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Economic Structures in Production Networks

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For Arthur J. Ehlmann,"The King"

Abstract

In contrast to the classic assumption of i.i.d.shocks, most economic activities in reality are connected and their influences propagate to others through economic networks in an uneven way. This dissertation studies how the connections among economic entities affect each individual firm, industry and city in the production network, and how the difference in production networks affects the performance of the system as a whole.

The first chapter, "**A tale of many cities: industrial networks and urban productivity**," examines how changes in urban industrial network structures explain the growth rates of labor productivity in cities. I formulate a multi-sector general equilibrium model with input-output (I-O) networks of firms within a city and trade across cities. A key input to this framework is an entropy-style city network entropy index. It serves as a concise summary of urban industrial structures and describes the concentration level of inputs for an average firm in a given city. The major theoretical result is that the improvement of urban industrial structures, indicated by an increase in city network entropy, leads to an increase in the urban labor productivity growth rate. This is because changes in city network entropy are results of both city-specific technological shocks and the evolution of the nationwide input-output structure. When these two forces align in a way that increases city network entropy, production activities in a city become better organized, and its labor productivity grows faster. In MSA-level data from BEA, I verify the theory by showing that changes in network entropy are positively correlated with a city's productivity growth. In two sets of counterfactuals, I demonstrate how the interaction between technology changes and urban industrial networks determine urban labor productivity. First, I find that the presence of urban industrial networks explains 54.7% of the variance in changes in urban labor productivity caused by local sectoral shocks. Second, I demonstrate that the variance in city network entropy can explain 45.3% of

the variance in growth rates of urban labor productivity caused by shifts in the national I-O structure.

The second chapter, "**Industrial Productivity and Urban Human Capital Spillovers**," demonstrates that human capital spillovers in urban industrial networks are important factors to explain the productivity variance in industries across cities. I propose a novel human capital network index to measure the level of spillovers from skilled workforce to the urban environment of an industry. For a target industry, the index achieves high values not only when the city has a large quantity of educated workers, but also when local human capital is concentrated on industries that are economically close to the target industry. The investigation of human capital spillovers' impact on industries also requires the knowledge of city-specific industrial productivity. Therefore I build a general equilibrium model with multiple industries within cities and competitive trade among firms across cities. Then I calibrate the model with the U.S. data to acquire city-specific industrial productivity. With the calibrated industrial productivity and human capital network index data, empirical analysis are done to verify the existence and extent of human capital spillovers in cities.

The empirical results show that there are three factors that decide the influence of urban human capital spillover on the productivity of an industry: 1) the general quantity of educated workforce in the city, 2) the concentration of human capital in an industry's input-output network, 3) the ability of an industry to absorb the spillover. While the majority of sectors benefit from a more educated urban environment, certain industries experience negative human capital spillover from the rest of the city.

The third chapter, "**Credit Risks in Production Network**," explores the empirical evidence that traces credit risk propagation in a inter-sectoral input-output (IO) network. The major finding is that after ranking the supplier-industries for a specific target industry based on the weight of input shares from IO tables,

the average probability for firms in an industry to default, get delisted and go bankrupt has higher and more significant correlations with more important supplier-industries than with less important supplier industries. This relationship is robust to controls and both linear and nonlinear specifications of the model. Results suggest that credit conditions are related to production network links and may be consistent with a theory in which there is direct propagation possibly via supplier trade credit or other financial links.

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Chapter 1

A Tale of Many Cities: Industrial Networks and Urban Productivity

Introduction

Why do cities have different rates of labor productivity growth? Back in 2001, Springdale, AR and Flint, MI were both manufacturing towns with the employment sizes of approximately 200,000. What made GDP per worker grow by 71% in Springdale, AR between 2001 and 2016 but only by 25% in Flint, MI? Answers to these questions are essential to understand the wide economic inequality across regions in the U.S. However, the majority of urban development theories that focus on urban agglomeration and city sizes have trouble explaining why cities with similar sizes, such as Springdale and Flint, have divergent economic paths.

Therefore this paper seeks a new answer to this question, and explores the relationship between the labor productivity of cities and urban industrial network structures. A city is more than the sum of its parts. The way a city's industrial activities are organized and inter-connected matters for its performance, so I propose a concise and tractable framework to describe city industrial network structures and to study their connections to urban labor productivity.

To model cities with inter-connected industries in an open economy, I build a multi-sector, multi-city general equilibrium model that features Leontief input-output (I-O) network production in every firm, as in Acemoglu et al. (2012a), as well as open trade and Bertrand price competition among firms across different cities, as in Eaton and Kortum (2002). This framework generates a Shannon's entropy style city network entropy index as an observable summary of the urban industrial structure. City network entropy describes the concentration level of input linkages for an average firm in a city. Firms with a few very important inputs have higher values of entropy than firms that rely equally on a large number of inputs. A city has higher aggregate city network entropy if a larger share of its firms is in sectors with high industry-level input linkage concentration. Figure 1.1 illustrates the difference between the high and the low level of city network entropy measures.

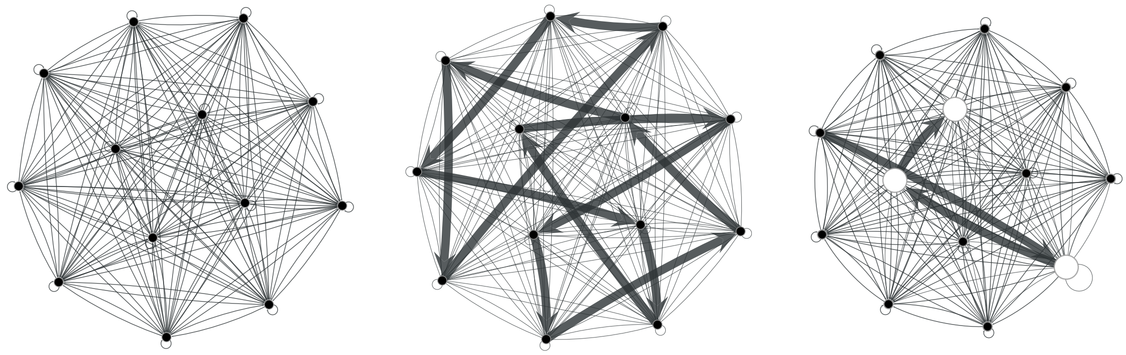
The main theoretical result of this model is that the improvement of urban industrial structures, indicated by an increase in city network entropy, leads to an increase in urban labor productivity growth rate. The intuition behind this lies in the fact that firms grow faster if they can improve technology to utilize a larger share of more productive inputs. In any production process, it is often the case that some inputs are relatively more productive than others. Because we cannot directly measure the productivity of each input, observing firms in an industry putting more weight on a particular input, namely increasing industry-level entropy, suggests that they have been able to shift the production process toward a more productive input. If a city has a larger share of this type of more productive firms, namely a higher city network entropy, its economic activities are better structured, the city thus has more potential for labor productivity growth.

In addition, the other crucial theoretical insight is that when economies trade, the foundational theorem of Hulten (1978) breaks down. The contribution of an industry's TFP to the aggregate city productivity is not only determined by its share in city level GDP, but also by its position in the urban industrial network, which is described by the city network entropy measure.

With this model, I then identify the two major forces that drive the changes in urban industrial network structures: local technology shocks in TFP and innovations in the national I-O network. On the one hand, local technology breakthroughs in individual sectors increase the competi-

Figure 1.1: Examples of Networks with Different Levels of Entropy

Node sizes in these graphs represent the output shares of sectors. Edge thickness represents the weights on inputs. Graph 1.1a shows the situation of lowest city network entropy with uniform edges among sectors with equal shares. Graph 1.1b shows the situation that all industries have equal output shares and only high weights on one input edge. This generates a high city network entropy value. Graph 1.1c shows a city that has high output shares on 3 industries with high industry-level entropy. Therefore this graph has a city entropy value between that of Graph 1.1a and Graph 1.1c.



(a) Low Entropy (b) High Entropy (c) Intermediate Entropy

tiveness of local industries in national trade. As a result, productive firms have bigger export markets than their counterparts in other cities, and increasingly larger local output shares. However, not every local technology breakthrough improves the local economy equally. If the nationwide I-O structure shifts in a way that makes locally productive industries less important nationally, a local productivity increase will have less impact on the local economy. For example, if a town becomes very efficient at making calculators, while the rest of the businesses in the country all replace calculators with computers in their production process, the local growth of the calculator industry will only have a limited positive influence on the city's aggregate productivity. This paper shows that only when local technology changes and national I-O structural innovations are aligned in a way that improves urban industrial structures will the labor productivity of cities grow faster. This improvement of urban industrial structures can be captured by an increase in the city network entropy measure.

To test the theoretical results empirically, I use data on 372 American Metropolitan Statistical Areas (MSAs) to show that, in the period of 2001 to 2016, increases in city network entropy are indeed strong predictors of higher local productivity growth rates. The positive correlation between them is statistically significant and robust to various controls and different time horizon specifications. Nevertheless, the lack of valid instrument prohibits the paper from providing vigorous causal inference for this statistical correlation.

In counterfactuals, first the model is calibrated with the same data as the empirical analysis. Then I demonstrate that the impact of local technological shocks and national I-O shocks on city productivity varies with urban industrial network structures. The first set of counterfactuals show that due to the existence of urban production networks, not every local industry's technological improvement contributes the same to the productivity growth of cities. Local technological improvements in industries with high entropy values boost the city productivity proportionally more than local technological improvements in industries with low entropy values. In particular, I find that 54.7% of the variance in labor productivity's elasticity to local sectoral shocks can be explained by the presence of urban industrial networks for Flint and Springdale. The second set of counterfactuals illustrate that with different industrial network structures, shifts in the national I-O structure can have opposite impacts on the labor productivity of cities. Assume the importance of an industry rises in the national I-O structure. If a city has a large number of firms highly dependent on this industry's inputs, its aggregate productivity gets a boost whereas in a case that a city has a large number of firms that use only a small amount of inputs from this industry, such change dampens the urban aggregate productivity. In simulations, the variance in city network entropy measures can explain 45.28% of the variance in the growth rates of urban labor productivity caused by shifts in the national I-O structure.

The contribution of this paper is twofold. On the theoretical side, it is the first to show the interaction among production networks in an open economy. It demonstrates how shapes of industrial networks can summarize the effects of technological changes on the productivity of cities. That is, when regional productivity innovations interact with the national I-O network in a way that increases network entropy, they improve a city's economic structures and its overall productivity. This new observation provides the potential for further work in endogenous city industrial network formation. Also, it is the first to show how Hulten's theorem breaks down in an open economy with competitive trade. On the quantitative side, this paper puts forward a tractable and computationally straightforward method to account for the impact of national and regional technological changes on urban productivity growth through industrial networks.

The remainder of the paper is organized as follows. After a brief discussion of related literature, Section 1.1 presents a multi-city multi-sector model of city production with trade and derives the city network entropy measure. Section 1.2 further discusses the interpretation of city network entropy and its relationship with technological changes. Section 1.3 takes the model to data and verifies the positive correlation between city network entropy and urban labor productivity growth empirically. In Section 1.4, I demonstrate how to calibrate the model and the simulations predict the impact of different types of technological changes on urban productivity through production network. Section 1.5 concludes.

Related literature

A large number of papers in the urban literature use agglomeration effects to show that city size determines productivity (Duranton and Puga, 2004). These theories are excellent at justifying the status quo of the continued exceptional performance of behemoth cities, but only provide limited explanations to why some average cities develop into big metropolises while others gradually decline and disappear. Also, traditional urban development models usually only use abstract aggregate city specific parameters to describe the heterogeneity of cities (Desmet and Rossi-Hansberg, 2013; Redding and Sturm, 2008; Au and Henderson, 2006). These aggregate parameters are relatively hard to track and need complex algorithms to solve for. In addition, changes in these city-specific parameters are often modeled as 'aggregate shocks' in residuals, which are hard to interpret economically. My paper develops a novel network measure to explain the heterogeneity in cities in terms of urban industrial structures. It is empirically easy to track. More importantly, I show that explaining the uniqueness of cities in terms of differences in urban industrial structures is economically meaningful– it improves the understanding of how technological changes affect regional economies differently.

The industrial network framework used in this paper is an extension of the multi-sector model with I-O linkages from Acemoglu et al. (2012a). Their paper puts multi-sector production with inter-sectoral intermediate inputs into a competitive general equilibrium to demonstrate how sectoral idiosyncratic shocks can lead to aggregate fluctuations through input-output linkages. While their paper discusses the behavior of a single closed network, my work studies multiple networks connected through inter-city trade. Also, they focus on the origins of aggregation fluctuation, whereas this paper mainly investigates the aggregate productivity changes caused by changes in the network.

The city level network entropy measure used in this paper is closely related to the network measures that Herskovic (2018) developed for financial market analysis. He shows that portfolio risks are correlated with portfolio sensitivity to input-output network sparsity and network concentration of the aggregate economy. My work is more focused on interactions among multiple city networks, and how various types of network changes can influence the economic system differently.

The general equilibrium framework used in this paper is also closely related to the international trade literature. Eaton and Kortum (2002) explains how comparative advantage affects the performance of cities through competitive markets. Although in their paper each city has a continuum of firms and each firm competes in its own nation-wide market, their model can only generate differences in city performance from abstract city-level productivity parameters, due to the lack of intra-city network structures. By combining their framework with city-level input-output network structures, this paper is capable of explaining differences in city growth with more observable city characteristics and more concrete reasoning. Arkolakis et al. (2015) proposed a systematic method of analyzing trade models and their work provided important tools for this paper to combine the model of Eaton and Kortum (2002) with network analysis.

In addition, Caliendo and Parro (2015) and Caliendo et al. (2017) used a similar framework, combining Eaton and Kortum (2002) and Acemoglu et al. (2012a), to investigate welfare effects of trade and estimate regional and sectoral productivity changes. While they focus on comparing welfare changes of the map as a whole through changes of different types of fundamentals, my work focuses on the change in the production network itself and its impact on the productivity growth of individual cities. Also, the model at the firm level in this paper is different from those previously used in a way that is essential for the derivation of the network entropy measure.

1.1 Model

This section introduces a general equilibrium model that features a map of numerous cities and has production networks of firms in cities, and trade networks among firms across different cities. Each city has a discrete set of sectors. Similarly to the set up in Eaton and Kortum (2002), each sector has a continuum of firms and firms in different cities trade competitively on national markets. In order to show that firms are inter-connected with each other through an input-output network, the model also has the novel feature that each firm takes other industries' inputs into its own production process. When describing any input-output and trade related variables, this paper always puts characteristics of suppliers, i.e., the origins, as superscripts and characteristics of buyers, i.e., the destinations, as subscripts to distinguish these two.

1.1.1 Firms' Problem

Consider a discrete set of cities $\mathbf{C} = \{1, \dots C\}$ that each one of them has a set of industries $\mathbf{N} = \{1, \dots n\}$. In each industry or sector, there is a continuum Ω of firms that each produces one variety of goods that can be used either for final consumption or as intermediate inputs to the production of other industries on national markets. Here Ω is normalized to 1 for every industry. Note that it is possible not every variety is actually produced in a specific city. An industry in a city varies exogenously in its firms' productivity for each good $\omega \in \Omega$. The output of a firm producing variety ω in industry i of city c, i.e. $q_{i,c}(\omega)$, is assumed to have the following Cobb-Douglas form:

$$q_{i,c}(\omega) = A_{i,c}(\omega) \left(l_{i,c}(\omega)\right)^r \left(I_{i,c}(\omega)\right)^{1-r}, \tag{1.1}$$

$$where \ \log I_{i,c}(\omega) = \sum_{j \in \mathbf{N}} d_i^j \int_{\Omega} \log q_{i,c}^j(\omega') d\omega'. \quad (1.2)$$

$l_{i,c}(\omega)$ is the quantity of labor input and r is the share of labor input. To simplify the presentation, I set r the same for every firm in the baseline model presented in this section. $I_{i,c}(\omega)$ is the collection of intermediate inputs that the producer of ω in industry i of city c needs from a national market. Specifically, $q_{i,c}^j(\omega')$ represents the amount of variety ω' in industry j that the producer of ω in $\{i, c\}$ wants to buy.

In Equation (1.2), d_i^j indicates the share of expense on good j in the total intermediate input expenditure of any firm in industry i. Therefore $d_i^j \in [0, 1]$. It shows the relative importance of different intermediate inputs in industry i's production process. Let $\mathcal{D}$ represent the matrix where $\mathcal{D}_{ji} = d_i^j$, then every column of $\mathcal{D}$ sums to 1. Note that $\mathcal{D}$ is the nationwide I-O matrix that doesn't vary from city to city. This assumption is reasonable considering the fact that standardized modern industrial production varies little from location to location in the basic formula within a country, i.e. cars produced both in Detroit and San Antonio need the similar ratio of rubber for tires and steel for car bodies.

$A_{i,c}(\omega)$ in Equation (1.1) is the productivity of variety ω. Assume it is a random variable drawn from the following city-industry specific Frechet distribution:

$$Pr(A_{i,c}(\omega) \le A) = exp\left\{-T_{i,c}A^{-\theta}\right\}, \quad (1.3)$$

where $\theta > 1$ governs the distribution of productivity across goods within an industry of a city. $T_{i,c} > 0$ is a measure of the aggregate productivity of industry i in city c. A larger $T_{i,c}$ indicates a higher probability of larger values of $A_{i,c}$.

To simplify the setup, here I assume all firms in the same city c pay wage w_c to every unit of labor they hire. Therefore the total amount of disposable expenditure that people working in city c receive is $E_c = w_c \sum_{i \in \mathbf{N}} \int_{\Omega} l_{i,c}(\omega) d\omega$.

The price of variety ω of good j in city c is determined by the Bertrand price competition on the national market, just as in Eaton and Kortum (2002). Namely firms only purchase inputs from the producers anywhere offering the lowest price. Therefore the price of variety ω of j that firms in

$\{i,c\}$ actually end up paying for is:

$$p^j_{i,c}(\omega) = \min_{c' \in \mathbf{C}} p^{j,c'}_{i,c}(\omega), \tag{1.4}$$

where $p^{j,c'}_{i,c}(\omega)$ is the price any firm in $\{i,c\}$ needs to pay to variety ω's producer in $\{j,c'\}$. In the case of $p^{j,c'}_{i,c}(\omega) > p^j_{i,c}(\omega)$, there is no trade between $\{i,c\}$ and $\{j,c'\}$ on variety ω. $p^{j,c'}_{i,c}(\omega)$ is a product between the out-of-factory price of good ω in $\{j,c'\}$ and an iceberg trade cost $\tau^{c'}_c$ between the two locations, i.e. $p^{j,c'}_{i,c}(\omega) = p_{j,c'}(\omega)\tau^{c'}_c$. Here $\tau^{c'}_c \in [1,+\infty)$, $\tau^c_c = 1$. I use $\mathcal{T}$ to represent the $n \times n$ trade costs matrix where $\mathcal{T}_{c',c} = \tau^{c'}_c$. Suppose the market of each industry in each city is perfectly competitive, the out-of-factory price of a good at its origin is simply its marginal cost. By solving the cost minimization problem of firms, the out-of-factory price of a good can be written as :

$$p_{i,c}(\omega) = \frac{w^r_c \tilde{p}^{1-r}_{i,c}}{A_{i,c}(\omega)(1-r)^{1-r} r^r}, \tag{1.5}$$

where $\tilde{p}_{i,c}$ is the intermediate input price index for any firm in $\{i,c\}$ with the functional form of $\log \tilde{p}_{i,c} = \sum_{j \in \mathbf{N}} d^j_i \int_\Omega \log p^j_{i,c}(\omega) d\omega - d^j_i \log d^j_i$. The detailed derivation process of marginal cost and the price index $\tilde{p}_{i,c}$ is in the appendix. Now we can rewrite the price of any variety of good j in city c as:

$$p^j_{i,c}(\omega) = \min_{c' \in \mathbf{C}} \frac{w^r_{c'} \tilde{p}^{1-r}_{j,c'}}{A_{j,c'}(\omega)(1-r)^{1-r} r^r} \tau^{c'}_c. \tag{1.6}$$

From the above expression, we can see that $p^j_{i,c}(\omega) = p^j_{i',c}(\omega)$ $\forall i,\ i' \in \mathbf{N}$. In other words, the purchasing price of a good in a city is the same for every local firm. Therefore, as an abuse of notation, I simplify this actual purchase price of good j for any i in city c as $p^j_c(\omega)$.

With this set up, when facing a wage w_c and a set of intermediate input prices $\{p^j_{i,c}(\omega)\}$, a firm's objective is to choose the quantity of labor input $l_{i,c}(\omega)$ and a set of intermediate inputs $\{q^j_{i,c}(\omega)\}$ for production and eventually to maximize its profit.

1.1.2 Households' Problem

In this world, a representative household chooses the city that offers the highest utility to live in. If the household lives in location c, it maximizes its utility by choosing the amount of different goods across the entire span of industry to consume, but at the same time it suffers from the disutility of living in a congested city. The households' utility maximization problem in location c can be presented in the form:

$$\max_{c_{i,c}, i \in \mathbf{N}} \eta \left(\sum_{i \in \mathbf{N}} \frac{1}{n} \int_{\Omega} \log c_{i,c}(\omega) d\omega \right) - (1-\eta) \log L_c + \log u_c, \tag{1.7}$$

$$s.t. \sum_{i \in \mathbf{N}} \int_{\Omega} c_{i,c}(\omega) p_c^i(\omega) d\omega = E_c. \tag{1.8}$$

Here $\frac{1}{n}$ is the preference weight of products from a sector, and it equals the number of industries in **N**. To simplify the presentation, preference weights for all industries are set as the same here. Choosing heterogeneous preference weights will not alter the major conclusions in this paper. L_c is city c's population size and $-(1-\eta)\log L_c$ represents the disutility from the crowdedness of the city. $\eta \in [0, 1]$ measures the relative importance between consumption and disutility. u_c is the city-specific preference that cannot be explained by the wage and consumption patterns of the household.

As mentioned in the previous subsection, E_c is the wage income of the city residents. Households living in the city c provide all their labor L_c for the local production, i.e. $L_c = \sum_{i \in \mathbf{N}} \int_{\Omega} l_{i,c}(\omega) d\omega = \sum_{i \in \mathbf{N}} L_{i,c}$. Therefore, the household income has this simple form of $E_c = w_c L_c$.

The CES preference yields a Cobb-Douglas style consumption price index $p_{i,c}$ such that $\log p_{i,c} = \int_{\Omega} \log p_c^i(\omega) d\omega$ for industry i in city c. Correspondingly, I can write the consumption index of industry i in city c as $C_{i,c}$ where $\log C_{i,c} = \int_{\Omega} \log c_{i,c}(\omega) d\omega$.

1.1.3 General Equilibrium Conditions

Given a set of $\mathcal{D}$, $\mathcal{T}$, $\{u_c\}$ and $\{T_{i,c}\}$, the general equilibrium is a set of goods prices $\{p_{i,c}\}$, wages $\{w_c\}$ and population sizes $\{L_{i,c}\}$ that:

- maximize firms' profits in every city and every industry;
- maximize households' welfare in every city;
- clear the labor market in each city:

$$E_c = \sum_{i \in \mathbf{N}} E_{i,c} = w_c \sum_{i \in \mathbf{N}} L_{i,c} = w_c L_c; \tag{1.9}$$

- clear goods markets in each city and each industry, which means the total revenue of an industry in a city is equal to the income earned from trade:

$$\int_{\Omega} q_{i,c}(\omega) p_c^i(\omega) d\omega = \sum_{j \in \mathbf{N}} \sum_{c' \in \mathbf{C}} \int_{\Omega} (q_{j,c'}^{i,c}(\omega) + c_{c'}^{i,c}(\omega)) p_c^i(\omega) d\omega, \tag{1.10}$$

  where $c_{c'}^{i,c}$ represents the total amount of consumption goods in city c' that comes from industry i in city c;
- balance trade in each city, which means the total expenditure is equal to the income earned from trade, i.e.

$$\sum_{i \in \mathbf{N}} \int_{\Omega} q_{i,c}(\omega) p_c^i(\omega) d\omega = \sum_{i \in \mathbf{N}} \sum_{c' \in \mathbf{C}} \sum_{j \in \mathbf{N}} \int_{\Omega} (q_{i,c}^{j,c'}(\omega) + c_{j,c}(\omega)) p_{i,c}^j(\omega) d\omega; \tag{1.11}$$

- and equalize welfare across cities, such that in equilibrium nobody wants to move, i.e.

$$\frac{w_c^{\eta} L_c^{\eta-1} u_c}{P_c} = \frac{w_{c'}^{\eta} L_{c'}^{\eta-1} u_{c'}}{P_{c'}}, \ \forall c, c' \in \mathbf{C}, \tag{1.12}$$

  where $P_c = \left(n \prod_{i \in \mathbf{N}} p_{i,c}\right)^{\frac{1}{n}}$ is city c's consumption price index.

1.1.4 Trade Shares and Output Shares

The following couple of lemmas provide expressions for trade shares and output shares in equilibrium of this model and they are indispensable tools for further analysis. These results extend the insights of Arkolakis et al. (2015) and Burres (1994) about spacial equilibrium into an economy with input-output production networks among firms.

Lemma 1. *In equilibrium, the share of goods needed from industry j in c' in* *the total consumption of city c is:*

$$\pi_c^{j,c'} = \frac{T_{j,c'}\left(w_{c'}^r \tilde{p}_{j,c'}^{1-r}\tau_{c',c}\right)^{-\theta}}{\Phi_c^j},$$

where $\Phi_c^j = \sum_{c'\in\mathbf{C}} T_{j,c'}\left(w_{c'}^r \tilde{p}_{j,c'}^{1-r}\tau_{c',c}\right)^{-\theta}$.

This expression of trade share $\pi_c^{j,c'}$ shows that if firms in $\{j,c'\}$ become more productive, namely, when $T_{j,c'}$ is higher, the export shares of their goods to every city in city c's aggregate output increase as well. The proof of Lemma 1 is in the appendix.

This expression of trade shares $\pi_c^{j,c'}$ offers a way to rewrite the values of local sales. Let $y_{i,c}$ represent the total output from $\{i,c\}$ and y_c be the total output of city c. First order conditions of firms' problem show that the demand of $\{i,c\}$ for good j is a constant share of the output of $\{i,c\}$, i.e. $d_i^j(1-r)q_{i,c}p_c^i$. First order conditions of households' problem indicate that the consumption demand of city c's household for good j is $\frac{E_c}{n}$. Let $x_c^{j,c'}$ denote the sales that industry j in city c receive from city c', then the decomposition of $x_c^{j,c'}$ into final consumption and industrial inputs can be written as:

$$\begin{aligned} x_c^{j,c'} &= \sum_{i\in\mathbf{N}}\left(c_{i,c}^{j,c'} + q_{i,c}^{j,c'}\right)p_{i,c}^{j,c'} \\ &= \left(\frac{E_c}{n} + \sum_{i\in\mathbf{N}} d_i^j(1-r)y_{i,c}\right)\pi_c^{j,c'}. \end{aligned} \tag{1.13}$$

Substituting the above expression back to the market clearing condition (1.10), I can rewrite the output for an exporting $\{i,c\}$ as:

$$\begin{aligned} y_{i,c} &= \sum_{c'\in\mathbf{C}}\left(\frac{rE_{c'}}{n} + \sum_{j\in\mathbf{N}} d_j^i(1-r)y_{j,c'}\right)\pi_c^{j,c'} \\ y_{i,c} &= \sum_{c'\in\mathbf{C}}\left(\frac{r\sum_{j\in\mathbf{N}} y_{j,c'}}{n} + \sum_{j\in\mathbf{N}} d_j^i(1-r)y_{j,c'}\right)\pi_c^{j,c'} \\ l_{i,c}w_c &= \sum_{c'\in\mathbf{C}}\frac{T_{i,c}}{\Phi_{c'}^i}\left(w_c^r\tilde{p}_{i,c}^{1-r}\tau_{c'}^c\right)^{-\theta}\left(\frac{r\sum_{j\in\mathbf{N}} l_{j,c'}w_{c'}}{n} + \sum_{j\in\mathbf{N}} d_j^i(1-r)l_{j,c'}w_{c'}\right) \\ l_{i,c}w_c &= T_{i,c}\left(w_c^r\tilde{p}_{i,c}^{1-r}\right)^{-\theta}\sum_{c'\in\mathbf{C}}\frac{(\tau_{c'}^c)^{-\theta}}{\Phi_{c'}^i}\left(\frac{r\sum_{j\in\mathbf{N}} l_{j,c'}w_{c'}}{n} + \sum_{j\in\mathbf{N}} d_j^i(1-r)l_{j,c'}w_{c'}\right), \end{aligned} \tag{1.14}$$

where $\Phi_c^j = \sum_{c' \in \mathbf{C}} T_{j,c'} \left(w_{c'}^r \tilde{p}_{j,c'}^{1-r} \tau_{c',c} \right)^{-\theta}$. This set of equations from the balance of trade forms the basis for the results in Lemma 2.

The following lemma shows that the share of different industries in a city is a function of the nationwide input-output structure $\mathcal{D}$, and the relative competitiveness of its industries in the national market. Again, the formal proof of Lemma 2 is in the appendix.

Lemma 2. *Let $\nu_c = [\nu_{1,c},\, \nu_{2,c},\, \ldots, \nu_{n,c}]'$ be the vector of industry output shares in city c where $\nu_{i,c} = \dfrac{y_{i,c}}{\sum_{i \in \mathbf{N}} y_{i,c}}$. Then the industry output share vector of any city can be written as:*

$$\nu_c = (I - (1-r)\pi_c^c \cdot \mathcal{D})^{-1} \cdot (\frac{r}{n}\pi_c^c \cdot + X_c), \tag{1.15}$$

where X_c is the vector of the export market's relative size for each industry in c, such that each coordinate is in the form:

$$X_{i,c} = \frac{\sum_{c' \in \mathbf{C} \setminus c} \left(\frac{r \sum_{j \in \mathbf{N}} y_{j,c'}}{n} + \sum_{j \in \mathbf{N}} d_j^i (1-r) y_{j,c'} \right) \pi_{c'}^{i,c}}{y_{i,c}}.$$

I is an $n \times n$ identity matrix and π_c^c is the $n \times n$ diagonal matrix that the ith element on the diagonal is $\pi_c^{i,c}$.

Lemma 2 indicates that output share $\nu_{i,c}$ is increasing in $\pi_c^{i,c}$ and $X_{i,c}$. As shown in Lemma 1, $\pi_c^{i,c'}$ is increasing in $T_{i,c} \forall c' \in \mathbf{C}$. Therefore, $\nu_{i,c}$ is increasing in $T_{i,c}$ whereas $\nu_{j,c}$ is decreasing in $T_{i,c}$ for any $j \in \mathbf{N} \setminus i$.

When there is no trade across cities, namely $\pi_c^c = I$ and $X_{i,c} = 0$. The expression of ν_c reduces to $\nu_c = \frac{r}{n} (I - (1-r) \cdot D)^{-1} \cdot I$, the same as in Acemoglu et al. (2012a) and Burres (1994). This makes the industry structure of each city the same. However, when cities are open to trade, their output share distributions are influenced by both the levels of local productivity $\{T_{i,c}\}$, as well as the national input-output network $\mathcal{D}$. Therefore, one important insight from this model is that competitive trade is one of the major reasons for cities in a country to have heterogeneous industrial compositions.

1.1.5 Industrial Network and Urban Productivity

Following the above results of output shares ν_c and trade shares $\{\pi_c^{i,c'}\}$, this section derives the expression of city-level productivity, i.e. gross output per worker, in the form of model fundamentals and studies its composition.

Theorem 1. *Let g_c be the gross output per worker of city c. Assume $\log \boldsymbol{T}_c = [\log T_{1,c}, \log T_{2,c}, ..., \log T_{n,c}]'$, and $\log \boldsymbol{p}_c = [\log p_{1,c}, \log p_{2,c}, ..., \log p_{n,c}]'$. In equilibrium the expected log labor productivity of a city $\log g_c$ can be written as:*

$$\log g_c = \underbrace{\frac{1}{r}\nu_c' \cdot m(\log \boldsymbol{T}_c)}_{\text{Industry Component}} + \underbrace{\sum_{i \in \boldsymbol{N}} \nu_{i,c} \sum_{j \in \boldsymbol{N}} d_i^j \log d_i^j}_{\textbf{city network entropy NE}TTc} + \underbrace{\frac{1}{r}\left[(I - \pi_c^{c-1})'\nu_c + \frac{r}{n}\boldsymbol{1} + \pi_c^{c-1} X_c\right]' \cdot \log \boldsymbol{P}_c}_{\text{Price Component}} + const. \tag{1.16}$$

where $N_c^E = \sum_{i \in \boldsymbol{N}} \nu_{i,c} \sum_{j \in \boldsymbol{N}} d_i^j \log d_i^j$ is the city network entropy measure of c and m is a nonlinear increasing function of $\log \boldsymbol{T}_c$.

$m(\log \mathbf{T}_c)$ in the first component of city-level productivity is independent from the input-output relationship in a city. The curviture of m depends on the competitiveness of city c in inter-city trade. When there is no trade across cities, $m(\log \mathbf{T}_c)$ goes to linear, i.e. $\log \mathbf{T}_c$. The formal proof is in the appendix.

Theorem 1 points out that the aggregate city-level labor productivity can be decomposed into three factors: (1) a weighted sum of the productivity of individual industries, (2) a urban industrial structure component in the form of *city network entropy* NE_c, and (3) the price effects determined by the competition on the national market. In the absence of intra-city production network, NE_c vanishes from the equation of Theorem 1 while the price component and the industry component stay. Therefore, NE_c represents the synergy created by the production coordination among firms in a city. While it is an endogenous component in the system, it captures movements in $\{T_{i,c}\}$ through $\{\nu_{i,c}\}$ as well as movements in $\mathcal{D}$. Section 1.2 will elaborate on the interpretation of NE_c.

