordinate system,

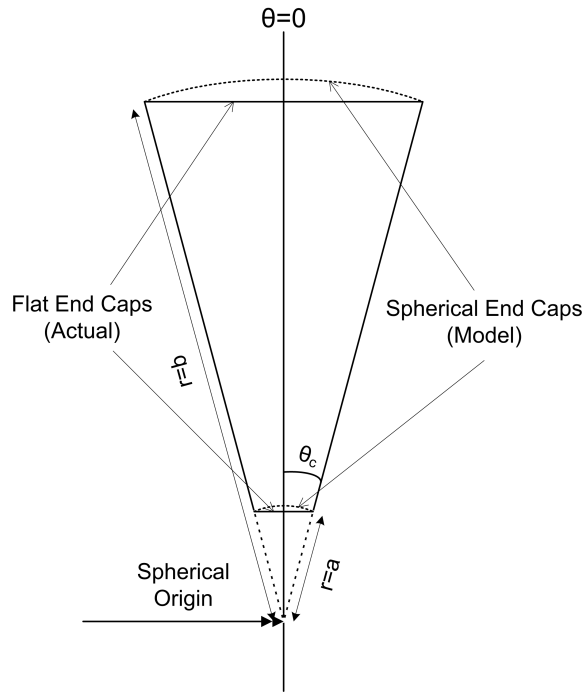


Figure B.2: Geometry in the spherical coordinate system (in the  $\hat{\theta}$ - $\hat{r}$  plane) for the conical resonator, showing the actual and modeled end-caps at  $r = a$  and  $r = b$ .

letting  $\bar{A} = \hat{r}A_r$  and  $\bar{F} = \hat{r}F_r$ , are as follows [102, eq. 6-26]:

$$E_r = \frac{1}{j\omega\epsilon} \left( \frac{\partial^2}{\partial r^2} + k^2 \right) A_r \quad (\text{B.3})$$

$$E_\theta = \frac{-1}{r \sin \theta} \frac{\partial F_r}{\partial \phi} + \frac{1}{j\omega\epsilon r} \frac{\partial^2 A_r}{\partial r \partial \theta} \quad (\text{B.4})$$

$$E_\phi = \frac{1}{r} \frac{\partial F_r}{\partial \theta} + \frac{1}{j\omega\epsilon r \sin \theta} \frac{\partial^2 A_r}{\partial r \partial \phi} \quad (\text{B.5})$$

$$H_r = \frac{1}{j\omega\mu} \left( \frac{\partial^2}{\partial r^2} + k^2 \right) F_r \quad (\text{B.6})$$

$$H_\theta = \frac{1}{r \sin \theta} \frac{\partial A_r}{\partial \phi} + \frac{1}{j\omega\mu r} \frac{\partial^2 F_r}{\partial r \partial \theta} \quad (\text{B.7})$$

$$H_\phi = \frac{-1}{r} \frac{\partial A_r}{\partial \theta} + \frac{1}{j\omega\mu r \sin \theta} \frac{\partial^2 F_r}{\partial r \partial \phi}. \quad (\text{B.8})$$

### B.2.1 TM Polarization Boundary Conditions

The fields that are transverse magnetic to the  $r$  dimension, according to (B.3)-(B.8), must have  $F_r = 0$ ; correspondingly, let  $A_r = H_0 \psi_{mnp}^{TM}(r, \theta, \phi)$ . Considering the  $E_\phi$  component and substituting

$F_r$  and  $A_r$  into (B.5) yields the following result:

$$E_\phi = H_0 \frac{m}{j\omega\epsilon r \sin \theta} \left[ \frac{\partial}{\partial r} \hat{B}_\nu \left( \frac{w'_{np} r}{a} \right) \right] P_\nu^m(\cos \theta) \begin{Bmatrix} -\sin m\phi \\ \cos m\phi \end{Bmatrix}. \quad (\text{B.9})$$

Enforcing the boundary condition that  $E_\phi(r, \theta, \phi)|_{\theta=\theta_c} = 0$  for all  $\phi$  and  $r$  in the cavity, we see that the following expression must be true:

$$P_\nu^m(\cos \theta)|_{\theta=\theta_c} = 0. \quad (\text{B.10})$$

This boundary condition will be enforced by choosing the appropriate combinations of  $m$  and  $\nu$  and will be explained in Section B.4.

Enforcing the boundary condition that  $E_\phi(r, \theta, \phi)|_{r=a \atop r=b} = 0$ , the following condition must be true:

$$\frac{\partial}{\partial r} \hat{B}_\nu \left( \frac{w'_{np} r}{a} \right) \Big|_{r=a \atop r=b} = 0. \quad (\text{B.11})$$

The function  $\hat{B}_\nu(x)$  is a linear combination of two functions that are solutions to Bessel's differential equation in spherical coordinates:

$$\hat{B}_\nu(x) = \alpha \hat{J}_\nu(x) + \beta \hat{N}_\nu(x), \quad (\text{B.12})$$

where  $\hat{J}_\nu(x)$  is the spherical Bessel function of the first kind,  $\hat{N}_\nu(x)$  is the spherical Bessel function of the second kind, and  $\alpha$  and  $\beta$  are constants. These spherical Bessel functions can be related to the cylindrical Bessel functions,  $J_\nu(x)$  and  $N_\nu(x)$ , via the following expressions [102, App. D]:

$$\hat{J}_\nu(x) = \sqrt{\frac{\pi x}{2}} J_{\nu+\frac{1}{2}}(x) \quad (\text{B.13})$$

$$\hat{N}_\nu(x) = \sqrt{\frac{\pi x}{2}} N_{\nu+\frac{1}{2}}(x). \quad (\text{B.14})$$

Since the partial derivative with respect to  $r$  of the function in (B.12) must be zero when  $r = a$  and  $r = b$  in order to satisfy the  $E_\phi = 0$  boundary condition, the constants  $\alpha$  and  $\beta$  must be chosen such that the same choice of the eigenvalue  $k_r = w'_{np}/a$  causes the boundary condition to be enforced at both  $r = a$  and  $r = b$ . Accordingly, the following choices for  $\alpha$  and  $\beta$  will enforce the boundary

conditions at  $r = a$ , independent of the value of  $k_r = w'_{np}/a$  (similar to [102, Prob. 6-5]):

$$\alpha = 1, \beta = \frac{\hat{J}'_\nu\left(\frac{w'_{np}a}{a}\right)}{\hat{N}'_\nu\left(\frac{w'_{np}a}{a}\right)}, \quad (\text{B.15})$$

where  $\hat{J}'_\nu(x)$  and  $\hat{N}'_\nu(x)$  are the derivatives of the first and second kind spherical Bessel functions. This choice for  $\alpha$  and  $\beta$  leaves  $k_r = w'_{np}/a$  free to choose so that the boundary condition at  $r = b$  is enforced, as will be described in Section B.4.

### B.2.2 TE Polarization Boundary Conditions

As it was the case for the TM polarization, the boundary condition  $E_\phi(r, \theta, \phi)|_{\theta=\theta_c} = 0$  must also hold for the TE polarizations for all  $\phi$  and  $r$  within the cavity. Equation (B.2) can be substituted into (B.8) with  $F_r = E_0\psi_{mnp}^{TE}(r, \theta, \phi)$  and  $A_r = 0$  to find the tentative expression for the  $E_\phi$  field:

$$E_\phi = \frac{E_0}{r} \hat{B}_\nu\left(\frac{w_{np}r}{a}\right) \left[ \frac{\partial}{\partial \theta} P_\nu^m(\cos \theta) \right] \begin{Bmatrix} \cos m\phi \\ \sin m\phi \end{Bmatrix}. \quad (\text{B.16})$$

One can see from the expression above that in order for the boundary condition  $E_\phi(r, \theta, \phi)|_{\theta=\theta_c} = 0$  to be true, the following restriction must be observed by choosing the Legendre orders,  $m$ , and degrees,  $\nu$ , appropriately (as outlined in the next section):

$$\left. \frac{\partial}{\partial \theta} P_\nu^m(\cos \theta) \right|_{\theta=\theta_c} = 0. \quad (\text{B.17})$$

To enforce the boundary condition that  $E_\phi(r, \theta, \phi)|_{r=a} = 0$ , the tentative expression for  $E_\phi$  in (B.16) suggests that the following must be true:

$$\hat{B}_\nu\left(\frac{w_{np}r}{a}\right) \Big|_{r=a} = 0, \quad (\text{B.18})$$

where  $\hat{B}_\nu(x)$  is the spherical Bessel function equal to a linear combination of the first and second kind spherical Bessel functions, as in (B.12). Similar to the TM solutions, if we let

$$\alpha = 1, \beta = \frac{\hat{J}_\nu\left(\frac{w_{np}a}{a}\right)}{\hat{N}_\nu\left(\frac{w_{np}a}{a}\right)}, \quad (\text{B.19})$$

then  $E_\phi$  will automatically be zero at  $r = a$ ; the eigenvalue  $k_r = w_{np}/a$  can then be chosen to make  $E_\phi = 0$  additionally at  $r = b$ , satisfying the boundary conditions on all walls for the TE polarization fields.