1.1.6 The Breakdown of Hulten's Theorem

One crucial implication of Theorem 1 is that the foundational theorem of Hulten (1978) breaks down in an open economy setting.

Hulten's theorem states that technological changes in one sector can only affect the aggregate productivity growth to the extent that is proportional to this sector's output share. In other words, in a closed economy where firms do not take inputs from other industries, an industry with an increasing level of productivity can only boost its' output to the extent that its local buyers and consumers can absorb. Hulten's theorem can be presented in the closed economy version of this model. According to Theorem 1's decomposition, when d_i^j goes to 0 for $j \neq i$ and d_i^i goes to 1, changes in $\mathbf{T}_{i,c}$ only affect the output per work through the industry component and its influence is restricted to its output share $\nu_{i,c}$, which is invariant for a given $\mathcal{D}$. More detailed mathematical analysis of cities in autarky is in the appendix.

However Hulten's theorem fails to hold in the open economy setting of the model. In an open economy, local industries are not confined by the local urban economic size and structure anymore. On the contrary, increasingly productive firms gain an increasing amount of customers across all cities from trade. Thanks to the increasing size of the export market, these firms produce disproportionately more than their original share of the output. As a result, from the channel of inter-city comparative advantage, productivity shocks of more productive industries in an open economy have a disproportionately higher impact on the local economy.

In math, this simply indicates that changes in $\mathbf{T}_{i,c}$ also move the industry output shares ν_c in the industry component of Theorem 1's decomposition. When output shares change in a city, workers switch sectors they work for. If more workers end up in more productive sectors in the economy, the labor productivity of the city grows faster whereas if more workers move to less productive sectors, the city level labor productivity growth slows down. This effect of industrial restructuring caused by $T_{i,c}$ is incorporated in the city network entropy part of Theorem 1. In a word, we cannot consider aggregate productivity shocks as a simple linear summation of sectoral shocks anymore. Urban industrial structures also determine the influence of sectoral shocks on the aggregate economy.

1.2 Network Entropy

The second component in the decomposition of Theorem 1, i.e.

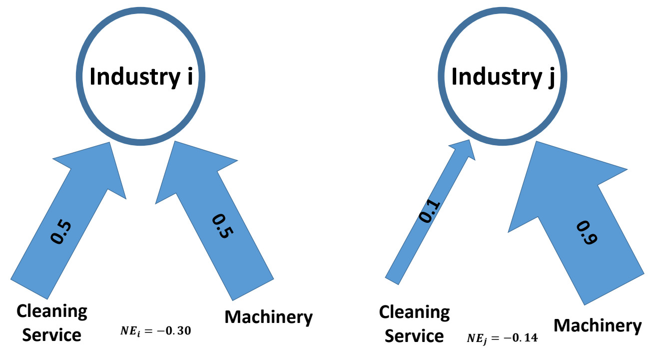
$$NE_c = \sum_{i \in \mathbf{N}} \nu_{i,c} \cdot \underbrace{\sum_{j \in \mathbf{N}} d_i^j \log d_i^j}_{\text{industry-level entropy } NE_i}, \tag{1.17}$$

is defined as *city network entropy*. City network entropy describes the concentration of input linkages of an average firm in a city. It is the weighted average of industry-level input entropy where the weights are local output shares. The input-output relationships of an urban production network can be represented as a graph where each industry is a node, output shares are node sizes, and the input flow between any pair of industries is an arc connecting two nodes. Weights on arcs indicate the importance of different inputs to a sector. Under this setup, city network entropy measures the inequality in input weight distribution. In other words, the stronger input linkages with a few sectors that firms have, the higher city network entropy is. Figure 1.1 illustrates three examples of urban industrial structures with different levels of city network entropy.

To further understand the connection between network entropy measures and the productivity of industries, I decompose the city network entropy NE_c into a weighted sum of industry-level network entropy measures $NE_i = \sum_{j \in \mathbf{N}} d_i^j \log d_i^j$. Figure 1.2 illustrates the difference between high and low industry-level entropy measures in an example with two inputs. Because industry j has a very high concentration of input demand from one of the inputs, it has a high value of NE_j, whereas industry i equally distributes input needs on two types of inputs, so it has a low NE_i. Note that due to the logarithm function insde NE_i, it is always non-positive and is bounded by $[-\log n, 0]$. Therefore, a high industry-level entropy means the absolute value of NE_i is closer to 0. If an industry only relies on itself for capital inputs, then the value of industry-level network entropy goes to 0. This is the opposite to the definition of Shannon's entropy. The composition of NE_c indicates that a city has higher aggregate city network entropy if a larger share of its firms is in sectors with high industry-level entropy.

Figure 1.2: Understanding Industry-Level Entropy Measure

Industry j highly concentrates input usage on machinery, so firms in j have a higher industry-level entropy of than firms in i.



One core claim of this paper is that a higher value of industry-level entropy NE_i indicates a more efficient and productive production process for an industry. Mathematically, this is because under Cobb-Douglas technology, higher concentration of input weights in the vector $\{d_i^1, ...d_i^n\}$ makes the marginal production cost of firms in industry i lower. Equation (1.6) shows that the marginal cost of production of a firm in $\{i, c\}$ has this form:

$$MC_{i,c} = \frac{w_c^r \tilde{p}_{i,c}^{1-r}}{A_{i,c}(\omega)(1-r)^{1-r} r^r} = \frac{w_c^r (\prod_{j \in \mathbf{N}} p_{j,c}^{d_i^j})^{1-r}}{A_{i,c}(\omega)(1-r)^{1-r} r^r} \cdot \frac{1}{\underbrace{\prod_{j \in \mathbf{N}} d_i^{j\, d_i^j}}_{\exp(NE_i)}} .$$

Without considering the price effect in the denominator $w_c^r \tilde{p}_{i,c}^{1-r}$, an increase in NE_i means a decline in marginal cost of every unit of the output so firms can produce more with the same amount of expense. In the appendix, I demonstrate in a two-industry example that the output per worker for firms in $\{i, c\}$ are also increasing in NE_i.

The economic intuition behind this is that: firms that are better at adopting more productive types of inputs grow faster. Every industry has relatively less productive input varieties with limited values of marginal product, such as cleaning service or security service, as well as relatively more productive input varieties with high values of the marginal product, such as computers for Microsoft or the automated production lines for Ford or GE. However, data limitations prevent us from measuring the productivity of each input directly. What can be observed is that firms in an industry change their input structures. Over time, observing firms in an industry putting more weights on a particular set of inputs indicates that firms in this industry have been able to shift their production process towards a set of more productive inputs. In other words, this industry becomes more productive.

I must point out that the above economic intuition of the connection between the industry-level productivity and its input entropy is only valid if different input categories have different levels of marginal product or contributions to the production process, and all industries have some varieties of inputs with low levels of marginal product in their production. In other words, if d_i^j has equal values for every j while the domain size **N** changes, NE_i will vary as well. In this case the above theory of "more productive inputs" is invalid. However, this paper only considers the situation of a fixed domain size NE_i. Also, in reality it is never the case that every dollar of different input categories contribute the same to the production process. Nearly all firms need some indispensable inputs such as utility services, cleaning services or security services, to keep the business going. However, these inputs are usually not the most productive elements of the business.

As a city, higher output weights on high entropy industries render a larger value of NE_c. Because a higher value of NE_c indicates that a larger part of the city produces relatively more efficiently, it is not hard to see why Theorem 1 indicates a positive correlation between NE_c and log output per worker in a city.

1.2.1 Technological Changes in Urban Industrial Networks

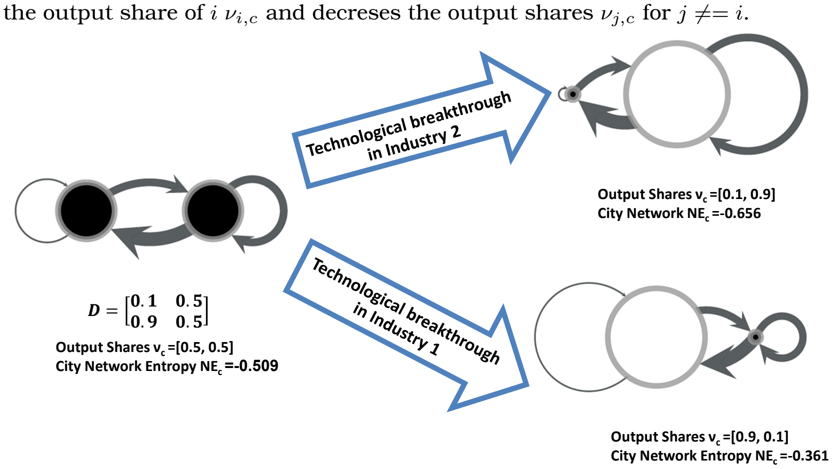
After having a basic understanding of the city network entropy measure, a natural question to ask is that what causes changes in city network entropy, which is an endogenous object in the model. This section will demonstrate how two exogenous forces, namely local technological shocks in $\{T_{i,c}\}$, and innovations in the national input-output network $\mathcal{D}$, can affect industrial network structures of cities, thus shifting urban labor productivity.

Local Technological Changes

First, let's consider a local technological breakthrough in industry i in city c, expressed as an increase in $T_{i,c}$. In this case, Lemmas 1 and 2 indicate that the output share of industry i in city c, namely $\nu_{i,c}$, goes up while shares of other industries decrease. The change of network entropy can go either way. If industry i is a comparatively more productive industry, then it has a higher-than-average industry-level entropy NE_i. Therefore, an increase in the share of i increases the city network entropy measure NE_c and boosts the aggregate city productivity. However, if industry i is less productive than the average, namely it has a lower-than-average industry-level entropy NE_i, an increase in i's output share only drags down NE_c and dampens the magnitude of city-level productivity growth.

Figure 1.3: Effect of Local Technological Changes in NE_c

The ith column of $\mathcal{D}$ represents the input shares into industry i. Local technological breakthrough in $T_{i,c}$ eventually increases the output share of i $\nu_{i,c}$ and decreses the output shares $\nu_{j,c}$ for $j \neq = i$.



In the example given by Figure 1.3, industry 1 has higher entropy than industry 2, which indicates that industry 1 is more efficient in the production process. Although increases in $T_{1,c}$ and $T_{2,c}$ both boost the value of the industry component in Equation (1.16), their effects on NE_c are different. Increase in $T_{2,c}$ pulls up the output share of industry 2 and shifts more labor in the city to a sector that is less efficient and lowers the entropy of the city as a whole. Therefore, the entropy measure decreases from -0.509 to -0.656. This indicates that the city gets structurally less efficient in production. Similarly, an increase in $T_{1,c}$ boosts the entropy measure from -0.509

to -0.361. As a result, a technological breakthrough in industry 1 makes the city grow faster than a technological breakthrough in industry 2.

The above analysis implies that due to the existence of intra-city production networks, not every local industry's technological improvement contributes the same to the city productivity growth. Local technological improvements in industries with high NE_i boost the city productivity more than local technological improvements in industries with low NE_i.

It is straightforward to understand that a changing $T_{i,c}$ represents shifts of relative competitiveness of city c in industry i products' production and changes in city c's share in product i's national market, but how can one show the change of relative competitiveness of one industry to another industry in the economy?

Nationwide Input-Output Structure Changes

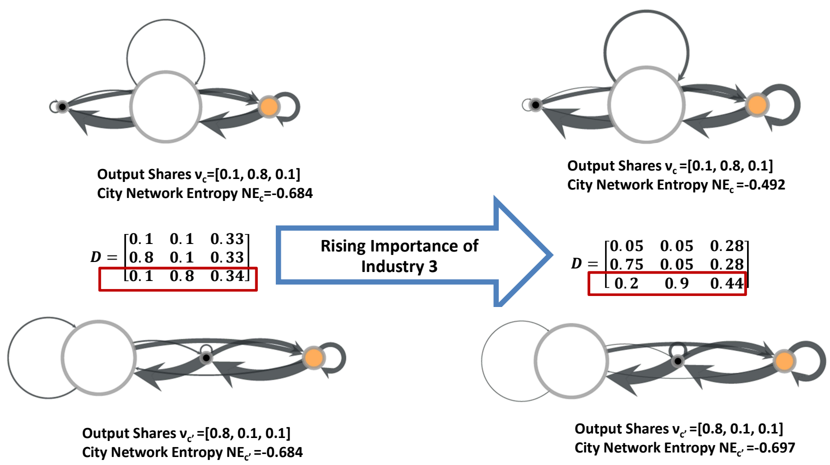
The second type of technological change considered here is the nationwide rise in the importance of industry i. Because products from different industries are not entirely substitutable, if a nationwide technology breakthrough in industry i is represented in the form of an increase in $T_{i,c}$ for all cities $c \in \mathbf{C}$, it can only affect the market sizes of other industries through very limited price competition within the sector i. Instead, I define a nationwide technological breakthrough as the situation that businesses need industry i's products more than any other industry j's products for reasons other than price concerns. In math this means the ith row of $\mathcal{D}$ increases whereas the jth row of $\mathcal{D}$ decreases. An example of such nationwide rising importance of an industry is that an increasing number of retail shops today start to use the Internet for business while keeping their brick and mortar stores. This change in input type is not only for the reason that an increasing online presence can save business costs, but also for the sake that a substantial amount of t customers shift to the Internet and businesses have to adapt to such changes.

According to the output share formulation in Lemma 2, increase in the ith row in $\mathcal{D}$ brings up the the output share of industry i in every city, i.e. $\nu_{i,c}\,\forall\, c \in \mathbf{C}$. This is a consequence of the rising total market size of industry i in the whole country's economy. Such nationwide technological changes in

$\mathcal{D}$ affect city network entropy measures in a way different from local productivity adjustments. An increase in d^i_j boosts the entropy levels of industries that originally use a large proportion of i's inputs and suppresses the entropy levels of sectors that use little input from i. In other words, if d^i_j is very small, $\sum_{i\in\mathbf{N}} d^i_j \log d^i_j$ decreases as d^i_j moves up, whereas for industry i with very high d^i_j originally, $\sum_{i\in\mathbf{N}} d^i_j \log d^i_j$ gets even higher if d^i_j rises. The economic intuition behind is that firms that know how to efficiently incorporate sector i's inputs into their production process benefit comparatively more from a technology breakthrough of industry i. Therefore if a city happens to host

Figure 1.4: Effect of National I-O Structure Change in $\mathcal{D}$

The ith row of $\mathcal{D}$ represents the input shares from industry i to every other industry. Nationwide technological breakthrough in industry 3 means the 3rd row of $\mathcal{D}$ increases. Graphs on the top rows are the results of city c. Graphs on the bottom rows are the results for city c'. The rising importance of industry 3 boosts the entropy for city c, who more heavily rely industry 3's inputs originally, but lowers the entropy of city c', who doesn't not need much of industry 3's inputs originally.



a large share of firms that depend heavily on industry i, its productivity will benefit from the industry-wide technological breakthrough of industry i indirectly through the local production network. On the other hand, for cities that are dominated by industries with very small input demands from industry i, the growth of their aggregate productivity will suffer from their 'suboptimal' network structures.

Figure 1.4 gives an example of two cities with different emphasis on their economic structures. City c in the top row has a very big industry 2, and 80% of its output is from that sector. City c' in the bottom row has an emphasis on industry 1 where 80% of its output comes from. When industry 3's relative importance in the economy rises, the third row of $\mathcal{D}$ increases.

As a result, industry 2 and 3's industry-level entropy measures increase whereas industry 1's industry-level entropy declines. According to the analysis above, this implies that industry 1's most important inputs, i.e. the ones from industry 2, have declining marginal products relative to other inputs, therefore the production efficiency of industry 1 declines. As a result, the productivity of the bottom row city c' suffers from the nationwide structural change with a decrease of network entropy from -0.684 to -0.697, while the productivity of the top row city c benefits from it, with an increase in city network entropy from -0.684 to -0.492.

The takeaway here is that a nationwide I-O structure change affects the productivity of cities differently. Assume the importance of industry i increases, i.e. values of the ith row in $\mathcal{D}$ increases. If a city has a large number of firms highly dependent on i's inputs, its aggregate productivity gets a boost. If a city has a large number of firms that use a small amount of i's inputs, such change dampens its aggregate productivity.

1.3 Empirical Evidence

The previous sections demonstrate how in theory changes in the network structure of cities are signaled by changes in city network entropy measures, affect growth of urban labor productivity. In this section, I use data from 372 MSAs across the U.S. between 2001 and 2016 to verify this relationship empirically.

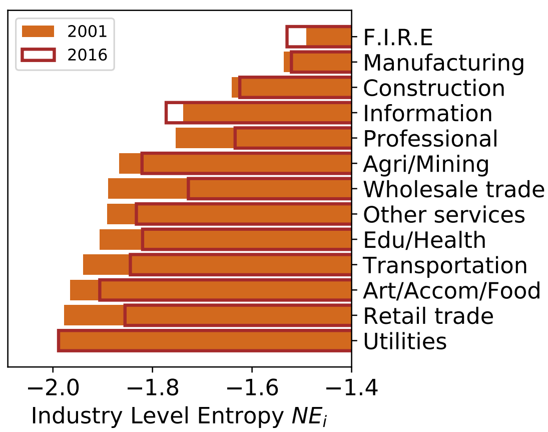
1.3.1 Data

To construct the city network entropy measures, I use the direct use tables from the I-O accounts data[1] provided by Bureau of Economic Analysis (BEA) for the matrix $\mathcal{D}$. Because the focus of this paper is on domestic private sectors of the economy, industries related to government activities are taken out. After removing other components not related to intermediate inputs, such as employee compensation, tax and operating surplus, these I-O tables are normalized such that each column sums to 1.

[1]The full name of the table is: Industry-by-Commodity Total Requirements, After Redefinitions.

Based on these I-O data, Figure 1.5 shows industry-level entropy measures calculated by Equation (1.17) from 2001 to 2016 for 13 industries that cover the whole private sector in the U.S. These 13 industries are defined by 2-digit North American Industry Classification System (NAICS) codes. In theory, the entropy interpretation and the model conclusion are not affected by how fine the industry definitions are or the number of industry categories included in the industry space. However, the regional industrial output data used later in this section has coarser industrial classifications than I-O tables. In order to match the output data with I-O tables and to minimize the appearance of 0 in output weight vectors ν_c in the counterfactual section, only 13 industries are used in the analysis. The analysis in the

Figure 1.5: National I-O Network Entropy by Industries, 2001-2016



High NE_i means NE_i is closer to 0. The industries are ranked from high to low based on the industry-level entropy measure in 2001. According to the graph, F.I.R.E and manufacturing always have the highest NE_i values while utilities always have the lowest NE_i. F.I.R.E. refers to finance, insurance, real estate, rental services. Professional refers to professional and business services, which include legal, accounting, scientific, technical and computer services, as well as business management.

previous section points out that a high industry-level NE_i is an indicator of relatively high industry-wise productivity. Therefore according to Figure 1.5, in 2016 the top four sectors with the most efficient input structures in the U.S. economy are manufacturing, F.I.R.E[2], construction, and professional and business services[3]. The network structure theory proposed

[2]F.I.R.E. refers to finance, insurance, real estate, rental and leasing

[3]Professional and business services include: legal services, computer systems design and related services, miscellaneous professional, scientific, and technical services, management of companies and enterprises, administrative and support services, waste management and remediation services.

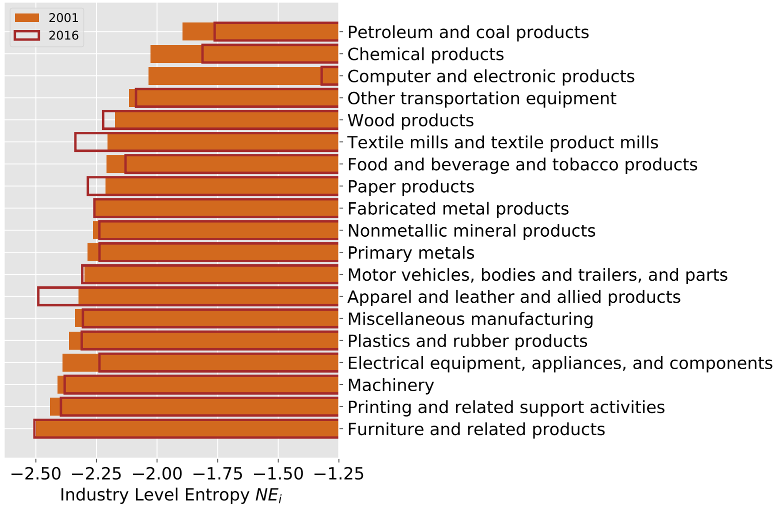
in the previous sections implies that cities with increasing output shares in these sectors are more likely to enjoy faster city aggregate productivity growth. In general, the industry-level entropy measures are relatively stable in the period. Exceptions are professional and business services, which has risen substantially. Also the entropy level of information and F.I.R.E has dropped in the same period of time. This signals improving productivity in the professional and business services sector, but the opposite in information and finance. In the appendix, Figure 24 shows the changes of industry-level network entropy for 13 industries from 1997 to 2016. Together with Figure 1.5, these two graphs imply that while some industries' entropy ranking changed dramatically in this period, the general entropy ranking for majority of industries are largely stable.

One potential problem is that the empirical relationship between industry productivity levels and entropy values can become weaker if the industry classification level used gets finer. Figures 25 and 26 in the appendix exhibit the ranks and changes over time of industry-level entropy measures under a finer definition of industries with 66 categories [4]. Under this 66 category classification, Figure 1.6 shows the input entropy changes for 3 digits industries within only the manufacturing sector between 2001 and 2016. We can see that the range of entropy values within finer classification of the manufacturing sector is even bigger than that of the 13 industry classification. These graphs together show that network entropy differences persist no matter how fine the industrial classification level is.

The other concern is that high industry entropy levels are just the consequence of strong self-supply effect. If industries that mainly supply to themselves are considered as structurally optimal, then it undermines the importance of an urban industrial network. In fact, not all high entropy industries are the biggest suppliers of themselves. The data from these 13 industry I-O tables shows that the top suppliers of construction and retail trade are not themselves but these sectors still possess relatively high levels of network entropy. Also, this self-supply phenomenon diminishes with a finer definition of industries. For example, although the two-digit manufac-

[4]The 66 industry categories are based on the finer total requirement input-output tables for 71 industries provided by BEA. After excluding the government sectors, 66 industries are left to cover the full span of the private sector.

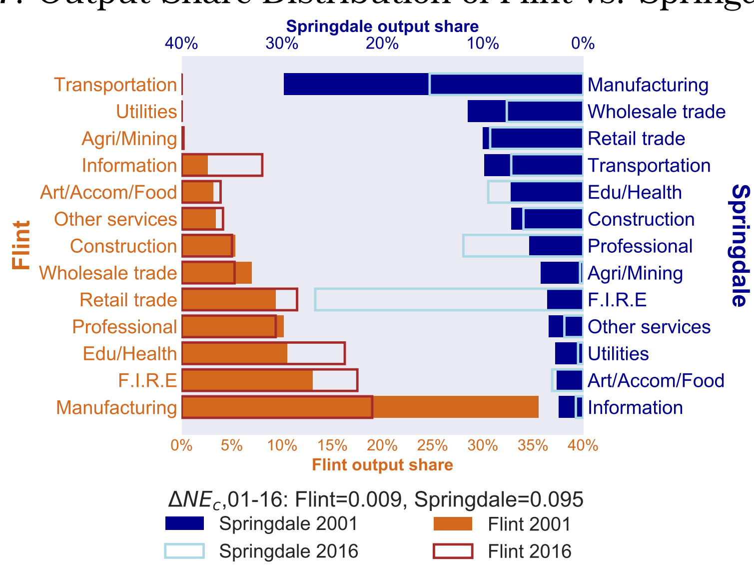
Figure 1.6: Industry Network Entropy for Manufacturing, 2001-2016



High NE_i means NE_i is closer to 0. The industries are ranked from high to low based on their 2001 industry-level entropy. All industries are in the manufacturing sector and belong to the total requirement tables for 71 indutries from BEA.

turing sector is the largest supplier of itself, quite a few three-digit manufacturing industries, such as 324 petroleum and coal products and 325 chemical products, are not the largest suppliers of themselves. Meanwhile, these finer sub-sectors of manufacturing still have relatively high entropy levels. Therefore, the fact that certain sectors are the biggest suppliers of themselves under a coarser definition of industries should not erode the value of network entropy in explaining the urban economic structures.

Figure 1.7: Output Share Distribution of Flint vs. Springdale, 01-16



Now let's come back to Flint, MI and Springdale, AR. Figure 1.7 shows the output share distributions of Flint and Springdale in 2001 and 2016. It is obvious that in 2001 both cities were typical manufacturing towns with around 30% of their outputs in the manufacturing sector. In 2016, the share of the manufacturing industry in both towns went down substantially. On the one hand, Springdale managed to build a new pillar for its economy– the financial sector. Also, professional and business services went through a fast expansion in Springdale. Both these industries were among the top 5 industries with the highest industry-level entropy. Especially, professional and business services had one of the largest rises of industry-level entropy in the national I-O table. Therefore, the city network entropy of Springdale has risen sharply. According to my theory, this indicates that Springdale managed to reshape its industrial structure to include more highly productive firms. As a result, the labor productivity in Springdale grew fast. On the other hand, Flint did not manage to build a new focus for its economy. The sectors with the largest rises in output shares were education and health care, as well as information. However, on the national I-O table, education and health care were not among the ones with high industry-level entropy. Also, the information sector has experienced the largest decline in industry-level entropy nationally. Therefore, the city network entropy of Flint only rose about 10% of the magnitude in Springdale. This indicates that Flint failed to restructure its economy to be in align with productive sectors in the U.S. and it didn't build a new core industry to compete on the national market. Consequently it grew comparatively slower. As shown here, all the above analysis can be summarized by the observable changes in city network entropy. Therefore city network entropy is a very effective tool to tell the story of changes in urban industrial structures.

1.3.2 Regression Analysis

In addition to the anecdotal evidence of Flint and Springdale, regression analysis is also presented in this section to support this theory of industrial structure and urban productivity in data. Due to the lack of valid controls to build a proper causal inference strategy, all the regressions in this section are only aimed at empirically verifying the correlation between city network entropy and urban productivity. I use employment and output data for MSAs at 2-digit NAICS level provided by BEA to construct city labor productivity measures.

The decomposition of log output per worker in Theorem 1 is used as the basic formulation of this empirical exercise, i.e.

$$\%\Delta Productivity_c = \Delta NE_c + controls + \epsilon_c. \tag{1.18}$$

From the point of view of a single city, firms take both the price levels $\mathbf{P}_c$ and local productivity levels $\mathbf{T}_c$ as given. Therefore, the industry and price components of Theorem 1 are exogenous and can move either in the same or opposite direction as the network components NE_c. As a result they are either captured by the controls or embedded in the residual term ϵ_c. In other words these two components are not likely to affect the statistical significance of the elasticity of labor productivity to the city network entropy.

1.3.3 Baseline Results

In the basic formulation, percentage changes of GDP per worker are used as the measure of city productivity changes according to the expression of productivity in Theorem 1. Basic controls include city-level employment size, year and location fixed effects. I run regressions in the form of Equation (1.18) for productivity and network entropy measures in 1-year, 3-year, 8-year and 15-year time span. Results are presented in the first 3 columns of Table 1.1 and they show that the coefficients of growth in NE_c are positive and statistically significant at 1% significance level across different time spans. The adjusted R^2 of 1-year and 3-year specifications are relatively low whereas the 8-year regression has a higher R^2 of 0.125. This implies that this network model of cities has a lower degree of correlation in the short term. It is consistent with the nature of industrial structural changes. Namely, improvements in production technology, either locally or nationally, have a greater impact over the long term. Also, the 15-year model has low R^2. This may be caused by the financial crisis and economic recessions in this period of time. Both network entropy and output per

Table 1.1: Productivity Growth v.s. Changes in NE_c over Different Time Horizons

variable name	1 yr growth	3 yr growth	8 yr growth	15 yr growth	8 yr growth 2	15 yr growth 2	2001-2008	2009-2016	2-period pooled
Intercept	**0.027*****	**0.102*****	**0.258*****	**0.44*****	**0.406*****	**0.481*****	**0.21*****	**0.148*****	**0.323*****
	(0.004)	(0.016)	(0.023)	(0.013)	(0.046)	(0.026)	(0.03)	(0.013)	(0.072)
NE_c growth	**0.598*****	**0.98*****	**1.526*****	**1.488*****	**0.922*****	**1.092*****	**1.582*****	**1.296*****	**1.265****
	(0.101)	(0.155)	(0.333)	(0.266)	(0.319)	(0.346)	(0.327)	(0.281)	(0.596)
employment	-0.0	-0.0	-0.0	-0.0	0.0	-0.0	-0.0	0.0	0.0
	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)
top ind share					**−0.823*****	-0.12	-0.072	0.062	**−0.801****
					(0.217)	(0.087)	(0.085)	(0.057)	(0.355)
tradable sec share growth					0.032	0.101	**0.162***	0.086	0.13
					(0.061)	(0.081)	(0.087)	(0.058)	(0.142)
city & year FE	YES	YES	YES	NO	YES	NO	NO	NO	YES
R^2	0.049	0.074	0.125	0.081	0.214	0.09	0.167	0.128	0.242
Observation	5580	4836	2976	372	2976	372	372	372	744

Table 1.2: Alternative Productivity Measure v.s. Changes in NE_c over Different Time Horizons

variable name	1 yr growth	3 yr growth	8 yr growth	15 yr growth	8 yr growth 2	15 yr growth 2	2001-2008	2009-2016	2-period pooled
Intercept	0.003	**0.026****	**0.055****	**−0.121*****	**0.176*****	-0.023	**0.163*****	**0.105*****	**0.264*****
	(0.003)	(0.013)	(0.024)	(0.014)	(0.035)	(0.027)	(0.03)	(0.013)	(0.075)
NE_c growth	**0.554*****	**0.835*****	**1.056*****	**1.763*****	**0.567****	**1.429*****	**1.692*****	**1.13*****	**1.355****
	(0.088)	(0.14)	(0.228)	(0.28)	(0.237)	(0.355)	(0.333)	(0.284)	(0.614)
employment	**−0.0***	**−0.0*****	**−0.0*****	-0.0	**−0.0***	-0.0	-0.0	0.0	0.0
	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)
top ind share					**−0.673*****	**−0.396*****	-0.074	0.022	**−0.776****
					(0.123)	(0.089)	(0.087)	(0.057)	(0.367)
tradable sec share growth					0.024	-0.045	**0.148***	**0.108***	0.133
					(0.054)	(0.083)	(0.089)	(0.059)	(0.144)
city & year FE	YES	YES	YES	NO	YES	NO	NO	NO	YES
R^2	0.048	0.061	0.072	0.098	0.138	0.144	0.171	0.116	0.239
Observation	5580	4836	2976	372	2976	372	372	372	744

Dependent variable: GDP per worker growth residual
Standard errors in the parentheses are culstering with respect to city and year
***p<0.01, **0.01≤p<0.05, *0.05≤p<0.1

worker data are likely to move nonlinearly, so linear trend estimates may not fit well here. In addition, as the time span of growth increases, the magnitude of the coefficient for ΔNE_c increases as well.

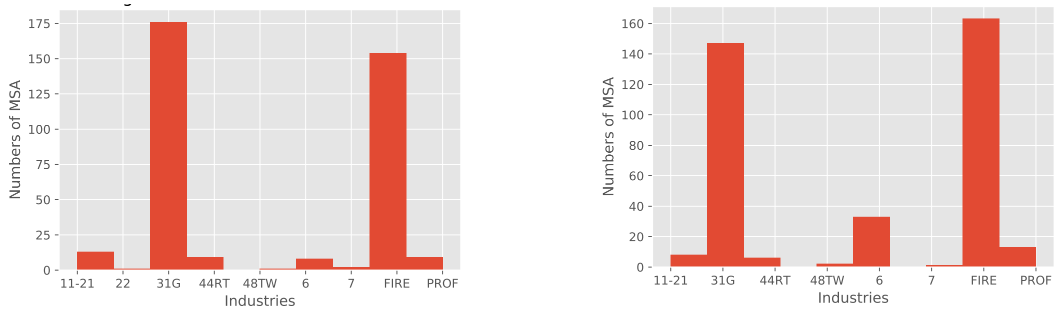
1.3.4 Robustness Check

This significantly positive correlation between city network entropy changes and urban productivity changes is robust to various model specifications as well. The rest of the columns from Tables 1.1 and 1.2 contain results from three types of robustness checks: additional controls, alternative time spans of the growth period and alternative dependent variables.

The first type of additional controls is designed to check whether the effect of industrial network structures on urban productivity is dominated by shares of the largest industries in cities. Because the largest sectors in cities are often the most productive ones as well, it is possible that the aggregate city productivity is entirely driven by the size of the largest industry and how the rest of the economy is organized has little influence on the aggregate urban labor productivity. To eliminate such possibility, I add the share of the largest industry in a city as a control variable in the regression and the results are reported in the 5th and 6th columns of Table 1.1. Based on Figure 1.8, F.I.R.E and manufacturing (31G) were most frequently shown leading industries of cities in data, so share changes of these two industries and fixed effects of the leading industries' categories are also considered as controls. Results of these experiments are in Table 4 of the appendix. In general, adding any one of these controls for leading industries does not take away the statistical significance of the change in entropy measure in the regression. Therefore, the network effects are not dominated by the shares of the largest industries in city economy.

The second type of additional controls is designed to examine whether the share of the tradable sector in a city's economy affects the relationship between its network entropy and productivity. I calculate the tradable proportion of a city's economy by multiplying the shares of traded goods share in each industry category calculated by Delgado et al. (2014) with local output weights. Then changes of tradable sector shares of cities are added as controls in regression showed in the 5th to 10th columns of Table 1.1.

Figure 1.8: Largest Industries in 372 MSAs



(a) Year 2001 (b) Year 2016

Because the coefficients of changes in network entropy are all statistically significant and positive, effects of network entropy on productivity survive the influence by the size of the tradable sectors.