### B.3 Complete Field Solutions

The complete field solutions can be found by updating the TM and TE mode functions in (B.1) and (B.2) with the information obtained from applying the boundary conditions in the last section:

$$\psi_{mnp}^{TM}(r, \theta, \phi) = \left[ \hat{J}_\nu \left( \frac{w'_{np} r}{a} \right) + \frac{\hat{J}'_\nu(w'_{np})}{\hat{N}'_\nu(w'_{np})} \hat{N}_\nu \left( \frac{w'_{np} r}{a} \right) \right] P_\nu^m(\cos \theta) \begin{Bmatrix} \cos m\phi \\ \sin m\phi \end{Bmatrix} \quad (\text{B.20})$$

$$\psi_{mnp}^{TE}(r, \theta, \phi) = \left[ \hat{J}_\nu \left( \frac{w_{np} r}{a} \right) + \frac{\hat{J}_\nu(w_{np})}{\hat{N}_\nu(w_{np})} \hat{N}_\nu \left( \frac{w_{np} r}{a} \right) \right] P_\nu^m(\cos \theta) \begin{Bmatrix} \cos m\phi \\ \sin m\phi \end{Bmatrix}. \quad (\text{B.21})$$

To obtain the complete field expressions, the above mode functions need to be substituted into the spherical field vector potential expressions in (B.3)-(B.8); for the TM polarization, let  $A_r = H_0 \psi_{mnp}^{TM}(r, \theta, \phi)$  and  $F_r = 0$ :

$$E_r = \frac{H_0}{j\omega\epsilon} \left( \omega^2 \mu \epsilon + \frac{\partial^2}{\partial r^2} \right) \left[ \hat{J}_\nu \left( \frac{w'_{np} r}{a} \right) + \frac{\hat{J}'_\nu(w'_{np})}{\hat{N}'_\nu(w'_{np})} \hat{N}_\nu \left( \frac{w'_{np} r}{a} \right) \right] P_\nu^m(\cos \theta) \begin{Bmatrix} \cos m\phi \\ \sin m\phi \end{Bmatrix} \quad (\text{B.22})$$

$$E_\theta = \frac{H_0}{j\omega\epsilon r} \frac{\partial}{\partial r} \left[ \hat{J}_\nu \left( \frac{w'_{np} r}{a} \right) + \frac{\hat{J}'_\nu(w'_{np})}{\hat{N}'_\nu(w'_{np})} \hat{N}_\nu \left( \frac{w'_{np} r}{a} \right) \right] \frac{\partial}{\partial \theta} P_\nu^m(\cos \theta) \begin{Bmatrix} \cos m\phi \\ \sin m\phi \end{Bmatrix} \quad (\text{B.23})$$

$$E_\phi = H_0 \frac{m}{j\omega\epsilon r \sin \theta} \frac{\partial}{\partial r} \left[ \hat{J}_\nu \left( \frac{w'_{np} r}{a} \right) + \frac{\hat{J}'_\nu(w'_{np})}{\hat{N}'_\nu(w'_{np})} \hat{N}_\nu \left( \frac{w'_{np} r}{a} \right) \right] P_\nu^m(\cos \theta) \begin{Bmatrix} -\sin m\phi \\ \cos m\phi \end{Bmatrix} \quad (\text{B.24})$$

$$H_r = 0 \quad (\text{B.25})$$

$$H_\theta = H_0 \frac{m}{r \sin \theta} \left[ \hat{J}_\nu \left( \frac{w'_{np} r}{a} \right) + \frac{\hat{J}'_\nu(w'_{np})}{\hat{N}'_\nu(w'_{np})} \hat{N}_\nu \left( \frac{w'_{np} r}{a} \right) \right] P_\nu^m(\cos \theta) \begin{Bmatrix} -\sin m\phi \\ \cos m\phi \end{Bmatrix} \quad (\text{B.26})$$

$$H_\phi = \frac{-H_0}{r} \left[ \hat{J}_\nu \left( \frac{w'_{np} r}{a} \right) + \frac{\hat{J}'_\nu(w'_{np})}{\hat{N}'_\nu(w'_{np})} \hat{N}_\nu \left( \frac{w'_{np} r}{a} \right) \right] \frac{\partial}{\partial \theta} P_\nu^m(\cos \theta) \begin{Bmatrix} \cos m\phi \\ \sin m\phi \end{Bmatrix}. \quad (\text{B.27})$$

Likewise for the TE polarization, let  $F_r = E_0 \psi_{mnp}^{TE}(r, \theta, \phi)$  and  $A_r = 0$ :

$$E_r = 0 \quad (\text{B.28})$$

$$E_\theta = E_0 \frac{-m}{r \sin \theta} \left[ \hat{J}_\nu \left( \frac{w_{np} r}{a} \right) + \frac{\hat{J}_\nu(w_{np})}{\hat{N}_\nu(w_{np})} \hat{N}_\nu \left( \frac{w_{np} r}{a} \right) \right] P_\nu^m(\cos \theta) \begin{Bmatrix} -\sin m\phi \\ \cos m\phi \end{Bmatrix} \quad (\text{B.29})$$

$$E_\phi = \frac{E_0}{r} \left[ \hat{J}_\nu \left( \frac{w_{np} r}{a} \right) + \frac{\hat{J}_\nu(w_{np})}{\hat{N}_\nu(w_{np})} \hat{N}_\nu \left( \frac{w_{np} r}{a} \right) \right] \frac{\partial}{\partial \theta} P_\nu^m(\cos \theta) \begin{Bmatrix} \cos m\phi \\ \sin m\phi \end{Bmatrix} \quad (\text{B.30})$$

$$H_r = \frac{E_0}{j\omega\mu} \left( \omega^2 \mu \epsilon + \frac{\partial^2}{\partial r^2} \right) \left[ \hat{J}_\nu \left( \frac{w_{np} r}{a} \right) + \frac{\hat{J}_\nu(w_{np})}{\hat{N}_\nu(w_{np})} \hat{N}_\nu \left( \frac{w_{np} r}{a} \right) \right] P_\nu^m(\cos \theta) \begin{Bmatrix} \cos m\phi \\ \sin m\phi \end{Bmatrix} \quad (\text{B.31})$$

$$H_\theta = \frac{E_0}{j\omega\mu r} \frac{\partial}{\partial r} \left[ \hat{J}_\nu \left( \frac{w_{np} r}{a} \right) + \frac{\hat{J}_\nu(w_{np})}{\hat{N}_\nu(w_{np})} \hat{N}_\nu \left( \frac{w_{np} r}{a} \right) \right] \frac{\partial}{\partial \theta} P_\nu^m(\cos \theta) \begin{Bmatrix} \cos m\phi \\ \sin m\phi \end{Bmatrix} \quad (\text{B.32})$$

$$H_\phi = E_0 \frac{m}{j\omega\mu r \sin \theta} \frac{\partial}{\partial r} \left[ \hat{J}_\nu \left( \frac{w_{np} r}{a} \right) + \frac{\hat{J}_\nu(w_{np})}{\hat{N}_\nu(w_{np})} \hat{N}_\nu \left( \frac{w_{np} r}{a} \right) \right] P_\nu^m(\cos \theta) \begin{Bmatrix} -\sin m\phi \\ \cos m\phi \end{Bmatrix}. \quad (\text{B.33})$$

These results reduce to and match the results obtained for the  $TE_{011}$  mode in [119] where investigators solved for the conical  $TE_{011}$  mode (because of its desirable current patterns) to investigate the conductivity of some super-conducting films as a function of temperature.

Solving for the orders of the spherical Bessel functions ( $\nu$ ), the degrees of the Legendre functions (also  $\nu$ ) and the orders of the Legendre functions ( $m$ ) and the solutions of the eigenfrequencies for the conical cavity will be considered in the following section.

## B.4 Eigenvalue Solutions and Resonant Frequencies

Through the process of applying the boundary conditions in the previous section, one can already start to see how the eigenfrequencies are derived for the conical cavity. The boundary condition  $E_\phi(r, \theta, \phi)|_{\theta=\theta_c} = 0$  for both the TM and TE polarizations restricts the choice of the  $m$  and  $\nu$  via both (B.10) and (B.17). The Legendre function orders  $m$  are chosen from the set of counting numbers ( $m = 0, 1, 2, 3, \dots$ ). For each order considered, the non-integer Legendre function degrees are found such that (B.10) and (B.17) are true, restricting the search to  $\nu > m$ , since  $P_\nu^m = 0$  for  $\nu < m$  [102, App.E]; there are an infinite number of allowable degrees in each order of the Legendre function, which are also infinite in number. Since the the Legendre function of non-integer degree does not have a closed form solution, the allowable degree in each chosen order must be found transcendently, evaluating (B.10) and (B.17) using a root finding algorithm in a mathematic solution tool



| TM Legendre Degrees, $\nu$ |         |         |         |         |         |         |         |         |         |          |
|----------------------------|---------|---------|---------|---------|---------|---------|---------|---------|---------|----------|
| $m$                        | $n = 1$ | $n = 2$ | $n = 3$ | $n = 4$ | $n = 5$ | $n = 6$ | $n = 7$ | $n = 8$ | $n = 9$ | $n = 10$ |
| 0                          | 16.1989 | 37.8355 | 59.5990 | 81.3910 | 103.194 | 125.002 | 146.813 | 168.626 | 190.440 | 212.255  |
| 1                          | 26.1157 | 48.2254 | 70.1159 | 92.0336 | 113.889 | 135.732 | 157.568 | 179.400 | 201.229 | 223.056  |
| 2                          | 35.1842 | 57.9680 | 80.2069 | 102.263 | 124.235 | 146.161 | 168.058 | 189.938 | 211.804 | 233.662  |
| 3                          | 43.8428 | 67.3113 | 89.9060 | 112.184 | 134.308 | 156.345 | 178.328 | 200.274 | 222.195 | 244.098  |
| 4                          | 52.2505 | 76.3780 | 99.3428 | 121.863 | 144.160 | 166.326 | 188.410 | 210.437 | 232.425 | 254.383  |
| 5                          | 60.4853 | 85.2391 | 108.575 | 131.347 | 153.828 | 198.331 | 220.449 | 242.512 | 264.534 | 286.524  |
| 6                          | 68.5922 | 93.9401 | 117.641 | 140.669 | 163.340 | 185.796 | 208.111 | 230.327 | 252.472 | 274.564  |
| 7                          | 76.5999 | 102.512 | 126.570 | 149.853 | 172.719 | 195.328 | 217.767 | 240.086 | 262.319 | 284.485  |
| 8                          | 84.5279 | 110.977 | 135.383 | 158.920 | 181.980 | 204.745 | 227.312 | 249.738 | 272.062 | 294.306  |
| 9                          | 92.3902 | 119.352 | 144.097 | 167.883 | 190.138 | 214.060 | 236.757 | 259.293 | 281.711 | 304.037  |