Next, to verify that the network effect on productivity is robust throughout the business cycle and the recent financial crisis, regression results with additional controls in the time span of 2001-2008 and 2009-2016 are presented separately in the 7th and 8th columns of Table 1.1. The data from before the financial crisis and after the crisis are pooled together for the regression in the 9th column. The results show that although the magnitude of elasticity of productivity to network entropy varies before and after the financial crisis, the coefficients of $NE_c\ g$ are always significant and positive.

In addition, I construct an alternative definition to the dependent variable and repeat all the regressions in Table 1.1. The outcomes are presented in Table 1.2. The alternative defintion of the dependent variable is a measure of 'productivity growth residual'. To calculate the residuals, first a simulated Bartik instrument of city productivity is constructed from multiplying the vector of national average industry output levels with the vector of city-specific output shares i.e. $\sum_{i \in \mathbf{S}} \nu_{i,c} \bar{g}_i$ where $\bar{g}_i$ is a national average output per worker of industry i. The 'productivity growth residual' is obtained by subtracting the changes of a city-specific Bartik instrument from the city level log GDP per worker, i.e. $\%\Delta g_c - \%\Delta \sum_{i \in \mathbf{S}} \nu_{i,c} \bar{g}_i$.

The idea behind this alternative dependent variable is that in the absence of intermediate input networks, the log productivity decomposition in Theorem 1 reduces to

$$\log g_c = \frac{1}{r} \nu_c' \cdot \log \mathbf{T}_c + \frac{1}{r} \nu_c' \cdot \log \mathbf{P}_c + const.$$

In this situation, the sectoral output share vector ν_c is a function of local preference weights and inter-city trade costs. Because in a competitive market local shocks cannot affect the market price vector $\mathbf{P}_c$, local technological changes only affect city output through changes in $\mathbf{T}_c$ and ν_c from the industry component. If assume shocks to $T_{i,c}$ are independent across industries and cities, then changes in output per worker of city c can be decomposed as

$$\%\Delta output/worker = \%\Delta\nu_c + \%\Delta\mathbf{T}_c.$$

While we can observe changes in ν_c, $\%\Delta\mathbf{T}_c$ is unobserved and can be consider as the residual of $\%\Delta output/worker - \%\Delta\nu_c$. If the network effect of city production does not exist, the part of changes in output per worker that cannot be explained by variations in ν_c should be independent from the network entropy measure NE_c.

Table 1.2 represents a positive and statistically significant correlation between city productivity residual changes and network entropy changes as well, although the magnitude of coefficients declines under the alternative dependent variables.

1.4 Counterfactuals

In this section, I first calibrate the multi-sector network model to replicate the current employment and wage distribution across American cities. Then with the calibrated model, simulation are carried out to illustrate how the network entropy measure captures the impact of different types of technological changes on city labor productivity.

1.4.1 Calibration

The following lemma provides a basis for the calibration.

Lemma 3. *If the set of $\mathcal{D}$ and $\mathcal{T}$, $\{u_c\}$ and productivity levels $\{T_{i,c}\}$ satisfies certain regularity conditions, there exists a unique solution of $\{w_c\}$, $\{p_{i,c}\}$,$\{L_{i,c}\}$ that solves:*

$$L_{i,c}w_c = T_{i,c}\left(w_c^r\tilde{p}_{i,c}^{1-r}\right)^{-\theta}\sum_{c'\in\mathbf{C}}\frac{(\tau_{c'}^c)^{-\theta}}{\Phi_{c'}^i}\left(\frac{r\sum_{j\in S}l_{j,c'}w_{c'}}{n}+\sum_{j\in\boldsymbol{N}}d_j^i(1-r)L_{j,c'}w_{c'}\right) \tag{1.19}$$

$$p_{i,c} = \Phi_c^j \overline{\theta}^{-1} \cdot \frac{exp\left\{\dfrac{\theta}{\gamma}\right\}}{(1-r)^{1-r} r^r} \tag{1.20}$$

$$\frac{w_c^\eta L_c^{\eta-1} u_c}{P_c} = \frac{w_{c'}^\eta L_{c'}^{\eta-1} u_{c'}}{P_{c'}}, \ \forall c, c' \in \boldsymbol{C}, \tag{1.21}$$

where $\Phi_c^j = \sum_{c' \in \boldsymbol{C}} T_{j,c'} \left(w_{c'}^r \tilde{p}_{j,c'}^{1-r} \tau_c^{c'} \right)^{-\theta}$ *and* $P_c = \left(n \prod_{i \in \boldsymbol{N}} p_{i,c} \right)^{\frac{1}{n}}$.

Equation (1.19) is derived from goods market clearing conditions in Equation (1.14). The derivation of Equation (1.20) is in the appendix. The equalization of welfare across city gives the last set of Equation (1.21). The equilibrium definition set of Equation (1.19) to (1.21) shows that there exists solutions of technology parameters $\{T_{i,c}\}$ and utility preference residuals $\{u_{i,c}\}$ given information of employment $\{L_{i,c}\}$, wage $\{w_c\}$, trade cost $\mathcal{T}$, I-O network data $\mathcal{D}$, proper values for productivity parameter θ and labor share r. Although in the absence of trade flow data solutions for $\{T_{i,c}\}$ and $\{u_{i,c}\}$, all simulations in this section are calculated with percentage changes in productivity, so the absolute values of $\{T_{i,c}\}$ and $\{u_{i,c}\}$ will not affect the results.

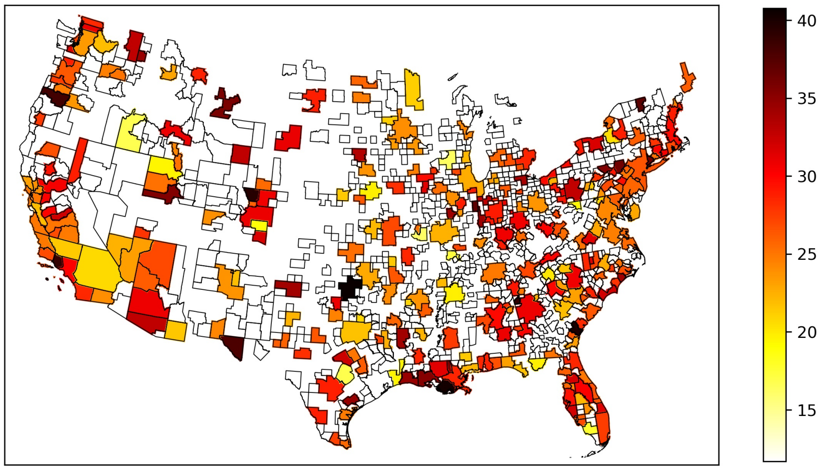
I use 2016 employment, wage data of MSAs and national input-output data from BEA to calibrate the model. The iceberg cost matrix $\mathcal{T}$ is estimated by the fast marching algorithm and structural estimation process described by Allen and Arkolakis (2014), from U.S. highway, rail and navigable water networks data (NDC, 1999; CTA, 2003; NHPN, 2005). Only cities that can be identified in the trade cost estimation, are considered in the calibration. Therefore the number of cities in the calibration is reduced to 303, slightly lower than the number of cities in the empirical exercises[5]. The labor share r is set to 0.388 for the baseline calibration to match the average labor share across industries in the U.S. in 2016.

The lack of trade flow data among industries and cities prevents me from estimating the comparative advantage parameter θ by using ratios of trade flows, as in Eaton and Kortum (2002). Therefore, I take several estimated values of θ from Eaton and Kortum (2002) and Caliendo and Parro (2015) for the calibration exercise. Figure 1.9 shows the city-specific preference

[5]Because the iceberg trade cost matrix estimation does not include data from Alaska and Hawaii, there is no MSA from these two states in the counterfactuals. However, 4 MSAs from these states are included in the empirical regressions in the previous section.

calibrated with the aggregate elasticity of 4.55 from Caliendo and Parro (2015) as θ. Different choices of θ do not affect the relative rankings of preference estimates.

Figure 1.9: City Specific Preference u_c in 2016 of 303 MSAs, with θ=4.55



1.4.2 Counterfactuals

With the calibrated model, I can test how cities with different network structures react to technological changes differently. The analysis in Section 1.2 shows that there are two types of technological changes that can affect city labor productivity through urban industrial networks, namely local technological changes in $\{T_{i,c}\}$ and national I-O relationship changes in $\mathcal{D}$. I will discuss the counterfactual results of these two types of technological changes respectively in this section.

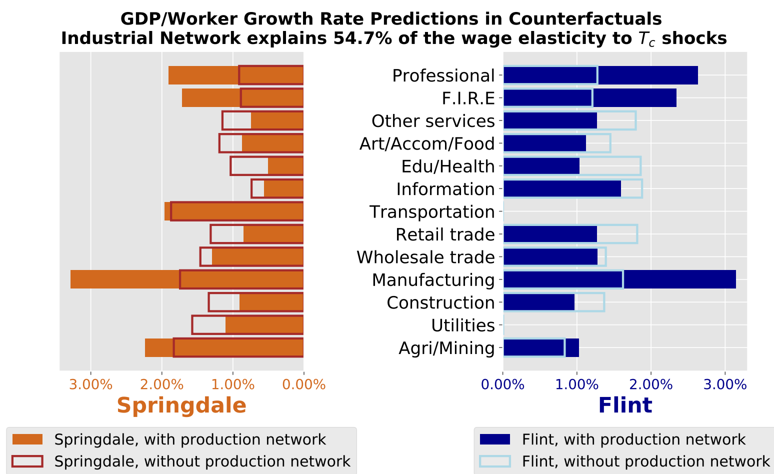
Local Technological Changes

To test the conclusion that not every local technology breakthrough improves local economy equally, I increase $T_{i,c}$ 10 times for a single industry i in a single city, and record changes in GDP per worker of all cities. This exercise is repeated for both Springdale and Flint, and every one of their 13 industries. Furthermore, to quantify the role of production networks in this process, I turn the network off in every city on the map by raising the labor share r to 1 and redo the experiments of increasing $T_{i,c}$. Another way of turning off the production network is to reduce the I-O matrix $\mathcal{D}$ to an identity matrix. In this case production functions become decreasing-return-to-scale and a firm's marginal cost is a function of the output. This dramatically increases the difficulty to solve for the market clearing conditions in Equation (1.19). Therefore, only one way for shut downing the production network is used here.

As in most geography models, wage and employment are underidentified without proper normalization. Therefore, in the simulation, the total size of employment is fixed at the 2016 level, and the wage of Midland, TX is normalized to 1[6]. Output per worker changes at city-level can be obtained from the new wage and labor distribution. Figure 1.10 shows the results of these experiments[7].

Figure 1.10: Effect of Local Technological Changes on NE_c

Each bar represents the percentage change in city level GDP/worker due to a 10-fold increase in $T_{i,c}$ for the corresponding industry on the y axis. Solid bars are GDP/worker growth rates with city production networks, while hollow bars are GDP/worker growth rates without city production networks.



The figures show that in the presence of city production networks, local technological breakthroughs in high entropy industries, such as professional and business services, F.I.R.E. as well as manufacturing, augment levels of city output per worker way more than their counterparts in models without city production networks. Whereas increases in $T_{i,c}$ for low entropy

[6]In the original data, the wage of Midland TX is the median of 303 cities. Therefore calculating the relative distance of other cities' wage to Midland TX is a good way to understand the dispersion of wages in the country.

[7]Shocks of Flint's transportation and utilities sectors have the minimum impact in the results. This is caused by the 0 employment in these sectors in BEA's original dataset.In simulations, I add one worker to these two industries to guarantee the convergence of the model. I took MSAs with more than 2 missing industries out of the sample in counterfactuals.

industries, such as utilities and retail trade, lead to smaller increases in city labor productivity in models with production network than in models without production network. Due to variations in the industry and price components of the productivity decomposition equation in Theorem 1, these cities have slightly different growth results with respect to local technological shocks in other sectors. In general, the existence of city production networks increases the variance of city productivity caused by different types of local technological shocks for 54.7% in these two cities.

Nationwide I-O Structure Changes

The next set of counterfactuals demonstrate the second type of technological changes – the rising importance of a specific industry nationwide. To depict such changes in the national I-O structure, I add 0.2 to every element of the ith row of the I-O matrix $\mathcal{D}$. Note that 0.2 is chosen arbitrarily. Although the magnitude of changes in $\mathcal{D}$ is likely exaggerated, these counterfactuals are aimed at showing the impact of nationwide I-O structure shifts more clearly. Smaller changes have effects with the same signs on urban labor productivity. The values of all the other rows decrease, but the relative proportions of them are kept the same. As discussed in Section 1.2.1, if the rise of industry i increases the industry-level entropy of most firms in a city, the aggregate productivity of the location will grow faster. On the contrary, if the rise of industry i decreases the industry-level entropy of most firms in a city, the city grows slower.

Figure 1.11: Changes of GDP/Worker Caused by the Increased Importance of the Utility Sector

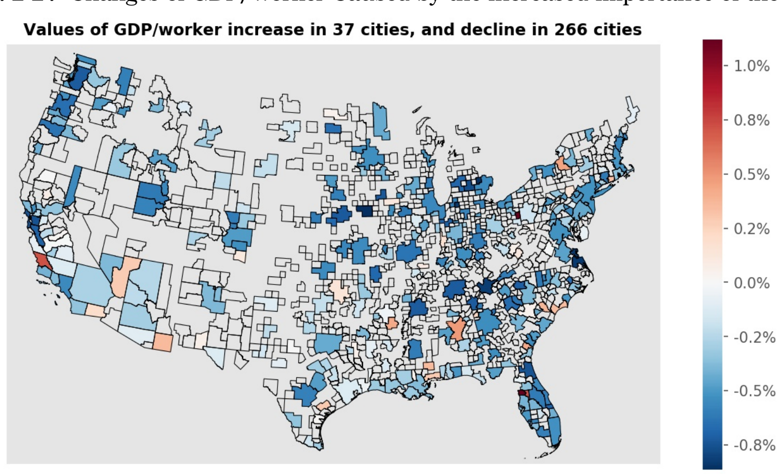
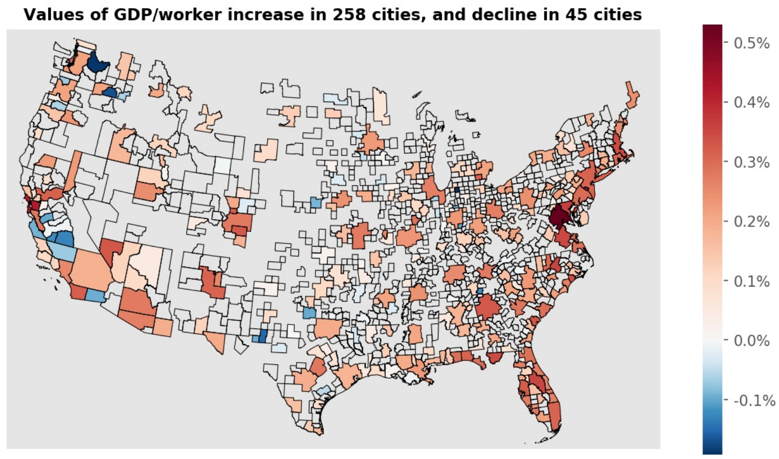


Figure 1.12: Changes of GDP/Worker by the Increased Importance of Professional and Business services



Figures 1.11 and 1.12 contrast the impact of two very different industries. If the utility sector rises, the majority of cities experience a decline in their aggregate productivity (Figure 1.11), whereas in the case of a growing professional and business services sector, the productivity of most cities grows (Figure 15). These outcomes indicate, for most firms in the country, that the output from the utility sector currently counts for small input shares and is not among the most productive ingredients in the production process. Therefore, when the importance of the utility sector rises, network entropy indices drop and most cities grow slower. On the contrary, most firms either are in the professional and business services sector themselves or rely on relatively large shares of inputs from the professional and business services sector. Therefore, when the importance of the professional and business service sector rises, the productivity of most cities benefit from this change. Figures 13 to 23 in the appendix and Table 1.3 summarizes the impact from other types of nationwide I-O network changes. These exercises demonstrate my analysis that there are relatively more productive input varieties and less productive ones in most production processes under this framework.

Although the impact of various nationwide I-O structure changes on cities can be dramatically different, it turns out the city network entropy measure can capture directions of urban productivity shifts reasonably well. I construct R^2 out of the following formulation from all 13 counterfactual exercises:

$$\%\Delta GDP/worker = \alpha + \beta \Delta NE_c + City\, Fixed\, Effect + Experiment\, Fixed\, Effect.$$

Table 1.3: The Impact of Rising Importance of Industries on Labor Productivity of U.S. Cities

Sector of Rising Importance	Agri/Mining	Utilities	Construction	Manufacturing	Wholesale trade	Retail trade
No. of cities growing	12	36	196	229	237	245
No. of cities declining	290	266	106	73	65	57

Sector of Rising Importance	Transportation	Information	Edu/Health	Art/Accom/Food	Other services	F.I.R.E	Professional
No. of cities growing	242	221	218	259	212	191	257
No. of cities declining	60	81	84	43	90	111	45

The rising importance of industry i is indicated by adding 0.2 to each element of the ith row of the I-O matrix $\mathcal{D}$. Every simulation has 303 MSAs. The Baseline city always has normalized growth rate of 0.

The result shows that the variance of city network entropy alone explains 45.28% of the variance in the growth rate of urban labor productivity. Also, β is positive and statistically significant.

1.5 Conclusion

In this paper, I build a multi-sector multi-city model with inter-sector intermediate input networks and inter-city trade to show that a city network entropy measure can describe changes in city industrial structures. Also, changes in city industrial network structures can be caused by both local productivity shocks and innovations in the national input-output structure. When these two types of forces move in a way that increases the city network entropy measure, a city organizes its production better and consequently improves its labor productivity. This feature exists in models either with or without trade component. In the presence of trade, the interaction between network structures and sectoral productivity breaks down Hulten's theorem.

With the national I-O matrices, city and industry-specific output and employment data from BEA, I generate empirical evidence to support the significant relationship between changes in city network entropy and urban productivity growth implied by the model. In counterfactuals, I demonstrate how local sectoral technology progress and nationwide I-O structure changes can affect city productivity differently for cities with different network structures. Specifically, the existence of city production networks accounts for 54.7% of the variance of the labor productivity's elasticity to local technological shocks in the two sample cities. Also, the variance in city network entropy can explain 45.28% of the variance in the growth rate of urban labor productivity caused by shifts in the national I-O structure.

Appendices

.1 Proofs

Proof of Lemma 1. Because $p_{j,c'}^{i,c} = \frac{w_c^r \tilde{p}_{i,c}^{1-r}}{A_{i,c}(1-r)^{1-r} r^r} \tau_{c'}^c$, I can derive the probability distribution of $p_{j,c'}^{i,c}$, i.e. the sales price of $\{i, c\}$ to any $\{j, c'\}$:

$$
\begin{aligned}
G_{j,c'}^{i,c}(p) &= Pr\left\{p_{j,c'}^{i,c}(\omega) \le p\right\} \\
&= Pr\left\{\frac{w_c^r \tilde{p}_{i,c}^{1-r}}{A_{i,c}(\omega)(1-r)^{1-r} r^r} \tau_{c'}^c \le p\right\} \\
&= Pr\left\{A_{i,c}(\omega) \ge \frac{w_c^r \tilde{p}_{i,c}^{1-r}}{(1-r)^{1-r} r^r p} \tau_{c'}^c\right\} \\
&= 1 - exp\left\{-T_{i,c}\left(\frac{w_c^r \tilde{p}_{i,c}^{1-r}}{(1-r)^{1-r} r^r p} \tau_{c'}^c\right)^{-\theta}\right\}.
\end{aligned} \tag{22}
$$

According to Equation (1.6), the probability distribution of good j's purchase price for any i in city c is:

$$
\begin{aligned}
G_{i,c}^j(p) &= Pr\left\{\min_{c' \in \mathbf{C}} P_{i,c}^{j,c'}(\omega) \le p\right\} \\
&= 1 - \prod_{c' \in \mathbf{C}} \left(Pr\left\{P_{i,c}^{j,c'}(\omega) \ge p\right\}\right) \\
&= 1 - \prod_{c' \in \mathbf{C}} \left(1 - G_{i,c}^{j,c'}(p)\right) \\
&= 1 - \prod_{c' \in \mathbf{C}} exp\left\{-T_{j,c'}\left(\frac{w_{c'}^r \tilde{p}_{j,c'}^{1-r}}{p(1-r)^{1-r} r^r} \tau_{c',c}\right)^{-\theta}\right\} \\
&= 1 - \exp\left\{-p^\theta \sum_{c' \in \mathbf{C}} T_{j,c'}\left(\frac{w_{c'}^r \tilde{p}_{j,c'}^{1-r}}{(1-r)^{1-r} r^r} \tau_{c',c}\right)^{-\theta}\right\} \\
&= 1 - \exp\left\{-p^\theta \tilde{\Phi}_c^j\right\},
\end{aligned}
$$

where $\tilde{\Phi}_c^j = \sum_{c' \in \mathbf{C}} T_{j,c'}\left(\frac{w_{c'}^r \tilde{p}_{j,c'}^{1-r}}{(1-r)^{1-r} r^r} \tau_{c',c}\right)^{-\theta}$. Because the value of $G_{i,c}^j(p)$ is the same for every i in the same city, I simplify the notation to $G_c^j(p)$.

With all these calculations now I can define $\pi_{i,c}^{j,c'}$, the probability that a specific city c' becomes the provider of good j for all industries in city c, as:

$$
\pi_{i,c}^{j,c'} = Pr\left\{p_{i,c}^{j,c'}(\omega) \le \min_{k \in \mathbf{C} \backslash c'} p_{i,c}^{j,k}(\omega)\right\}
$$

$$
\begin{aligned}
&= \int_0^\infty Pr\left\{\min_{k\in\mathbf{C}\setminus c'} p_{i,c}^{j,k} \geq p\right\} dG_{i,c}^{j,c'}(p) \\
&= \int_0^\infty \prod_{k\in\mathbf{C}\setminus c'} (1 - G_{i,c}^{j,k}(p)) dG_{i,c}^{j,c'}(p) \\
&= \int_0^\infty \prod_{k\in\mathbf{C}\setminus c'} exp\left\{-T_{j,k}\left(\frac{w_k^r \tilde{p}_{j,k}^{1-r}}{p(1-r)^{1-r}r^r}\tau_{k,c}\right)^{-\theta}\right\} dG_{i,c}^{j,c'}(p) \\
&= \int_0^\infty \prod_{c'\in\mathbf{C}} exp\left\{-T_{j,c'}\left(\frac{w_{c'}^r \tilde{p}_{j,c'}^{1-r}}{p(1-r)^{1-r}r^r}\tau_{c',c}\right)^{-\theta}\right\} \cdot \left(-T_{j,c'}\theta\left(\frac{w_{c'}^r \tilde{p}_{j,c'}^{1-r}}{(1-r)^{1-r}r^r}\tau_{c',c}\right)^{-\theta}\right) p^{\theta-1}dp \\
&= T_{j,c'}\left(\frac{w_{c'}^r \tilde{p}_{j,c'}^{1-r}}{(1-r)^{1-r}r^r}\tau_{c',c}\right)^{-\theta} \int_0^\infty exp\left\{-p^\theta \tilde{\Phi}_c^j\right\} \cdot \theta p^{\theta-1}dp \\
&= \frac{T_{j,c'}\left(\frac{w_{c'}^r \tilde{p}_{j,c'}^{1-r}}{(1-r)^{1-r}r^r}\tau_{c',c}\right)^{-\theta}}{\tilde{\Phi}_c^j}\left(-exp\left\{-p^\theta \tilde{\Phi}_c^j\right\}\Big|_0^\infty\right) \\
&= \frac{T_{j,c'}\left(\frac{w_{c'}^r \tilde{p}_{j,c'}^{1-r}}{(1-r)^{1-r}r^r}\tau_{c',c}\right)^{-\theta}}{\tilde{\Phi}_c^j}.
\end{aligned}
$$

Again, because the value of $\pi_{i,c}^{j,c'}$ does not depend on i, I can define the probability of $\{j, c'\}$ become the supplier of good j in city c as:

$$
\pi_c^{j,c'} = \pi_{i,c}^{j,c'} = \frac{T_{j,c'}\left(\frac{w_{c'}^r \tilde{p}_{j,c'}^{1-r}}{(1-r)^{1-r}r^r}\tau_{c',c}\right)^{-\theta}}{\tilde{\Phi}_c^j}.
$$

We can see that the probability for any $\{j, c'\}$ becomes an exporter does not depend on the specific price it charges.

By the law of large numbers, $\pi_{i,c}^{j,c'}$ can be viewed as the fraction of good j from city c' sold to industry i in city c. Next I am going to prove that $\pi_{i,c}^{j,c'}$ is also the fraction of expenditure on intermediate inputs that industry i in city spend on good j specifically from city c'. The idea is that any city winning the bid for exporting product j to industry i in c has exactly the same price distribution.

$$
\begin{aligned}
Pr\left\{p_{i,c}^{j,c'}(\omega) \leq \rho \middle| p_{i,c}^{j,c'}(\omega) \leq \min_{k\in\mathbf{C}\setminus c'} p_{i,c}^{j,k}(\omega)\right\} &= \frac{\int_0^\rho Pr\left\{\min_{k\in\mathbf{C}\setminus c'} p_{i,c}^{j,k}(\omega) \geq p\right\} dG_{i,c}^{j,c'}(p)}{Pr\left\{p_{i,c}^{j,c'} \leq \min_{k\in\mathbf{C}\setminus c'} p_{i,c}^{j,k}\right\}} \\
&= \frac{1}{\pi_{i,c}^{j,c'}} \int_0^\rho \prod_{k\in\mathbf{C}\setminus c'} (1 - G_{i,c}^{j,k}(p)) dG_{i,c}^{j,c'}(p)
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1}{\pi_{i,c}^{j,c'}} \frac{T_{j,c'}\left(\frac{w_{c'}^{r}\tilde{p}_{j,c'}^{1-r}}{(1-r)^{1-r}r^{r}}\tau_{c',c}\right)^{-\theta}}{\tilde{\Phi}_c^j}\left(-exp\left\{-p^{\theta}\tilde{\Phi}_c^j\right\}|_0^{\rho}\right)\\
&= \frac{1}{\pi_{i,c}^{j,c'}} \frac{T_{j,c'}\left(\frac{w_{c'}^{r}\tilde{p}_{j,c'}^{1-r}}{(1-r)^{1-r}r^{r}}\tau_{c',c}\right)^{-\theta}}{\tilde{\Phi}_c^j}\left(1-exp\left\{-\rho^{\theta}\tilde{\Phi}_c^j\right\}\right)\\
&= 1-exp\left\{-\rho^{\theta}\tilde{\Phi}_c^j\right\}\\
&= G_c^j(\rho).
\end{aligned}
$$

The expression of this price distribution is independent from the origin city c'. The importer will need to pay statistically the same price to any exporter that happens to offer the lowest price bid. Therefore the fraction of goods j that industry i in c bought from c', $\pi_{i,c}^{j,c'}$ is also the fraction of expenditure that city c spending on good j specifically from city c'. □

Derivation of Equation (1.20). The industry level price in city c for industry i is:

$$
\begin{aligned}
\log p_{i,c} &= \int_{\Omega} \log p_c^i(\omega)d\omega\\
\log p_{i,c} &= \int_{\Omega} \log p_c^i(\omega)dG_c^i(p)\\
\log p_{i,c} &= \int_0^{\infty} \log p \frac{d\left(1-\exp\left\{-p^{\theta}\tilde{\Phi}_c^i\right\}\right)}{dp}dp\\
\log p_{i,c} &= \int_0^{\infty} \log p \frac{d\left(1-\exp\left\{-\exp(\theta\log p+\log\tilde{\Phi}_c^i)\right\}\right)}{d\log p}d\log p\\
\log p_{i,c} &= \int_{-\infty}^{\infty} t \frac{d\left(1-\exp\left\{-\exp(\theta t+\log\tilde{\Phi}_c^i)\right\}\right)}{dt}dt.
\end{aligned}
$$

$1-\exp\left\{-\exp(\theta t+\log\tilde{\Phi}_c^i)\right\}$ is the CDF of t with gumbel distribution, i.e. $t \sim Gumbel(-\frac{\log\tilde{\Phi}_c^i,}{\theta}, \frac{1}{\theta})$, so the right hand side is simply the mean of t, $-\frac{\log\tilde{\Phi}_c^i,}{\theta}-\frac{\gamma}{\theta}$, where γ is the Euler's constant. Then I can further write the price index as:

$$
\begin{aligned}
\log p_{i,c} &= -\frac{\log\tilde{\Phi}_c^i}{\theta}-\frac{\gamma}{\theta}\\
p_{i,c} &= \Phi_c^{i^{-\frac{1}{\theta}}}\exp\left\{\frac{\theta}{\gamma}\right\}
\end{aligned}
$$

$$p_{i,c} = \left\{ \sum_{c' \in \mathbf{C}} T_{i,c'} \left(\frac{w_{c'}^r \tilde{p}_{i,c'}^{1-r}}{(1-r)^{1-r} r^r} \tau_{c',c} \right)^{-\theta} \right\}^{-\dfrac{1}{\theta}} \exp\left\{ \frac{\theta}{\gamma} \right\}.$$

Therefore, if $p_{i,c}$ is deterministic conditional on all the parameters and thus $\tilde{p}_{i,c}$ is also deterministic. $\square$

Derivation of Equation (1.5). The lagrangian of firms' cost minimization problem is:

$$\mathcal{L} = w_c l_{i.c}(\omega) + \sum_{j \in \mathbf{N}} \int_{\Omega} p_c^j(\omega) q_{i,c}^j(\omega') d\omega' + \lambda \left[\log q_{i,c}(\omega) - \log \left(A_{i,c} l_{i.c}(\omega)^r I_{i,c}(\omega)^{1-r} \right) \right]$$
$$\textit{where } \log I_{i,c}(\omega) = \sum_{j \in \mathbf{N}} d_i^j \int_{\Omega} \log q_{i,c}^j(\omega') d\omega'.$$

FOC w.r.t. $q_{i,c}^j(\omega)$ gives:

$$\begin{aligned}
p_c^j(\omega) q_{i,c}^j(\omega) &= d_i^j \cdot \lambda(1-r) \\
\log q_{i,c}^j(\omega) &= \log d_i^j + \log(\lambda(1-r)) - \log p_c^j(\omega) \qquad (23) \\
\int_{\Omega} \log q_{i,c}^j(\omega) d\omega &= \log d_i^j + \log(\lambda(1-r)) - \int_{\Omega} \log p_c^j(\omega) d\omega \\
\sum_{j \in \mathbf{N}} d_i^j \int_{\Omega} \log q_{i,c}^j(\omega) d\omega &= \sum_{j \in \mathbf{N}} d_i^j \log d_i^j - \sum_{j \in \mathbf{N}} d_i^j \int_{\Omega} \log p_c^j(\omega) d\omega + \log(\lambda(1-r)) \\
\log I_{i,c} &= \log(\lambda(1-r)) - \log \tilde{p}_{i,c} \\
I_{i,c}(\omega) &= \lambda(1-r) \tilde{p}_{i,c}^{-1}. \qquad (24)
\end{aligned}$$

FOC w.r.t. $l_{i.c}(\omega)$ gives:

$$l_{i.c}(\omega) = \lambda r {w_c}^{-1}.$$

The ratio of the above two equations gives:

$$\begin{aligned}
\frac{I_{i,c}(\omega)}{l_{i.c}(\omega)} &= \frac{(1-r) w_c}{r \tilde{p}_{i,c}} \\
I_{i,c}(\omega) &= \frac{(1-r) w_c}{r \tilde{p}_{i,c}} l_{i.c}(\omega) \qquad (25) \\
l_{i.c}(\omega) &= \frac{r \tilde{p}_{i,c}}{(1-r) w_c} I_{i,c}(\omega). \qquad (26)
\end{aligned}$$

Substitute Equation (25) and Equation (26) back to the production function

$q_{i,c} = A_{i,c} l_{i.c}(\omega)^r I_{i,c}^{1-r}$, I get:

$$l_{i.c}(\omega) = \left(\frac{(1-r)w_c}{r\tilde{p}_{i,c}}\right)^{r-1} \frac{q_{i,c}}{A_{i,c}}$$
$$I_{i,c}(\omega) = \left(\frac{(1-r)w_c}{r\tilde{p}_{i,c}}\right)^{r} \frac{q_{i,c}}{A_{i,c}}.$$

Now I can write out the total cost and marginal cost of production:

$$\begin{aligned}
TC_{i,c} &= w_c l_{i.c}(\omega) + \sum_{j\in\mathbf{N}} \int_{\Omega} p_c^j(\omega) q_{i,c}^j(\omega')d\omega' \\
&= w_c \left(\frac{(1-r)w_c}{r\tilde{p}_{i,c}}\right)^{r-1} \frac{q_{i,c}}{A_{i,c}} + \tilde{p}_{i,c} I_{i,c}(\omega) \\
&= w_c \left(\frac{(1-r)w_c}{r\tilde{p}_{i,c}}\right)^{r-1} \frac{q_{i,c}}{A_{i,c}} + \tilde{p}_{i,c} \left(\frac{(1-r)w_c}{r\tilde{p}_{i,c}}\right)^{r} \frac{q_{i,c}}{A_{i,c}} \\
&= \frac{q_{i,c} w_c^r \tilde{p}_{i,c}^{1-r}}{A_{i,c}(1-r)^{1-r} r^r}. \\
MC_{i,c} &= \frac{\partial TC_{i,c}}{\partial q_{i,c}} \\
&= \frac{w_c^r \tilde{p}_{i,c}^{1-r}}{A_{i,c}(1-r)^{1-r} r^r}.
\end{aligned}$$

□

Proof of Lemma 2. Let $Yc = [y_{1,c},\ y_{2,c},\ ...,y_{n,c}]'$ be the output vector of city c. In equilibrium, the labor market clearing conditions implies that $\sum_{c\in\mathbf{C}} y_{i,c} = \frac{E_c}{r}$. Because $y_{i,c} = \sum_{c'\in\mathbf{C}} x_{c'}^{i,c}$, Equation (1.13) shows the relationship among $y_{i,c}$ as:

$$\begin{aligned}
y_{i,c} &= \sum_{c'\in\mathbf{C}} x_{c'}^{i,c} \\
&= \sum_{c'\in\mathbf{C}} \sum_{j\in\mathbf{N}} \left(d_j^i(1-r)y_{jc'} + \frac{rE_{c'}}{n}\right) \frac{\pi_{c'}^{i,c}}{\tau_{c,c'}} \\
&= \sum_{c'\in\mathbf{C}} \sum_{j\in\mathbf{N}} \left(d_j^i(1-r)y_{j,c'} + \frac{r\sum_{j\in\mathbf{N}} y_{j,c'}}{n}\right) \frac{\pi_{c'}^{i,c}}{\tau_{c,c'}}
\end{aligned}$$

If I separate the intra-city trade component from inter-city trade components, I can rewrite the above equation into:

$$y_{i,c} = \pi_c^{i,c}\left(\sum_{j\in\mathbf{N}} d_j^i(1-r)y_{j,c} + \frac{r\sum_{j\in\mathbf{N}} y_{j,c}}{n}\right) + \sum_{c'\in\mathbf{C}\setminus c}\sum_{j\in\mathbf{N}}\left(d_j^i(1-r)y_{j,c'} + \frac{r\sum_{j\in\mathbf{N}} y_{j,c'}}{n}\right)\frac{\pi_{c'}^{i,c}}{\tau_{c,c'}}$$

$$y_{i,c} = \pi_c^{i,c}\left(\sum_{j\in\mathbf{N}} d_j^i(1-r)y_{j,c} + \frac{r\sum_{i\in\mathbf{N}} y_{j,c}}{n}\right) + Export$$

In vector form:

$$Y_c = (1-r)\pi_c^c\cdot D\cdot Y_c + \frac{r}{n}Y_c'\cdot\mathbf{1}\cdot\pi_c^c + X_c$$
$$\nu_c = (1-r)\pi_c^c\cdot D\cdot\nu_c + \frac{r}{n}\pi_c^c + X_c$$
$$\nu_c = \left(I-(1-r)\pi_c^c\cdot D\right)^{-1}\cdot\left(\frac{r}{n}\pi_c^c\cdot + X_c\right)$$

□

Proof of Theorem 1. According, city-wide labor market clearing conditions, $L_{i,c} = \nu_{i,c}L_c$. Also, the F.O.C. of firms' problem gives the expression for output of city c as $y_{i,c} = \frac{1}{r}w_cL_{i,c}$. Therefore, I can use wage to represent GPD per worker in this model as: $g_c = \frac{y_{i,c}}{L_{i,c}} = \frac{w_c}{r}$. According to the local market's competitive pricing mechanism in Equation (1.5), I can rewrite wage in city c as $w_c = (1-r)^{\frac{1-r}{r}} p_{i,c}(\omega)^{\frac{1}{r}}\tilde{p}_{i,c}^{\frac{r-1}{r}} A_{i,c}(\omega)^{\frac{1}{r}}$. Now I can rewrite g_c as a function of good prices and productivity:

$$g_c(\omega) = \frac{w_c}{r}$$
$$g_c(\omega) = (1-r)^{\frac{1-r}{r}} p_{i,c}(\omega)^{\frac{1}{r}}\tilde{p}_{i,c}^{\frac{r-1}{r}} A_{i,c}(\omega)^{\frac{1}{r}}$$
$$\log g_c(\omega) = \frac{1-r}{r}\log(1-r) + \frac{1}{r}\log A_{i,c}(\omega) + \frac{r-1}{r}\log\tilde{p}_{i,c} + \frac{1}{r}\log p_{i,c}(\omega). \tag{27}$$

There is a chance for any firm to be not competitive enough such that it does not produce at all. For these firms log output per worker is negative infinity thus meaningless in the analysis. Therefore, the measure of industrial productivity, i.e. expected log GDP per worker for industry i in city c, is only calculated for firms that are producing. It is not hard to prove that any firm that is competitive enough to export must be the local supplier of its own city as well. Therefore, the out-ot-factory prices of producing firms follow the sales price distribution of its own city and industry. In other words, $p_{i,c}(\omega)$ has the same price distribution as importing prices $p_c^i(\omega)$. Therefore in expectation $\mathbb{E}\left(\log p_{i,c}(\omega)\right) = \log p_{i,c}$

Because $A_{i,c}$ follows a Frechet distribution, $\log A_{i,c}$ follows a Gumbel distribution, i.e.$\log A_{i,c} \sim Gumbel(\frac{\log T_{i,c}}{\theta}, \frac{1}{\theta})$ and $\mathbb{E}\left(\log A_{i,c}\right) = \frac{\log T_{i,c}}{\theta} + \frac{\gamma}{\theta}$, where γ is the Euler's constant. Here I call the CDF and PDF of $\log A_{i,c}(\omega)$ as $F(\log A_{i,c}(\omega))$ and $f\left(\log A_{i,c}(\omega)\right)$. Assume the market price for variety ω in $\{i, c\}$ is p, then ω only produces when it has productivity $A_{i,c}(\omega)$ higher than certain threshold $A^*_{i,c}(p)$ From the derivation of $G^{i,c}_{j,c'}$ in Equation (22) I can see that $A^*_{i,c}(p) = \frac{w_c^r \tilde{p}_{i,c}^{1-r}}{(1-r)^{1-r} r^r p}$. Therefore the conditional expectation of log TFP of a surviving firm can be expressed as $\mathbb{E}_A\left(\log A_{i,c} | \log A_{i,c} > A^*_{i,c}(p)\right) = \int_{A^*_{i,c}}^{\infty} \frac{x f(x)}{1 - F(A^*_{i,c})} dx$. Also, the conditional expectation of log TFP on all the surviving firms in $\{i, c\}$ is $\mathbb{E}_p\left(\mathbb{E}_A\left(\log A_{i,c} | \log A_{i,c} > A^*_{i,c}(p)\right)\right) = \int_0^{\infty} \int_{A^*_{i,c}}^{\infty} \frac{x f(x)}{1 - F(A^*_{i,c})} dx dG_c^i(p)$. It is not hard to notice that for a fixed p, $\mathbb{E}_A\left(\log A_{i,c} | \log A_{i,c} > A^*_{i,c}(p)\right) = \int_{A^*_{i,c}}^{\infty} \frac{x f(x)}{1 - F(A^*_{i,c})} dx$ is an increasing function of $\log T_{i,c}$. When there are a large number of industries and locations, technology change in $\{i, c\}$ has little impact of national market input price and local labor wages, thus little influence on $A^*_{i,c}$. However, an increase in $\log T_{i,c}$ shifts the distribution of $\log A_{i,c}$ to the right, therefore the conditional mean increases. Because this is true for any value of p, it is also true to $\mathbb{E}_p\left(\mathbb{E}_A\left(\log A_{i,c} | \log A_{i,c} > A^*_{i,c}(p)\right)\right)$. As a result, here I can use an increasing function $m(\log T_{i,c})$ to represent $\mathbb{E}_p\left(\mathbb{E}_A\left(\log A_{i,c} | \log A_{i,c} > A^*_{i,c}(p)\right)\right)$.

With the above analysis, I can generate city level productivity measure by integrating over the variety space Ω, $\log A_{i,c} = \frac{\log T_{i,c}}{\theta} + \theta\gamma$, where γ is the Euler's constant. and $\log p_{i,c}$ has probability of π_c^c to match its own import price of p_c^i. Therefore the expected log productivity of city c can be rewritten as:

$$\log g_c = \frac{1-r}{r}\log(1-r) + \frac{1}{r}m(\log T_{i,c}) + \frac{r-1}{r}\log \tilde{p}_{i,c} + \frac{\log p_c^i}{r}$$

$$\log g_c = \frac{1-r}{r}\log(1-r) + \frac{1}{r}\sum_{i\in\mathbf{N}} \nu_i m(\log T_{i,c}) + \frac{r-1}{r}\sum_{i\in\mathbf{N}} \nu_i \left(\sum_{j\in\,\mathbf{N}} d_i^j \log p_c^j - d_i^j \log d_i^j\right) + \sum_{i\in\mathbf{N}} \nu_i \frac{\log p_c^i}{r}$$

$$\log g_c = \frac{1-r}{r}\log(1-r) + \frac{1}{r}\sum_{i\in\mathbf{N}}\nu_i m(\log T_{i,c}) + \frac{1}{r}\sum_{i\in\mathbf{N}}\left(\nu_i - (1-r)\sum_{j\in\mathbf{N}} d^i_j \nu_j\right)\log p^i_c + \frac{1-r}{r}\sum_{i\in\mathbf{N}}\nu_i \sum_{j\in\mathbf{N}} d^j_i \log d^j_i$$

$$\log g_c = \frac{1}{r}\nu'_c \cdot m(\log T_c) + \frac{1}{r}\left[(I-(1-r)D)\nu_c)\right]' \cdot \log \mathbf{P}_c + \frac{1-r}{r}NE_c + const$$

$$\log g_c = \frac{1}{r}\nu'_c \cdot m(\log T_c) + \frac{1-r}{r}NE_c + \frac{1}{r}\left[(I-\pi^{c-1}_c)'\nu + \frac{r}{n}\mathbf{1} + \pi^{c-1}_c X_c)\nu_c)\right]' \cdot \log \mathbf{P}_c + const$$

where $NE_c = \sum_{i\in\mathbf{N}}\nu_i \sum_{j\in\mathbf{N}} d^j_i \log d^j_i$ is the city network entropy. □

Derivation of $\mathbb{E}(\log A_{i,c})$. Because the CDF of $A_{i,c}$ is $\mathbb{P}(A_{i,c} \leq a) = exp\left\{-T_{i,c}a^{-\theta}\right\}$, I have:

$$\begin{aligned}
\mathbb{P}(\log A_{i,c} \leq b) &= exp\mathbb{P}\left(A_{i,c} \leq e^b\right) \\
&= exp\left\{-T_{i,c}exp\left\{b\right\}^{-\theta}\right\} \\
&= exp\left\{-exp\left\{\log T_{i,c}\right\}exp\left\{-\theta b\right\}\right\} \\
&= exp\left\{-exp\left\{\log T_{i,c} - \theta b\right\}\right\} \\
&= e^{-e^{-\theta\left(b-\frac{\log T_{i,c}}{\theta}\right)}}.
\end{aligned}$$

This CDF shows that $\log A_{i,c} \sim Gumbel(\frac{\log T_{i,c}}{\theta}, \frac{1}{\theta})$. Therefore, $\mathbb{E}(\log A_{i,c}) = \frac{\log T_{i,c}}{\theta} + \frac{\gamma}{\theta}$, where γ is the Euler's constant. □

Proof of Lemma 3. First, I want to show that for every w_c, the equation system $\tilde{p}^{1-\sigma}_{j,c'} = D \cdot \sum_{i\in\mathbf{N}}\sum_{c\in\mathbf{C}} a_{ij}\left(w^\gamma_c \tau_{c,c'}\right)^{1-\sigma} \cdot \left(\tilde{p}^{1-\sigma}_{i,c}\right)^{1-\gamma}$ has a solution of $\tilde{p}_{i,c}$.

By definition $\tau_{c,c'} \in [1,+\infty)\,\forall c,c' \in \mathbf{C}$, and $a_{ij} \in [0,1]\,, \forall i,\, j \in \mathbf{N}$. With normalization $w_c \in (0,+\infty)\,, \forall c \in \mathbf{C}$. Therefore, the coefficient array of the system $Da_{ij}\left(w^\gamma_c \tau_{c,c'}\right)^{1-\sigma}$ is bounded by $[0,+\infty)$. Proposition 1 in Allen et al. (2018) shows that for a given set of w_c there exists a unique combination of $\lambda > 0$ and a set of $\tilde{p}^{1-\sigma}_{i,c}$ that solves $\tilde{p}^{1-\sigma}_{j,c'} = \lambda \cdot \sum_{i\in\mathbf{N}}\sum_{c\in\mathbf{C}} Da_{ij}\left(w^\gamma_c \tau_{c,c'}\right)^{1-\sigma} \cdot \left(\tilde{p}^{1-\sigma}_{i,c}\right)^{1-\gamma}$ for all $i \in \mathbf{N}$ and $c \in \mathbf{C}$ and $\sum_{i,c}\tilde{p}^{1-\sigma}_{i,c} = B$ for some arbitrary normalization B. Now if I renormalize the solution $\tilde{p}^{1-\sigma}_{i,c}$ to $\hat{p}_{i,c} = \lambda^{1-\gamma}\tilde{p}^{1-\sigma}_{i,c}$, then $\hat{p}_{i,c}$ is always the unique solution of the system $\hat{p}_{j,c'} = \cdot \sum_{i\in\mathbf{N}}\sum_{c\in\mathbf{C}} Da_{ij}\left(w^\gamma_c \tau_{c,c'}\right)^{1-\sigma} \cdot \left(\hat{p}_{i,c}\right)^{1-\gamma}$.

Because for every w_c there exists a unique solution of $\tilde{p}_{i,c}$ solve the system, that means for every w^γ_c there exists a unique $w^\gamma_c \tilde{p}^{1-\gamma}_{i,c}$ that solves the system as well.

Next I want to show by the implicit function theorem the solution to the system $w^\gamma_c \tilde{p}^{1-\gamma}_{i,c}$ is a continuous mapping of w_c over the normalized space of w_c. Let $K_{ij,cc'}$ represent the coefficient array $Da_{ij}\tau^{1-\sigma}_{c,c'}$, $\hat{X}_c = w^{\gamma(1-\sigma)}_c$ and

$\hat{Y}_{i,c} = \left(w_c^{\gamma}\tilde{p}_{i,c}^{1-\gamma}\right)^{1-\sigma}$. Then the equation system for $\tilde{p}_{i,c}$ can be rewritten as

$$F_{j,c'}(\hat{X}_c, \hat{Y}_{j,c}) = \frac{\hat{Y}_{j,c'}}{\hat{X}_{c'}} - \sum_{c \in \mathbf{C}} K_{ij,cc'}\hat{Y}_{i,c} = 0$$

The Jacobian of F is in the form of:

$$J_Y = \begin{bmatrix} \frac{\partial F_{1,1}}{\partial \hat{Y}} & \cdots & \frac{\partial F_{1,C}}{\partial \hat{Y}} \\ \frac{\partial F_{2,1}}{\partial \hat{Y}} & \cdots & \frac{\partial F_{2,C}}{\partial \hat{Y}} \\ \vdots & \cdots & \vdots \\ \frac{\partial F_{n,1}}{\partial \hat{Y}} & \cdots & \frac{\partial F_{n,C}}{\partial \hat{Y}} \end{bmatrix}$$

where every $\dfrac{\partial F_{j,c'}}{\partial \hat{Y}}$ is a $C \times n$ matrix with the following form:

$$\frac{\partial F_{j,c'}}{\partial \hat{Y}_{i,c}} = -K_{ij,cc'} \; for \; i \neq j \; or \; c \neq c'$$
$$\frac{\partial F_{j,c'}}{\partial \hat{Y}_{j,c'}} = \frac{1}{\hat{X}_{c'}} - K_{jj,c'c'} \; otherwise.$$

the parameter matrix K with the form:

$$K = \begin{bmatrix} -K_{11,11} & \cdots & -K_{n1,11} & -K_{11,12} & \cdots & -K_{n1,12} & \cdots & -K_{11,1C} & \cdots & -K_{n1,1C} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \cdots & \vdots & \ddots & \vdots \\ -K_{11,C1} & \cdots & -K_{n1,C1} & -K_{11,C2} & \cdots & -K_{n1,C2} & \cdots & -K_{11,CC} & \cdots & -K_{n1,CC} \\ -K_{12,11} & \cdots & -K_{n2,11} & -K_{12,12} & \cdots & -K_{n2,12} & \cdots & -K_{12,1C} & \cdots & -K_{n2,1C} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \cdots & \vdots & \ddots & \vdots \\ -K_{12,C1} & \cdots & -K_{n2,C1} & -K_{12,C2} & \cdots & -K_{n2,C2} & \cdots & -K_{12,CC} & \cdots & -K_{n2,CC} \\ \vdots & & \vdots & \vdots & & \vdots & \cdots & \vdots & & \vdots \\ \vdots & & \vdots & \vdots & & \vdots & \cdots & \vdots & & \vdots \\ -K_{1S,11} & \cdots & -K_{nS,11} & -K_{1S,12} & \cdots & -K_{nS,12} & \cdots & -K_{1S,1C} & \cdots & -K_{nS,1C} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \cdots & \vdots & \ddots & \vdots \\ -K_{1S,C1} & \cdots & -K_{nS,C1} & -K_{1S,C2} & \cdots & -K_{nS,C2} & \cdots & -K_{1S,CC} & \cdots & -K_{nS,CC} \end{bmatrix}$$

represents the random trade barrier and productivity information that are exogenous to the model. It is reasonable to assume that in the status of nature K has measure 0 to make $det(J_Y) = 0$ for the given normalized space of w_c. Because every $\hat{X}_c$ has a corresponding solution of F, by implicit function theorem for almost all the given K I can find a function f that is continuous on every $\hat{X}_c$ that satisfies $\hat{Y}_{i,c} = f(\hat{X}_c)$ and $F\left(\hat{X}_c, f(\hat{X}_c)\right) = 0$. In other words, With some simple monotone transformation, I can show that there exists a continuous function g such that $w_c^{\gamma}\tilde{p}_{i,c}^{1-\gamma} = g(w_c)$ and $F(w_c, g(w_c)) = 0$. □

.2 Special Cases of Theorem 1, Cities in Autarky

This section discusses special cases of Theorem 1 and show how an open economy production model with trade breaks down Hulten's Theorem.

When there is no trade across cities, every firm on the map produces, and they only buy and sell to local producers and consumers. This indicates that industrial output share of a city is only affected by the nation-wide input-output structure and preference shares in consumption, which is set to $\frac{1}{n}$ here. The technological parameters in the industry component of Theorem 1 become linear, i,e. $m(\log T_{i,c}) = \frac{\log T_{i,c}}{\theta} + \frac{\gamma}{\theta}$. Also, without trade the input price for the same type of product is its output price. Therefore I can rewrite the expression of log output per worker into:

$$
\begin{aligned}
\log g_c &= \frac{1-r}{r}\log(1-r) + \frac{1}{r}\left(\frac{\log T_{i,c}}{\theta} + \frac{\gamma}{\theta}\right) + \frac{r-1}{r}\log\tilde{p}_{i,c} + \frac{\log p_c^i}{r} \\
\log g_c &= \frac{1-r}{r}\log(1-r) + \frac{1}{r}\sum_{i\in\mathbf{N}}\nu_i\left(\frac{\log T_{i,c}}{\theta} + \frac{\gamma}{\theta}\right) + \frac{r-1}{r}\sum_{i\in\mathbf{N}}\nu_i\left(\sum_{j\in\,\mathbf{N}} d_i^j \log p_c^j - d_i^j\log d_i^j\right) + \sum_{i\in\mathbf{N}}\nu_i\frac{\log p_c^i}{r} \\
\log g_c &= \frac{1-r}{r}\log(1-r) + \frac{1}{r}\sum_{i\in\mathbf{N}}\nu_i\left(\frac{\log T_{i,c}}{\theta} + \frac{\gamma}{\theta}\right) + \frac{1}{r}\sum_{i\in\mathbf{N}}\left(\nu_i - (1-r)\sum_{j\in\,\mathbf{N}} d_j^i\nu_j\right)\log p_c^i \\
&\quad + \frac{1-r}{r}\sum_{i\in\mathbf{N}}\nu_i\sum_{j\in\,\mathbf{N}} d_i^j\log d_i^j \\
\log g_c &= \frac{1}{r}\nu_c'\cdot\log T_c + \frac{1}{r}\left[(I-(1-r)D)\nu_c)\right]'\cdot\log\mathbf{P}_c + \frac{1-r}{r}NE_c + const \\
\log g_c &= \frac{1}{r}\nu_c'\cdot\log T_c + \frac{1-r}{r}NE_c + \frac{1}{n}\mathbf{1}\cdot\log\mathbf{P}_c + cons.
\end{aligned}
$$

In this situation, the elasticity of output per worker g_c to technological level T_c is $\frac{d\log g_c}{d\log T_c} = \nu_c$, simply the output share of sectors. Without trade, the result in Lemma 2 is reduced to

$$\nu_c = \frac{r}{n}\left(I - (1-r)\cdot D\right)^{-1} \cdot \mathbf{1}.$$

In other words, sectoral output shares ν_c solely depend on the input-output network structure D. This result exactly matches the foundational theorem of Hulten (1978).

However, the classic Hulten's theorem breaks down in an open economy setting. According to Lemma 2, when there is inter-city trade sectoral output shares of a city ν_c are affected by not only the network structure $\mathcal{D}$, but also the competitiveness of industries in the local market π_c^c and the relative size of the export market X_c. While network structure determines the proportion of the economy that is related to the production of a sector, comparative advantage of firms in trade determines the size of the economy that an industry have access to. In a closed economy, an industry with increasing productivity can only increase its' output to an extent that its local buyer and consumers can absorb. In contrast, an open economy makes local industries not confined by the local economic size and structure, and gives increasingly productive firms an increasing amount of customers across the country. As a result, productivity shocks of more productive industries in an open economy have a disproportionately higher impact on the local economy. I cannot consider aggregate productivity shocks as a simple linear summation of sectoral shocks anymore.

.3 Two-Industry Example for City Network Entropy

This section presents a 2-industry city model to help the understanding of why high level of industry input entropy NE_i indicates a higher level of production technology or productivity.

Lemma 4. *Assume in a two-industry city the total population is 1, and the production technology $A_{i,c} = 1$ for either industry i is also 1. The total budget, or total cost, of industry 1 and industry 2 together are fixed at B. Let l_i is the labor size of industry i, then the log output per worker of industry i can be* *expressed as:*

$$\log \frac{q_1}{l_1} = (1-r)NE_1 - (1-r)\left(d_1^1 \log p_1 + d_1^2 \log p_2\right) + const;$$
$$\log \frac{q_2}{l_2} = (1-r)NE_2 - (1-r)\left(d_2^1 \log p_1 + d_2^2 \log p_2\right) + const;$$

where $const = (1-r)\log(1-r)$.

Proof. Because labor is paid by an uniform wage and labor cost counts for a constant share of r in the output of both industries, l_1 and l_2 also equal to the output shares of two industries here. Because the labor cost also counts for a constant share of r in the budgets of both industries, the ratio of budget for these two industries are also their population ratio, i.e. $\frac{B_1}{B_2} = \frac{rB_1}{rB_2} = \frac{l_1 w}{l_2 w} = \frac{l_1}{1_2}$. In other words, the budget of industry 1 and **2** are $l_1 B$ and $l_2 B$ respectively.

Let's first look at industry 1. The first order conditions of the production optimization indicates that the value of intermediate input 1 has a constant share d_1^1 in the industry 1's budget, i,e. $q_1^1 p_1 = (1-r)d_1^1 B_1 = (1-r)d_1^1 l_1 B$. Therefore I can rewrite the quantity demanded from input 1 as $q_1^1 = \frac{d_1^1(1-r)l_1 B}{p_1}$. Similarly, $q_1^2 = \frac{d_1^2(1-r)l_1 B}{p_2}$. Next I can substitute these expressions back to the log form of the production function to get:

$$\log q_1 = \log\left(l_1^r\left(q_1^{1\,d_1^1} q_1^{2\,d_1^2}\right)^{1-r}\right)$$
$$\log q_1 = r\log l_1 + (1-r)\left(d_1^1 \log q_1^1 + d_1^2 \log q_1^2\right)$$
$$\log q_1 = r\log l_1 + (1-r)\left(d_1^1 \log \frac{d_1^1(1-r)l_1 B}{p_1} + d_1^2 \log \frac{d_1^2(1-r)l_1 B}{p_2}\right)$$
$$\log q_1 = r\log l_1 + (1-r)\left(d_1^1 \log d_1^1 + d_1^2 \log d_1^2 - d_1^1 \log p_1 - d_1^2 \log p_2 + \log(1-r)l_1 B\right)$$
$$\log q_1 = (1-r)NE_1 - (1-r)\left(d_1^1 \log p_1 + d_1^2 \log p_2\right) + \log l_1 + (1-r)\log B + (1-r)\log(1-r)$$
$$\log q_1 = (1-r)NE_1 - (1-r)\left(d_1^1 \log p_1 + d_1^2 \log p_2\right) + \log l_1 + const$$
$$\log \frac{q_1}{l_1} = (1-r)NE_1 - (1-r)\left(d_1^1 \log p_1 + d_1^2 \log p_2\right) + const.$$

The exact the same analysis process can be applied to industry 2 as well. □

This lemma shows that the productivity of industry i in terms of output per worker is affected by two factors: its input structure in the form of NE_i and a substitution effect of price component. To focus on how NE_i affects productivity, let's turn off the substitution effect of price first.

.3.1 Without Substitution Effect of Price

In Lemma 4, prices are constant. Now consider a even more restrictive case where I hold $p_1 = p_2$, and Lemma 4 reduces to:

$$\log \frac{q_i}{l_i} = (1-r)NE_i + const,\ \ for\, i \in [1,2].$$

The above expression shows without considering the substitution effect of price, the absolute quantity of an industry's output per worker increases in its industry level entropy NE_i. In other words, if the intermediate input share vector of industry 1 goes from $(0.5, 0.5)$ to $(0.1, 0.9)$, every worker in industry 1 can produce more.

.3.2 Substitution Effect of Price

Next let's consider the more general case that $p_1 \neq p_2$. It is not necessarily true that output per worker of industry 1 increases in NE_1 anymore. In fact when NE_i increases, if $d_1^1 \log \frac{p_1}{p_2}$ declines at a bigger magnitude industry 1's output may even decline in this case. Therefore I need to discuss what form of price normalization is reasonable here. In other words, how do I measure the wealth level of the economy when there is relative price?

In this two sector economy, the substitution effect of price works exactly the opposite ways on the production process of two different industries. For example, if $\frac{p_1}{p_2} > 1$, when the unit input efficiency of input 1 rises, industry 1's output increases where as the industry 2's output decreases. Also, if $\frac{p_1}{p_c} \neq 1$, the output of these two industries are valued differently in the economy. Therefore, merely considering the substitution effect of price on a single industry is not sufficient.

Because this paper focuses on the behavior of city level production, I consider the substitution effect of price on city level average wage in the following analysis.

Lemma 5. *Assume in a two-industry city the total population is 1, and the production technology $A_{i,c} = 1$ for either industry. The budget, or total cost, of industry 1 and industry 2 together are fixed at B. Let l_1 and l_2 be the shares of the population working for two industries,then the log total wage can be written as:*

$$\log w = (1-r)l_1 NE_1 + l_2 NE_2 + \frac{\log \boldsymbol{P}}{2} + \log rB,$$

where $\log \boldsymbol{P} = (\log p_1, \log p_2)'$*.*

Proof. In this model, workers in these two industries are paid the same wage, so $w = \dfrac{rq_1p_1}{l_1} = \dfrac{rq_2p_2}{l_2}$. Lemma 1 shows that in an autarky state, output share of a city sole depends on the input-output share matrix. Therefore, l_1 and l_2 are endogenous to changes in the input-output matrix. In addition, because the budget ratio of two industries are always the same as the ratio of their output in the city, here I use B to represent a city level budget. The budget of industry 1 and 2 are l_1B and l_2B respectively. With these revisions, log wage, as a measure of total wealth in the economy, can be written as:

$$\begin{aligned}
\log w &= l_1 \log \frac{rq_1p_1}{l_1} + l2 \log \frac{rq_2p_2}{l2} \\
\log w &= \log r + l_1 \left(r \log l_1 + (1-r)\left(NE_1 - d_1^1 \log p_1 - d_1^2 \log p_2 + \log l_1 B\right) + logp_1 - \log l_1\right) \\
&\quad + l_2 \left(r \log l_2 + (1-r)\left(NE_2 - d_2^1 \log p_1 - d_2^2 \log p_2 + \log l_2 B\right) + \log p_2 - \log l_2\right) \\
\log w &= (1-r)(l_1 NE_1 + l_2 NE_2) + (l_1 \log p_1 + l_2 \log p_2) \\
&\quad - (1-r)\left(l_1 d_1^1 p_1 + l^1 d_1^2 p_2 + l_2 d_2^1 p_2 + l_2 d_2^2 p_2)\right) + \log B + \log r \\
\log w &= (1-r)N^s + \mathbf{L} \cdot \log \mathbf{P} - (1-r)\mathbf{D} \cdot \mathbf{L} \log \mathbf{P} + \log B + \log r,
\end{aligned}$$

where $\mathbf{L} = (l_1,\ l_2)', \log \mathbf{P} = (\log p_1, \log p_2)'$ and D is the input output matrix of this city. According to lemma 1, in an autarky economy, $\mathbf{L} \cdot \log \mathbf{P} - (1-r)\mathbf{D} \cdot \mathbf{L} \log \mathbf{P} = \dfrac{\log \mathbf{P} \cdot}{2} = \dfrac{\log(p_1p_2)}{2}$. Therefore, log wage can be further simplified as:

$$\log w = (1-r)N^s + \frac{\log \mathbf{P} \cdot}{2} + \log rB.$$

□

This expression of log wage indicates that if I normalize p_1p_2 to a constant, the substitution effect of prices from different sectors cancel out and the wage level of a city is only positively affected by its network entropy.

Because wage is always share r of the total output. The total output of the city also increases in network entropy. Because $\frac{\log \mathbf{P}\cdot}{2}$ is also the expression of the consumer consumption price index in the city, it is also reasonable to measure wage and output increase relatively to the people's cost of living in the city.

.4 Additional Table

Table 4: 8 year GDP/worker Growth with Leading Industries as control

variable name	base line	2 ind share	top ind	top ind share
Intercept	**24.367***	**19.103***	**22.657***	**35.093***
	(0.0)	(0.0)	(0.0)	(0.0)
NE_c growth	**1.621***	**2.019***	**1.652***	**0.967***
	(0.0)	(0.0)	(0.0)	(0.0)
employment	-0.0	-0.0	-0.0	0.0
	(0.3025)	(0.2327)	(0.2574)	(0.6978)
year	**−0.012***	**−0.009***	**−0.011***	**−0.017***
	(0.0)	(0.0)	(0.0)	(0.0)
31G share change		0.001		
		(0.8488)		
FIRE share change		**−0.024***		
		(0.0)		
top industry share				**−0.938***
				(0.0)
city FE	Yes	Yes	Yes	Yes
top ind FE			Yes	
adjusted R^2	0.58	0.595	0.597	0.632
Observation	2970	2970	2970	2970

Dependent variable: GDP per worker growth residual
Standard errors in the parentheses are culstering with respect to city and year
***p<0.01, **0.01≤p<0.05, *0.05≤p<0.1
31G refers to the manufacturing sector. FIRE refers to finance, insurance, real estate and rental services.