Table B.2: Table of the allowable Legendre function degrees, equal to the spherical Bessel function orders, for the TM polarization modes, for  $\theta_c = 8.25^\circ$ .

| TE Legendre Degrees, $\nu$ |         |         |         |         |         |         |         |         |         |          |
|----------------------------|---------|---------|---------|---------|---------|---------|---------|---------|---------|----------|
| $m$                        | $n = 1$ | $n = 2$ | $n = 3$ | $n = 4$ | $n = 5$ | $n = 6$ | $n = 7$ | $n = 8$ | $n = 9$ | $n = 10$ |
| 0                          | 26.1157 | 48.2254 | 70.1159 | 92.0336 | 113.889 | 135.732 | 157.568 | 179.400 | 201.229 | 223.056  |
| 1                          | 12.3152 | 36.5346 | 58.7892 | 80.8011 | 102.730 | 124.619 | 146.487 | 168.343 | 190.189 | 212.030  |
| 2                          | 20.7548 | 46.0910 | 68.7489 | 90.9762 | 113.040 | 135.022 | 156.958 | 178.865 | 200.752 | 222.626  |
| 3                          | 28.7387 | 55.1951 | 78.3176 | 100.814 | 123.055 | 145.164 | 167.195 | 189.175 | 211.122 | 233.044  |
| 4                          | 36.5177 | 64.0097 | 87.6071 | 110.395 | 132.836 | 155.092 | 177.235 | 199.305 | 221.324 | 243.306  |
| 5                          | 44.1584 | 72.6193 | 96.6847 | 119.773 | 142.426 | 164.841 | 187.108 | 209.278 | 231.379 | 253.429  |
| 6                          | 51.7194 | 81.0745 | 105.595 | 128.984 | 151.855 | 174.436 | 196.835 | 219.112 | 241.302 | 263.429  |
| 7                          | 59.2176 | 89.4079 | 114.368 | 138.055 | 161.147 | 183.899 | 206.435 | 228.825 | 251.109 | 273.316  |
| 8                          | 66.6677 | 97.6425 | 123.027 | 147.008 | 170.320 | 193.245 | 215.922 | 238.428 | 260.811 | 283.101  |
| 9                          | 74.0792 | 105.795 | 131.590 | 155.858 | 179.389 | 202.489 | 225.308 | 247.932 | 270.416 | 292.794  |

Table B.3: Table of the allowable Legendre function degrees, equal to the spherical Bessel function orders, for the TE polarization modes, for  $\theta_c = 8.25^\circ$ .

such as Mathematica. The first root degree  $\nu$  greater than  $m$  is represented with mode index triplet  $(m, n = 1, p)$ , the second with  $(m, n = 2, p)$  and so-forth. A table of the first several allowable degrees within the first several integer orders of the Legendre function that satisfy (B.10) and (B.17) for  $\theta_c = 8.25^\circ$  are shown in Tables B.2 and B.3 for the TM and TE polarizations, respectively.

One can see from (B.9) and (B.16) that the allowable degrees of the Legendre function become the allowable orders of the spherical Bessel functions. As discussed in the previous section, the boundary condition at  $r = a$  is automatically satisfied by the choice of  $\alpha$  and  $\beta$ ; however, the boundary condition at  $r = b$  is satisfied by choosing the eigenvalue  $k_r = w_{np}/a$  such that

$$\hat{J}'_\nu\left(\frac{w'_{np}b}{a}\right) + \frac{\hat{J}'_\nu(w'_{np})}{\hat{N}'_\nu(w'_{np})} \hat{N}'_\nu\left(\frac{w'_{np}b}{a}\right) = 0, \quad (\text{B.34})$$

for the TM polarization modes and

$$\hat{J}_\nu\left(\frac{w_{np}b}{a}\right) + \frac{\hat{J}_\nu(w_{np})}{\hat{N}_\nu(w_{np})} \hat{N}_\nu\left(\frac{w_{np}b}{a}\right) = 0, \quad (\text{B.35})$$

for the TE polarization modes. Once again, there are an infinite number of eigenvalues that satisfy this requirement. The first value of  $w'_{np}$  or  $w_{np}$  that causes the above conditions to be true for the  $n^{\text{th}}$  allowable order of the spherical Bessel function is given the mode triplet  $p = 1$ , the second  $p = 2$  and so-forth. As with the Legendre function of non-integer degree, the non-integer order Bessel functions do not have a closed form solution, and a transcendental solution must be solved using a computational tool to find the required roots. Once the eigenvalues of the spherical Bessel functions are known, the resonant frequencies of the conical resonator quickly follow for the TM and TE polarization modes:

$$f_r^{TM}(m, n, p) = \frac{\nu}{2\pi} \frac{w'_{np}}{a} \quad (\text{B.36})$$

$$f_r^{TE}(m, n, p) = \frac{\nu}{2\pi} \frac{w_{np}}{a}. \quad (\text{B.37})$$

where  $\nu$  is the speed of light in the medium contained in the cavity. Note that the conical resonator still exhibits a degeneracy in the  $\hat{\phi}$  dimension (except for the  $m = 0$  index modes), given by the dual sine and cosine function options in (B.20) and (B.21). The fundamental resonance for the conical cavity is the  $TE_{111}$  mode ( $\nu = 12.312$  and  $w_{np} = 3.616$  for the cavity in Fig. B.2); the resonant frequency for the  $TE_{111}$  mode is 433.5 MHz in this cavity.

The frequency response in the conical cavity resonator shown in Fig. B.2 was measured using a vector network analyzer by inserting short probes through the side-walls of the cavity near  $r = b$ . The  $S_{21}$  response measured by the network analyzer is shown in Fig. B.3, with vertical dashed-line labels added to indicate where the theory outlined above predicts resonances, using the eigenvalues given in Tables B.2 and B.3. The roots of the spherical Bessel function are found using a root finding algorithm in MATLAB<sup>®</sup> according to (B.34) and (B.35). One can see that the fit between the predicted value and the actual references is fairly good, and the errors are likely due to the assumption in the theoretical analysis that the end caps are rounded in shape—the actual end caps are flat.

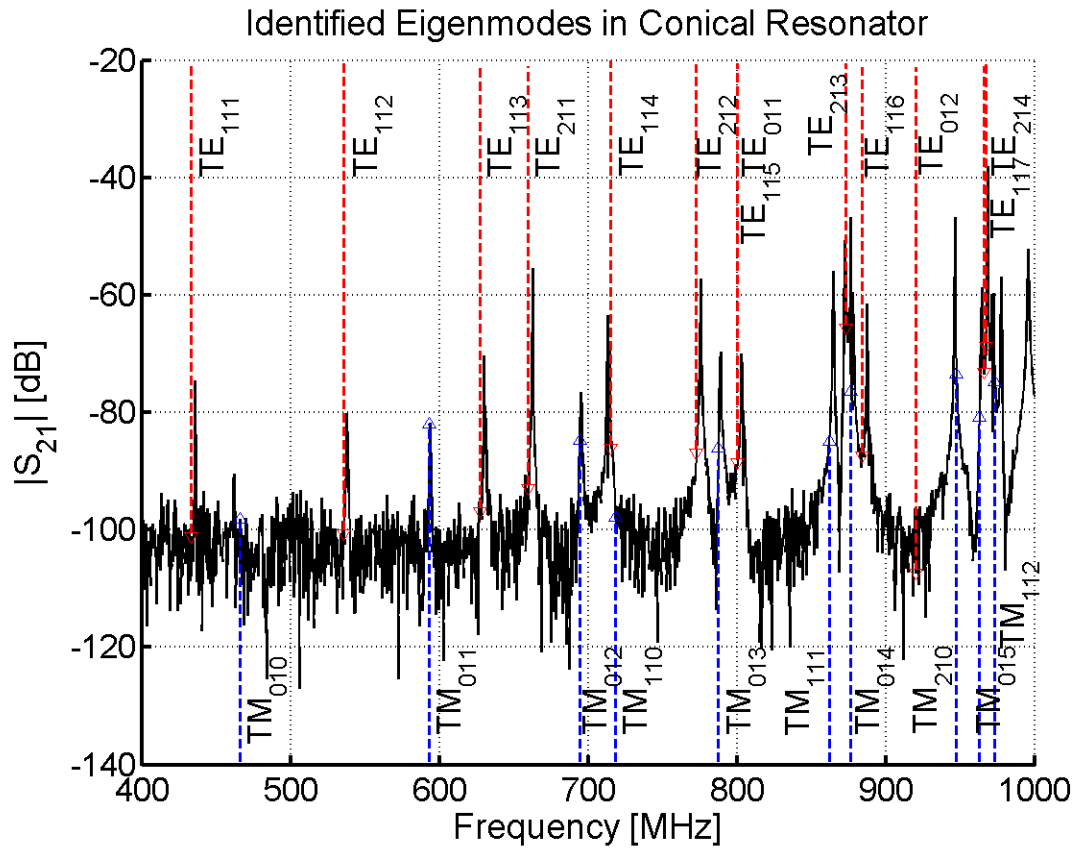


Figure B.3: Magnitude  $S_{21}$  response measured in the conical cavity resonator from 400 MHz to 1.0 GHz. Dashed lines and labels represent eigenfrequency predictions found by solving for the Legendre/Bessel eigenvalues.