.5 Additional Figures

Figure 13: Changes of GDP/Worker Caused by the Increased Importance of the Agriculture/Mining Sector in the Network

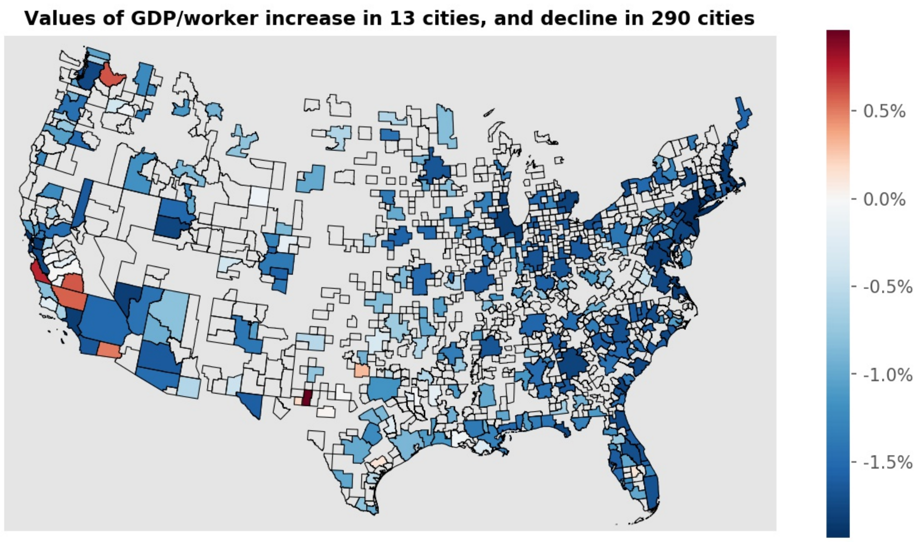


Figure 14: Changes of GDP/Worker Caused by the Increased Importance of the Construction Sector in the Network

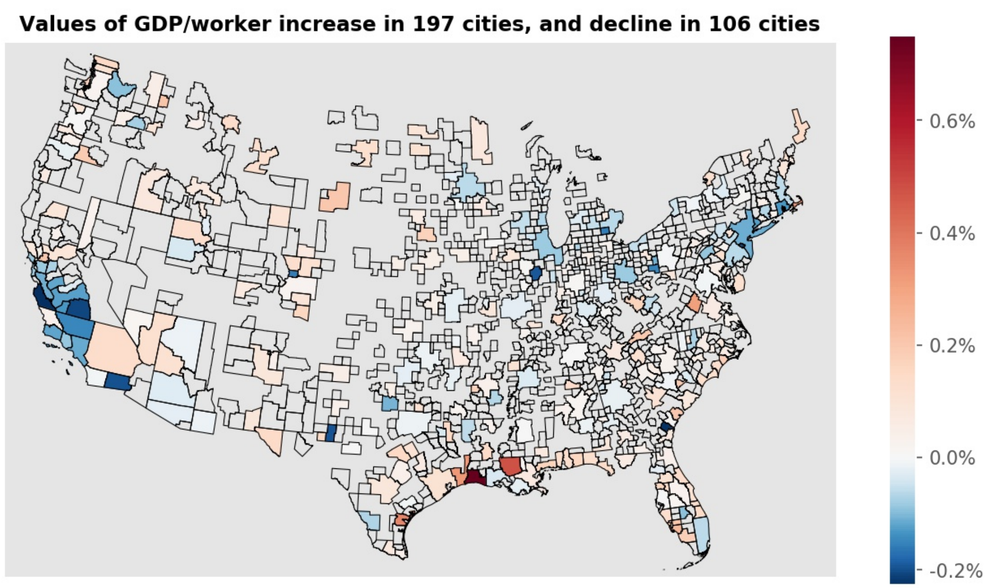


Figure 15: Changes of GDP/Worker Caused by the Increased Importance of the Manufacturing Sector in the Network

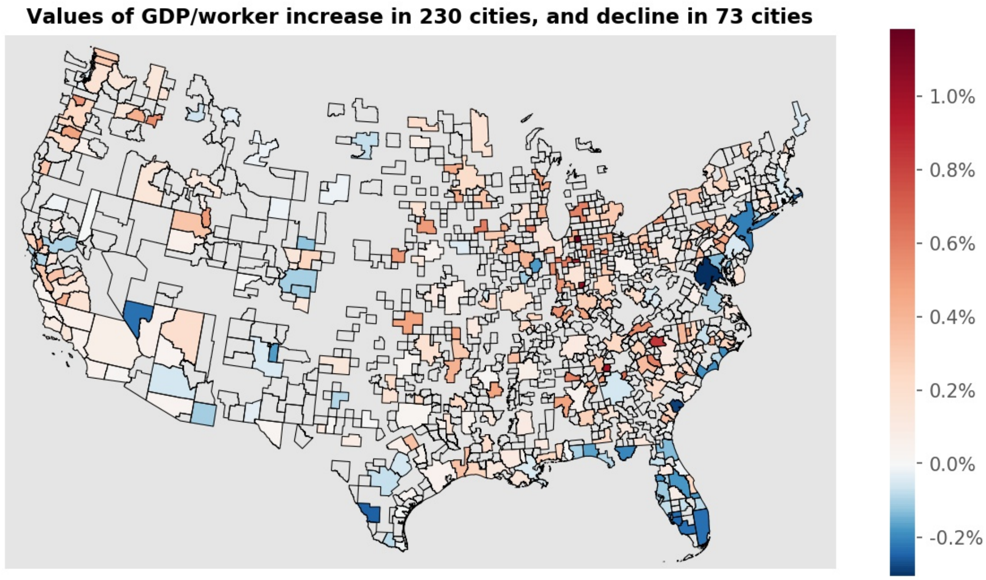


Figure 16: Changes of GDP/Worker Caused by the Increased Importance of the Wholesale Trade Sector in the Network

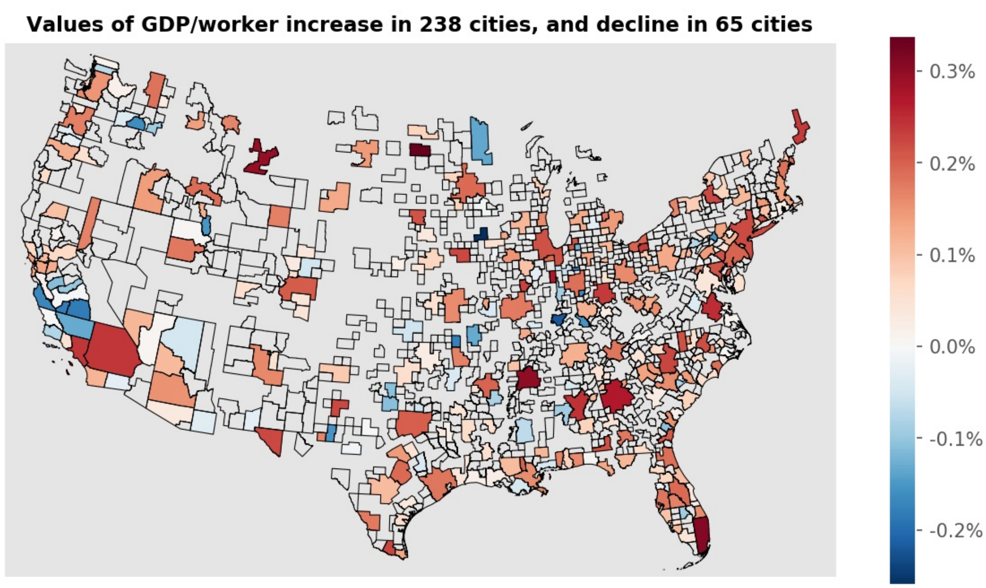


Figure 17: Changes of GDP/Worker Caused by the Increased Importance of the Retail Trade Sector in the Network

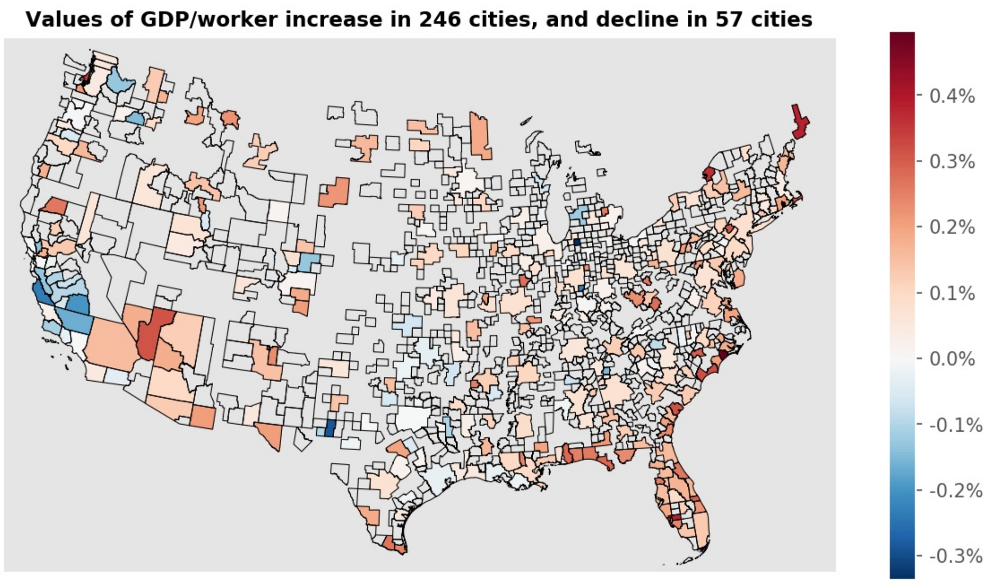


Figure 18: Changes of GDP/Worker Caused by the Increased Importance of the Transportation/Warehouse Sector in the Network

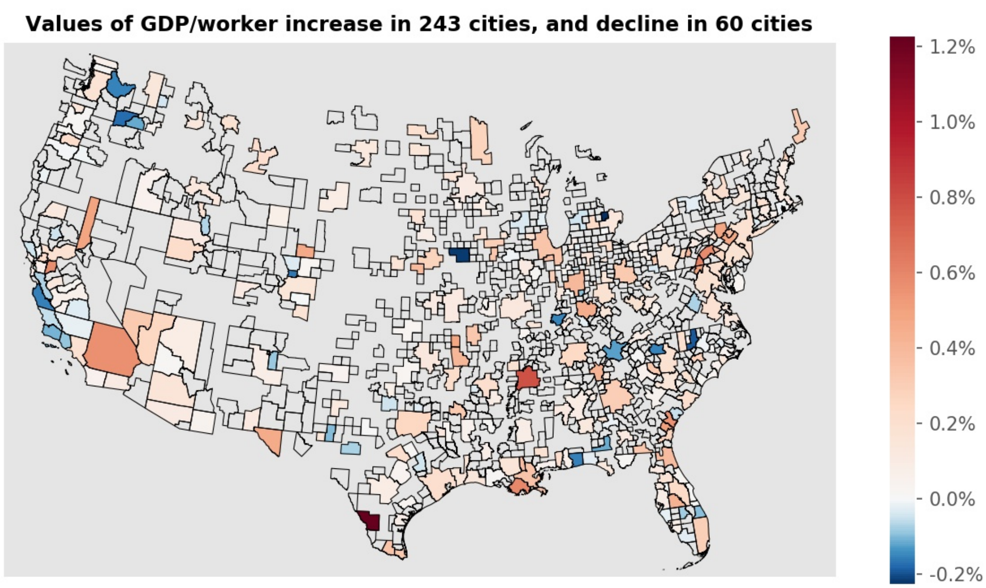


Figure 19: Changes of GDP/Worker Caused by the Increased Importance of the Information Sector in the Network

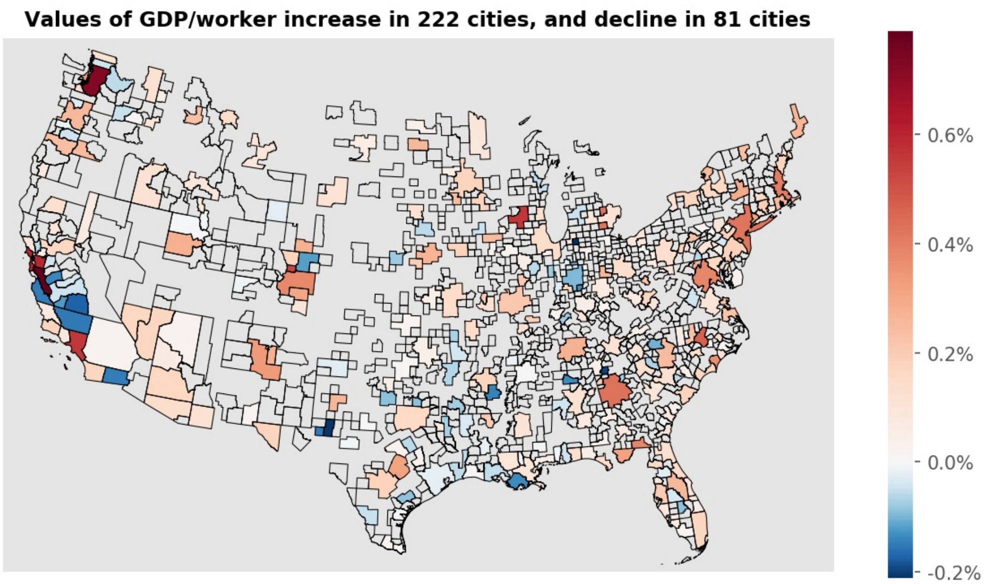


Figure 20: Changes of GDP/Worker Caused by the Increased Importance of the Edudcation/Healthcare/Social Assistant Sector in the Network

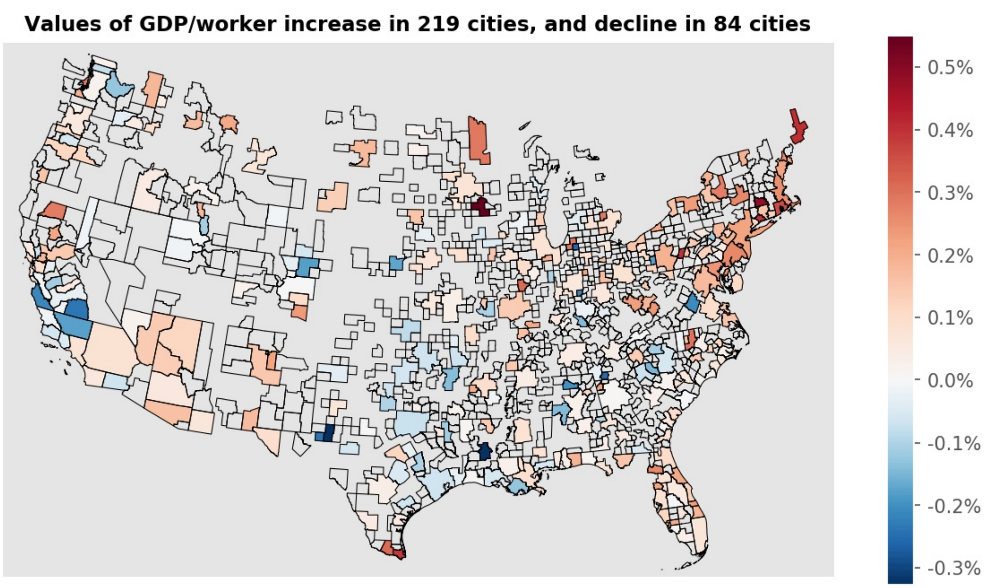


Figure 21: Changes of GDP/Worker Caused by the Increased Importance of the Art/Entertainment/Accommodation/Food Sector in the Network

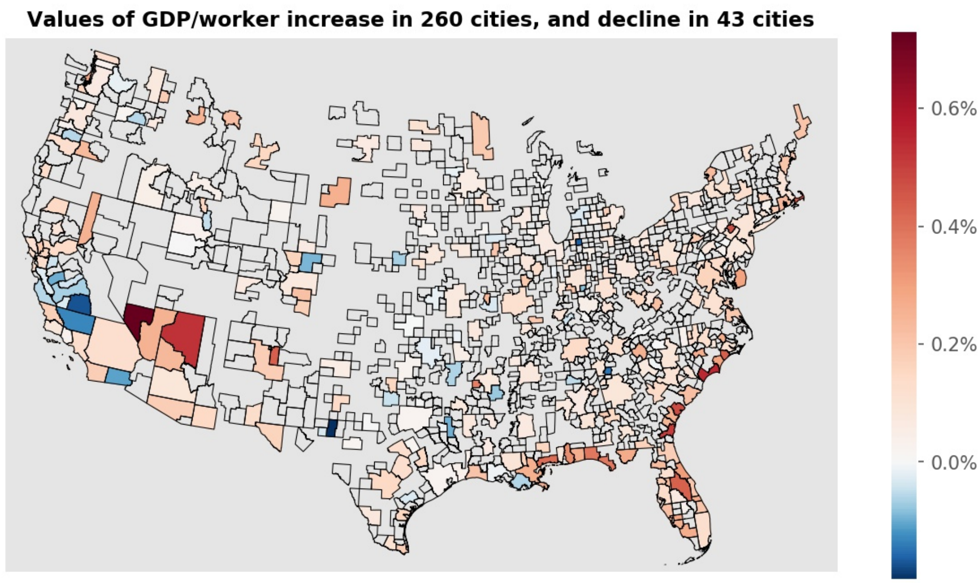


Figure 22: Changes of GDP/Worker Caused by the Increased Importance of the Other Services Sector in the Network

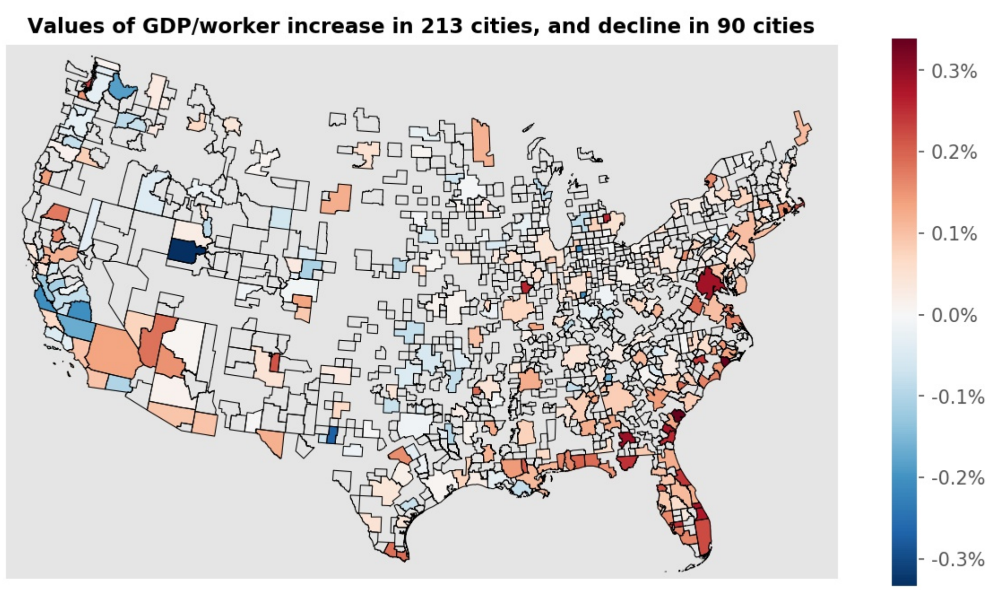


Figure 23: Changes of GDP/Worker Caused by the Increased Importance of the F.I.R.E Sector in the Network

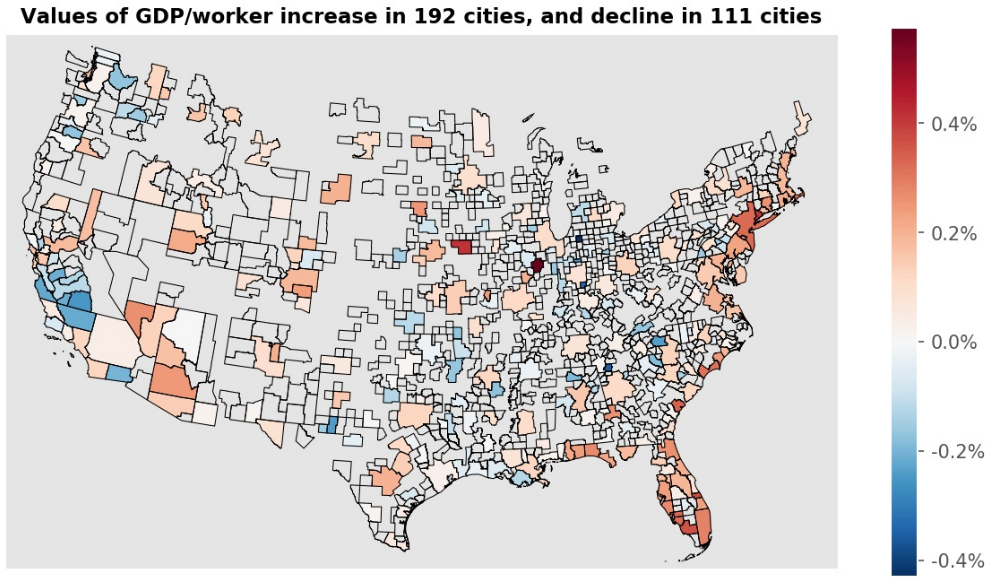


Figure 24: Changes of Industry Level Entropy of 13 industries, 1997-2016

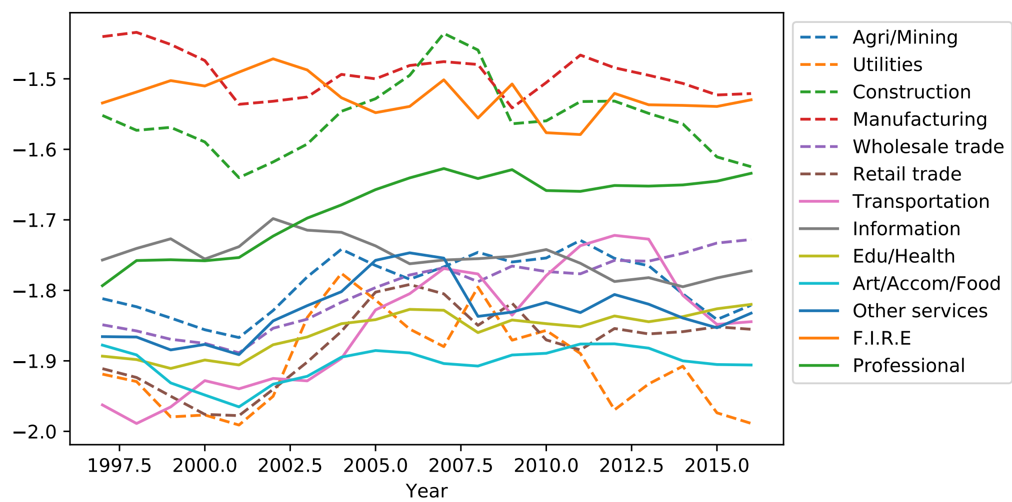


Figure 25: Changes of Industry Level Entropy of 66 industries, 1997-2016, part 1

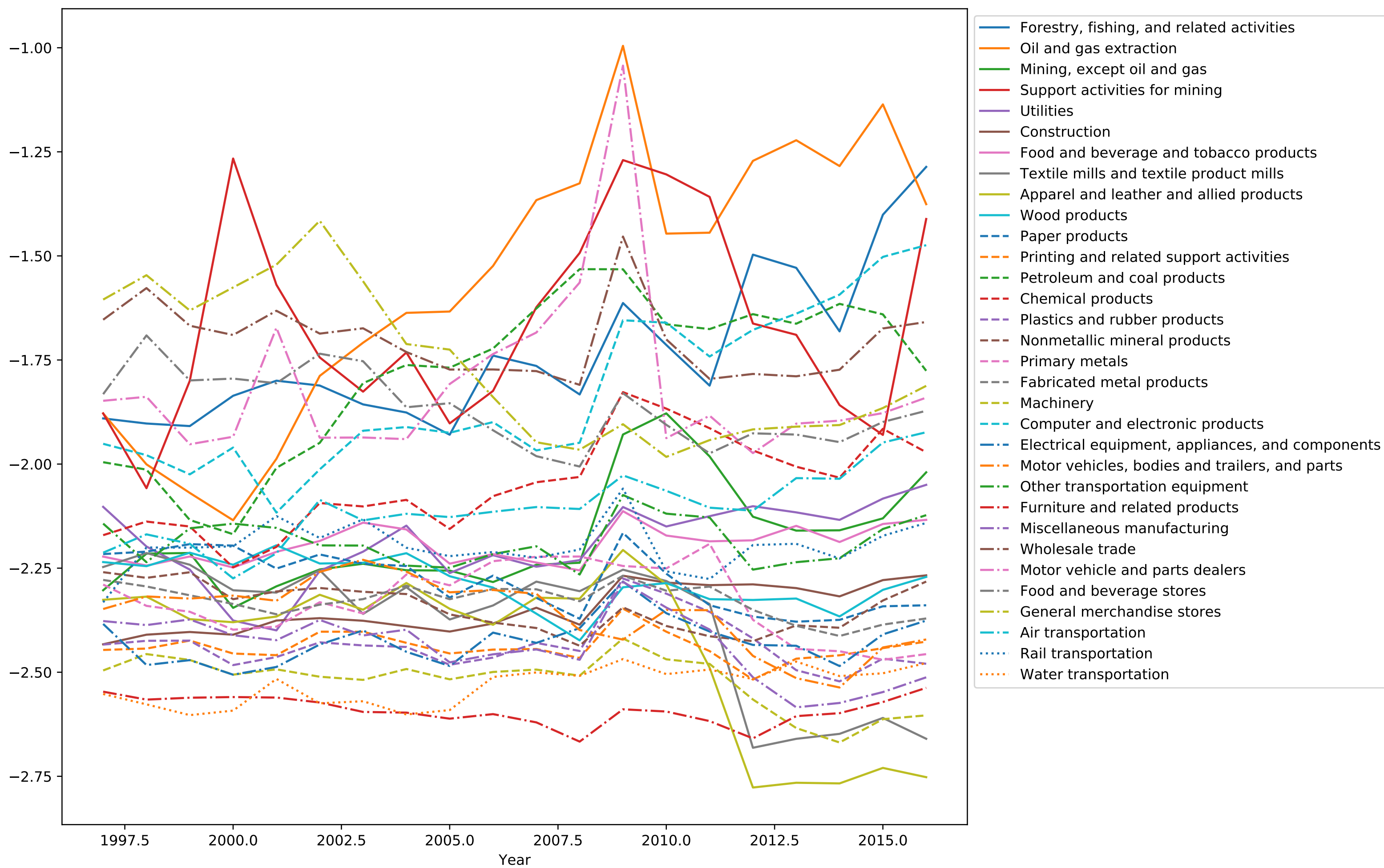
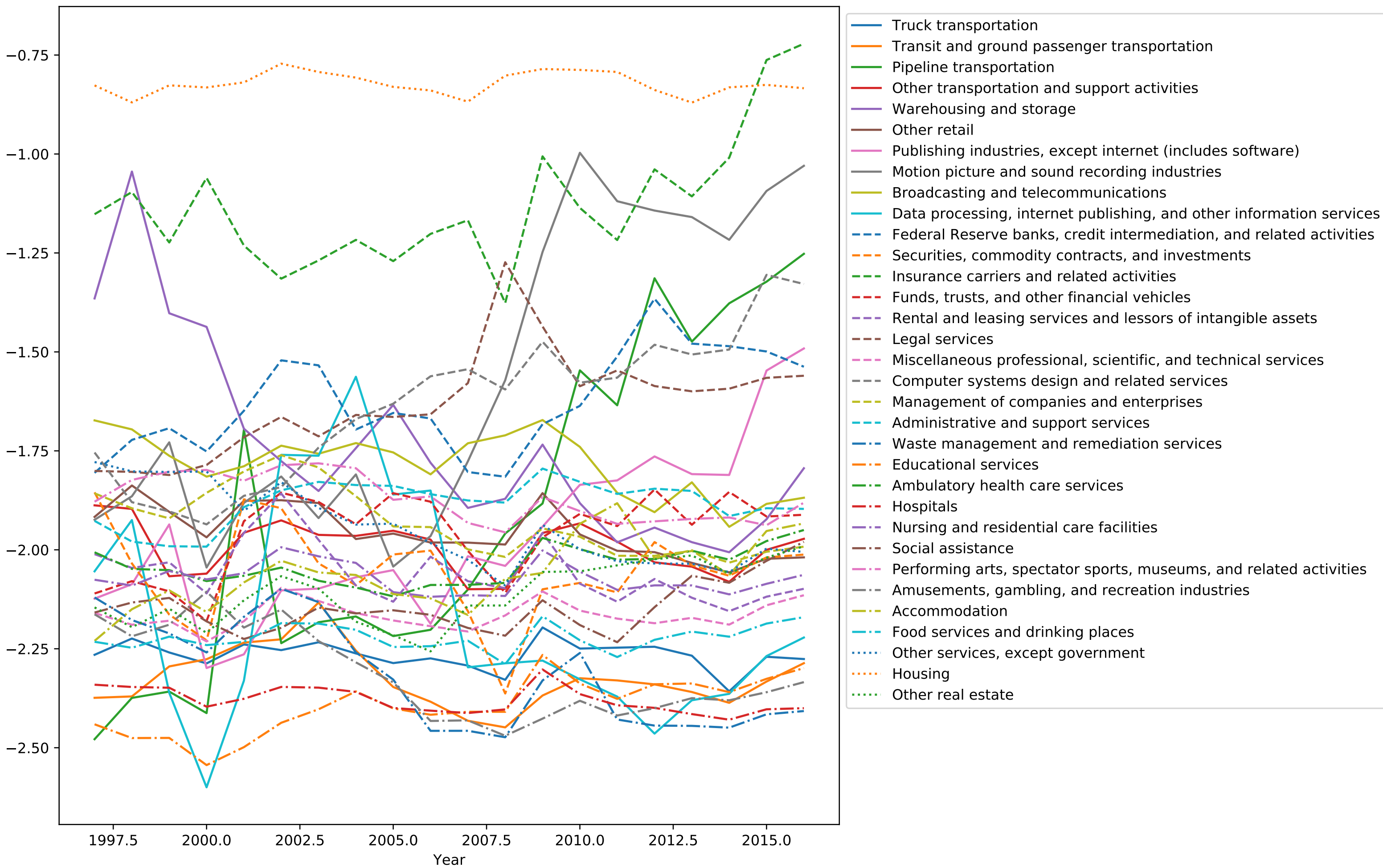


Figure 26: Changes of Industry Level Entropy of 66 industries, 1997-2016, part 2



Chapter 2

Industrial Productivity and Urban Human Capital Spillovers

Introduction

Why do productivity of industries vary significantly across cities? In Alfred Marshall's theory of industry agglomeration, he identified the three aspects of the urban space that affect productivity: 1. the closeness of goods in the supply chain; 2. the pooling of labor; 3. the knowledge spillover or human capital externalities (Marshall, 1920). This paper focuses on the last factor, the impact of urban human capital spillovers. Human capital spillovers refer to the simple idea that when employees interact more frequently with other skilled and educated workers in cities, information and knowledge spillover from workers to workers. As a result, skilled labor learn faster and their firms become more productive. In this paper, I develop a systematic and innovative way to measure the human capital spillovers occurred in the production networks of cities and quantify their impact on productivity of various industries.

The major contribution of this paper is creating a human capital network index to measure two aspects of the urban human capital environment for different industries: the quantity of skilled labor in the city, as well as the allocation of these talents within the urban production network. For a target industry, the index achieves high values not only when the city has a large quantity of educated workers, but also when this human capital is concentrated on industries that are "economically closer" to the target industry. To measure economic distances, this paper proposes a novel betweenness centrality measure. It is based on the assumption that the economic distance between any pair of industries for knowledge and information to travel back and forth is proportional to the frequency of business transactions between them.

After creating the index, in order o quantify the spillovers from the urban human capital environment to industrial productivity, the next step is to measure the productivity of firms in different locations. The lack of physical capital input information for industries other than manufacturing, makes it impractical to directly measure TFP values of firms in every industry. Therefore, this paper adopts a general equilibrium model with multiple industries within cities and competitive trade among firms across cities. The equilibrium conditions of this model render a closed form relationship between industry-location-specific TFP, local wage and labor allocation. Based on this relationship, I acquire industry-location-specific TFP values by calibrating the model to the U.S. economy with MSA-industry specific data from Bureau of Economic Analysis (BEA), trade flow data from Commodity Flow Survey (CFS) and individual-level labor market data from American Community Survey (ACS)[1].

Then regression analysis verifies that there is a significant correlation between human capital levels in the urban production environment and the productivity for most of industries. These results are robust to different specifications of human capital network measures, time specifications as well as various levels of industrial classifications.

These empirical results contribute two new economic insights. First, not only the aggregate quantity of human capital in cities, but also their allocations in the production network determines the magnitude of human capital spillovers among industries. Second, the ability of business to absorb urban human capital spillovers on productivity varies from industry to industry. While majority of sectors benefit from a more educated urban environment, certain industries experience negative human capital spillover from the rest of the city. Examples of such negative-spillover industries

[1]Indiviual-level ACS data used in this paper is acquired through Integrated Public Use Microdata Series (IPUMS)

are farming, forestry and fishing, mining, transportation and warehousing as well as some subcategories of manufacturing. A possible explanation here is that the large presence of human-capital-intensive sectors in town may lure away talents from industries with low intensity of human capital usage, and consequently reduces the quality of human capital in these industries. These results provide the evidence for the theory that cities with high human capital level are more prone to specialize in certain industries.

The remainder of the paper is organized as follows. After a brief discussion of related literature, Section 2.1 sets up a multi-city multi-sector environment of city production and explains the construction of the human capital network index as a novel measure of human capital spillovers in a single city. Section 2.2 further complete the general equilibrium model with labor and trade setup to provide theoretical support to calibrate city-industry specific productivity or TFP. To take the theory to data, first Section 2.3 elaborates on the details of the calibration strategy for industry-location-specific productivity based on the model in Section 2.2. Then in Section 2.4, empirical analysis combines the calibrated TFP values and human capital network index in the data, and sheds light on the differential impact of human capital spillovers on various industries in cities. Section 2.5 concludes. The Appendix contains all the proofs.

Related literature This paper is deeply rooted in Alfred Marshall's (Marshall, 1920) theory of agglomeration of economies. Since his time, the knowledge externalities or human capital spillovers, as one of the three aspects of urban agglomeration, has been extensively studied. Ellison et al. (2010) showed that industries that are more likely to coexist in the same regions tend to have both have stronger input-output connections and higher level of knowledge sharing, as measured by pattern citations. Moretti (2004a) empirically proved the existence of positive human capital spillovers from other industries to firms in the manufacturing sector in the same cities. Specifically, he adopted a network view of industries in the urban environment and found that this impact of human capital spillovers on manufacturing productivity declines as the economic distance between two industries increases.These papers empirically show that the locations of human capital in the urban input-output production network matters for the human capital spillover that manufacturing firms receive. My paper further explore the network view of the economy and extends the study of human capital spillovers beyond manufacturing. This helps us to acquire a more complete understanding of the wide variation in the relationship between different industries and knowledge spillovers in cities. The approach to empirically measure network effect is also influenced by Acemoglu et al. (2012a).

The empirical method used to construct the human capital network index is an extension of recent applications graph theory in social science works. Newman (2005) originally proposed the concept of betweeness centrality to describe the influence a node has over the spread of information through the network. Blöchl et al. (2011) adapted this betweeness centrality for vertex analysis in input-output networks. In the context of production network, high betweeness nodes are the ones where shocks linger the longest while propagating through the economy. Human capital spillovers in nature are resonating propagation of knowledge among different industries. In other words information bounces back and forward frequently among nodes, stead of passing through only once. Therefore, simple indegree counting cannot capture the intensity of this random-walk-style behavior of knowledge. As a result, I further modify the betweeness centrality developed by these two papers to account for the economic distance between any pair of industries that is meaningful for human capital spillovers.

In order to calibrate industry-city-specific TFP, I construct the multisector trade model with Bertrand price competition that is heavily influenced by Eaton and Kortum (2002). Allen and Arkolakis (2014) and Arkolakis et al. (2015) provide methods to identify and estimate this type of general equilibrium models. However, their methods rely on trade flow data among different locations. Due to the lack trade flow data among cities, I adapt their estimation process into calibration with parameter values taken from Caliendo and Parro (2015) and Caliendo et al. (2017).

2.1 Human Capital in Production Networks

This section reveals the construction of human capital index in a multi-city and multi-industry environment that measures the human capital spillover level in a single city. Consider a set of city **C** $= \{1, ..., C\}$ on the map. Each city has a set of industries **N** $= \{1, ..., N\}$ in it. In each industry or sector, there is a continuum of firms of measure Ω.

2.1.1 Environment

Each firm produces a distinct final good $\omega \in \Omega$ for household consumption. A typical firm here uses Cobb-Douglas technology to combine four elements to produce a unit of good: 1) physical capital, 2) unskilled labor, 3) skilled labor and 4) firm's specific technology or TFP. Therefore, the production function of variety producer $\omega \in \Omega$ in industry i of city c has the following expression:

$$q_{i,c}(\omega) = A_{i,c}\left(\mathcal{H}_{i,c}\right) \cdot l_{i,c}(\omega)^{\alpha_{i,c}} h_{i,c}(\omega)^{\beta_{i,c}} k_{i,c}(\omega)^{1-\alpha_{i,c}-\beta_{i,c}}. \tag{2.1}$$

$q_{i,c}(\omega)$ is the output level of this firm while $k_{i,c}(\omega)$ refers to the amount of capital input. $l_{i,c}(\omega)$ is the total amount of labor input from workers without college degrees, whereas $h_{i,c}(\omega)$ is the total amount of labor input from workers with college degrees or more education. The rest of the paper refers $l_{i,c}(\omega)$ as unskilled labor and $h_{i,c}(\omega)$ as skilled labor. The total amount of skilled and unskilled labor in the whole industry i in city c are represented as $L_{i,c} = \int_{\omega\in\Omega} l_{i,c}(\omega)d\omega$ and $H_{i,c} = \int_{\omega\in\Omega} h_{i,c}(\omega)d\omega$ respectively. $\alpha_{i,c}$ and $\beta_{i,c}$ are the shares of input values in the output. Here I assume constant return to scale to simplify the estimation process later on. $A_{i,c}$ is a random variable that represents firm-specific TFP and is discussed below.

The main focus of this paper is the allocation of human capital, namely the allocation of skilled labor $h_{i,c}(\omega)$ and $H_{i,c}$. This is because the key assumption here is that human capital not only directly contribute to the production process of their employers but also spread out knowledge and information to whomever they are in contact with during the production process, and eventually indirectly affect the technology and productivity of other firms that have business relationship with their employers in the same geographic location. To exemplify this idea, I propose a human capital index $\mathcal{H}_{i,c}$ to measure of human capital density in the same city c as firms in industry i, but do not directly work for industry i. Also, the density function of $A_{i,c}$ is affected by the value of $\mathcal{H}_{i,c}$. Specifically, assume $A_{i,c}(\mathcal{H}_{i,c})$ follows a Frechet distribution, as in Eaton and Kortum (2002), i.e.

$$Pr(A_{i,c} \leq A) = exp\left\{-T_{i,c}(\mathcal{H}_{i,c})\, A^{-\theta}\right\}, \tag{2.2}$$

where $\theta > 1$ governs the distribution of productivity across goods within an industry of a city. $T_{i,c} > 0$ is a measure of the aggregate productivity of industry i in city c. In this paper $T_{i,c}$ is called as the fundamental productivity. A larger fundamental productivity $T_{i,c}$ indicates a higher probability of larger realization of the random variable $A_{i,c}$. Assume $T_{i,c}$ is a function of the human capital spillovers in the city, then it has the following form:

$$T_{i,c} = T_i + T_c + \theta_i \mathcal{H}_{i,c} + \epsilon_{i,c}; \tag{2.3}$$

where $\mathcal{H}_i$ is the human capital network index of industry i in city c and it is discussed in detail below. $\epsilon_{i,c}$ is the idiosyncratic productivity shock to technology in industry i of city. Assume $\epsilon_{i,c}$ is normally distributed with finite variance and mean 0. θ_i represents the ability of industry i to absorb the human capital spillover in a city.

2.1.2 Human Capital Network Index

The human capital network index $\mathcal{H}_{i,c}$ is designed to measure the quality of human capital in industry i's local environment in city c. It is based on the observation of Moretti (2004a) about the relationship "economic distance" and human capital spillovers. Namely, as the "economic distance" between two industries decreases, the externality of business partner's human capital on firms' productivity gets stronger. Therefore $\mathcal{H}_{i,c}$ is given the following functional form:

$$\mathcal{H}_{i,c} = \sum_{j \in \mathbf{N} \setminus \{i\}} D_{ji} \cdot H_{i,c}, \tag{2.4}$$

where $H_{i,c}$ represents the total amount of skilled labor in industry i in city c, and D_{ji} measures the "economical distance" between industry j and industry i. Two industries are considered "economically close" if they have frequent business contact and a large amount of input-output transactions. As a result, D_{ji} is bigger and the human capital level of one industry will have bigger indirectly effect on the other one. I use **D** to denote the matrix of economic distance where the value at coordinate $\{j, i\}$ is D_{ji}. In order to systematically measure economic distance in data, the following concept of betweenness is adopted to calculate **D**.

2.1.3 Betweenness

The frequent exchange of goods, ideas and capital among firms makes economic relationships often be described as networks. Among various types of economic networks, this paper focuses on the most frequently studied input-output networks (Acemoglu et al., 2012a). A densely connected input-output network can be described as a direct graph. In such a graph each industry is a node and each arc represents an input-output relationship between a pair of nodes. One economic activity is defined as the movement of resource from one node to another node through the direct arc between these two. In one period, an infinite number of economic activities can happen in an input-output graph. A normalized input-output matrix is a summary of all the arch weights of such a graph. Let M denote this matrix, then entry m_{ji} represents the probability for one unit of output in industry j ends up in industry i after one economic activity. In other words, Each row of M has the sum of 1. Because each row of the normalized input-output matrix can also be interpreted as the probability of one unit resource transiting from the source industry to a certain target industry, M can also be called a transition matrix.

Human capital spillovers in nature are resonating propagations of knowledge and information that move back and forward among workers in the production process, whereas the input-output matrix M only describes the one-time transition probability of economic activities in the network. Therefore, input-output matrix M itself is not the best choice to evaluate the importance of human capital spillovers properly. The graph theory concept of betweenness centrality measures the frequency that a node or an arch been visited in this type of resonating propagation process (Jackson, 2010). Therefore, this paper considers betweenness centrality as a better way to evaluate the importance of arches in the spillover process. Traditionally betweenness centrality measures the influence a node over the spread of information through the network. The following method of calculating betweenness for an arch is modified from Blöchl et al. (2011)'s method of counting vertex centrality in input-output networks and Newman (2005)' s random walk betweenness measure.

Let M_{-h} represent a new transition matrix by deleting the h-th row and column of the normalized input-output matrix M. If a unit of input travels from the source node industry s and eventually ends up in the target industry $h \neq s$, the probability of being at node $j \neq s$ after r steps should be $\{(M_{-h})^r\}_{sj}$, namely the value at the coordinates $\{s, j\}$ of the matrix $(M_{-h})^r$. Also, the probability of this unit input using the edge $\{j,\, i\}$ immediately after r steps is $\{(M_{-h})^r\}_{sj} \cdot m_{ji}$. Therefore, I can write the number of times for a unit of input to pass through edge j, i on its way traveling from source node industry s to the target industry h as:

$$\begin{aligned} B_{ji}^{sh} &= \sum_{r=1}^{\infty} \{(M_{-h})^r\}_{sj} \cdot m_{ji} \\ &= m_{ji} \sum_{r=1}^{\infty} \{(M_{-h})^r\}_{sj} \\ &= m_{ji} \left\{(I - M_{-h})^{-1}\right\}_{sj}, \end{aligned}$$

where I is an identity matrix. With the above definition of B_{ji}^{sh}, in this paper the betweenness measure of the input-output edge $\{j,\, i\}$ is defined as the average number of times that a unit of good passing through edge $\{j,\, i\}$, regardless of direction, across all source-target pairs:

$$D_{j,\,i} = \frac{\sum_{s \in \mathbf{N}} \sum_{h \in \mathbf{N} \setminus \{s\}} B_{ji}^{sh} + B_{ij}^{sh}}{2N(N-1)}. \tag{2.5}$$

It is worth to notice that $D_{j,\,i}$ and $D_{i,\,j}$ have the same value under this definition. $D_{j,\,i}$ has a domain of $[0, +\infty)$. Bigger the value of $D_{j,\,i}$ is, more frequently this edge is used by any unit of goods in the production network. A betweenness measure $D_{j,i}$ tells how important the input-output relationship between j, i in the resonating knowledge spillover process is relative to all other input-output pairs in the matrix D. In contrast, in the original input-output M, it is only meaningful to compare $M_{j,i}$ with other elements on the jth row. In other words the matrix of $\{D_{j,i}\}$ is a more comprehensive measure of economic distance that can be compared across the entire economy. Therefore the economic distance matrix $\mathbf{D}$ is defined as $\{D_{j,i}\}$. Note that a "shorter" economic distance in this paper is actually represented by a bigger value in the matrix $\mathbf{D}$.

With this definition of betweenness, the human capital network index $\mathcal{H}_{i,c} = \sum_{j \in \mathbf{N} \setminus \{i\}} D_{ji} \cdot H_j$ can be interpreted as a measure of frequency for human capital traveling in the input-output environment of industry i in city c. Because the value of $\{D_{j,i}\}$ can be compared with any other value throughout the whole matrix of $\mathbf{D}$, $\mathcal{H}_{i,c}$ can be used to compare the intensity of human capital environment for any pair of industries in any two locations.

2.1.4 Illustrative Example

To help readers to understand the feature of the betweenness measure and the human capital index newly defined in this paper, this section presents an illustrative example.

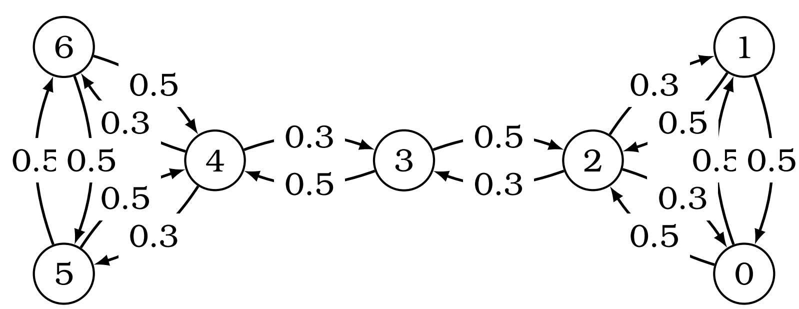


Figure 2.1: Input-Output Network with 7 Industries

Figure 2.1 visualizes a simple normalized input-output network with seven industries. In this graph, each direct arch, namely an arch with direction and an arrowhead, represents an input-output relationship from a source industry to a target industry. The number on the arch represents the probability for one unit of goods from the source industry goes to the target industry. The lack of arch between two nodes indicates an input-output relationship of weight zero. For simplicity, equal probabilities are assigned to all the arches coming from the same source. As a result, this graph is fully symmetric.

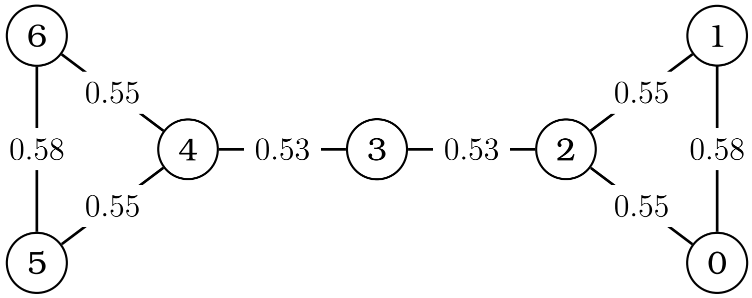


Figure 2.2: Betweenness Measure

Figure 2.2 shows the betweenness measure for this network calculated from Equation (2.5). The values on arches between the pairs $\{0,1\}$ and $\{5,6\}$ are higher than arches between the pairs $\{3,4\}$ and $\{2,3\}$. This indicates that resources in this network are more frequently to travel repeatedly on arches between between the pairs $\{0,1\}$ and $\{5,6\}$ than the arches in the middle. If we consider knowledge or ideas as a kind of resource that can be spread out among workers along supply chains back and forth, then the more frequently knowledge or information repeatedly visits an arch, the more likely it spills over to the production process of the source or target nodes. Under this assumption, the above betweenness measure is very good for the purpose of measuring knowledge spillover.

Table 2.1: Human Capital Network Index for the Example Graph

Nodes	0	1	2	3	4	5	6
HC Index	1.1242	1.1242	1.618	1.0513	1.618	1.1242	1.1242

Assuming in every industry there is a measure 1 of human capital, then table 2.1 presents the results for human capital network index calculated based on Equation (2.4). Industry 4 and 2 have the highest human capital index value of 1.62, while industry 3 has the lowest human capital index value of 1.12. This indicates that there is stronger presence of human capital spillover in the urban environment of industry 4 and 2 than that of industry 3. Intuitively this makes sense because according to the transition matrix, any random walk of information or knowledge would spend a lot of time within the fully connected subgraph on both sides but rarely passes through the central node 3. Note that in this example, the amount of human capital in each industry is the same, so the difference of human capital environment for industries is entirely caused by the different ways that they are connected in the network.

2.2 General Equilibrium Model

After establishing the method of measuring human capital spillovers, the next step is to measure the productivity values of every industry in every city in order to study the relationship of these two. This section presents the rest of the general equilibrium model, the definition of the equilibrium and important properties of the model to provide a framework to calibrate the location-industry-specific productivity of firms.

2.2.1 Firms' Problem

With the production function shown in Equation (2.1), the objective of a firm is to maximize its profit by choosing the amount of skilled and unskilled labor inputs as well as capital inputs, after paying for all the production costs, i.e.

$$\max_{l_{i,c}(\omega),h_{i,c}(\omega),k_{i,c}(\omega)} p_{i,c}(\omega)q_{i,c}(\omega) - l_{i,c}(\omega)W_c^L - h_{i,c}(\omega)W_c^H - k_{i,c}(\omega)r. \tag{2.6}$$

W_c^H and W_c^L are the city-wide wages for two different types of labor. I Assume firms get capital from a friction-less national market at an universal interest rate of r. $p_{i,c}(\omega)$ is the output price on the national market for good ω from industry i in city c.

2.2.2 Households' Problem

In every city c, a representative household provides a total amount of $L_c = \sum_{i\in\mathbf{N}} L_{i,c}$ unskilled labor and $H_c = \sum_{i\in\mathbf{N}} H_{i,c}$ skilled labor to local firms to earn wages W_c^L and W_c^H. Also, the household get interest payment of the amount K_c from the capital market. Assume the interest revenue for city c is from a pool of national capital income and is proportionally to its population size. then K_c can be written as:

$$K_c = \sum_{c \in \mathbf{C}} \sum_{i \in \mathbf{N}} \int_{\Omega} k_{i,c}(\omega) r d\omega \cdot \frac{H_c + L_c}{\sum_{c' \in \mathbf{C}} H_{c'} + L_{c'}}.$$

The household spend their wages and capital income on all variety of goods to maximize the following utility function.

$$\max_{c_{i,c}(\omega), i \in \mathbf{N}} \prod_{i \in \mathbf{N}} C_{i,c}^{a_i} \tag{2.7}$$

$$s.t. \sum_{i \in \mathbf{N}} \int_{\Omega} c_{i,c}(\omega) p_{i,c}(\omega) d\omega = W_c^H H_c + W_c^L L_c + K_c. \tag{2.8}$$

$$where,\ C_{i,c} = \left(\int_{\Omega} c_{i,c}(\omega)^{\frac{\sigma - 1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma - 1}}. \tag{2.9}$$

Here a_i is the value share of industry i's output in the final consumption. $c_{i,c}(\omega)$ is the local consumption amount of variety ω in industry i and $p_{i,c}(\omega)$ is its price.

The price of variety ω of good j in city c is determined by Bertrand competition on the national market, just as in Eaton and Kortum (2002). Namely consumers in $\{i, c\}$ only purchase goods from the producer offering the lowest price. Therefore the price of variety ω of industry j that consumers in city c actually end up paying for is:

$$p_c^j(\omega) = \min_{c' \in \mathbf{C}} p_c^{j,c'}(\omega), \tag{2.10}$$

where $p_c^{j,c'}(\omega)$ is the price consumers in city c need to pay to variety ω's producer in $\{j, c'\}$. In the case of $p_c^{j,c'}(\omega) > p_c^j(\omega)$, there is no trade between consumers in $\{c\}$ and firms from $\{j, c'\}$ on variety ω. $p_c^{j,c'}(\omega)$ is a product between the out-of-factory price of good ω in $\{j, c'\}$ and an iceberg trade cost of $\tau_c^{c'}$, i.e. $p_{i,c}^{j,c'}(\omega) = p_{j,c'}(\omega)\tau_c^{c'}$. Here $\tau_c^{c'} \in [1, +\infty)$, $\tau_c^c = 1$. I use $\mathcal{T}$ to represent the $n \times n$ trade costs matrix. The $\{c', c\}$ coordinate of $\mathcal{T}$ has value $\tau_c^{c'}$. Suppose the market of each industry in each city is perfectly competitive, the out-of-factory price of a good at its origin is simply its marginal cost. By solving the cost minimization problem of firms, the out-of-factory price of a good can be written as :

$$p_{i,c}(\omega) = \frac{{W_c^L}^{\alpha_{i,c}} {W_c^H}^{\beta_{i,c}} r^{1-\alpha_{i,c}-\beta_{i,c}}}{A_{i,c}\left(\mathcal{H}_{i,c}\right) \alpha_{i,c}^{\alpha_{i,c}} \beta_{i,c}^{\beta_{i,c}} (1-\alpha_{i,c}-\beta_{i,c})^{1-\alpha_{i,c}-\beta_{i,c}}}. \tag{2.11}$$

The detailed derivation process of the marginal cost is in the appendix. Now the price of any variety of good j in city c can be rewritten as:

$$p_c^j(\omega) = \min_{c' \in \mathbf{C}} \frac{{W_c^L}^{\alpha_{i,c'}} {W_c^H}^{\beta_{i,c'}} r^{1-\alpha_{i,c'}-\beta_{i,c'}}}{A_{i,c'}\left(\mathcal{H}_{i,c'}, \omega\right) \alpha_{i,c'}^{\alpha_{i,c'}} \beta_{i,c'}^{\beta_{i,c'}} (1-\alpha_{i,c'}-\beta_{i,c'})^{1-\alpha_{i,c'}-\beta_{i,c'}}} \tau_c^{c'}. \tag{2.12}$$

Note that $p_{i,c}(\omega)$ is the out-of-factory price from the source city c whereas $p_c^j(\omega)$ is the final consumption price that consumers in the destination city c pay. Also, if firm ω in $\{i,c\}$ is actually competitive enough to sell to any location in the country, it will first become the supplier of its own city, because it doesn't have to pay a transportation cost in the local competition. Therefore the local sales price $p_c^j(\omega) = p_{i,c}(\omega)$ for any firm in $\{i,c\}$ that is producing.

2.2.3 General Equilibrium Conditions

Given a set of $\mathcal{T}$, $\{L_{i,c}\}$, $\{T_{i,c}\}$, $\{\alpha_{i,c}\}$ and $\{\beta_{i,c}\}$, the general equilibrium of this model is a set of $\{W_c^L\}$, $\{W_c^H\}$, r and $\{p_{i,c}\}$ such that

- maximize firms' profits in every city and every industry;
- maximize households' welfare in every city;
- clear the labor market in each city;
- clear the capital market nationally;
- clear goods markets in each city and each industry, which means the total income of an industry in a city is equal to the income earned from trade, i.e.

$$\int_\Omega q_{i,c}(\omega) p_c^i(\omega) d\omega = \sum_{j\in\mathbf{N}} \sum_{c'\in\mathbf{C}} \int_\Omega c_{c'}^{j,c}(\omega) p_{c'}^j(\omega) d\omega; \tag{2.13}$$

where $c_{c'}^{j,c}$ represents the total amount of consumption goods in city c' that comes from industry j in city c;

- balance trade in each city, which means the total expenditure is equal to the income earned from trade, i.e.

$$\sum_{i\in\mathbf{N}}\int_{\Omega} q_{i,c}(\omega)p_c^i(\omega)d\omega=\sum_{i\in\mathbf{N}}\sum_{c\in\mathbf{C}}\int_{\Omega} c_{i,c}(\omega)p_c^i(\omega)d\omega. \tag{2.14}$$

2.2.4 Trade Shares

Following Arkolakis et al. (2015), this model generates an expression of trade shares. Knowing trade shares is essential to find the closed form expression for calculating the fundamental productivity of industries. The following lemma represents the trade share expression. This expression deviates from Arkolakis et al. (2015) by having two different types of wage inputs.

Lemma 6. *In equilibrium, the share of goods sold from c' in the consumption of j in city c is*

$$\pi_c^{j,c'}=\frac{T_{j,c'}\left(\mathcal{H}_{j,c'}\right)\left(\dfrac{{W_{c'}^L}^{\alpha_{j,c'}}{W_{c'}^H}^{\beta_{j,c'}}r^{1-\alpha_{j,c'}-\beta_{j,c'}}}{\alpha_{j,c'}^{\alpha_{j,c'}}\beta_{j,c'}^{\beta_{j,c'}}(1-\alpha_{j,c'}-\beta_{j,c'})^{1-\alpha_{j,c'}-\beta_{j,c'}}}\tau_c^{c'}\right)^{-\theta}}{\Phi_c^j}.$$

where $\Phi_c^j=\sum_{c'\in\mathbf{C}}T_{j,c'}\left(\mathcal{H}_{j,c'}\right)\left(\dfrac{{W_{c'}^L}^{\alpha_{j,c'}}{W_{c'}^H}^{\beta_{j,c'}}r^{1-\alpha_{j,c'}-\beta_{j,c'}}}{\alpha_{j,c'}^{\alpha_{j,c'}}\beta_{j,c'}^{\beta_{j,c'}}(1-\alpha_{j,c'}-\beta_{j,c'})^{1-\alpha_{j,c'}-\beta_{j,c'}}}\tau_c^{c'}\right)^{-\theta}$.

The proof of Lemma 6 is in the appendix. Knowing trade shares $\pi_c^{j,c'}$ provides a way to rewrite values of the local sales. Let $y_{i,c}$ represent the total output from $\{i,c\}$ and y_c be the total output of city c. First order conditions of the households' problem indicate that the consumption demand of city c's household for good j is $a_j\left(W_c^H H_c+W_c^L L_c+K_c\right)$. Denote the trade flow from industry j in city c' to city c as $x_c^{j,c'}$, then the trade flow can expressed in the following form:

$$x_c^{j,c'}=a_j\left(W_c^H H_c+W_c^L L_c+K_c\right)\pi_c^{j,c'}. \tag{2.15}$$

Substituting the above expression back to the market clearing condition

(2.13), I can rewrite the output expression for industry $\{i, c\}$ as:

$$y_{j,c'} = a_j \sum_{c \in \mathbf{C}} \left(W_c^H H_c + W_c^L L_c + K_c \right) \pi_c^{j,c'}$$

$$y_{j,c'} = a_j \sum_{c \in \mathbf{C}} \left(\sum_{i \in \mathbf{N}} (\alpha_{i,c} + \beta_{i,c}) y_{i,c} + K_c \right) \pi_c^{j,c'}$$

$$y_{j,c'} = a_j \sum_{c \in \mathbf{C}} \left(\sum_{i \in \mathbf{N}} (\alpha_{i,c} + \beta_{i,c}) y_{i,c} + K_c \right) \sum_{c' \in \mathbf{C}} T_{j,c'} \left(\mathcal{H}_{j,c'} \right) \frac{\left(\frac{{W_{c'}^L}^{\alpha_{j,c'}} {W_{c'}^H}^{\beta_{j,c'}} r^{1-\alpha_{j,c'}-\beta_{j,c'}}}{\alpha_{j,c'}^{\alpha_{j,c'}} \beta_{j,c'}^{\beta_{j,c'}} (1 - \alpha_{j,c'} - \beta_{j,c'})^{1-\alpha_{j,c'}-\beta_{j,c'}}} \tau_c^{c'} \right)^{-\theta}}{\Phi_c^j}, \tag{2.16}$$

where $K_c = \left(\sum_{c \in \mathbf{C}} (1 - \alpha_{i,c} - \beta_{i,c}) \, y_{i,c} \right) \cdot \left(\frac{L_c + H_c}{\sum_{c' \in \mathbf{C}} L_{c'} + H_{c'}} \right)$. Equation (2.16) provides a method to extract the value of fundamental productivity $T_{j,c'} \left(\mathcal{H}_{j,c'} \right)$. As long as all other parameters and variables in the equations are known, values of $T_{j,c'} \left(\mathcal{H}_{j,c'} \right)$ are identified by solving a nonlinear equation system.

2.3 Calibration

This sections provides details of calibration to acquire the industry-location-specific fundamental productivity values $T_{j,c'} \left(\mathcal{H}_{j,c'} \right)$. According to Equation (2.16), if knowing wage W_c^L and W_c^H, labor input shares $\alpha_{i,c}$ and $\beta_{i,c}$, ice berg trade costs $\mathcal{T}$, and comparative advantage parameter θ, we can solve for fundamental technology $T_{i,c}$ for every location and every industry.

The calibration is limited to the U.S. data in 2016. I use industry-level GDP in 381 Metropolitan Statistical Areas (MSAs) in 2016 for $y_{i,c}$. The data is provided by Bureau of Economic Analysis (BEA). In total 20 industries by NAICS classification are considered in the baseline calibration and they cover the entire span of private sectors in the U.S. BEA also provides industry-MSA-level labor income and employment data. Ratios between labor income and GDP are used as the capital input shares $1 - \alpha_{i,c} - \beta_{i,c}$. I obtain skilled labor input shares α_i for each MSA in the sample from 2016's American Community Survey (ACS). Specifically in this paper, skilled labor is defined as the population 25 years old and over, with associate's degree, bachelor's degree, graduate or professional degree, while population in the same age range with less education is categorized as unskilled labor. First, with ACS data I compute the labor income ratios between two education groups in each city, i.e. $\frac{\alpha_{i,c}}{\beta_{i,c}}$. Then the capital input ratios obtained previously help to back out the values of $\beta_{i,c}$ and $\alpha_{i,c}$ respectively. ACS samples also provide ratios of employment between two education groups. Applying the sample ratios on the regional employment data from BEA generates estimates of two types of employment, i.e. L_c and H_c. After knowing input shares and employment shares of different types of labors and the total employment and income values, I can calculate local wages W_c^L and W_c^H as well.

The iceberg trade cost matrix $\mathcal{T}$ is estimated by the same method as in Allen and Arkolakis (2014). First, I used fast marching method (FMM) and U.S. highway, rail and navigable water networks data (NDC, 1999; CTA, 2003; NHPN, 2005) to estimate the instantaneous trade costs for any point on the map. Then I use the Commodity Flow Survey (CFS) data of trade shares in different states to estimate trade costs between any two points on the map. Due to the lack of trade follow data among cities. The iceberg trade cost is estimated by state level trade flows from 2012 CFS. Then I calculate centers of each MSA on the map and simulate the trade cost matrix among cities through the estimated trade cost function. Because the I use the aggregate trade flows without industry breakdown for the estimation, the trade cost matrix is the same for every industry.

The lack of trade flow data among industries and cities makes it impossible to estimate the comparative advantage parameter θ with the same method as in Eaton and Kortum (2002), by using ratios of trade flows. Therefore, I take several estimated values of θ from Eaton and Kortum (2002) and Caliendo and Parro (2015) for the calibration exercise. Additional results from different choices of θ is in the appendix. Also, the interest rate r is set to 0.5% to match the Fed rate in 2016.

Only cities with complete employment and wage data, and which also can be identified in the trade cost estimation, are considered in the calibration. Therefore the number of cities in the calibration is reduced to 303. Putting all these values back to equation 2.16, I get a set of unique solution for

$T_{j,c'}$ ($\mathcal{H}_{j,c'}$). They are the model estimates of the fundamental productivity for each industry in each city in 2016.

2.4 Empirical Evidence for Human Capital Spillovers

To verify the existence of human capital spillover on industrial productivity in cities, it is also necessary to construct the human capital network index $\mathcal{H}_{i,c}$. The economic distance matrix D is calculated from the total requirement tables from the input-output accounts [2] provided by BEA. The human capital quantity values $H_{i,c}$ are imputed from the skilled worker wage, city-level total labor income and skilled human capital input shares.

2.4.1 Baseline Model

With the definition of fundamental productivity $\{T_{i,c}\}$ in Equation (2.3) and the calibrated values of $\{T_{i,c}\}$ from the general equilibrium model, regressions with the following formula can be used to check the relationship between city human capital spillover and productivity of industries:

$$T_{i,c} = b_0 + \theta_i(\mathcal{H}_{i,c} \cdot I_i) + city\,FE + industry\,FE + \epsilon_{i,c}. \tag{2.17}$$

Here I_i is an industry dummy matrix. θ_i here can be interpreted as the ability of industry i to absorb the human capital spillovers in any city. City and industry fixed effects cannot eliminate all the confounders in the regression. Therefore strictly speaking these correlations cannot be directly interpreted as casual relationships. However they provide supportive evidence for the impact of human capital spillovers on productivity.

The first column of Table 2.2 reports the estimates of θ_i from this baseline model.

The results show that the human capital network index has significant correlations with industry-level TFP in 19 out of 20 sectors at 1% level in the data. Among all the significant θ_i values, 15 industries have positive θ_i. This indicates that most of business are more productive in cities with

[2]The full name of the table is: Industry-by-Commodity Total Requirements, After Redefinitions.

Table 2.2: The Effect of City Human Capital Spillovers on Industrial Productivity for 20 Industries

values of θ_i by industry	baseline, 2016	alternative $\mathcal{H}_{i,c}$ 2016 1	alternative $\mathcal{H}_{i,c}$ 2016 2	2005-2016
Farm	$\mathbf{-0.149}$***	$\mathbf{-0.014}$***	$\mathbf{-0.073}$***	$\mathbf{-0.007}$***
	(0.019)	(0.001)	(0.006)	(0.001)
Forestry Fishing	$\mathbf{-0.285}$***	$\mathbf{-0.053}$***	-0.168	$\mathbf{-0.025}$***
	(0.059)	(0.005)	(0.053)	(0.005)
Mining	$\mathbf{-0.26}$***	$\mathbf{-0.033}$***	-0.16	-0.007
	(0.064)	(0.006)	(0.051)	(0.005)
Utilities	-0.042	-0.002	-0.011	0.012
	(0.043)	(0.006)	(0.041)	(0.004)
Construction	**0.214***	**0.023***	**0.223***	**0.022***
	(0.023)	(0.003)	(0.023)	(0.002)
Manufacturing	0.024	-0.002	**0.028***	0.001
	(0.008)	(0.001)	(0.007)	(0.001)
Wholesale Trade	**0.117***	**0.013***	**0.153***	**0.023***
	(0.022)	(0.003)	(0.022)	(0.002)
Retail Trade	**0.04***	**0.002***	**0.001***	$\mathbf{-0.0}$***
	(0.004)	(0.001)	(0.0)	(0.0)
Transportation Warehousing	$\mathbf{-0.121}$*	-0.004	-0.028	0.007
	(0.047)	(0.004)	(0.032)	(0.003)
Information	**0.352***	**0.028***	**0.263***	**0.028***
	(0.02)	(0.002)	(0.011)	(0.001)
Finance, Insurance	**0.45***	**0.016***	**0.488***	**0.026***
	(0.026)	(0.002)	(0.026)	(0.002)
Real Estate	**1.439***	**0.178***	**1.461***	**0.108***
	(0.089)	(0.012)	(0.083)	(0.007)
Professional Service	**0.65***	**0.039***	**0.65***	**0.046***
	(0.024)	(0.003)	(0.023)	(0.002)
Management	**0.351***	-0.003	**0.318***	**0.028***
	(0.079)	(0.008)	(0.044)	(0.003)
Admin Waste Mngmnt	**6.878***	**0.419***	**5.379***	**0.305***
	(0.387)	(0.054)	(0.329)	(0.04)
Education Service	**0.137***	**0.015***	**0.151***	**0.018***
	(0.009)	(0.001)	(0.009)	(0.001)
Health Care Social Assistant	**0.238***	**0.015***	**0.183***	0.002
	(0.011)	(0.002)	(0.008)	(0.001)
Art Entertianment Recreation	**0.695***	**0.056***	**0.695***	**0.062***
	(0.069)	(0.008)	(0.066)	(0.006)
Accommondation Food Service	**0.32***	-0.002	**0.244***	-0.001
	(0.033)	(0.004)	(0.022)	(0.003)
Other Services	**0.263***	0.0	**0.226***	$\mathbf{-0.003}$*
	(0.02)	(0.002)	(0.019)	(0.001)
Year Fixed Effects				Yes
City Fixed Effects	Yes	Yes	Yes	Yes
Industry Fixed Effects	Yes	Yes	Yes	Yes
R^2	0.326	0.303	0.281	0.268
Observations	6969	6969	6969	83628

Dependent variable: fundamental productivity $T_{i,c}$ by Industry and MSA.
θ_i measures the effect of urban human capital spillovers on the TFP of industries in the city.
A bigger θ_i implies a stronger correlation between TFP and human capital spillovers.
Standard errors in the parentheses are culstering with respect to city and year
***p<0.001, **0.001≤p<0.005, *0.005≤p<0.01

higher human capital network index $\mathcal{H}_{i,c}$. Due to the way the human capital network index is constructed, $\mathcal{H}_{i,c}$ can have higher values for two reasons. Firstly, there may be a larger stock of skilled labor in the overall city, i.e. $H_{i,c}$ values are bigger in general. Secondly, skilled workers are more concentrated in industries that do more frequent transactions with industry i, i.e. $H_{j,c}$ values are bigger for industry j with higher $D_{i,j}$ values. Under either case, higher values of $\mathcal{H}_{i,c}$ indicate higher levels of human capital in the urban environment that industries reside in. Therefore positive θ_i shows that most industries benefit from being in an urban environment with skilled workers.