## B.5 Eigenfrequency Solutions using the WKB method

The transcendental solution of finding the eigenfrequencies for the conical resonator using the relations given in (B.10), (B.17), (B.34) and (B.35) can be very tedious, and requires some fairly advanced mathematical tools, such as Mathematica<sup>®</sup> and MATLAB<sup>®</sup>. An easier solution to finding the resonant frequencies of the conical resonator is to utilize the Wentzel-Kramers-Brillouin method (WKB method), which has been applied to problems in quantum mechanics (e.g. see references [122, 123]), and was originally developed by Wentzel, Kramers, Brillouin [124].

### B.5.1 The WKB Approximation Method

The WKB method, in short, provides a method to find approximate solutions to the differential equation of the form

$$\frac{d^2y}{dx^2} + f(x)y = 0. \quad (\text{B.38})$$

Notice that this differential equation has the same form as the wave equation in a particular dimension,  $u$ :

$$\nabla^2\psi(u) + k_u^2(u)\psi(u) = 0, \quad (\text{B.39})$$

The WKB solution assumes that  $k_u^2(u)$  is no longer a constant, but is rather a slowly varying function over some dimension,  $u$ . Notice that if  $f(x)$  is constant, the solution to the differential equation of (B.38) would be  $y = e^{j\phi(x)}$ ; substituting this into the differential expression gives

$$-(\phi')^2 + j\phi'' + f(x) = 0. \quad (\text{B.40})$$

If we assume that  $\phi''$  is small, then  $\phi' = \pm\sqrt{f}$  and  $\phi = \pm\int\sqrt{f}dx$ . Thus, the WKB method specifies that in order for  $f(x)$  be slowly varying, it must obey the following inequality [122]:

$$\frac{1}{2}\left|\frac{f'(x)}{\sqrt{f(x)}}\right| \ll |f(x)|. \quad (\text{B.41})$$

From the differential expression in (B.38), it is known that if  $f(x) < 0$ , the solutions to the equation will be in the form of a decaying or growing exponential, and if  $f(x) > 0$ , the solutions will be of the harmonic functions, i.e. a linear combination of sines and cosines. If we let  $f(x) < 0$  for  $x < x_0$  and  $f(x) > 0$  for  $x > x_0$  and assume that the solution for  $x < x_0$  must be of the decaying exponential form, the WKB method gives the solution to (B.38) as

$$\frac{2}{\sqrt[4]{f(x)}} \cos \left[ \int_{x_0}^{x'} \sqrt{f(x')} dx' - \frac{\pi}{4} \right]. \quad (\text{B.42})$$

Notice from this expression that the solutions to WKB method will be oscillatory for  $f(x) > 0$  and exponential for  $f(x) < 0$ .

### B.5.2 Applying the WKB Method to the Conical Cavity Resonator

Consider now the wave equation (Helmholtz equation) in cylindrical coordinates, whose solution for a longitudinal component of the electric or magnetic field is assumed to be of the product form  $E_z(\rho, \phi, z) = R(\rho)\Phi(\phi)Z(z)$ . Substitution of this product solution into the cylindrical Helmholtz equation and dividing by the same assumed solution yields

$$\frac{1}{\rho R(\rho)} \frac{d}{d\rho} \left( \rho \frac{dR(\rho)}{d\rho} \right) + \frac{1}{\rho^2 \Phi(\phi)} \frac{d^2 \Phi(\phi)}{d\phi^2} + \frac{1}{Z(z)} \frac{d^2 Z(z)}{dz^2} + k^2 = 0. \quad (\text{B.43})$$

which can be separated into three differential equations, one for each dimension in the cylindrical coordinate system. The differential expression for the  $\hat{z}$ -dimension is

$$\frac{d^2 Z(z)}{dz^2} + k_z^2 Z(z) = 0, \quad (\text{B.44})$$

which is of the form of (B.38), and  $k_z^2$  is analogous to  $f(x)$ . Separation of (B.43) into each dimensional component leads to the dispersion relationship,  $k^2 = k_\rho^2 + k_z^2$ , and it can be seen that  $k_z^2$  is a constant since  $k^2 = \omega^2 \mu \epsilon$  and  $k_\rho^2 = (x_{np}/a)^2$ , where  $x_{np}$  is the  $p^{\text{th}}$  zero of the governing  $n^{\text{th}}$  order cylindrical Bessel function of the first kind, and  $a$  is the radius of the cylindrical resonator, which is constant over the length of a *cylindrical* resonator. Now imagine that the radius of the cylindrical resonator was not a constant, but rather was a *linearly varying parameter with z*, so the cavity takes on a truncated conical shape. According to the cross sectional diagram shown in Fig. B.4, the cylin-

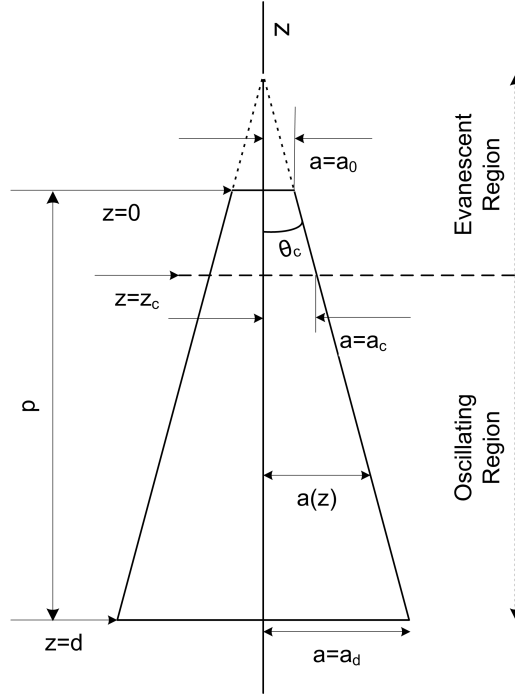


Figure B.4: Cross-sectional diagram of the geometry used to apply the WKB method in the conical resonator.

drical resonator takes on a conical shape, where  $a(z) = a_0 + z \tan \theta_c$ , and  $a_0$  is the radius at  $z = 0$  and  $\theta_c$  is the half-cone-angle of the right-angle cone. Substituting this expression for the radius as a function of  $z$  into the cylindrical dispersion relation and solving for  $k_z$  gives

$$k_z(z) = \sqrt{k^2 - \left( \frac{x_{np}}{a + z \tan \theta_c} \right)^2} = \sqrt{k^2 - k_p^2(z)}. \quad (\text{B.45})$$

As is well known in waveguides, if  $k_z^2 > 0$ —i.e.  $k^2 > k_p^2$ —the solutions to (B.44) will be the harmonic functions (modes resonate in the cavity); likewise, if  $k_z^2 < 0$ —i.e.  $k^2 < k_p^2$ —the solutions to (B.44) will be the growing and decaying exponentials (modes are cutoff and evanesce in the resonator). The radius at which  $k^2 = k_p^2$  can then be called the *cutoff radius* and is given by  $a_c = x_{np}/k$ . Thus, modes in this quasi-cylindrical conical resonator whose eigenvalues are associated with  $x_{np}$  will resonate for radii greater than  $a_c$  and will evanesce for radii less than  $a_c$ . This cutoff radius can be solved for a particular point along the  $z$ -axis,  $z_c = (a_c - a_0)/(\tan \theta_c)$ ; this point is called the *turning-point*, i.e. the point at which propagating energy in the resonator turns back and propagates in the opposite direction, as if it were reflected. Since  $k_z^2$  is now defined as a function of

$z$ , and  $z_c$  is the boundary value between exponential solution regions and harmonic solution regions (analogous to  $x_0$  in the discussion of the WKB solutions above), one can clearly see that the fields in the quasi-cylindrical conical resonator have the same form as the expressions in (B.42) and that the WKB method can be applied to find an approximate solution for (B.44).

Since metallic waveguides and resonators have two sets of field solutions, one whose electric field is transverse to the propagation direction (TE polarization) and the other whose magnetic field is transverse to the propagation direction (TM polarization), it needs to be determined which of these solutions corresponds to the solution in (B.42). Consider the  $\hat{z}$  component of the electric ( $E_z$ ) or magnetic ( $H_z$ ) field, corresponding to the longitudinal TM and TE field solutions, respectively, in the cylindrical resonator of radius  $a$  with perfectly reflecting boundaries ( $\Gamma = 1$ ) on each end ( $z = 0$  and  $z = d$ ) that form the cavity of length  $d$ :

$$E_z^{TM}(\rho, \phi, z) = \frac{H_0}{j\omega\epsilon} \left( \omega^2\mu\epsilon - \left( \frac{q\pi}{d} \right)^2 \right) J_n \left( \frac{x_{np}\rho}{a} \right) \left\{ \begin{matrix} \sin n\phi \\ \cos n\phi \end{matrix} \right\} \cos \frac{q\pi z}{d} \quad (\text{B.46})$$

$$H_z^{TE}(\rho, \phi, z) = \frac{E_0}{j\omega\mu} \left( \omega^2\mu\epsilon - \left( \frac{q\pi}{d} \right)^2 \right) J_n \left( \frac{x'_{np}\rho}{a} \right) \left\{ \begin{matrix} \sin n\phi \\ \cos n\phi \end{matrix} \right\} \sin \frac{q\pi z}{d}. \quad (\text{B.47})$$

From (B.44), we know that the solution of the  $z$ -dependance of the field components  $E_z$  and  $H_z$  in the cavity are the harmonic functions,  $e^{jk_z z}$  and  $e^{-jk_z z}$ , i.e. a forward propagating and reverse propagating wave. The total field  $z$ -dependance is a superposition of the two fields:  $Ae^{jk_z z} + Be^{-jk_z z}$ . The forward and reverse propagating waves meet at the metallic boundary created by the end cap at  $z = 0$ , where the lossless boundary conditions ( $|A| = |B|$ ) for  $E_z$  in (B.46) require that  $A = B$ , making the superposition of the two waves proportional to  $\cos(k_z z)$ . Likewise, the boundary conditions on  $H_z$  in (B.47) at  $z = 0$  require that  $A = -B$ , creating a superposition of the two waves that is proportional to  $\sin(k_z z)$ . Then it is clear that the solution in (B.42) corresponds to the TM polarization, which is proportional to the cosine function. To create a corresponding solution for the TE polarization that is proportional to the sine function, the solution in (B.42) needs to have a phase offset of  $\pi/2$  added to the argument of the cosine.