Four industries, including farming, forestry and fishing, mining, as well as transportation and warehousing, have significantly negative θ_i. This indicates that higher human capital levels in the urban environment are associated with lower producty in these industries. This is reasonable because these industries in general have smaller demand of highly educated workers in their production process. More productive workers in these industries are more easily drawn by other business in a high human capital environment. In other words, even if some college educated labor work for these industries, on average their quality may be lower than their counterparts in other human-capital-intensive industries. Because the model considers only two skill-levels, it cannot tell the different qualities of workers within the same education group. As a result, the type of 'crowd-out' effect is picked up by negative θ_i.

One potential concern for this explanation of negative θ_i is that these industries may be so concentrated in specific locations that their productivity is determined by natural geographical conditions instead of their relative positions in urban human capital networks. For example, farming, forestry and fishing can only be more prosperous if there are such natural resources nearby. They may not be affected by the quality of human capital in their suppliers or customers. There are two treatments of the empirical exercise can solve this issue. First, all the regressions in Table 2.2 include city fixed effects in the regression. Therefore the location specific attributes are controlled. values of θ_i are only meant to explain the variation in industrial productivity from city-level average by the variation in human capital network index from its city-level average. Second, the sample used in regressions only includes cities with wage, labor and GDP data in the full spans of 20 industries and all rural area are excluded in the analysis, therefore the fundamental economic structures of these cities are not so different to an extent they are completely deprived of certain sectors.

2.4.2 Robustness Checks

In this section, additional regressions are carried out to check the robustness of the results.

The previous section elaborates on the argument that there can be two potential sources for higher $\mathcal{H}_{i,c}$ values, either higher total quantity of human capital in the city, or better allocations of human capital in the city that is specifically beneficial to industry i. A reasonable concern is that if the quantity aspect of human capital spillovers dominates the allocation aspect of human capital spillovers, then the network structure of human capital allocation will have trivial impact on productivity. Then instead of a complicated human capital network index, simple aggregate measures of human capital are sufficient to account for the spillovers.

To eliminate this concern, I use alternative definitions of $\mathcal{H}_{i,c}$ to run the same regression as in Equation (2.17). The first alternative is to use the skilled labor input shares $\alpha_{i,c}$ to replace $H_{i,c}$ in the calculation of $\mathcal{H}_{i,c}$. In the second alternative construction, the educated labor shares $\dfrac{L_{i,c}}{L_{i,c}+H_{i,c}}$ are used to replace $H_{i,c}$ in $\mathcal{H}_{i,c}$. The second and the third columns of Table 2.2 show the results. For most industries, the magnitude and significance level of θ_i decline while R^2 values for the entire regressions decrease as well. However, the majority of θ_i values are still statistically significant. This indicates that while the aggregate quantity of skilled labor does explain part of the urban human capital spillovers, the allocations of these educated workers in the production network also matter significantly.

To show that this relationship between urban human capital spillovers and industrial productivity is robust over time. I expand the calibration of fundamental productivity $T_{i,c}$ to BEA and ACS data in the time period from 2005 to 2016. Because CFS data is not available at yearly basis, only

Table 2.3: Effect of City Human Capital Spillover on Industrial Productivity for 15 Manufacturing Subsectors Only

values of θ_i by industry	baseline, 2016	alternative $\mathcal{H}_{i,c}$ 2016 1	alternative $\mathcal{H}_{i,c}$ 2016 2
Food, Beverage & Tobacco	**0.403*** **	**0.042*** **	**0.04*** **
	(0.076)	(0.006)	(0.006)
Textile, Apparel & Leather	**0.42* **	0.07	**0.078*** **
	(0.172)	(0.023)	(0.022)
Wood & Paper	**0.313*** **	**0.044*** **	**0.061*** **
	(0.06)	(0.011)	(0.01)
Printing	0.356	-0.006	0.074
	(0.234)	(0.026)	(0.025)
Petroleum & Coal	-1.059	-0.001	**−0.071*** **
	(0.374)	(0.018)	(0.017)
Chemical	**0.365*** **	**0.028*** **	**0.028*** **
	(0.045)	(0.005)	(0.005)
Plastics & Rubber	0.14	0.045	0.038
	(0.238)	(0.015)	(0.014)
Nonmetallic Mineral	0.127	**0.112*** **	**0.139*** **
	(0.284)	(0.027)	(0.026)
Machinary	**0.47*** **	**0.077*** **	**0.074*** **
	(0.099)	(0.005)	(0.004)
Primary & Fabricated Metal	**0.345*** **	**0.141*** **	**0.123*** **
	(0.076)	(0.01)	(0.009)
Computer & Electronics	**0.231*** **	**0.043*** **	**0.052*** **
	(0.048)	(0.006)	(0.006)
Electrical Equipment	**0.301* **	**0.045*** **	**0.04*** **
	(0.137)	(0.011)	(0.011)
Transportation Equipment	**0.295*** **	**0.048*** **	**0.042*** **
	(0.026)	(0.003)	(0.002)
Furniture	0.515	**0.103*** **	**0.125*** **
	(0.304)	(0.031)	(0.029)
Miscellaneous Manufacturing	**0.401* **	0.031	0.056
	(0.183)	(0.02)	(0.018)
City Fixed Effects	Yes	Yes	Yes
Industry Fixed Effects	Yes	Yes	Yes
R^2	0.356	0.211	0.189
Observations	4545	4545	4545

Dependent variable: GDP per worker by Industry and MSA.
θ_i measures the effect of urban human capital spillover on the TFP of industries in the city.
A bigger θ_i implies a stronger correlation between TFP and human capital spillovers.
Standard errors in the parentheses are culstering with respect to city and year
***p<0.001, **0.001≤p<0.005, *0.005≤p<0.01

2012 CFS data is used in these additional calibrations. Therefore these TFP estimates are less reliable. With the new estimates, I run the regression in Equation (2.17) with additional year fixed effects. The results show that in spite of slight decline in the magnitude of θ_i estimates and R^2 values, the majority of θ_i estimates remain significant and keep the same signs the baseline results.

In the baseline result, the only industry with insignificant θ_i is manufacturing. This seems to contradict the results of Moretti (2004a). However, due to the wide variety of business under the umbrella named manufacturing, it is very likely that while high-end manufacturing business can indeed benefit greatly from an more educated urban environment, some other manufacturing business may experience similar "crowd-out" effect as the ones mentioned previously. Therefore, I break down the manufacturing sector to 15 finer NAICS categories, and redo the calibrations and regressions with only manufacturing data. Because BEA has only employment and wage breakdowns for 20 industries at MSA level, I have to impute employment and wage values for these new classifications only relying on ACS 5% samples. As a results more industries in more MSAs end up with 0 employment and output. When solving the general equilibrium model, I put 1 worker in all these 0 employment cells. I used state average wages and input ratios for these industry-city combinations as well. Therefore the results are less reliable and not directly comparable to results in Table 2.2. There results are presented in Table 2.3.

According to the baseline results in first column of Table 2.3, 10 out 15 manufacturing sectors have significant θ_i. Also the variation among values of θ_i for different industries is big. This can explain why θ_i values for manufacturing in Table 2.2 are mostly not significant. The 2nd and 3rd columns of Table 2.3 repeat the experiments with alternative $\mathcal{H}_{i,c}$ definitions. Similar to Table 2.2 results, the magnitude and significance level of coefficient estimates, as well as overall R^2 in these two columns decrease.

2.5 Conclusion

This paper explores the relationship between human capital spillovers in cities and regional difference of industrial productivity in the U.S. To measure the human capital spillover faced by a specific industry in the city, I develop a novel human capital network index. It is a weighted average of skilled labor force level of all other industries in the same city where the weights are modified betweenness measures between the target industry and source industry. The index is high for an industry not only when the general level of human capital level in the city is high but also when human capital is more concentrated in economically closer nodes in town for an industry. Therefore this index evaluates both the quantity and the allocation of human capital in a city for this specific industry. To measure the industrial productivity in different locations, I build a general equilibrium model with multi-sector production in cities and competitive trade across cities and then calibrated the model with MSA level data in the U.S.

With both the measure of urban human capital spillovers and industrial productivity, this paper conduct empirical regressions to show that there are three factors that decide the influence of urban human capital spillover on the productivity of an industry: 1) the general quantity of educated workforce in the city, 2) the concentration of human capital in an industry's input-output network 3) the ability of an industry to absorb the spillovers. Specifically, the ability of business to absorb urban human capital spillovers on productivity varies from industry to industry. While majority of sectors benefit from a more educated urban environment, certain industries, such as farming, forestry and fishing, mining, transportation and warehousing as well as some subcategories of manufacturing, experience negative human capital spillover from the rest of the city. A possible explanation here is that the large presence of human-capital-intensive sectors in town may lure away talents from industries with low intensity of human capital usage and decrease the human capital quality in these industries. This result provides evidence that cities with high human capital level are more prone to specialize in certain industries.

This work provides evidence to show how fundamental productivity of industries are influenced by human capital allocation in different geographic locations. In turn, Yu (2018) shows that the structure of firms' productivity in an urban industrial network affect labor allocation across sectors and cities. Together these two works lay the foundation for analyzing the endogenous evolution of urban industrial structures and human capital migration. Future work in this direction will shed more light on the dynamics of geographic equality in the U.S. economy.

Appendices

Derivation of Equation (2.11). The industry level price in city c for industry i is:

$$\log p_{i,c} = \int_{\Omega} \log p_c^i(\omega) d\omega$$
$$\log p_{i,c} = \int_{\Omega} \log p_c^i(\omega) dG_c^i(p)$$
$$\log p_{i,c} = \int_0^{\infty} \log p \frac{d\left(1-\exp\left\{-p^{\theta}\tilde{\Phi}_c^i\right\}\right)}{dp} dp$$
$$\log p_{i,c} = \int_0^{\infty} \log p \frac{d\left(1-\exp\left\{-\exp(\theta \log p + \log \tilde{\Phi}_c^i)\right\}\right)}{d \log p} d \log p$$
$$\log p_{i,c} = \int_{-\infty}^{\infty} t \frac{d\left(1-\exp\left\{-\exp(\theta t + \log \tilde{\Phi}_c^i)\right\}\right)}{dt} dt$$

$1-\exp\left\{-\exp(\theta t + \log \tilde{\Phi}_c^i)\right\}$ is the CDF of t with gumbel distribution, i.e. $t \sim Gumbel(-\frac{\log \tilde{\Phi}_c^i,}{\theta}, \frac{1}{\theta})$, so the right hand side is simply the mean of t, $-\frac{\log \tilde{\Phi}_c^i,}{\theta} - \frac{\gamma}{\theta}$, where γ is the Euler's constant. Then I can further write the price index as:

$$\log p_{i,c} = -\frac{\log \tilde{\Phi}_c^i}{\theta} - \frac{\gamma}{\theta}$$
$$p_{i,c} = {\Phi_c^i}^{-\frac{1}{\theta}} \exp\left\{\frac{\theta}{\gamma}\right\}$$
$$p_{i,c} = \frac{{W_c^L}^{\alpha_{i,c}} {W_c^H}^{\beta_{i,c}} r^{1-\alpha_{i,c}-\beta_{i,c}}}{A_{i,c}\left(\mathcal{H}_{i,c}\right) \alpha_{i,c}^{\alpha_{i,c}} \beta_{i,c}^{\beta_{i,c}} (1-\alpha_{i,c}-\beta_{i,c})^{1-\alpha_{i,c}-\beta_{i,c}}}.$$

Therefore, if $p_{i,c}$ is deterministic conditional on all the parameters and thus $\tilde{p}_{i,c}$ is also deterministic. □

Proof of Lemma 6. Because $p_{j,c'}^{i,c}(\omega) = \frac{{W_c^L}^{\alpha_{i,c}} {W_c^H}^{\beta_{i,c}} r^{1-\alpha_{i,c}-\beta_{i,c}}}{A_{i,c}\left(\mathcal{H}_{i,c}, \omega\right) \alpha_{i,c}^{\alpha_{i,c}} \beta_{i,c}^{\beta_{i,c}} (1-\alpha_{i,c}-\beta_{i,c})^{1-\alpha_{i,c}-\beta_{i,c}}} \tau_{c'}^c$, we can derive the probability distribution of $p_{j,c'}^{i,c}$, i.e. the sales price of $\{i, c\}$ to any $\{j, c'\}$:

$$G_{j,c'}^{i,c}(p) = Pr\left\{p_{j,c'}^{i,c}(\omega) \le p\right\}$$

$$
\begin{aligned}
&= Pr\left\{\frac{W_c^{L\,\alpha_{i,c}} W_c^{H\,\beta_{i,c}} r^{1-\alpha_{i,c}-\beta_{i,c}}}{A_{i,c}\left(\mathcal{H}_{i,c},\omega\right)\alpha_{i,c}^{\alpha_{i,c}}\beta_{i,c}^{\beta_{i,c}}(1-\alpha_{i,c}-\beta_{i,c})^{1-\alpha_{i,c}-\beta_{i,c}}}\tau_{c'}^{c} \leq p\right\} \\
&= Pr\left\{A_{i,c}\left(\mathcal{H}_{i,c},\omega\right) \geq \frac{W_c^{L\,\alpha_{i,c}} W_c^{H\,\beta_{i,c}} r^{1-\alpha_{i,c}-\beta_{i,c}}}{\alpha_{i,c}^{\alpha_{i,c}}\beta_{i,c}^{\beta_{i,c}}(1-\alpha_{i,c}-\beta_{i,c})^{1-\alpha_{i,c}-\beta_{i,c}}p}\tau_{c'}^{c}\right\} \\
&= 1 - exp\left\{-T_{i,c}\left(\mathcal{H}_{i,c}\right)\left(\frac{W_c^{L\,\alpha_{i,c}} W_c^{H\,\beta_{i,c}} r^{1-\alpha_{i,c}-\beta_{i,c}}}{\alpha_{i,c}^{\alpha_{i,c}}\beta_{i,c}^{\beta_{i,c}}(1-\alpha_{i,c}-\beta_{i,c})^{1-\alpha_{i,c}-\beta_{i,c}}p}\tau_{c'}^{c}\right)^{-\theta}\right\}. \qquad (18)
\end{aligned}
$$

From Equation (2.12), the probability distribution of good j's purchase price for any i in city c is:

$$
\begin{aligned}
G_{i,c}^{j}(p) &= Pr\left\{\min_{c'\in\mathbf{C}} P_{i,c}^{j,c'}(\omega) \leq p\right\} \\
&= 1 - \prod_{c'\in\mathbf{C}}\left(Pr\left\{P_{i,c}^{j,c'}(\omega) \geq p\right\}\right) \\
&= 1 - \prod_{c'\in\mathbf{C}}\left(1 - G_{i,c}^{j,c'}(p)\right) \\
&= 1 - \prod_{c'\in\mathbf{C}} exp\left\{-T_{j,c'}\left(\mathcal{H}_{j,c'}\right)\left(\frac{W_{c'}^{L\,\alpha_{j,c'}} W_{c'}^{H\,\beta_{j,c'}} r^{1-\alpha_{j,c'}-\beta_{j,c'}}}{\alpha_{j,c'}^{\alpha_{j,c'}}\beta_{j,c'}^{\beta_{j,c'}}(1-\alpha_{j,c'}-\beta_{j,c'})^{1-\alpha_{j,c'}-\beta_{j,c'}}p}\tau_{c}^{c'}\right)^{-\theta}\right\} \\
&= 1 - \exp\left\{-p^{\theta}\sum_{c'\in\mathbf{C}} T_{j,c'}\left(\mathcal{H}_{j,c'}\right)\left(\frac{W_{c'}^{L\,\alpha_{j,c'}} W_{c'}^{H\,\beta_{j,c'}} r^{1-\alpha_{j,c'}-\beta_{j,c'}}}{\alpha_{j,c'}^{\alpha_{j,c'}}\beta_{j,c'}^{\beta_{j,c'}}(1-\alpha_{j,c'}-\beta_{j,c'})^{1-\alpha_{j,c'}-\beta_{j,c'}}}\tau_{c}^{c'}\right)^{-\theta}\right\} \\
&= 1 - \exp\left\{-p^{\theta}\Phi_c^{j}\right\},
\end{aligned}
$$

where $\Phi_c^j = \sum_{c'\in\mathbf{C}} T_{j,c'}\left(\mathcal{H}_{j,c'}\right)\left(\frac{W_{c'}^{L\,\alpha_{j,c'}} W_{c'}^{H\,\beta_{j,c'}} r^{1-\alpha_{j,c'}-\beta_{j,c'}}}{\alpha_{j,c'}^{\alpha_{j,c'}}\beta_{j,c'}^{\beta_{j,c'}}(1-\alpha_{j,c'}-\beta_{j,c'})^{1-\alpha_{j,c'}-\beta_{j,c'}}}\tau_{c}^{c'}\right)^{-\theta}$. Because the value of $G_{i,c}^{j}(p)$ is the same for every i in the same city, I simplify the notation to $G_c^j(p)$. With all these calculations now we can define $\pi_{i,c}^{j,c'}$, the probability that a specific city c' becomes the provider of good j for all industries in city c, as:

$$
\begin{aligned}
\pi_{i,c}^{j,c'} &= Pr\left\{p_{i,c}^{j,c'}(\omega) \leq \min_{k\in\mathbf{C}\setminus c'} p_{i,c}^{j,k}(\omega)\right\} \\
&= \int_0^{\infty} Pr\left\{\min_{k\in\mathbf{C}\setminus c'} p_{i,c}^{j,k} \geq p\right\} dG_{i,c}^{j,c'}(p)
\end{aligned}
$$

$$= \int_0^\infty \prod_{k \in \mathbf{C} \backslash c'} (1 - G_{i,c}^{j,k}(p)) dG_{i,c}^{j,c'}(p)$$

$$= \int_0^\infty \prod_{k \in \mathbf{C} \backslash c'} exp\left\{ -T_{j,k} (\mathcal{H}_{j,k}) \left(\frac{W_k^{L\alpha_{j,k}} W_k^{H\beta_{j,k}} r^{1-\alpha_{j,k}-\beta_{j,k}}}{\alpha_{j,k}^{\alpha_{j,k}} \beta_{j,k}^{\beta_{j,k}} (1 - \alpha_{j,k} - \beta_{j,k})^{1-\alpha_{j,k}-\beta_{j,k}} p} \tau_c^k \right)^{-\theta} \right\} dG_{i,c}^{j,c'}(p)$$

$$= \int_0^\infty exp\{\Phi_c^j\} \cdot \left(-T_{j,c'} \theta \left(\frac{W_{c'}^{L\alpha_{j,c'}} W_{c'}^{H\beta_{j,c'}} r^{1-\alpha_{j,c'}-\beta_{j,c'}}}{\alpha_{j,c'}^{\alpha_{j,c'}} \beta_{j,c'}^{\beta_{j,c'}} (1 - \alpha_{j,c'} - \beta_{j,c'})^{1-\alpha_{j,c'}-\beta_{j,c'}}} \tau_c^{c'} \right)^{-\theta} \right) p^{\theta-1} dp$$

$$= T_{j,c'} (\mathcal{H}_{j,c'}) \left(\frac{W_{c'}^{L\alpha_{j,c'}} W_{c'}^{H\beta_{j,c'}} r^{1-\alpha_{j,c'}-\beta_{j,c'}}}{\alpha_{j,c'}^{\alpha_{j,c'}} \beta_{j,c'}^{\beta_{j,c'}} (1 - \alpha_{j,c'} - \beta_{j,c'})^{1-\alpha_{j,c'}-\beta_{j,c'}}} \tau_c^{c'} \right)^{-\theta} \int_0^\infty exp\left\{ -p^\theta \Phi_c^j \right\} \cdot \theta p^{\theta-1} dp$$

$$= \frac{T_{j,c'} (\mathcal{H}_{j,c'}) \left(\frac{W_{c'}^{L\alpha_{j,c'}} W_{c'}^{H\beta_{j,c'}} r^{1-\alpha_{j,c'}-\beta_{j,c'}}}{\alpha_{j,c'}^{\alpha_{j,c'}} \beta_{j,c'}^{\beta_{j,c'}} (1 - \alpha_{j,c'} - \beta_{j,c'})^{1-\alpha_{j,c'}-\beta_{j,c'}}} \tau_c^{c'} \right)^{-\theta}}{\Phi_c^j} \left(-exp\left\{ -p^\theta \Phi_c^j \right\} \Big|_0^\infty \right)$$

$$= \frac{T_{j,c'} (\mathcal{H}_{j,c'}) \left(\frac{W_{c'}^{L\alpha_{j,c'}} W_{c'}^{H\beta_{j,c'}} r^{1-\alpha_{j,c'}-\beta_{j,c'}}}{\alpha_{j,c'}^{\alpha_{j,c'}} \beta_{j,c'}^{\beta_{j,c'}} (1 - \alpha_{j,c'} - \beta_{j,c'})^{1-\alpha_{j,c'}-\beta_{j,c'}}} \tau_c^{c'} \right)^{-\theta}}{\Phi_c^j}.$$

Again, because the value of $\pi_{i,c}^{j,c'}$ does not depend on i, we can define the probability of $\{j, c'\}$ become the supplier of good j in city c as:

$$\pi_c^{j,c'} = \frac{T_{j,c'} (\mathcal{H}_{j,c'}) \left(\frac{W_{c'}^{L\alpha_{j,c'}} W_{c'}^{H\beta_{j,c'}} r^{1-\alpha_{j,c'}-\beta_{j,c'}}}{\alpha_{j,c'}^{\alpha_{j,c'}} \beta_{j,c'}^{\beta_{j,c'}} (1 - \alpha_{j,c'} - \beta_{j,c'})^{1-\alpha_{j,c'}-\beta_{j,c'}}} \tau_c^{c'} \right)^{-\theta}}{\Phi_c^j}.$$

We can see that the probability for any $\{j, c'\}$ becomes an exporter does not depend on the specific price it charges.

By the law of large numbers, $\pi_{i,c}^{j,c'}$ can be viewed as the fraction of good j from city c' sold to industry i in city c. Next I am going to prove that $\pi_{i,c}^{j,c'}$ is also the fraction of expenditure on intermediate inputs that industry i in city spend on good j specifically from city c'. The idea is that any city winning the bid for exporting product j to industry i in c has exactly the same price distribution.

$$Pr\left\{ p_{i,c}^{j,c'}(\omega) \leq \rho \middle| p_{i,c}^{j,c'}(\omega) \leq \min_{k \in \mathbf{C} \backslash c'} p_{i,c}^{j,k}(\omega) \right\}$$

$$
\begin{aligned}
&= \frac{\int_0^{\rho} Pr\left\{\min_{k\in \mathbf{C}\backslash c'} p_{i,c}^{j,k}(\omega) \geq p\right\} dG_{i,c}^{j,c'}(p)}{Pr\left\{p_{i,c}^{j,c'} \leq \min_{k\in \mathbf{C}\backslash c'} p_{i,c}^{j,k}\right\}} \\
&= \frac{1}{\pi_{i,c}^{j,c'}} \int_0^{\rho} \prod_{k\in \mathbf{C}\backslash c'} (1 - G_{i,c}^{j,k}(p)) dG_{i,c}^{j,c'}(p) \\
&= \frac{1}{\pi_{i,c}^{j,c'}} \frac{T_{j,c'}\left(\mathcal{H}_{j,c'}\right)\left(\frac{{W_{c'}^L}^{\alpha_{j,c'}} {W_{c'}^H}^{\beta_{j,c'}} r^{1-\alpha_{j,c'}-\beta_{j,c'}}}{\alpha_{j,c'}^{\alpha_{j,c'}} \beta_{j,c'}^{\beta_{j,c'}} (1-\alpha_{j,c'}-\beta_{j,c'})^{1-\alpha_{j,c'}-\beta_{j,c'}}} \tau_c^{c'}\right)^{-\theta}}{\Phi_c^j} \left(-exp\left\{-p^{\theta}\Phi_c^j\right\}\Big|_0^{\rho}\right) \\
&= \frac{1}{\pi_{i,c}^{j,c'}} \frac{T_{j,c'}\left(\mathcal{H}_{j,c'}\right)\left(\frac{{W_{c'}^L}^{\alpha_{j,c'}} {W_{c'}^H}^{\beta_{j,c'}} r^{1-\alpha_{j,c'}-\beta_{j,c'}}}{\alpha_{j,c'}^{\alpha_{j,c'}} \beta_{j,c'}^{\beta_{j,c'}} (1-\alpha_{j,c'}-\beta_{j,c'})^{1-\alpha_{j,c'}-\beta_{j,c'}}} \tau_c^{c'}\right)^{-\theta}}{\Phi_c^j} \left(1 - exp\left\{-\rho^{\theta}\Phi_c^j\right\}\right) \\
&= 1 - exp\left\{-\rho^{\theta}\Phi_c^j\right\} \\
&= G_c^j(\rho).
\end{aligned}
$$

The expression of this price distribution is independent from the origin city c'. The importer will need to pay statistically the same price to any exporter that happens to offer the lowest price bid. Therefore the fraction of goods j that industry i in c bought from c', $\pi_{i,c}^{j,c'}$ is also the fraction of expenditure that city c spending on goods j specifically from city c'.

□

Chapter 3

Credit Risks in Production Network

Introduction

In recently years, the theory that idiosyncratic credit risks can propagate or spill over through a network of firms and result in macroeconomic downturn has been frequently discussed. While many researches focus on modeling financial contagion in banking networks in theory and simulating possible results, very little valid empirical evidence is presented for financial contagion in the production network. In data, how can we trace credit risk propagation in an observable production network? What new insight do empirical measures of relationship between credit risk and network structure bring to us about the exact mechanism of financial risk propagation in production networks?

To answer these questions, this paper investigates the empirical evidence of credit risk contagion specifically in an inter-sectoral input-output (IO) production network. The logic behind our empirical strategy is simple. If financial stress does propagate through the production network, we should observe *stronger* credit risk spillovers between industries that are economically close than the spillovers between two industries that are economically distant. The measure of economic distance in this paper is defined as the strength of supplier-customer relationships and is numerically described by weights between nodes in the IO tables. Supplier industries that provide inputs counting for a larger share of the production cost are considered to be economically closer to the customer industries. Credit risks are measured by the probability for public firms to acquire default ratings, get delisted or go bankrupt.

Consistent with our theory, the empirical results show that the correlation of credit risks between customer industries and supplier industries declines as the economic distance between the two sides increases. This relationship is robust to both linear and nonlinear specifications of models. Also, the results are robust after adding conventional accounting variables for bankruptcy and default predictions as controls. Another finding is that industrial average credit risks have a higher level of comovement than industrial average accounting variables across sectors. Due to the absence of additional instruments, we cannot rule out unobserved confoundess or provide a robust causal inference to determine the direction of contagion. Nevertheless, these results are the first to record the credit risks contagion in a production network in data. They provide the potential for new insights into the financial contagion process.

To explain the cause of these observed correlations between economic distance and risk contagion, we conjecture that the trade credit interdependency among firms along the supply chains is the main mechanism for credit risk spillovers through a production network. Trade credit is the single most important source for short-term finance for firms (Petersen and Rajan, 1997). Also, nonpayment of trade credits is among the top causes for firm bankruptcy according to Boissay (2006). Besides, trade credit is not diversified for lenders as customers of firms usually concentrate in very specific sectors of the economy. Due to all these characteristics of trade credits, when sectoral productivity or demand shock hits, if a large number of downstream buyers in a specific sector experience high level of financial stress and are incapable of paying their debt back at the same time, their suppliers, who also tend to concentrate in a few sectors, will suddenly become insolvent as well. In addition, we think this type of financial contagion upstream is likely to influence major suppliers that are economically closer to customers way more significant than minor suppliers that are economically more distant. The reason is twofold. Firstly, buyers have stronger incentive to borrow from major suppliers with whom the transactions cover a larger amount of production cost. Secondly, buyers usually maintain longer and more stable business relationship with major suppliers. As a result, major suppliers have more information about their customers and thus are more likely to lend them. Both of these reasons can lead to inter-sectoral contagion of credit risks.

The remainder of the paper is organized as follows. After a brief discussion of related literature, Section 3.1 puts forward the baseline regression model and Section 3.2 describes the data used in the empirical analysis. Section 3.3 presents all the empirical results that show the inter-sectoral credit risk comovement in a network and interpret the results. In Section 3.4, we present our conjecture abour the mechanism that drives the empirical results for network spillover of credit risks from the previous part. Finally, section 3.5 concludes.

Related literature This paper is a novel application of network analysis on the study of credit risks and financial contagion. The idea that idiosyncratic shocks can propagate through networks and eventually create aggregate fluctuations has been extensively explored by many economists both theoretically and empirically. Acemoglu et al. (2012b) first built a theoretical framework for input-output production network analysis in the macroeconomic system. Later many work, such as Herskovic (2018) and Acemoglu et al. (2015), further extended these frameworks to the analysis of financial markets. This paper applied their views of production network contagion specifically to the topic of credit risks. The regression method used in this paper to detect spillover in a network is first used in Moretti (2004b) to study the network spillovers of human capital in cities.

As to credit risks literature, on the one hand, there are a large number of empirical studies that focus on bankruptcy and default predictions through regressions, such as Chava and Jarrow (2004), Das et al. (2007), Duffie et al. (2007) as well as Campbell et al. (2008). Their common assumptions are either firm-level risks are independent to each other or they have limited dependency described by conditional probability. The key difference of our work compared to these paper is that we use an observable input-output network to measure the correlation of risks. On the other hand, many theory works try to depict the exact process and mechanism for the financial contagion to happen inside a network of firms and banks and how macroeconomic cycles can be created in the process. Among them, studies such as Allen and Gale (2000) focus on the risk propagation through banks whereas other research, including Kiyotaki and Moore (1998), Boissay (2006) and Gatti et al. (2010) also pay attention trade credit in their analysis. This paper tries to bridge these two genres of credit risks literature, and provide robust empirical evidence to support the theory framework of financial contagion in credit networks.

In addition, classic theories about incentive and information asymmetry behind trade credit, such as Smith (1987) and Petersen and Rajan (1997), have inspired our conjecture of how trade credit assists risk spillovers in the production network in this paper.

3.1 Econometric Model

The central assumption behind all the empirical analysis of this paper is that if financial distress and credit risks do spillover and propagate through the input-output network, then industries with closer economic distance should have higher levels of correlations of default probabilities than industries with longer economic distance. Let $D_{i,k,t}$ denote the occurrence of a high credit risk event of firm i in industry k in year t, then we can summarize this assumption into the following equation:

$$D_{i,k,t} = \beta_0 + \beta_1 DS^1_{k,t} + \beta_2 DS^2_{k,t} + ... + \beta_n DS^n_{k,t} + controls_{i,k,t} + \epsilon_{i,k,t}, \qquad (3.1)$$

where $DS^n_{k,t}$ is the probability of high credit risk events, such as bankruptcy or default, of the nth largest supplier for firms in industry k in year t. To avoid endogeneity and identification issues frequently mentioned in the discussion of peer effects, such as Angrist (2014) and Manski (1993), we exclude a firm's own industry from its supplier default rate variables $DS^n_{k,t}$. In this case, none of the peers from the same industry as the target firm on the left hand side ends up on the right hand side of the regression. In addition, because $DS^n_{k,t}$ have the same values for firms in the same industries at a specific time, these variables are essentially a way to dissect year and industry fixed effects. Therefore dummies for year and industry fixed effects are not included in regressions anymore.

We also incorporate a selection of firm-level accounting variables common in empirical bankruptcy and default prediction literature as controls for time-varying, city and firm specific shocks, i.e. $controls_{i,k,t}$ in the equation. LASSO regressions help us to pick five of the most significant variables from the ones used in Campbell et al. (2008) and Chava and Jarrow (2004) for this paper. They are return on assets (ROA), total liabilities on assets (TL), the current to total liabilities ratio (SHORT), interest coverage ratio (ICR) and cash flow ratio (LIQ). Following the treatment of previous papers, these variables are winsorized, standardized and a subset of them are also logarithmized. $\epsilon_{i,k,t}$ is the residual time-varying, city and firm specific idiosyncratic shock that cannot be explained by these controls.

Following our assumption, the key to identification of the network effect is that as n increases, the economic distance between two industries increases, therefore we should observe the significance level and the magnitude of β_n both decline. This idea of using credit risk spillovers declining with economic distance to verify the network effect of financial stress is inspired by Moretti (2004b)'s work of measuring human capital spillovers in production networks. Similar to his paper, we adopt input-output tables as a measure of economic distance. Specifically, the economic distance between a firm and its suppliers is in proportion to the value of inputs that each supplier's industry provides to the firm's own industry.

Also, these regressions are designed to extract correlations between group averages, between R^2, instead of the standard R^2, is a better measurement here. More specifically, if the production network structure indeed affects the strength of industry-wise credit risk correlations, we should observe a significant level of the R^2 *between* values, instead of the standard R^2, in these regressions.

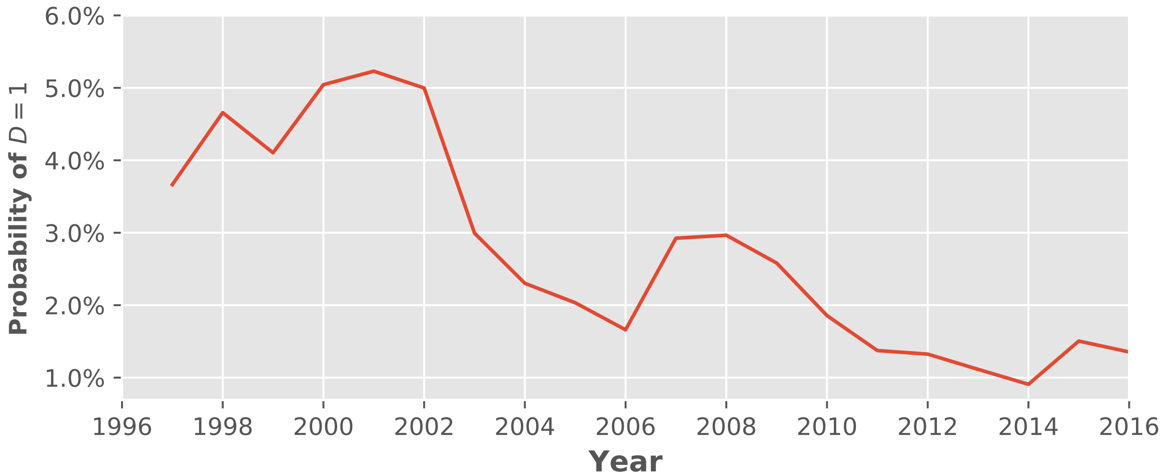
As to the threat to the identification strategy, while plenty of confounders that both move the default risks of supplier industries and customers, they do not directly impact the economic distance measures by IO tables. Input-output relationships recorded in the U.S. IO tables are relatively stable in the last 15 years so economic distance can be considered as exogenous to business cycles and economic shocks. Therefore as long as β_n has an declining order as n increases, the existence of network effect can be verified. However, the caveat of this method lies in its incapability to support any causal interpretation of the network effect that can be captured by it. In other words, the empirical results cannot directly make any statement on the direction of credit risks contagion in the network, whether it is from customer industries to supplier industries or the other way around.