### B.5.3 Closed-form for the Resonant Frequencies

Now, the WKB solution will be applied to the conical resonator to find its resonant frequencies. Consider the longitudinal fields of the TM and TE polarizations in the cylindrical resonator discussed previously. As can be seen from the  $z$ -dependance in (B.46) and (B.47), in order to satisfy the boundary conditions at the end caps, the argument of the cosine function in (B.42) (for TM field solutions—the TE solutions are governed by the sine function), must be an integer multiple of  $\pi$  when  $z = d$ . Let the argument of the cosine in in (B.42) for  $x > x_0$  be  $\Phi(z)$ , replacing  $f(x)$  with  $k_z^2(z)$  (i.e. the waveguide equivalent propagation constant) and  $x_0$  by  $z_c$  (i.e. the turning point):

$$\Phi(z) = \int_{z=z_c}^z k_z(z') dz' + \vartheta, \quad \vartheta = \begin{cases} +\pi/4 \rightarrow \text{TE} \\ -\pi/4 \rightarrow \text{TM} \end{cases}. \quad (\text{B.48})$$

The variable  $\vartheta$  in the above expression selects the appropriate phase value corresponding to the TM and TE field solutions as discussed previously. To meet the resonance condition,  $\Phi(z = d) = \pi q$ ,  $q = 1, 2, 3 \dots$ ; accordingly, a concise resonance condition can be written as

$$\int_{z=z_c}^{z=d} k_z(z') dz' + \vartheta = q \pi, \quad \vartheta = \begin{cases} +\pi/4 \rightarrow \text{TE} \\ -\pi/4 \rightarrow \text{TM} \end{cases} \quad (\text{B.49})$$

and substituting in  $k_z(z)$  as defined in (B.45),

$$\int_{z=z_c}^{z=d} \sqrt{k^2 - \left( \frac{x_{np}}{a + z' \tan \theta_c} \right)^2} dz' + \vartheta = q \pi, \quad \vartheta = \begin{cases} +\pi/4 \rightarrow \text{TE} \\ -\pi/4 \rightarrow \text{TM} \end{cases}. \quad (\text{B.50})$$

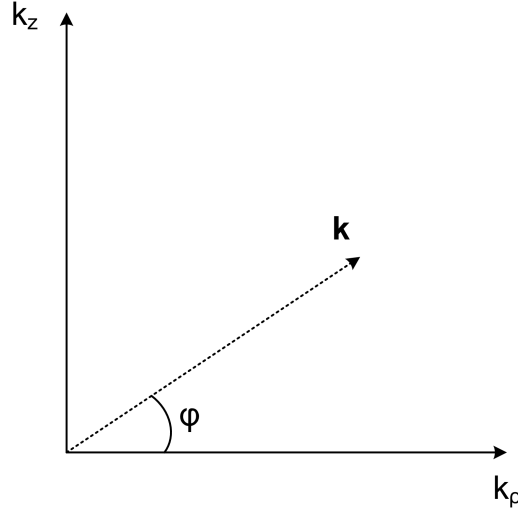
Evaluating the integral in the above expression is now required. Rewriting and letting  $\alpha = x_{np}/k \tan \theta_c$  and  $\beta = a/\tan \theta_c$ :

$$\int_{z=z_c}^{z=d} k \sqrt{1 - \left( \frac{x_{np}}{\tan \theta_c} \right)^2 \left( \frac{1}{\frac{a}{\tan \theta_c} + z'} \right)^2} dz' = \int_{z=z_c}^{z=d} k \sqrt{1 - \alpha^2 \left( \frac{1}{\beta + z'} \right)^2} dz'. \quad (\text{B.51})$$

Changing variables to let  $x = \beta + z$ , the integral becomes

$$\int_{x=\beta+z_c}^{x=\beta+d} k \sqrt{1 - \alpha^2 \left( \frac{1}{x'^2} \right)} dx' = \int_{x=\beta+z_c}^{x=\beta+d} \frac{k}{x'} \sqrt{x'^2 - \alpha^2} dx'. \quad (\text{B.52})$$



Figure B.5: Illustration of the  $\hat{k}$ -space for the conical cavity resonator.

The above integral can be solved using trigonometric substitution, letting  $x = \alpha \sec \varphi$ , thus  $dx = \alpha \sec \varphi \tan \varphi d\varphi$ :

$$k \int_{\varphi_c}^{\varphi_d} \frac{\sqrt{\alpha^2 \sec^2 \varphi' - \alpha^2}}{\alpha \sec \varphi'} \alpha \sec \varphi' \tan \varphi' d\varphi' = k\alpha \int_{\varphi_c}^{\varphi_d} \tan^2 \varphi' d\varphi' = \frac{x_{np}}{\tan \theta_c} [\tan \varphi - \varphi]_{\varphi_c}^{\varphi_d}, \quad (\text{B.53})$$

where  $\varphi_c$  corresponds to the point  $z = z_c$  and  $\varphi_d$  corresponds to the point  $z = d$ . Examining the relation between  $\varphi$  and  $z$ , it can be seen that

$$\cos \varphi = \frac{\alpha}{x} = \frac{\alpha}{\beta + z} = \frac{x_{np}/k}{b + z \tan \theta_c} = \frac{k_\rho(z)}{k}. \quad (\text{B.54})$$

If we consider the two-dimensional  $k$ -space made by  $k_\rho$  and  $k_z$ , then the  $k$ -vector at any distance along the  $z$ -axis in the conical resonator is shown in Fig. B.5, whose magnitude is  $k = \sqrt{k_\rho^2 + k_z^2}$  and whose phase angle is given by  $\varphi$ . Thus, at  $z = z_c$ , at which point a given mode becomes cutoff and  $k_\rho = k$ , the phase of the  $k$ -vector,  $\varphi$ , is zero, making  $\varphi_c = 0$  and  $\varphi_d = \cos^{-1}(k_\rho/k) = \cos^{-1}(x_{np}/ka_d)$ . Substituting the solution for the integral into the resonance condition given in (B.49), the closed-form solution of the resonance condition is obtained:

$$\frac{x_{np}}{\tan \theta_c} [\tan \varphi_d - \varphi_d] + \vartheta = q\pi, \quad \vartheta = \begin{cases} +\pi/4 \rightarrow \text{TE} \\ -\pi/4 \rightarrow \text{TM} \end{cases}, \quad (\text{B.55})$$

where  $\varphi_d = \cos^{-1}(x_{np}/ka_d)$ . Even though this expression still requires a transcendental solution to find the allowable wavenumbers  $k$  (and thereby the resonant frequencies), corresponding to the  $TE_{npq}$  or  $TM_{npq}$  mode triplets, it is simple and can be solved using much less demanding methods than the transcendental solutions involved the Legendre functions and spherical Bessel functions of non-integer order. Unlike the solutions using the Legendre and Bessel functions, which assumed spherical end caps on the conical resonator, this solution assumes that the resonator has flat end caps, suggesting that the resonant frequency estimates might be closer to the measured resonances in the cone.

It is important to note that as higher order Bessel mode families are considered (increasing  $x_{np}$ ), there will be a point at which modes do not cutoff before reaching  $z = 0$ . That is, for  $k > x_{np}/a$  and equivalently,  $f > (\nu x_{np})/(2\pi a)$ , the limits of the integral solution in (B.53) become  $\varphi_d = \cos^{-1}(x_{np}/ka_d)$  and  $\varphi_0 = \cos^{-1}(x_{np}/ka_0)$  for the upper and lower integration limits, respectively. Furthermore, since there is no longer a mode cutoff with adjacent evanescent field solutions, the  $\pm\pi/4$  phase shift in (B.55) derived from the WKB method is no longer required. A modified resonance condition for resonances where  $f > (\nu x_{np})/(2\pi a)$  is given as follows:

$$\frac{x_{np}}{\tan \theta_c} [\tan \varphi_d - \varphi_d - \tan \varphi_0 + \varphi_0] = q\pi, \quad (\text{B.56})$$

Since in this discussion we are only exploring the low-order modes near the fundamental resonance, resonance frequencies that require this modified calculation will not be presented.