3.2 Data

To test the above theory, we used public firm data of credit risks and accounting information, as well as industry level data of input-output relationships between 1971 and 2016 in the empirical analysis.

Events that represent high levels of credit risks and financial distress, such as default and bankruptcy, occur at a low frequency. Therefore, in order to capture as many such events as possible, this paper uses three types of events, namely default, delisting and bankruptcy, to construct the credit risk flags D for public firms. In a given year, D flag is 1 for a firm if within 12 months after its 10K statement release dates, any one of the three types of events happens. Otherwise D flag has value 0. Notice that when D flag is 1 for a firm in a specific year, there can still be up to a one-year lag between the 10K statement release date and the date that an actual D event happens. Among these three types of events, the default category consists of firms getting Moody's S&P credit rating of D in the Compustat database. Delisting events refer to firms being delisted from stock exchanges under the codes of 400 (liquidations) or 500 (dropped) in the database of Center for Research in Security Prices (CRSP). There are multiple sources of bankruptcy events. The first one is the chapter 7 and 11 bankruptcy data set cleaned and used by Chava and Jarrow (2004), Chava (2014) and Alanis et al. (2018). This data set covers bankruptcy events for public firms from 1964 to 2014. The second is from Bankruptcydata.com, which covers bankruptcy events up to 2018. Due to the limit of D flags' source data, only public firms are included in the analysis.

Figure 3.1: Occurrence of Bankruptcy/Default/Delisting Events, 1997-2016



There is a time lag between $D = 1$ and an actual D event happens. For example, in 2007 $D = 2.13\%$. It means the probability for an average firm to go bankrupt or default or delist within next 12 months is 2.13%

The industrial network data used here are input-output tables by 71 industries provided by Bureau of Economic Analysis (BEA). Industries here are defined by the first 3 digit of North American Industry Classification System (NAICS) codes. BEA only offers these tables in the period of 1997 to 2016, so we limit the analysis to only these years. We exclude government sectors, the banking sector [1] and utility sector [2] from the analysis because their financing and credit mechanism are different and we want to focus on private sectors. Besides, the warehousing and storage industry[3]is free from any default, delisting or bankruptcy event observed during the data period, thus being removed as well. Eventually 66 private industries are left in the data set. The IO tables are normalized to only include these industries. Figure 3.1 shows the average annual probability for any one of the three types of D events to happen during this period of time for firms in these 53 industries. We can see that D flags peak twice around 2001 for the recession following the bust of the dot-com bubble, as well as the great recession in 2008.

[1]NAICS code: 523 524 525
[2]NAICS code: 22
[3]NAICS code 493

Input-Output tables provide us meanings to measure the economic distance between any two industries, but among many types of IO tables, which ones to choose? This paper specifically focuses on ranking the major customers and suppliers of a specific industry in the network, so only two types of IO tables are used here. We use the total requirements tables to find the most important suppliers for an industry.[4] For a pair of source and target industries, a number X in a total requirements table means that X dollars of direct and indirect inputs are required from the source industry to produce one dollar of output in the target industry. Therefore the top Kth most important suppliers for an industry in this study are defined as the source industries with the largest K weights for the same target industry in the total requirements tables. We extract information of the most important customers for an industry from the use tables.[5] These tables show the total value of direct intermediate inputs used by each industry to produce its output. The top Kth most important customers for an industry are defined as the target industries with the largest K weights for the same source industry in the use tables. However, it is challenging to rank customer-industries merely based on use tables. Many firms produce multiple products across multiple industries while their NAICS codes are only associated with their primary products. The total requirement tables provide a convenient way to identify and rank supplier-industries because it counts for the value of inputs both from primary product producers and secondary product producers to one dollar of output of the target industry. In other words,these tables reflect the relationships between industries more accurately.[6] However, no such normalized table exists to show the direct and indirect domestic customer decomposition of sellers. The use and domestic supply tables provided by BEA are before industry redefinition, so many entries have value 0. More importantly they do not reallocate outputs. Therefore it is difficult for the normalized version of these tables to accurately identify and rank customer-industries for target firms. Therefore, the baseline analysis will focus on the network relationships derived

[4]The full name of the tables are: Industry-by-Commodity Total Requirements, After Redefinitions.

[5]The full name of the tables are: The Use of Commodities by Industries.

[6]The total use tables (after redefinition) reallocate the production of secondary products from the producing industry to the industry for which the product is primary.

from the total requirement table whereas the results from the use tables are included in the robustness check. In this paper, we consider major suppliers and customers to an industry to have closer economic distance to the target industry, whereas minor suppliers and customers have longer economic distance to it.

The firm-level accounting variables that are used as controls are also from Compustat. Tables 3.1 and 3.2 show the summary statistics of these accounting variables before any manipulation for all public firms in the dataset and only firms with $D = 1$ respectively.

Table 3.1: Summary Statistics of Accounting Variables of All Firms

Variables	count	mean	std	min	25%	50%	75%	max
ROA	178083	-0.524	67.881	-25884.808	-0.063	0.029	0.073	2369.429
TL	178083	1.41	73.012	0.001	0.363	0.529	0.699	25968.974
SHORT	178083	0.568	0.271	0.001	0.344	0.55	0.804	2.233
ICR	178083	613.252	10866.695	0.0	23.956	59.033	154.427	2066413
LIQ	178083	0.997	3.358	0.0	0.092	0.296	0.853	498

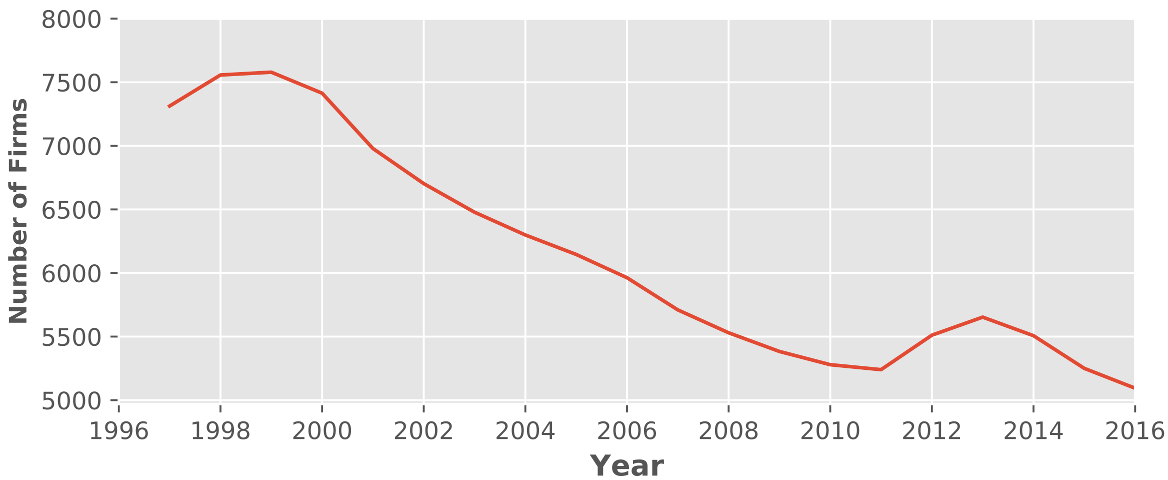
Table 3.2: Summary Statistics of Accounting Variables of Firms with D flags

Variables	count	mean	std	min	25%	50%	75%	max
ROA	4744	-0.777	4.567	-249	-0.707	-0.263	-0.066	11.566
TL	4744	2.416	91.827	0.017	0.589	0.835	1.119	6324
SHORT	4744	0.644	0.307	0.001	0.366	0.71	0.943	1
ICR	4744	226.124	2711.568	0.001	8.991	23.386	58.432	147715
LIQ	4744	0.488	1.756	0.0	0.035	0.117	0.409	81.146

$D = 1$ refers to an occurrence of a default, delisting or bankruptcy event for a firm.
Accounting Variable Definitions: $ROA = \frac{Net\ Income}{Assets}$, $TL = \frac{Total\ Liabilities}{Assets}$, $SHORT = \frac{Current\ Liabilities}{Total\ Liabilities}$, $ICR = \frac{Revenue}{Interest\ Expenses}$, $LIQ = \frac{Cash}{CurrentLiabilities}$.

To guarantee the consistency and comparability of the results, only firms with all the five chosen accounting variables during the data period are kept in the analysis. Eventually we used the data of 12590 firms. Figure 3.2 is the annual firm count in the data period.

Figure 3.2: Number of Firms in the Dataset, 1997-2016



Only public firms with all the accounting information in COMPUSTAT are included in the analysis.

3.3 Empirical Results

Baseline Results

The regression results for the baseline linear model in Equation (3.1) from Section 3.1 is presented in Table 3.3.

Regressions 0 to 4 show that among the top five supplier-industries of a firm, β_n, the coefficient of $DS^n_{k,t}$, decreases as n increases. This indicates that the correlations between the firm's credit risk and its suppliers' declines as the supplier-industries provide less and less inputs into the target firm's production process. Also, starting from the 6th largest suppliers, the coefficient of credit risks turns statistically insignificant. This declining rank of β_n is consistent in all 5 regressions so this result is stable to change of specifications. Although there are many unobserved the macro economic factors that can affect both $D_{i,k,t}$ and $DS^n_{k,t}$ at the same time, they are unlikely to affect the ranking of β_n. These results are supportive evidence for the existence of the contagion of credit risks in the production network. This is the first piece of empirical evidence that shows that sectoral linkages create contagion that can be observed at a macro level.

In addition, according to the between R^2 values of regressions 0 to 4

Table 3.3: Linear Correlations of Credit Risks in the Industrial Network

Variables	0	1	2	3	4	5	6	7	8	9	10
Ds1	0.441***	0.336***	0.271***	0.24***	0.23***		0.386***	0.296***	0.236***	0.21***	0.202***
	(0.076)	(0.066)	(0.058)	(0.053)	(0.055)		(0.073)	(0.066)	(0.058)	(0.054)	(0.055)
Ds2	0.407***	0.319***	0.249***	0.214***	0.199***		0.358***	0.283***	0.218***	0.188***	0.176***
	(0.088)	(0.079)	(0.071)	(0.073)	(0.067)		(0.075)	(0.066)	(0.06)	(0.062)	(0.057)
Ds3		0.243***	0.22***	0.201***	0.195***			0.208***	0.187***	0.171***	0.166***
		(0.055)	(0.051)	(0.047)	(0.048)			(0.051)	(0.048)	(0.043)	(0.044)
Ds4			0.178***	0.146***	0.135***				0.164***	0.137***	0.129***
			(0.028)	(0.024)	(0.026)				(0.028)	(0.023)	(0.026)
Ds5				0.126***	0.119***					0.107***	0.102***
				(0.026)	(0.024)					(0.025)	(0.023)
Ds6					0.051						0.039
					(0.038)						(0.033)
ROA						−0.029***	−0.026***	−0.026***	−0.026***	−0.026***	−0.026***
						(0.004)	(0.004)	(0.003)	(0.003)	(0.003)	(0.003)
SHORT						0.004***	0.004***	0.004***	0.004***	0.004***	0.004***
						(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
logICR						0.003***	0.002*	0.002*	0.002*	0.002*	0.002*
						(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
logLIQ						−0.004*	−0.003*	−0.003*	−0.003*	−0.003*	−0.003*
						(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
logTL						0.011***	0.011***	0.011***	0.011***	0.011***	0.011***
						(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
R^2 between	0.445	0.45	0.445	0.444	0.444	0.097	0.453	0.456	0.452	0.451	0.45
nobs	93306	93306	93306	93306	93306	94688	93306	93306	93306	93306	93306

Dependent variables are D flags for default, delisting and bankruptcy events.
Dsn represents the average default probability of the nth largest supplier industry of the target firm.
Controls include return on assets (ROA), total liabilities on assets (TL), the current to total liabilities ratio (SHORT), interest coverage ratio (ICR) and cash flow ratio (LIQ).
Standard errors in the parentheses are clustered with respect to industry and year.
***p<0.01, **0.01≤p<0.05, *0.05≤p<0.1

in Table 3.3, the D rates of supplier-industries alone explain around 46% of the variation of average industry level D rates of the target industries. These high between R^2 values indicate that the average probability of default, delisting or bankruptcy events in industries have very significant and observable comovement with each other.

In Table 3.3, regressions 6 to 10 demonstrate that after controlling for the accounting information of individual firms, such pattern of declining coefficients are still robust. All these financial variables are standardized at industry levels so they only represents firm-level idiosyncratic shocks. Therefore they are incapable of explaining the group difference of frequency of D events among different industries. The low between R^2 value of regression 5, which only includes accounting variables as regressors, verifies this.

Robustness Checks

Additional robustness checks are also done to verify the results. First, to understand the relative magnitude of these between R^2 values in Table 3.3, we use accounting variables of the top sixth largest supplier-industries, instead of the industry-level D rates, as regressors to run the same regressions, i.e.

$$D_{i,k,t} = \beta_0 + \beta_1 VarS^1_{k,t} + \beta_2 VarS^2_{k,t} + ... + \beta_n VarS^n_{k,t};$$

where $VarS$ can be one of the five accounting variables from supplier-industries, including return on assets (ROA), total liabilities on assets (TL), the current to total liabilities ratio (SHORT), interest coverage ratio (ICR) and cash flow ratio (LIQ).

Table 3.4 shows the results. The between R^2 of these regressions are significantly lower than the ones in Table 3.1. This indicates that comovements of D rates among industries in the production network are much stronger than the comovement between D rates of the target industries and other accounting variables of the supplier-industries.

Why can D flags of suppliers' industries capture the movements of D flags in the target industries much better than the more detailed accounting information of suppliers? One plausible explanation for such phenomenon is that at any point of time events such as default, delisting and bankruptcy have very skewed distributions, whereas accounting variables, especially after winsorization and standardization, often are normally distributed. Due to the limited sample size, these accounting variables have difficulty to effectively capture information needed to predict rare events with low frequency of occurrence among operating firms. This is a typical issue for imbalanced dataset classification problem(Chawla, 2009). However, default, delisting and bankruptcy events of different industries have similarly skewed distributions thus observations of these events are more likely to be correlated if they are indeed connected through the production network.

Next, to compare our results with analysis in conventional empirical credit risk literature with accounting variables, we also run regressions of a

Table 3.4: Credit Risks v.s. Alternative Accounting Variables in the Production Network

Variables	ROA	logTL	SHORT	logICR	logLIQ	ROA	logTL	SHORT	logICR	logLIQ
Ds1	-0.015	0.022*	0.015	0.015	-0.013	-0.012	0.016	0.017	0.015	-0.009
	(0.01)	(0.013)	(0.016)	(0.022)	(0.019)	(0.009)	(0.011)	(0.015)	(0.019)	(0.017)
Ds2	-0.017	0.018*	-0.011	-0.01	−0.023*	-0.011	0.019**	-0.011	-0.012	−0.023**
	(0.011)	(0.011)	(0.017)	(0.02)	(0.013)	(0.01)	(0.01)	(0.015)	(0.017)	(0.011)
Ds3	−0.024***	0.033***	−0.021*	−0.025***	−0.021***	−0.02***	0.025***	-0.016	−0.019**	−0.018***
	(0.007)	(0.01)	(0.013)	(0.009)	(0.007)	(0.007)	(0.009)	(0.012)	(0.008)	(0.006)
Ds4	−0.025***	0.027***	-0.01	−0.029***	−0.018**	−0.022***	0.023***	-0.008	−0.027***	−0.017***
	(0.006)	(0.007)	(0.008)	(0.009)	(0.008)	(0.006)	(0.006)	(0.007)	(0.007)	(0.006)
Ds5	−0.034***	0.04***	-0.013	−0.039***	−0.028**	−0.028***	0.034***	-0.011	−0.033***	−0.025**
	(0.007)	(0.01)	(0.012)	(0.013)	(0.014)	(0.006)	(0.008)	(0.011)	(0.011)	(0.011)
Ds6	−0.027***	0.018**	0.001	-0.016	−0.021***	−0.023***	0.015**	0.0	-0.015	−0.022***
	(0.007)	(0.008)	(0.01)	(0.011)	(0.008)	(0.007)	(0.008)	(0.008)	(0.01)	(0.006)
ROA							−0.027***	−0.029***	−0.028***	−0.028***
							(0.004)	(0.004)	(0.003)	(0.003)
SHORT						0.004***		0.004***	0.003**	0.003**
						(0.001)		(0.001)	(0.001)	(0.001)
logICR						0.003**	0.001		0.004***	0.004***
						(0.001)	(0.001)		(0.001)	(0.001)
logLIQ						-0.003	−0.005**	−0.004*		-0.002
						(0.002)	(0.002)	(0.002)		(0.002)
logTL						0.012***	0.009***	0.011***	0.012***	
						(0.002)	(0.002)	(0.002)	(0.002)	
R^2 between	0.255	0.253	0.044	0.046	0.113	0.285	0.285	0.125	0.13	0.191
nobs	94540	94540	94540	94540	94540	94540	94540	94540	94540	94540

Dependent variables are column names.
Column names are the chosen accounting variables from suppliers as regressors.
sn represents the industry average of the column accounting variable for the nth largest supplier-industry of a firm.
Controls include return on assets (ROA), total liabilities on assets (TL), the current to total liabilities ratio (SHORT), interest coverage ratio (ICR) and cash flow ratio (LIQ).
Standard errors in the parentheses are clustered with respect to industry and year.
***p<0.01, **0.01≤p<0.05, *0.05≤p<0.1

logistic version of the model, i.e.

$$D_{i,k,t} = \frac{1}{1 + exp\{\beta_0 + \beta_1 DS^1_{k,t} + \beta_2 DS^2_{k,t} + ... + \beta_n DS^n_{k,t} + controls_{i,k,t} + \epsilon_{i,k,t}\}}.$$

The results are presented in Table 3.5. The coefficients for $DS^1_{k,t}$ to $DS^4_{k,t}$ are consistently significant across all regressions and declining as n increases. Starting from $DS^5_{k,t}$, the coefficients turn to insignificant. Therefore, the evidence of network spillover persists in these nonlinear regressions, although the results are slightly less significant.

Finally, a natural question to ask here is that if such network relationship can be observed with customer-industries of target firms, can it also exist with major customer-industries of firms? In other words can we observe declining coefficients for this following regression:

$$D_{i,k,t} = \alpha_0 + \alpha_1 DC^1_{k,t} + \alpha_2 DC^2_{k,t} + ... + \alpha_n DC^n_{k,t} + controls_{i,k,t} + \epsilon_{i,k,t}; \quad (3.2)$$

Table 3.5: Logistic Regressions of Credit Risks in the Industrial Network

Variables	0	1	2	3	4	5	6	7	8
Ds1	**7.543*** **	**7.291*** **	**7.232*** **	**7.245*** **		**6.988*** **	**6.696*** **	**6.615*** **	**6.623*** **
	(0.529)	(0.534)	(0.535)	(0.535)		(0.57)	(0.575)	(0.577)	(0.578)
Ds2	**2.19*** **	**2.168*** **	**2.161*** **	**2.167*** **		**2.499*** **	**2.474*** **	**2.464*** **	**2.468*** **
	(0.302)	(0.306)	(0.307)	(0.306)		(0.322)	(0.327)	(0.328)	(0.328)
Ds3		**1.521*** **	**1.532*** **	**1.529*** **			**1.569*** **	**1.581*** **	**1.58*** **
		(0.267)	(0.267)	(0.267)			(0.295)	(0.295)	(0.295)
Ds4			**0.433* **	**0.429* **				**0.526** **	**0.524** **
			(0.238)	(0.238)				(0.252)	(0.252)
Ds5				-0.164					-0.098
				(0.178)					(0.184)
Intercept	**−3.746*** **	**−3.787*** **	**−3.8*** **	**−3.793*** **	**−3.836*** **	**−4.161*** **	**−4.202*** **	**−4.218*** **	**−4.214*** **
	(0.029)	(0.03)	(0.031)	(0.032)	(0.024)	(0.034)	(0.035)	(0.036)	(0.037)
ROA					**−0.611*** **	**−0.599*** **	**−0.598*** **	**−0.597*** **	**−0.597*** **
					(0.017)	(0.017)	(0.017)	(0.017)	(0.017)
SHORT					-0.006	0.002	0.002	0.002	0.002
					(0.019)	(0.019)	(0.019)	(0.019)	(0.019)
logICR					0.015	0.011	0.011	0.01	0.01
					(0.019)	(0.019)	(0.019)	(0.019)	(0.019)
logLIQ					**−0.186*** **	**−0.169*** **	**−0.167*** **	**−0.167*** **	**−0.167*** **
					(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
logTL					**0.104*** **	**0.126*** **	**0.129*** **	**0.129*** **	**0.129*** **
					(0.021)	(0.021)	(0.021)	(0.021)	(0.021)
Pseudo R^2	0.009	0.01	0.01	0.01	0.102	0.109	0.11	0.11	0.11
nobs	94731	94731	94731	94731	94731	94731	94731	94731	94731

Dependent variables are D flags for default, delisting and bankruptcy events.
Column names are the chosen accounting variables from suppliers as regressors.
sn represents the industry average of the column accounting variable for the nth largest supplier-industry of a firm.
Controls include return on assets (ROA), total liabilities on assets (TL), the current to total liabilities ratio (SHORT), interest coverage ratio (ICR) and cash flow ratio (LIQ).
Standard errors in the parentheses are clustered with respect to industry and year.
***p<0.01, **0.01≤p<0.05, *0.05≤p<0.1

where $DC_{k,t}^{n}$ is the default, bankruptcy and delisting probability of the nth largest cusotmer for firms in industry k in year t. Similar to the interpretation of β_n, the theory in Section 3.1 implies that the values of α_n should decline as n increases.

The challenging part of this regression is to pick and rank customer-industries in data. As discussed in th Section 3.2, it is difficult for the normalized version of IO tables from BEA to accurately identify and rank customer-industries for firms. Namely, more often than not public firms produce goods across multiple industries but use tables do not reclassify these productions. Under this concern, regression results from Equation (3.2) are expected to be weak. Table 3.6 shows the results of Equation (3.2) and it follows the same format as Table 3.3. Indeed, the declining coefficients are not observed in these results. Also R^2 between values are less significant. The reduction of observation numbers in this table compared to Table 3.3 is caused by the larger amount of 0's in the use IO tables. Better metric to measure economic distances between firms and their major customers can potentially solve this problem.

Table 3.6: Linear Correlations of Credit Risks with Customers in the Production Network

Variables	0	1	2	3	4	5	6	7	8	9	10
Dc1	0.393***	0.274**	0.211**	0.153**	0.099**		0.342***	0.238**	0.184**	0.132**	0.084**
	(0.145)	(0.111)	(0.096)	(0.073)	(0.049)		(0.123)	(0.094)	(0.08)	(0.061)	(0.04)
Dc2	0.085*	0.061	0.052	0.041	0.031		0.08*	0.058*	0.051*	0.04*	0.032*
	(0.05)	(0.038)	(0.034)	(0.027)	(0.021)		(0.042)	(0.031)	(0.028)	(0.022)	(0.017)
Dc3		0.287***	0.233***	0.178***	0.127***			0.254***	0.208***	0.158***	0.113***
		(0.068)	(0.049)	(0.042)	(0.025)			(0.061)	(0.046)	(0.041)	(0.027)
Dc4			0.161***	0.118**	0.08*				0.138***	0.1**	0.067*
			(0.06)	(0.053)	(0.046)				(0.052)	(0.045)	(0.038)
Dc5				0.23***	0.207***					0.206***	0.186***
				(0.058)	(0.049)					(0.053)	(0.045)
Dc6					0.243***						0.216***
					(0.048)						(0.042)
ROA						−0.029***	−0.026***	−0.026***	−0.025***	−0.025***	−0.025***
						(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)
SHORT						0.003***	0.004***	0.004***	0.004***	0.004***	0.004***
						(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
logICR						0.003**	0.002*	0.002*	0.002*	0.002*	0.002*
						(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
logLIQ						−0.004*	−0.004*	−0.004*	−0.004*	−0.004*	−0.004*
						(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
logTL						0.011***	0.011***	0.011***	0.011***	0.011***	0.011***
						(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
R^2 between	0.303	0.362	0.37	0.395	0.421	0.099	0.337	0.386	0.391	0.414	0.434
nobs	88433	88433	88433	88433	88433	94203	88433	88433	88433	88433	88433

Dependent variables are D flags for default, delisting and bankruptcy events.
Dcn represents the average default probability of the nth largest customer industry of the target firm.
Controls include return on assets (ROA), total liabilities on assets (TL), the current to total liabilities ratio (SHORT), interest coverage ratio (ICR) and cash flow ratio (LIQ).
Standard errors in the parentheses are clustered with respect to industry and year.
***p<0.01, **0.01≤p<0.05, *0.05≤p<0.1

In addition, to make sure the regression results are robust to different sources of bankruptcy events that are used to construct D flags for firm, regressions in Table 3.3 are duplicated for individual bankruptcy database separately. The declining significance levels and magnitudes of DS_n's coefficients are also observed in each set of these regressions.

3.4 Mechanism Discussion

The previous section presents robust evidence of credit risk spillovers in the production network. Namely, as the contribution of one industry's inputs in the production process of another industry increases, the probability for high financial stress events, including default, delisting and bankruptcy, of these two industries have a higher level of correlation. Then what can explain this type of credit risk contagion propagating thorough the production network?

Firms are inter-connected with each other not only through the demand and supply of real goods and services in the production process, but also through direct borrowing and lending in the form of trade credits. Therefore there can be two potential channels to cause the observed credit risk contagions contagion. One possible explanation is that demand and supply shocks of real goods and services may pass down the supply chain and put pressure on the profitability of suppliers or customers in the network. Financial distress of major suppliers or customers may harm firms ability to either make or sell goods for profit. Consequently, sudden drops in profit may push firms towards insolvency and eventually bankruptcy, default or delisting. If this is the main mechanism that is behind the network correlations observed in the previous section, we should observe the accounting variables measuring profitability of firms in the target industry correlated with the frequency of observing D flags in supplier and customer industries. More importantly, as the economic distance between supplier and customer firms gets shorter, such correlation should get stronger.

Table 3.4 shows that D rates of supplier industries can only explain 25% of the variance in return on assets(ROA), which is the main measure of profitability, of the target industries, whereas D rates of suppliers industries can explain up to 44.5% of the variance in D. Moreover, neither the magnitude nor the significance of coefficients for DS_1 to DS_6 follows a declining order. These results show that financial distress of supplier industries has relatively weak explanatory power to the changes in profitability of target industries. Table 3.7 shows extra regression results between accounting variables and D flags. Column 1 to 5 show that accounting variables of supplier industries also have relatively low explanatory power to the variance in D flags of the target industries. However according to column 6 to 10, they do explain the variance in accounting variables of the target firms better. There is also an absence of the declining order for both the magnitude and the significance of coefficients in every one of the regression in

Table 3.7: Linear Correlations of Credit Risks in the Industrial Network

Variables	D	D	D	D	D	ROA	SHORT	logICR	logLIQ	logTL
ROAs1	−0.022**					0.13**				
	(0.009)					(0.056)				
ROAs2	−0.018***					0.201***				
	(0.007)					(0.053)				
ROAs3	−0.026***					0.204***				
	(0.006)					(0.035)				
ROAs4	−0.017***					0.111***				
	(0.004)					(0.037)				
ROAs5	−0.015***					0.083***				
	(0.004)					(0.026)				
SHORTs1		0.022					0.255***			
		(0.015)					(0.066)			
SHORTs2		-0.007					0.034			
		(0.009)					(0.067)			
SHORTs3		−0.021**					0.109***			
		(0.009)					(0.03)			
SHORTs4		−0.016***					0.081***			
		(0.003)					(0.019)			
SHORTs5		−0.011***					0.06***			
		(0.004)					(0.01)			
logICRs1			0.005					0.189**		
			(0.018)					(0.083)		
logICRs2			-0.007					0.092		
			(0.012)					(0.065)		
logICRs3			−0.023***					0.157***		
			(0.008)					(0.041)		
logICRs4			−0.026***					0.027		
			(0.005)					(0.025)		
logICRs5			−0.027***					0.057		
			(0.006)					(0.044)		
logLIQs1				-0.008					0.228***	
				(0.014)					(0.076)	
logLIQs2				−0.022*					0.109***	
				(0.013)					(0.027)	
logLIQs3				−0.021***					0.103***	
				(0.008)					(0.036)	
logLIQs4				−0.013*					0.096***	
				(0.007)					(0.028)	
logLIQs5				-0.001					0.089***	
				(0.007)					(0.02)	
logTLs1					0.016*					0.221***
					(0.009)					(0.044)
logTLs2					0.024***					0.098
					(0.008)					(0.077)
logTLs3					0.031***					0.168***
					(0.007)					(0.039)
logTLs4					0.022***					0.07**
					(0.005)					(0.033)
logTLs5					0.022***					0.056*
					(0.008)					(0.03)
R^2 between	0.206	0.073	0.075	0.059	0.22	0.469	0.023	-0.008	0.088	0.431
nobs	88421	88421	88421	88421	88421	94203	94203	94203	94203	94203

Dependent variables are column names. They are all attributes of target firms.
Controls include return on assets (ROA), total liabilities on assets (TL), the current to total liabilities ratio (SHORT), interest coverage ratio (ICR) and cash flow ratio (LIQ).
Standard errors in the parentheses are clustered with respect to industry and year.
***p<0.01, **0.01≤p<0.05, *0.05≤p<0.1

Table 3.7. This implies the comovements among accounting variables, as well as between these accounting variables and D flags, are more likely to

be caused by other confounders in the macroeconomic environment other than the production network structure. The financial risks passing down through real goods and services transactions are not likely to be the main reason for the network correlation of financial distress we observed in the previous section.

Another potential answer for this network correlation lies in the heavy usage of trade credit in the production process. According to Petersen and Rajan (1997) and Boissay (2006), trade credit is the single most important source for firms to acquire short-term external finance. Moreover, large public firms that are typically presented in the COMPUSTAT dataset rely more heavily on trade credit than smaller private firms (Petersen and Rajan, 1997). When the financial stress of the downstream industries increases, the likelihood for their suppliers not being able to recollect accounts receivable and suddenly go insolvent increases as well. Therefore, high probability for customers to default, delist or go bankrupt indicates for higher risk for the suppliers to got through similar events as well.

In addition, we conjecture that firms are likely to rely more heavily on trade credit from the major suppliers than from less important suppliers. On the one hand, if a specific type of inputs counts for a very significant share of the production cost, it is more likely for the value of cash flow and the frequency of transactions with these suppliers to be much higher. Therefore, firms have more incentive borrow from these major suppliers that cover a larger amount of cost on their balance sheet than from smaller suppliers who provide miscellaneous minor parts through infrequent transactions for the production process. On the other hand, firms usually establish longer-term and more stable business relationships with their major suppliers. As a result, major suppliers have more information about the operation conditions of these business and have bigger control over the contracts. It is not surprising that they are more willing to lend a bigger share of trade credits to these customers. With this line of reasoning, firms' financial soundness is more integrated with their major suppliers than the minor ones. Therefore, we should observe a higher level of credit risk contagion from buyers to their major suppliers than to the minor ones. The regression results are consistent with this theory. However, more detailed firm level trade credit data that can be matched with bankruptcy, default and delisting events is indeed to verify this theory empirically. We will leave that for future work.

3.5 Conclusion

In this paper, we present the first empirical evidence that industry level credit risk is correlated in proportion to their linkages in the production network. specifically, if ranking the supplier-industries for a specific target industry based on the weights of input shares from IO tables, the credit risks for firms in an industry, measured by the average probability to default, delist and go bankrupt, has higher and more significant correlations with credit risks of more important supplier-industries than with less important supplier-industries. This relationship is robust to extra controls as well as both linear and nonlinear specifications of the regression models.

As to the cause for this type of financial contagion through the production network, our conjecture is that the credit risks of firms can pass to suppliers and customers either through the demand and supply of real good and services in the production process or through the heavy usage of trade credit. On the one hand, once customer or supplier industries default, firms may experience to either make or sell their products and consequently hurt their profit. Decline in profit reduce the ability for firms to pay back their debt. However empirical results provide weak support to this hypothesis that links firms' profitability to upstream or downstream firms' financial stress. On the other hand, once customers default or go bankrupt, accounts payables of the supplier firms will get written off and net worth of suppliers will shrink and therefore the chance for insolvency for upstream firms increase. Moreover, we argue that firms have incentive to rely more on trade credit when purchasing major inputs that counts for a larger share of production costs than buying minor categories of inputs. Therefore, the credit risk contagion should be higher for major supplier-industries than for minors ones. More firm level data of credit credit is indeed to verify the second hypothesis.

One important direction of future research is to use more detailed firm-level trade credit data to empirically verify the above theory of the specific way how trade credit creates contagion in the network. Also, a richer theoretical framework will be very helpful for simulating the propagation of financial stress caused by sectoral production shocks and demand shocks.

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