Recall that the WKB solution made the assumption in (B.41) in order for the approximations to be accurate. It is useful to know what this restriction is in terms of the quantities of the derived transcendental solution. First, we can write the derivative the WKB function  $f(z) = k_z^2(z)$  as follows, using the definition given in (B.45):

$$\frac{df(z)}{dz} = \frac{-2x_{np}^2 \tan \theta_c}{(a + z \tan \theta_c)^3}. \quad (\text{B.57})$$

The approximation inequality follows, substituting in the above derivative of the function and the

function itself from (B.45) into (B.41):

$$\frac{1}{2} \left| \frac{-2x_{np}^2 \tan \theta_c}{(a_0 + z \tan \theta_c)^3} \left[ k^2 - \left( \frac{x_{np}}{a_0 + z \tan \theta_c} \right)^2 \right]^{-\frac{1}{2}} \right| \ll \left| k^2 - \left( \frac{x_{np}}{a_0 + z \tan \theta_c} \right)^2 \right| \quad (\text{B.58})$$

$$\Rightarrow \frac{x_{np}^2 \tan \theta_c}{(a_0 + z \tan \theta_c)^3} \ll \left| \left( k^2 - \left( \frac{x_{np}}{a_0 + z \tan \theta_c} \right)^2 \right)^{\frac{3}{2}} \right| \quad (\text{B.59})$$

$$\Rightarrow \frac{\tan \theta_c k_p^3}{x_{np}} \ll \left| \left( k^2 - k_p^2 \right)^{\frac{3}{2}} \right| = |k_z^3| \quad (\text{B.60})$$

$$\Rightarrow \tan \theta_c \ll x_{np} \left( \frac{|k_z|}{k_p} \right)^3 \quad (\text{B.61})$$

$$\Rightarrow \tan \theta_c \ll x_{np} |\tan \varphi|^3, \quad -\frac{\pi}{2} \leq \varphi \leq \frac{\pi}{2}. \quad (\text{B.62})$$

Several qualifications can be noticed from the restriction given concisely in (B.62). The WKB approximation provides adequate solutions to the wave equation in the conical resonator whose half-cone angle,  $\theta_c$ , is small and the solutions are being considered for  $\varphi$  significantly larger than zero. Note that the dispersion angle,  $\varphi(z) = \cos^{-1}(k_p/k)$ , is equal to zero at the cutoff radius of a given propagating mode, and (B.62) suggests that the WKB method would have inaccurate solutions; however since  $\varphi$  is a function of  $z$ , the inequality of (B.62) can still hold for regions where the radius is considerably larger than the cutoff radius. Furthermore, when the modes being considered have a cutoff radius smaller than the small radius of the cone,  $a_0$ , then the modes never approach cutoff and there will be no turning point (since  $\varphi(z) > 0$  for all  $z$ ), and (B.62) suggests that the WKB approximations will become even more accurate. Lastly, the  $x_{np}$  term in the inequality shows that the WKB approximation should be more accurate for higher order modes (larger values of  $x_{np}$ ). Even though this condition does not rigorously hold very near the cutoff radius, the resonant frequencies derived from the WKB method are nevertheless very close to the values derived from the roots of the Legendre and Bessel functions in the previous sections.

#### B.5.4 Comparison of WKB Method to Legendre/Bessel Root Method and Measured Results

Table B.4 lists some of the lowest order resonant frequencies below 1.00 GHz of the conical resonator as derived from the Legendre and Bessel function roots, the WKB method and measurements

| Mode       | Legendre & Bessel Roots | WKB Method | Measured with VNA |
|------------|-------------------------|------------|-------------------|
| $TE_{011}$ | 800.63                  | 802.62     | N/A               |
| $TE_{012}$ | 920.17                  | 924.03     | N/A               |
| $TE_{111}$ | 433.50                  | 433.76     | 436.06            |
| $TE_{112}$ | 535.56                  | 537.35     | 538.00            |
| $TE_{113}$ | 627.71                  | 630.50     | 630.31            |
| $TE_{211}$ | 659.37                  | 659.94     | 662.69            |
| $TE_{212}$ | 772.85                  | 775.13     | 775.56            |
| $TE_{213}$ | 873.17                  | 876.52     | 872.69            |
| $TM_{010}$ | 465.92                  | 471.11     | 462.31            |
| $TM_{011}$ | 593.31                  | 596.59     | 594.00            |
| $TM_{012}$ | 694.38                  | 698.39     | 695.06            |
| $TM_{110}$ | 718.24                  | 724.90     | 714.31            |
| $TM_{111}$ | 862.10                  | 866.36     | 864.81            |
| $TM_{112}$ | 973.26                  | 978.21     | 972.44            |

Table B.4: Table of the resonant frequencies in MHz of the conical resonator with flat end-caps and half-cone angle  $\theta_c = 8.25^\circ$  measured using the vector network analyzer and predicted using the Legendre/Bessel root method and the WKB method.

using the vector network analyzer. One can see that the WKB method matches to both the measured and root derived resonant frequencies very well, often showing a match that is better than 1%. The resonant frequencies for the  $TE_{01p}$  modes are not listed here since the measurements were made using a linear probe antenna, inserted radially through the cone surface, and this type of antenna will not couple to the  $TE_{01p}$  mode family in that position.

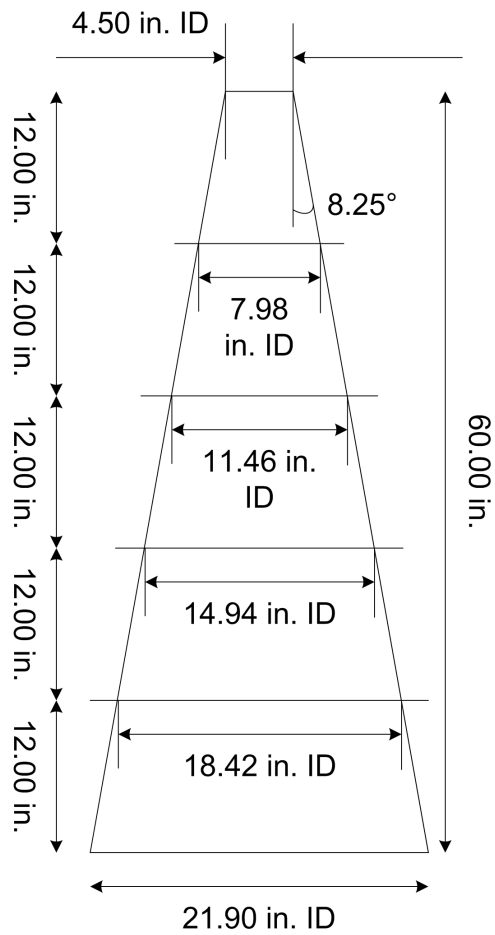
It should also be mentioned that both the WKB solution and the Legendre/Bessel root solution require approximations. The WKB method correctly assumes that the conical resonator has flat end-caps, but it requires a small half-cone angle to approximate the resonant frequencies. The Legendre/Bessel Root method assumes that the conical resonator has spherically-shaped end-caps (an approximation), but finds exact resonant frequency solutions under that assumption. It is not clear, however, from Table B.4 that one of these methods is clearly more accurate than the other, although both methods do predict the measured values very well. Manufacturing tolerances and geometric imperfections could also be cited for minor deviations from theory in the measured results.

## Appendix C

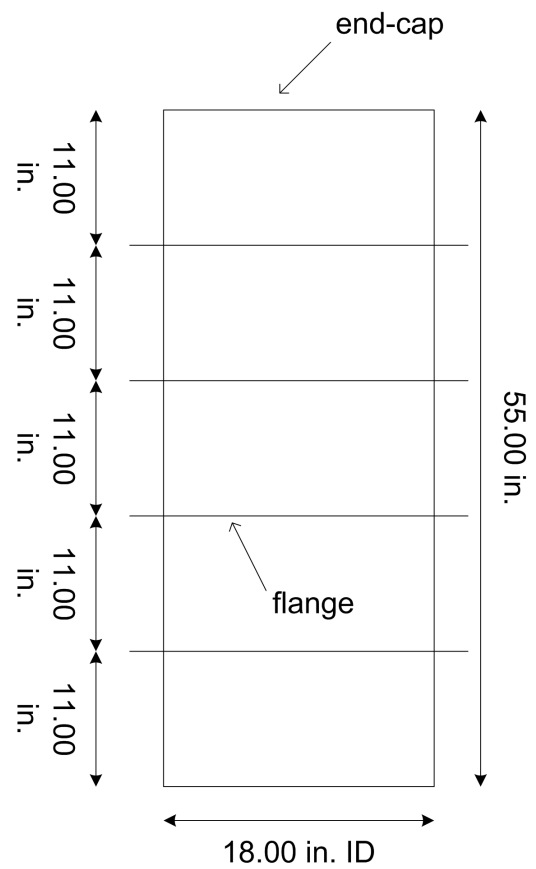
# Enclosed Space Platform Design

## Diagrams

The following diagrams provide the details of assembly and dimensions of the cylindrical and conical enclosed space platforms. Both of these enclosures were manufactured at a precision metal forming machine shop, which is capable of bending metal shapes to very precise tolerances. Each segment of the cylinder and cone was formed separately from 0.125 in. aluminum, and the appropriate flange or end-plate was added to both sides of each segment, as illustrated in Figs. C.2 through C.7. End-plates made of 0.250 in. aluminum are welded to the bottom segments of both designs in place of a flange. Figure C.8 shows the hoisting brackets that are welded to the top 0.250 aluminum end-cap that is welded to the top segment of both designs. Care was taken to ensure that weld seams were smooth as seen from the inside of each design so that the empty cone and cylinder were as geometrically ideal as possible for the purposes of measuring resonant modes in the frequency domain.

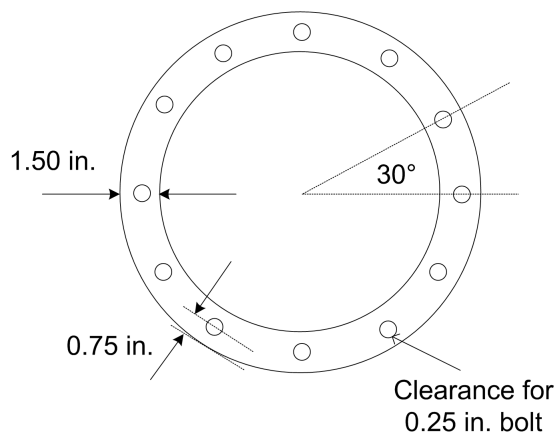
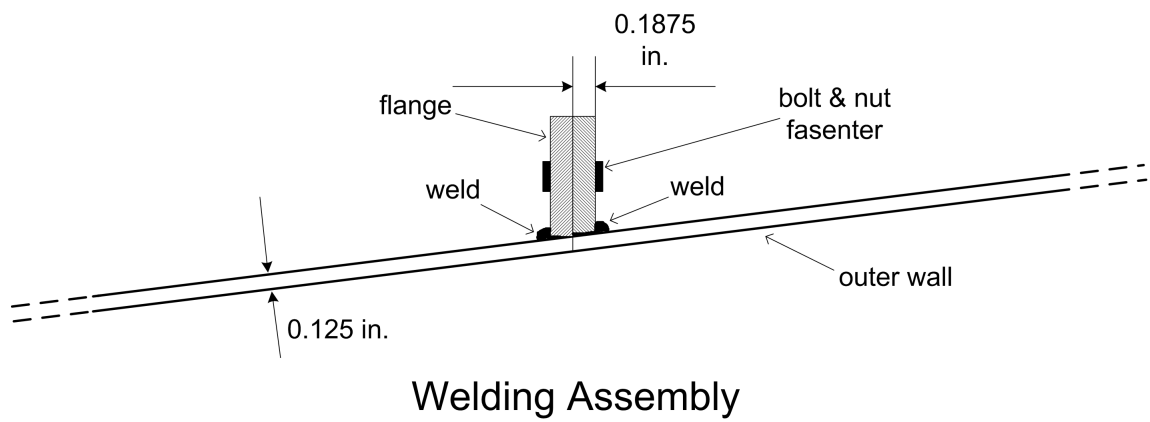


Cone Dimensions



Cylinder Dimensions

Figure C.1: Diagrams of the conical and cylindrical enclosed space platforms showing segment dimensions.



Flange Design

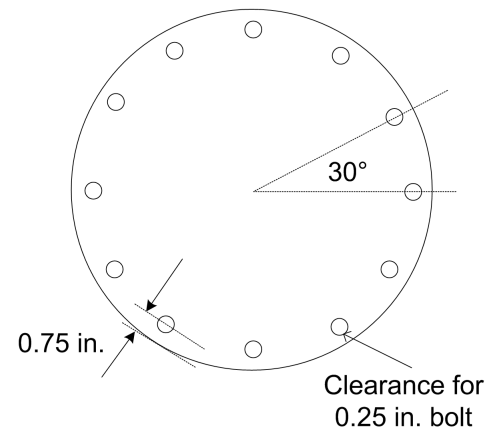
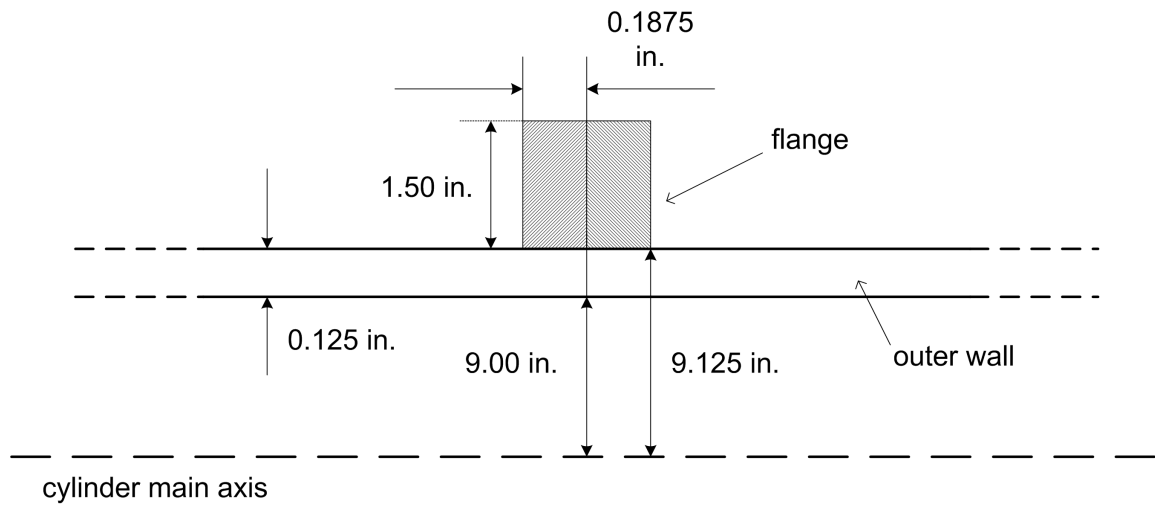


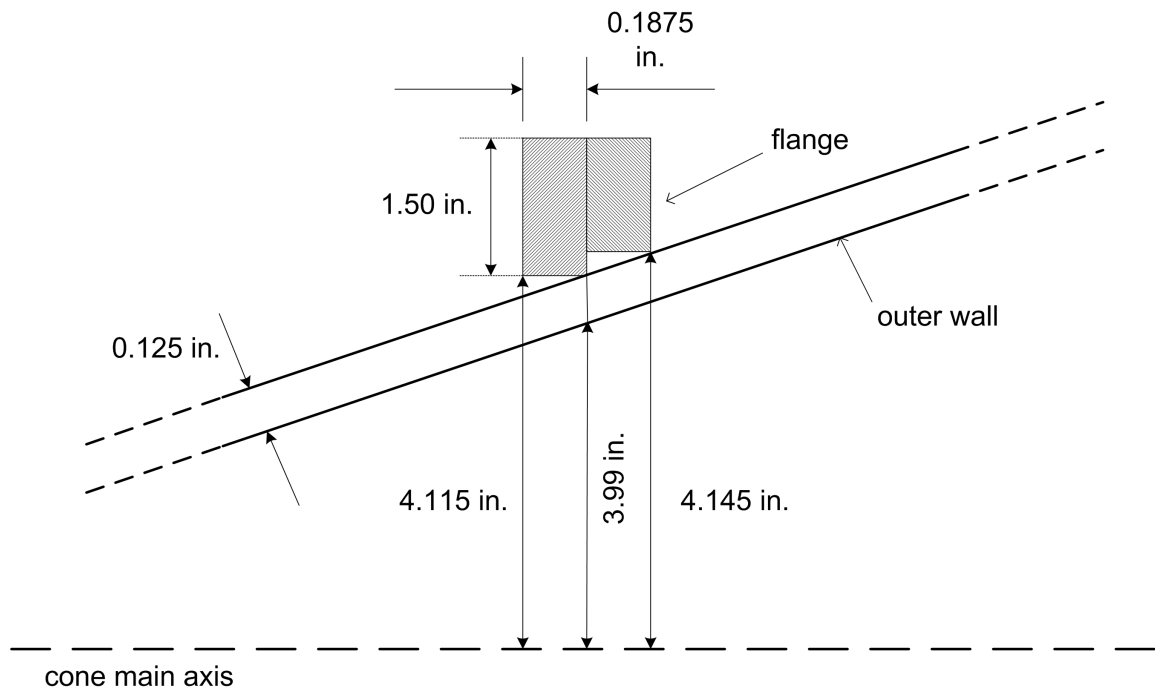
Plate Design

Figure C.2: Welding assembly diagram showing the interface between the flanges and the body of the individual conical or cylindrical segments. Cross section diagrams for the flanges and flange-plates are also included.



### Flange Interface: 18.00 in. ID

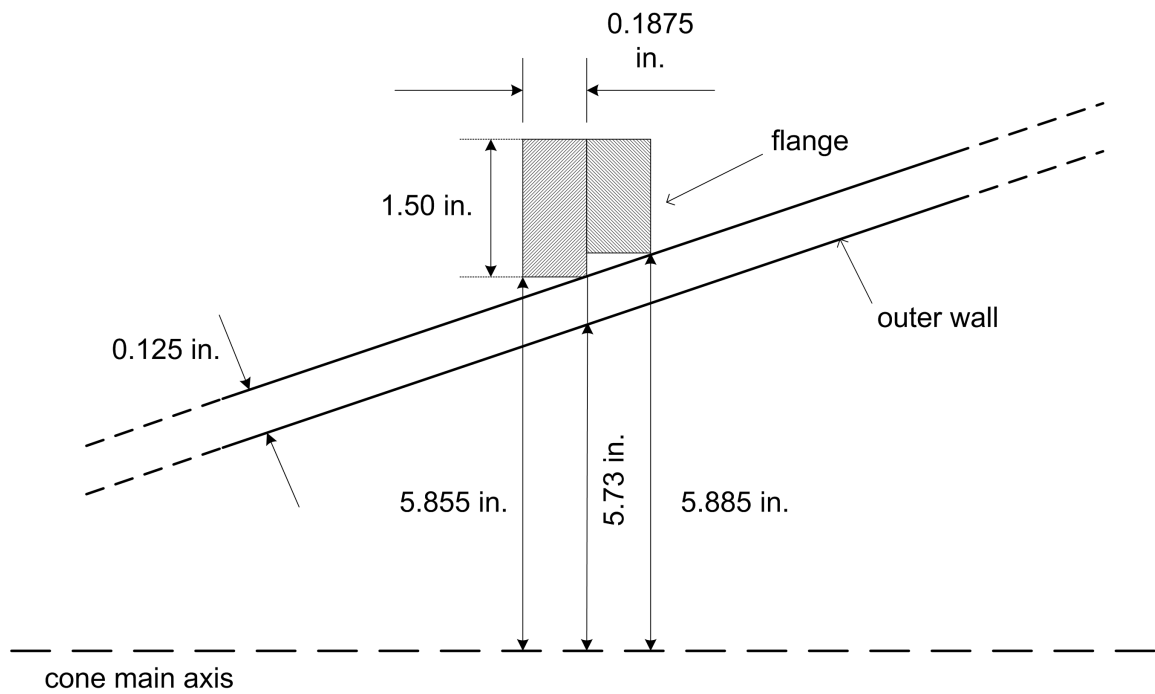
Figure C.3: Flange-body interface dimensions for all seams and segments of the cylindrical platform.



### Flange Interface: 7.98 in. ID

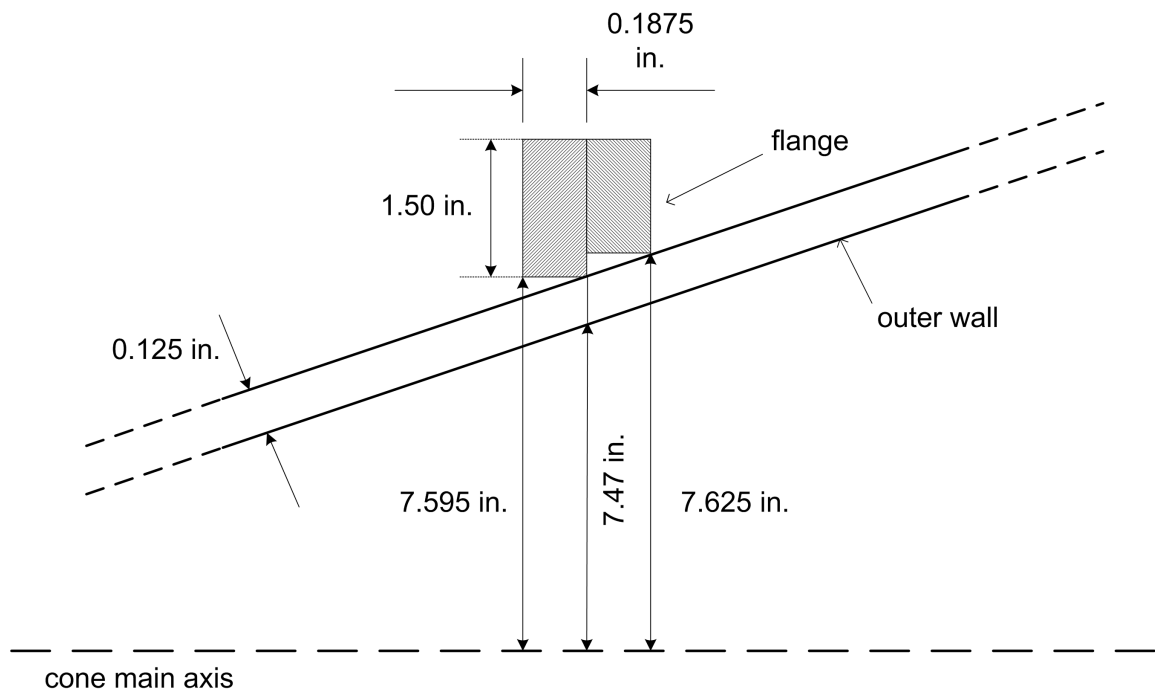
Figure C.4: Flange-body interface dimensions for the 7.98 in. OD conical segment interface.





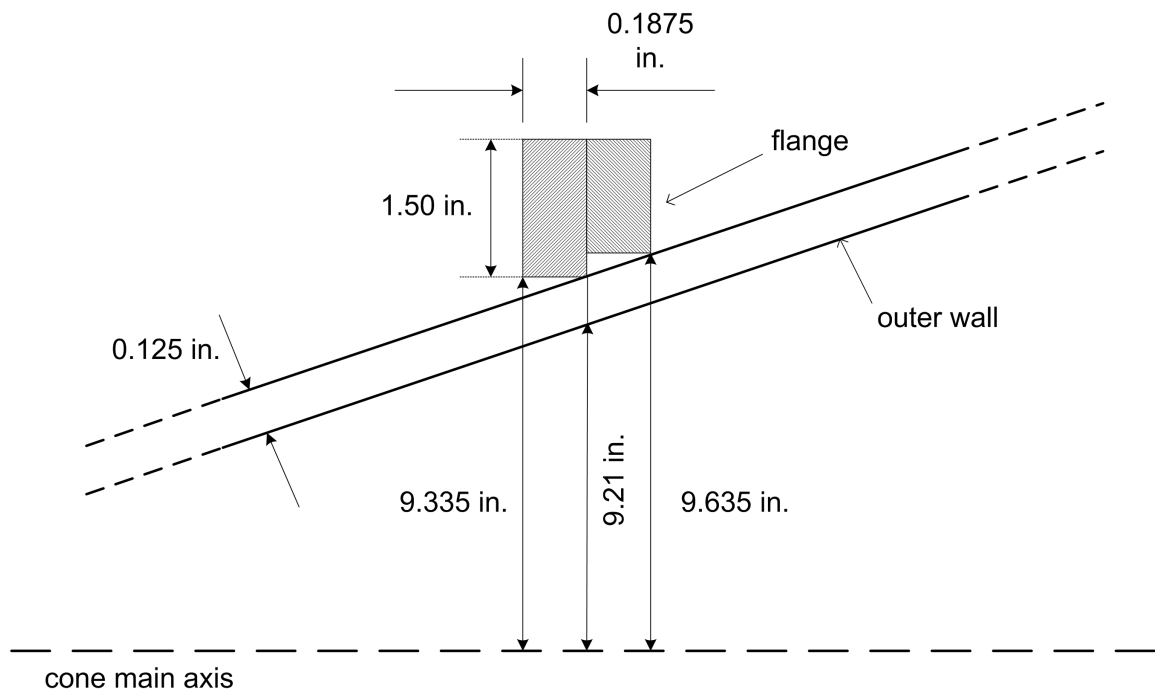
### Flange Interface: 11.46 in. ID

Figure C.5: Flange-body interface dimensions for the 11.46 in. OD conical segment interface.



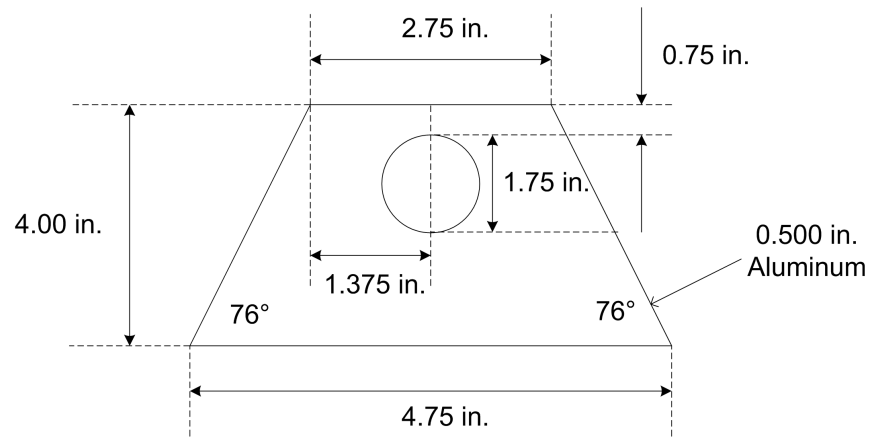
### Flange Interface: 14.94 in. ID

Figure C.6: Flange-body interface dimensions for the 14.94 in. OD conical segment interface.

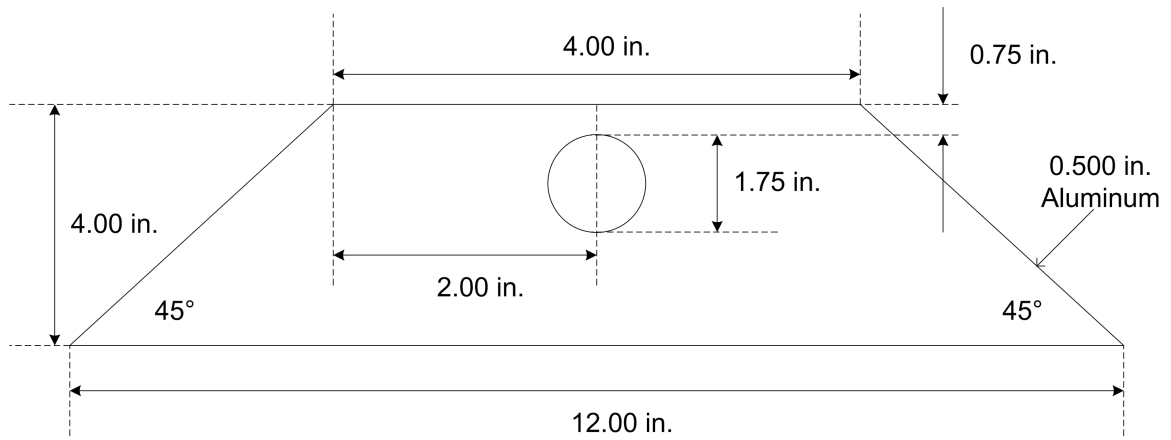


### Flange Interface: 18.42 in. ID

Figure C.7: Flange-body interface dimensions for the 18.42 in. OD conical segment interface.



Cone Hoisting Bracket



Cylinder Hoisting Bracket

Figure C.8: Hoisting bracket design for the conical and cylindrical platforms.

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